

Probing CP-violating contact interactions in $e^+e^- \rightarrow HZ$ with polarized beams

Kumar Rao¹ and Saurabh D. Rindani²

*Theory Group, Physical Research Laboratory
Navrangpura, Ahmedabad 380009, India*

Abstract

We examine very general four-point interactions arising due to new physics contributing to the Higgs production process $e^+e^- \rightarrow HZ$. We write all possible forms for these interactions consistent with Lorentz invariance. We allow the possibility of CP violation. Contributions to the process from anomalous ZZH and γZH interactions studied earlier arise as a special case of our four-point amplitude. Expressions for polar and azimuthal angular distributions of Z arising from the interference of the four-point contribution with the standard-model contribution in the presence of longitudinal and transverse beam polarization are obtained. An interesting CP-odd and T-odd contribution is found to be present only when both electron and positron beams are transversely polarized. Such a contribution is absent when only anomalous ZZH and γZH interactions are considered. We show how angular asymmetries can be used to constrain CP-odd interactions at a linear collider operating at a centre-of-mass energy of 500 GeV with transverse beam polarization.

¹Email address: kumar@prl.res.in

²Email address: saurabh@prl.res.in

1 Introduction

Despite the dramatic success of the standard model (SM), an essential component of SM responsible for generating masses in the theory, viz., the Higgs mechanism, as yet remains untested. The SM Higgs boson, signalling symmetry breaking in SM by means of one scalar doublet of $SU(2)$, is yet to be discovered. A scalar boson with the properties of the SM Higgs boson is likely to be discovered at the Large Hadron Collider (LHC). However, there are a number of scenarios beyond the standard model for spontaneous symmetry breaking, and ascertaining the mass and other properties of the scalar boson or bosons is an important task. This task would prove extremely difficult for LHC. However, scenarios beyond SM, with more than just one Higgs doublet, as in the case of minimal supersymmetric standard model (MSSM), would be more amenable to discovery at a linear e^+e^- collider operating at a centre-of-mass (cm) energy of 500 GeV. We are at a stage when such a linear collider, currently called the International Linear Collider (ILC), seems poised to become a reality [1].

Scenarios going beyond the SM mechanism of symmetry breaking, and incorporating new mechanisms of CP violation have also become a necessity in order to understand baryogenesis which resulted in the present-day baryon-antibaryon asymmetry in the universe. In a theory with an extended Higgs sector and new mechanisms of CP violation, the physical Higgs bosons are not necessarily eigenstates of CP [2, 3]. In such a case, the production of a physical Higgs can proceed through more than one channel, and the interference between two channels can give rise to a CP-violating signal in the production.

Here we consider in a general model-independent way the production of a Higgs mass eigenstate H through the process $e^+e^- \rightarrow HZ$. This is an important mechanism for the production of the Higgs, the other important mechanisms being $e^+e^- \rightarrow e^+e^-H$ and $e^+e^- \rightarrow \nu\bar{\nu}H$ proceeding via vector-boson fusion. $e^+e^- \rightarrow HZ$ is generally assumed to get a contribution from a diagram with an s -channel exchange of Z . At the lowest order, the ZZH vertex in this diagram would be simply a point-like coupling (Fig. 1). Interactions beyond SM can modify this point-like vertex by means of a momentum-dependent form factor, as well as by adding more complicated momentum-dependent forms of anomalous interactions considered in [4, 5, 6, 7, 8, 9, 10]. The corresponding diagram is shown in Fig. 2, where the anomalous ZZH vertex is denoted by a blob. There could also be a

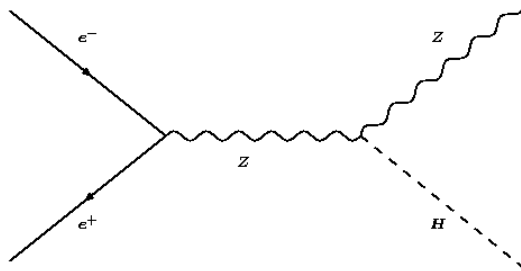


Figure 1: Higgs production diagram with an s -channel exchange of Z with point-like ZZH coupling.

diagram with a photon propagator and an anomalous γZH vertex, which we do not show separately. We consider here a beyond-SM contribution represented by a four-point coupling shown in Fig. 3. This is general enough to include the effects of the diagram in Fig. 2. Such a discussion would be relevant in studying effects of box diagrams with new particles, or diagrams with t -channel exchange of new particles, in addition to s -channel diagrams.

We write down the most general form for the four-point coupling consistent with Lorentz invariance. We do not assume CP conservation. We then obtain angular distributions for Z (and therefore for H) arising from the square of amplitude M_1 for the diagram in Fig. 1 with a point-like ZZH coupling, together with the cross term between M_1 and the amplitude M_2 for the diagram Fig. 3. We neglect the square of M_2 , assuming that this new physics contribution is small compared to the dominant contribution $|M_1|^2$. We include the possibility that the beams have polarization, either longitudinal or transverse. While we have restricted the actual calculation to SM couplings in calculating M_1 , it should be borne in mind that in models with more than one Higgs doublet this amplitude would differ by an overall factor depending on the mixing among the Higgs doublets. Thus our results are trivially applicable to such extensions of SM, by an appropriate rescaling of the coupling.

We are thus addressing the question of how well the form factors for the four-point e^+e^-HZ coupling can be determined from the observation of Z angular distributions in the presence of unpolarized beams or beams with

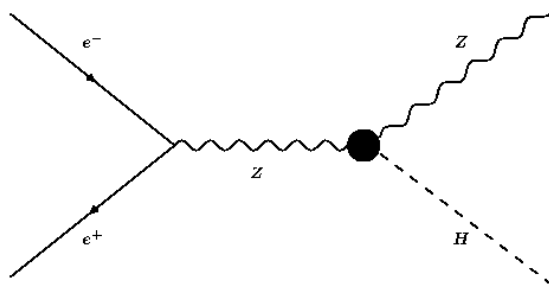


Figure 2: Higgs production diagram with an s -channel exchange of Z with anomalous ZZH coupling.

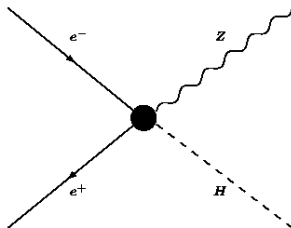


Figure 3: Higgs production diagram with a four-point coupling.

either longitudinal or transverse polarizations. A similar question taking into account a new-physics contribution which merely modifies the form of the ZZH vertex has been addressed before in several works [4, 5, 6, 7, 8, 9, 10]. Those works which do take into account four-point couplings, do not do so in all generality, but stop at the lowest-dimension operators [7]. Studies which include beam polarization in the context of a general VVH vertex are [4, 8, 10]. The approach we adopt here has been used for the process $e^+e^- \rightarrow \gamma Z$ in [11, 12] and for the process $Z \rightarrow b\bar{b}\gamma$ in [13]. A more general analysis of a one-particle inclusive final state is carried out in [14].

The four-point couplings, in the limit of vanishing electron mass, can be neatly divided into two types – chirality-conserving (CC) ones and chirality-violating (CV) ones. The CC couplings involve an odd number of Dirac γ matrices sandwiched between the electron and positron spinors, whereas the

CV ones come from an even number of Dirac γ matrices. In this work, we obtain angular distributions for both CC and CV couplings. However, since in practice, CV couplings are usually proportional to the fermionic mass (in this case the electron mass), we concentrate on the CC ones (see, however, [15]).

Polarized beams are likely to be available at a linear collider, and several studies have shown the importance of linear polarization in reducing backgrounds and improving the sensitivity to new effects [16]. The question of whether transverse beam polarization, which could be obtained with the use of spin rotators, would be useful in probing new physics, has been addressed in recent times in the context of the ILC [11, 15, 16, 17, 18, 20, 21]. In earlier work, it has been observed that polarization does not give any new information about the anomalous ZZH couplings when they are assumed real [10]. However, in our work, we find that there are terms in the differential cross section which are absent unless both electron and positron beams are transversely polarized. Thus, transverse polarization, if available at ILC, would be most useful in isolating such terms. This is particularly significant because these terms are CP violating. Moreover, one of them is even under naive CPT, and thus would survive even when no imaginary part is present in the amplitude. We discuss the ramifications of this in due course.

In the next section we write down the possible model-independent four-point e^+e^-HZ couplings. In Section 3, we obtain the angular distributions arising from these couplings in the presence of beam polarization. Section 4 deals with angular asymmetries which can be used for separating various form factors and Section 5 describes the numerical results. Section 6 contains our conclusions and a discussion.

2 Form factors for the process $e^+e^- \rightarrow HZ$

The most general four-point vertex for the process

$$e^-(p_1) + e^+(p_2) \rightarrow Z^\alpha(q) + H(k) \quad (1)$$

consistent with Lorentz invariance can be written as

$$\Gamma_{4\text{pt}}^\alpha = \Gamma_{\text{CC}}^\alpha + \Gamma_{\text{CV}}^\alpha, \quad (2)$$

where the chirality-conserving part $\Gamma_{\text{CC}}^\alpha$ containing an odd number of Dirac γ matrices is

$$\Gamma_{\text{CC}}^\alpha = -\frac{1}{M}\gamma^\alpha(V_1 + \gamma_5 A_1) + \frac{1}{M^3}\not{p}(V_2 + \gamma_5 A_2)k^\alpha - \frac{i}{M^3}\not{p}(V_3 + \gamma_5 A_3)(p_2 - p_1)^\alpha, \quad (3)$$

and the chirality violating part containing an even number of Dirac γ matrices is

$$\begin{aligned} \Gamma_{\text{CV}}^\alpha &= \frac{i}{M^2} [-(S_1 + i\gamma_5 P_1)k^\alpha - (S_2 + i\gamma_5 P_2)(p_2 - p_1)^\alpha] \\ &\quad - \frac{1}{M^4}\epsilon^{\mu\nu\alpha\beta} p_{2\mu} p_{1\nu} k_\beta (S_3 + i\gamma_5 P_3). \end{aligned} \quad (4)$$

In the above expressions, V_i , A_i , S_i and P_i are form factors, and are Lorentz-scalar functions of the Mandelstam variables s and t for the process eq. (1). For simplicity, we will only consider the case here when the form factors are constants. M is a parameter with dimensions of mass, put in to render the form factors dimensionless.

The expressions for the four-point vertices may be thought to arise from effective Lagrangians

$$\begin{aligned} \mathcal{L}_{\text{CC}} &= \frac{1}{M}\bar{\psi}\not{Z}(v_1 + \gamma_5 a_1)\psi\phi \\ &\quad + \frac{1}{M^3}\bar{\psi}\not{p}\not{Z}^\alpha(v_2 + \gamma_5 a_2)\psi\partial_\alpha\phi \\ &\quad + \frac{i}{M^3} [\partial_\alpha\bar{\psi}\gamma^\mu(v_3 + \gamma_5 a_3)\psi - \bar{\psi}\gamma^\mu(v_3 + \gamma_5 a_3)\partial_\alpha\psi] \phi\partial_\mu Z^\alpha, \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{L}_{\text{CV}} &= \frac{1}{M^2}\bar{\psi}(s_1 + i\gamma_5 p_1)\psi\partial_\alpha\phi Z^\alpha \\ &\quad + \frac{i}{M^2} [\partial_\alpha\bar{\psi}(s_2 + i\gamma_5 p_2)\psi - \bar{\psi}(s_2 + i\gamma_5 p_2)\partial_\alpha\psi] \phi Z^\alpha \\ &\quad + \frac{i}{M^4}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\bar{\psi}(s_3 + i\gamma_5 p_3)\partial_\nu\psi\partial_\beta\phi Z_\alpha, \end{aligned} \quad (6)$$

where the coupling constants v_i , a_i , s_i and p_i in the Lagrangians have been promoted to form factors in momentum space when writing the vertex functions Γ .

It may be appropriate to contrast our approach with the usual effective Lagrangian approach. In the latter approach, it is assumed that SM is an

effective theory which is valid up to a cut-off scale Λ . The new physics occurring above the scale of the cut-off may be parametrized by higher-dimensional operators, appearing with powers of Λ in the denominator. These when added to the SM Lagrangian give an effective low-energy Lagrangian where, depending on the scale of the momenta involved, one includes a range of higher-dimensional operators up to a certain maximum dimension. Our effective theory is not a low-energy limit, so that the form factors we use are functions of momentum not restricted to low powers. Thus, the M we introduce is not a cut-off scale, but an arbitrary parameter, introduced just to make the form factors dimensionless.

We thus find that there are 6 independent form factors in the chirality conserving case, and 6 in the chirality violating case. An alternative form for the Γ above would be using Levi-Civita ϵ tensors whenever a γ_5 occurs. The independent form factors then are then some linear combinations of the form factors given above. However, the total number of independent form factors remains the same.

Note that we have not imposed CP conservation in the above. The CP properties of the various terms appearing in the four-point vertices may be deduced from the CP properties of the corresponding terms in the effective Lagrangian. Thus, one can check that the terms corresponding to the couplings v_3 , a_3 , s_1 , p_2 and s_3 in the effective Lagrangian are CP violating. As a consequence, the terms corresponding to V_3 , A_3 , S_1 , P_2 and S_3 are CP violating, whereas the remaining are CP conserving. This conclusion assumes that the form factors are constants, since the couplings in the effective Lagrangian are constants. The conclusion can also be carried over when the form factors are arbitrary functions of s and even functions of $t - u \equiv \sqrt{s}|\vec{q}| \cos \theta$, where θ is the angle between \vec{q} and \vec{p}_1 (or constants). This is because in momentum space, $s \equiv (p_1 + p_2)^2$ is even under CP, whereas $t - u \equiv \sqrt{s}|\vec{q}| \cos \theta$ is odd under C and even under P, and thus odd under CP.

The expression for the amplitude for (1), arising from the SM diagram of Fig. 1 with a point-like ZZH vertex, is

$$M_{\text{SM}} = -\frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{m_Z}{s - m_Z^2} \bar{v}(p_2) \gamma^\alpha (g_V - \gamma_5 g_A) u(p_1), \quad (7)$$

where the vector and axial-vector couplings of the Z to electrons are given by

$$g_V = -1 + 4 \sin^2 \theta_W, \quad g_A = -1, \quad (8)$$

and θ_W is the weak mixing angle. This corresponds to the special case with the following form factors nonzero:

$$V_1 = \frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{M m_Z}{s - m_Z^2} g_V, \quad (9)$$

and

$$A_1 = -\frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{M m_Z}{s - m_Z^2} g_A. \quad (10)$$

As mentioned earlier, in other models with extra scalar doublets, the above expressions would be modified simply by a factor depending on the mixing among the doublets.

3 Angular distributions

Using the expression (7) for the SM contribution from the diagram (1), we now calculate the angular distribution arising from the square of the SM amplitude and from the interference between the SM amplitude and the amplitude arising from the four-point couplings of (3) or (4). We ignore terms bilinear in the four-point couplings, assuming that the new-physics contribution is small. We treat the two cases of longitudinal and transverse polarizations for the electron and positron beams separately.

We choose the z axis to be the direction of the e^- momentum, and the xz plane to coincide with the production plane. The positive x axis is chosen, in the case of transverse polarization, to be along the direction of the e^- polarization. We then define θ and ϕ to be the polar and azimuthal angles of the momentum \vec{q} of the Z .

We obtain, for the differential cross section with longitudinal polarization, the expression

$$\frac{d\sigma_L}{d\Omega} = \frac{d\sigma_L^{\text{SM}}}{d\Omega} + \frac{d\sigma_L^{\text{CC}}}{d\Omega}, \quad (11)$$

where

$$\frac{d\sigma_L^{\text{SM}}}{d\Omega} = \frac{\lambda^{1/2}}{64\pi^2 s} (1 - P_L \bar{P}_L) F^2 [g_V^2 + g_A^2 - 2g_V g_A P_L^{\text{eff}}] \left[1 + \frac{|\vec{q}|^2 \sin^2 \theta}{2m_Z^2} \right] \quad (12)$$

is the SM contribution, and

$$\frac{d\sigma_L^{\text{CC}}}{d\Omega} = \frac{\lambda^{1/2}}{64\pi^2 s} \frac{2F}{M} (1 - P_L \bar{P}_L)$$

$$\begin{aligned}
& \times \left[\left\{ (g_V - P_L^{\text{eff}} g_A) \text{Re} V_1 + (P_L^{\text{eff}} g_V - g_A) \text{Re} A_1 \right\} \left(1 + \frac{|\vec{q}|^2 \sin^2 \theta}{2m_Z^2} \right) \right. \\
& - \frac{\sqrt{s} q_0}{M^2} \left\{ [(g_V - P_L^{\text{eff}} g_A) \text{Re} V_2 + (P_L^{\text{eff}} g_V - g_A) \text{Re} A_2] \right. \\
& \quad \left. + [(g_V - P_L^{\text{eff}} g_A) \text{Im} V_3 + (P_L^{\text{eff}} g_V - g_A) \text{Im} A_3] \beta_q \cos \theta \right\} \\
& \left. \times \frac{|\vec{q}|^2}{2m_Z^2} \sin^2 \theta \right] \tag{13}
\end{aligned}$$

is the contribution of the chirality-conserving couplings. There is no contribution from the chirality-violating couplings for unpolarized or longitudinally polarized beams. In the above, we have used

$$F = \frac{m_Z}{s - m_Z^2} \left(\frac{e}{2 \sin \theta_W \cos \theta_W} \right)^2, \tag{14}$$

$$\lambda = 4|\vec{q}|^2 s = (s - m_H^2 - m_Z^2)^2 - 4m_H^2 m_Z^2, \tag{15}$$

$$\beta_q = \frac{|\vec{q}|}{q_0}, \tag{16}$$

and

$$P_L^{\text{eff}} = \frac{P_L - \bar{P}_L}{1 - P_L \bar{P}_L}. \tag{17}$$

For the case of transverse polarization, we assume that the spins of the electron and positron are both perpendicular to the beam direction, and also that they are parallel (or anti-parallel) to each other. When the beams are transversely polarized we obtain the differential cross section as

$$\frac{d\sigma_T}{d\Omega} = \frac{d\sigma_T^{\text{SM}}}{d\Omega} + \frac{d\sigma_T^{\text{CC}}}{d\Omega} + \frac{d\sigma_T^{\text{CV}}}{d\Omega}, \tag{18}$$

where

$$\begin{aligned}
\frac{d\sigma_T^{\text{SM}}}{d\Omega} &= \frac{\lambda^{1/2}}{64\pi^2 s} F^2 \left[(g_V^2 + g_A^2) \left(1 + \frac{|\vec{q}|^2}{2m_Z^2} \sin^2 \theta \right) \right. \\
&\quad \left. + P_T \bar{P}_T (g_V^2 - g_A^2) \frac{|\vec{q}|^2}{2m_Z^2} \sin^2 \theta \cos 2\phi \right] \tag{19}
\end{aligned}$$

is the SM contribution,

$$\begin{aligned}
\frac{d\sigma_T^{\text{CC}}}{d\Omega} &= \frac{\lambda^{1/2}}{64\pi^2 s} \frac{2F}{M} \left\{ [(g_V \text{Re}V_1 - g_A \text{Re}A_1) \right. \\
&+ \frac{|\vec{q}|^2}{2m_Z^2} \sin^2 \theta [(g_V \text{Re}V_1 - g_A \text{Re}A_1) \\
&+ P_T \bar{P}_T [(g_V \text{Re}V_1 + g_A \text{Re}A_1) \cos 2\phi - (g_V \text{Im}A_1 + g_A \text{Im}V_1) \sin 2\phi]] \\
&- \frac{s^{1/2} q_0}{M^2} \frac{|\vec{q}|^2}{2m_Z^2} \sin^2 \theta [(g_V \text{Re}V_2 - g_A \text{Re}A_2) \\
&+ P_T \bar{P}_T [(g_V \text{Re}V_2 + g_A \text{Re}A_2) \cos 2\phi - (g_V \text{Im}A_2 + g_A \text{Im}V_2) \sin 2\phi]] \\
&- \frac{s^{1/2} |\vec{q}|}{M^2} \frac{|\vec{q}|^2}{2m_Z^2} \cos \theta \sin^2 \theta [(g_V \text{Im}V_3 - g_A \text{Im}A_3) \\
&+ P_T \bar{P}_T [(g_V \text{Im}V_3 + g_A \text{Im}A_3) \cos 2\phi + (g_V \text{Re}A_3 + g_A \text{Re}V_3) \sin 2\phi]] \left. \right\} \quad (20)
\end{aligned}$$

is the contribution from the chirality-conserving couplings, and

$$\begin{aligned}
\frac{d\sigma_T^{\text{CV}}}{d\Omega} &= \frac{\lambda^{1/2}}{64\pi^2 s} \frac{F s^{1/2}}{M^4} |\vec{q}| \sin \theta \\
&\times \left(q_0 \left[- \{g_V \text{Re}S_1 \sin \phi + g_A \text{Im}S_1 \cos \phi\} (P_T - \bar{P}_T) \right. \right. \\
&+ \left. \{ -g_V \text{Re}P_1 \cos \phi + g_A \text{Im}P_1 \sin \phi \} (P_T + \bar{P}_T) \right] \\
&+ |\vec{q}| \cos \theta \left[\{g_A \text{Re}S_2 \cos \phi - g_V \text{Im}S_2 \sin \phi\} (P_T - \bar{P}_T) \right. \\
&- \left. \{g_A \text{Re}P_2 \sin \phi + g_V \text{Im}P_2 \cos \phi\} (P_T + \bar{P}_T) \right] \\
&+ \frac{1}{2} s^{1/2} \left[\{g_A \text{Re}S_3 \sin \phi + g_V \text{Im}S_3 \cos \phi\} (P_T - \bar{P}_T) \right. \\
&+ \left. \{g_A \text{Re}P_3 \cos \phi - g_V \text{Im}P_3 \sin \phi\} (P_T + \bar{P}_T) \right] \left. \right). \quad (21)
\end{aligned}$$

is the contribution from the chirality-violating couplings.

We now examine how the angular distributions in the presence of polarizations may be used to determine the various form factors.

4 Polarization and Angular asymmetries

The parametrizations we use for the new-physics interactions have 6 complex couplings (form factors) in the CC and case, and 6 in the CV case. Thus, there are 12 real parameters to be determined in each case. We start by

making a simplifying assumption that the form factors we have written down are only functions of s and $t - u$ (or equivalently $\cos\theta$). In that case, using the unpolarized distributions, which have approximately the same form as the SM distribution, viz., $A + B \sin^2\theta$, except for the V_3 and A_3 terms, which have a $\sin^2\theta \cos\theta$ dependence, it is not possible to determine separately all the terms. The terms proportional to $\sin^2\theta \cos\theta$ can be determined using a simple forward-backward asymmetry:

$$A_{\text{FB}}(\theta_0) = \frac{1}{\sigma(\theta_0)} \left[\int_{\theta_0}^{\pi/2} \frac{d\sigma}{d\theta} d\theta - \int_{\pi/2}^{\pi-\theta_0} \frac{d\sigma}{d\theta} d\theta \right], \quad (22)$$

where

$$\sigma(\theta_0) = \int_{\theta_0}^{\pi-\theta_0} \frac{d\sigma}{d\theta} d\theta, \quad (23)$$

and θ_0 is a cut-off in the forward and backward directions needed to keep away from the beam pipe, which could nevertheless be chosen to optimize the sensitivity. This asymmetry is odd under CP and is proportional to the combination $g_V \text{Im}V_3 - g_A \text{Im}A_3$. An observation of $A_{\text{FB}}(\theta_0)$ can thus determine that combination of parameters. It should be noted that only imaginary parts of V_3 and A_3 enter. This can be related to the fact that the CP-violating asymmetry $A_{\text{FB}}(\theta_0)$ is odd under naive CPT. It follows that for it to have a non-zero value, the amplitude should have an absorptive part.

We now treat the cases of longitudinally and transversely polarized beams.

Case(a) Longitudinal polarization:

The forward-backward asymmetry of eq. (22) in the presence of longitudinal polarization, which we denote by $A_{\text{FB}}^L(\theta_0)$, determines a different combination of the same couplings $\text{Im}V_3$ and $\text{Im}A_3$. Thus observing asymmetries with and without polarization, the two imaginary parts can be determined independently.

In the same way, a combination of the cross section for the unpolarized and longitudinally polarized beams can be used to determine two different combinations of the remaining couplings which appear in (13). However, one can get information only on the real parts of V_1, A_1, V_2 and A_2 , not on their imaginary parts.

With unpolarized or longitudinally polarized beams, it is not possible to get any information of the chirality-violating couplings, as they do not contribute.

Case(b) Transverse polarization:

In the case of the angular distribution with transversely polarized beams, there is a dependence on the azimuthal angle ϕ of the Z . Thus, in addition to ϕ -independent terms which are the same as those in the unpolarized case, there are terms with factors $\sin^2 \theta \cos 2\phi$, $\sin^2 \theta \sin 2\phi$, $\sin^2 \theta \cos \theta \cos 2\phi$ and $\sin^2 \theta \cos \theta \sin 2\phi$ in the case of CC couplings, and factors $\sin \theta \cos \phi$, $\sin \theta \sin \phi$, $\sin \theta \cos \theta \cos \phi$, $\sin \theta \cos \theta \sin \phi$ in the case of CV couplings. The ϕ -dependent terms in the CC case occur with the factor of $P_T \overline{P}_T$ and in the CV case with a factor of $P_T + \overline{P}_T$ or $P_T - \overline{P}_T$. Thus, in the CC case, both beams need to have transverse polarization for a nontrivial azimuthal dependence. In the CV case, it is possible to have ϕ dependence with either the electron or the positron beam polarized. We find that that with the possibility of flipping transverse polarization of one beam, it is possible to examine 4 types of angular asymmetries in each of CC and CV cases. Each angular asymmetry would enable the determination of a different combination of couplings.

We will concentrate on the CC case, as most theories permit only CC couplings, at least in the limit of $m_e = 0$. We further restrict ourselves here only to terms which involve a $\cos \theta$ factor, which gives rise to a forward-backward asymmetry, due to the fact that $\cos \theta$ changes sign under $\theta \rightarrow \pi - \theta$. These correspond to the case of CP violation.

We can then define two different asymmetries, which serve to measure two different combinations of CP-violating couplings:

$$\begin{aligned}
 A_{\text{FB}}^{\text{T}}(\theta_0) = & \frac{1}{\sigma(\theta_0)} \left[\int_{\theta_0}^{\pi/2} d\theta \left(\int_0^{\pi/2} d\phi - \int_{\pi/2}^{\pi} d\phi \right. \right. \\
 & \left. \left. + \int_{\pi}^{3\pi/2} d\phi - \int_{3\pi/2}^{2\pi} d\phi \right) \frac{d\sigma}{d\theta d\phi} \right. \\
 & - \int_{\pi/2}^{\pi-\theta_0} d\theta \left(\int_0^{\pi/2} d\phi - \int_{\pi/2}^{\pi} d\phi \right. \\
 & \left. \left. + \int_{\pi}^{3\pi/2} d\phi - \int_{3\pi/2}^{2\pi} d\phi \right) \frac{d\sigma}{d\theta d\phi} \right], \tag{24}
 \end{aligned}$$

and

$$\begin{aligned}
A'_{\text{FB}}{}^{\text{T}}(\theta_0) = & \frac{1}{\sigma(\theta_0)} \left[\int_{\theta_0}^{\pi/2} d\theta \left(\int_{-\pi/4}^{\pi/4} d\phi - \int_{\pi/4}^{3\pi/4} d\phi \right. \right. \\
& \left. \left. + \int_{3\pi/4}^{5\pi/4} d\phi - \int_{5\pi/4}^{7\pi/4} d\phi \right) \frac{d\sigma}{d\theta d\phi} \right. \\
& - \int_{\pi/2}^{\pi-\theta_0} d\theta \left(\int_{-\pi/4}^{\pi/4} d\phi - \int_{\pi/4}^{3\pi/4} d\phi \right. \\
& \left. \left. + \int_{3\pi/4}^{5\pi/4} d\phi - \int_{5\pi/4}^{7\pi/4} d\phi \right) \frac{d\sigma}{d\theta d\phi} \right], \tag{25}
\end{aligned}$$

The former is odd under naive time reversal, whereas the latter is even. The CPT theorem then implies that these would be respectively dependent on real and imaginary parts of form factors. The integrals in the above may be evaluated to yield

$$A_{\text{FB}}^{\text{T}}(\theta_0) = 3P_T \bar{P}_T \frac{|\vec{q}|^3 s^{1/2} (g_V \text{Re } A_3 + g_A \text{Re } V_3) \cos \theta_0 (\cos(2\theta_0) - 3)}{F(g_A^2 + g_V^2) M^3 \pi (12m_Z^2 + 5|\vec{q}|^2 - |\vec{q}|^2 \cos(2\theta_0))}, \tag{26}$$

and

$$A'_{\text{FB}}{}^{\text{T}}(\theta_0) = 3P_T \bar{P}_T \frac{|\vec{q}|^3 s^{1/2} (g_A \text{Im } A_3 + g_V \text{Im } V_3) \cos \theta_0 (\cos(2\theta_0) - 3)}{F(g_A^2 + g_V^2) M^3 \pi (12m_Z^2 + 5|\vec{q}|^2 - |\vec{q}|^2 \cos(2\theta_0))}. \tag{27}$$

We see that the two asymmetries A_{FB}^{T} and $A'_{\text{FB}}{}^{\text{T}}$ can measure, respectively, the combinations $g_V \text{Re } A_3 + g_A \text{Re } V_3$ and $g_A \text{Im } A_3 + g_V \text{Im } V_3$. The latter is dominated by $\text{Im } A_3$ which may also be determined using unpolarized beams. The former requires transverse polarization to measure.

It can be checked that if one considers only contributions from a modification of the ZZH vertex as in [4, 5, 6, 7, 8, 9, 10] A_{FB}^{T} and $A'_{\text{FB}}{}^{\text{T}}$ vanish. This result for A_{FB}^{T} is obtained in [10]. Thus, observation of a nonzero asymmetry would signal the presence of the CP-violating four-point interaction.

5 Numerical Results

We now obtain numerical results for the polarized cross sections, the asymmetries and the sensitivities of these asymmetries for a definite configuration of the linear collider. For our numerical calculations, we have made use of the following values of parameters: $m_Z = 91.19$ GeV, $\alpha = 1/128$, $\sin^2 \theta_W = 0.22$,

$M = 1$ TeV. It should be noted that the particular choice of M is simply for convenience, and is not simply related to any assumption about the scale of new physics – a change in M can always be compensated by corresponding changes in the form factors. For the parameters of the linear collider, we have assumed $\sqrt{s} = 500$ GeV, $P_L = 0.8$, $\bar{P}_L = -0.6$, $P_T = 0.8$, $\bar{P}_T = 0.6$, and an integrated luminosity $\int \mathcal{L} dt = 500 \text{ fb}^{-1}$. For most of our calculations we choose three values of the Higgs mass, $m_H = 150$ GeV, 200 GeV and 300 GeV.

We have assumed that the contribution of the Z exchange diagram of Fig. 1 is the same as that in SM. Since we are keeping open the possibility that the Higgs boson we are dealing with is not an SM Higgs, this assumption may not be correct. However, the modification for a Higgs of a different model will be multiplication by a certain overall factor depending on the mixing of the different Higgs bosons in the model. This can easily be taken care of while interpreting our results for such a model.

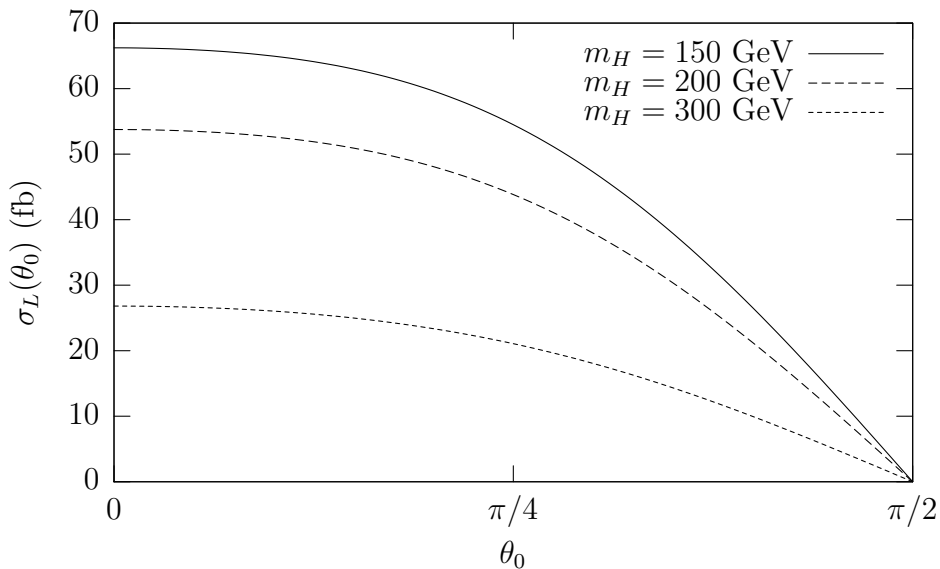


Figure 4: The SM cross section for longitudinally polarized beams with $P_L = 0.8$, $\bar{P}_L = -0.6$ integrated over the polar angle in the range $\theta_0 < \theta < \pi - \theta_0$ as a function of θ_0 for different values of m_H .

In Fig. 4 we have plotted the SM cross section in the presence of longitudinally polarized beams as a function of the cut-off θ_0 for three values

of m_H . Fig. 5 shows the corresponding plot for the SM cross sections with transversely polarized beams.

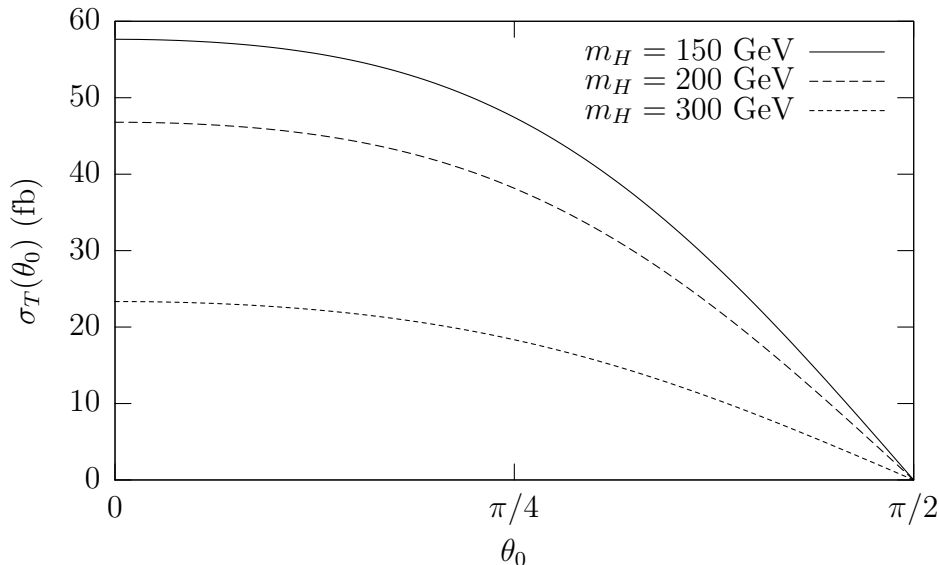


Figure 5: The SM cross section for transversely polarized beams with $P_T = 0.8$, $\overline{P}_T = 0.6$ integrated over the polar angle in the range $\theta_0 < \theta < \pi - \theta_0$ as a function of θ_0 for different values of m_H .

In Fig. 6 is plotted the forward-backward asymmetry $A_{\text{FB}}^L(\theta_0)$ with longitudinal polarization as a function of the cut-off θ_0 . Only the parameter $\text{Im } V_3$ is chosen nonzero, and to have the value 0.1. This choice is for illustration. Fig. 7 shows the same asymmetry for a fixed value of $m_H = 150$ GeV, for the combinations $\text{Im } V_3 = 0.1$, $\text{Im } A_3 = 0$ and $\text{Im } V_3 = 0$, $\text{Im } A_3 = 0.1$, and for values of \overline{P}_L differing in sign. The asymmetry depends on the relative signs of P_L and \overline{P}_L and is larger in magnitude when the relative signs are opposite.

In the case of transverse polarization, the two asymmetries A_{FB}^T and A'_{FB}^T are shown as functions of θ_0 in Figs. 8 and 9 for values of polarization $P_T = 0.8$ and $\overline{P}_T = 0.6$. In Fig. 8, the only nonzero parameter is $\text{Re } V_3 = 0.1$, whereas in Fig. 9, the only nonzero parameter is $\text{Im } V_3 = 0.1$.

In all the above figures, the dependence on the cut-off θ_0 is mild for small values of θ_0 . Hence the results will not be sensitive to the choice of θ_0 , if it is small.

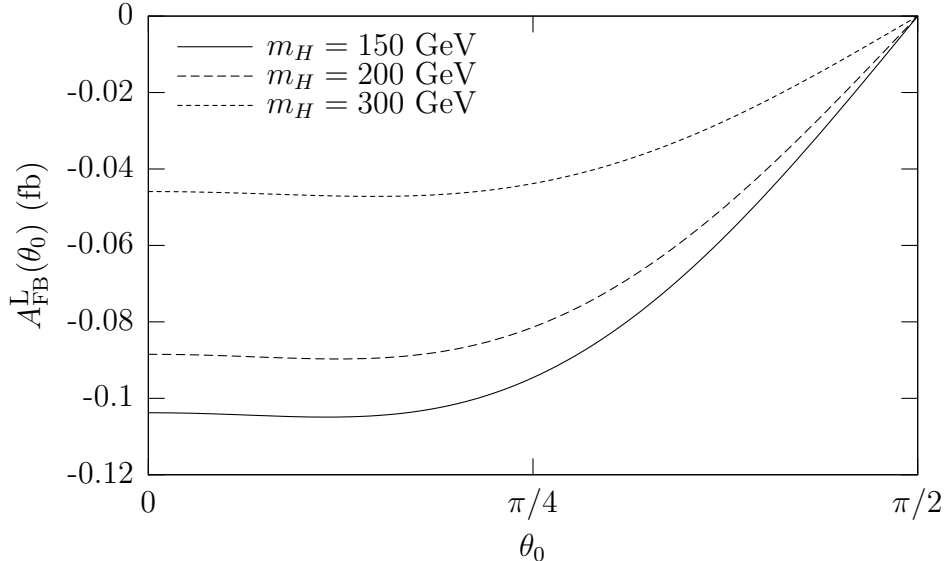


Figure 6: The forward-backward asymmetry for longitudinally polarized beams, $A_{\text{FB}}^L(\theta_0)$, for $\text{Im } V_3 = 0.1$ as a function of θ_0 . All other couplings are taken to be zero.

We now examine the accuracy to which each of the couplings can be determined for linear collider operating at $\sqrt{s} = 500$ GeV and with an integrated luminosity of 500 fb^{-1} . At the 90% confidence level (CL), the limit that can be placed on a parameter contributing linearly to a certain asymmetry A is given by $1.64/(A_1\sqrt{N_{\text{SM}}})$, where A_1 is the asymmetry for unit value of the parameter.

We first consider the determination of the parameter $\text{Re } V_3$ from a measurement of the asymmetry A_{FB}^T for a typical value of $\theta_0 = 45^\circ$. If the asymmetry is not observed, we find that the limit placed on $\text{Re } V_3$ is 3.9×10^{-2} for $m_H = 150$ GeV, 5.1×10^{-2} for $m_H = 200$ GeV, and 1.3×10^{-1} for $m_H = 300$ GeV. Since the combination which appears in the asymmetry is $g_V \text{Re } A_3 + g_A \text{Re } V_3$, it implies that the corresponding limits on $\text{Re } A_3$ will be a factor $|g_A/g_V| \approx 8.3$ higher. Thus, the asymmetry is more sensitive to $\text{Re } V_3$ because of a larger coupling g_A multiplying it.

The expression for $A_{\text{FB}}^{\prime T}$ is identical to that for A_{FB}^T , except that the factor $g_V \text{Re } A_3 + g_A \text{Re } V_3$ is replaced by $g_V \text{Im } V_3 + g_A \text{Im } A_3$. Thus, now it will be $\text{Im } A_3$ which will have the limits mentioned above for $\text{Re } V_3$, and $\text{Im } V_3$ will

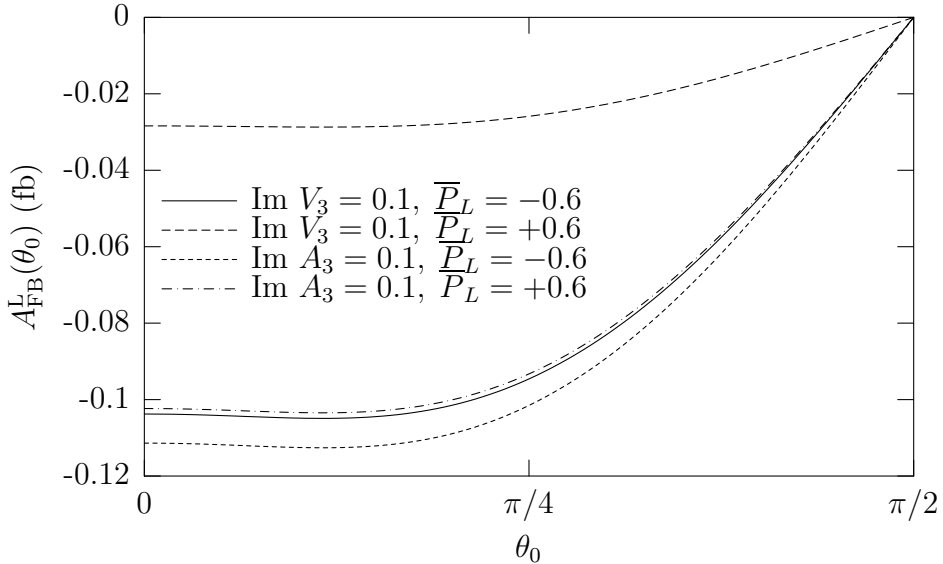


Figure 7: The asymmetry $A_{\text{FB}}^L(\theta_0)$ defined in the text for $m_H = 150$ GeV, $P_L = 0.8$, for two combinations of couplings, and for two values of \bar{P}_L differing in sign.

have limits which are a factor of about 8.3 larger.

It should be borne in mind that the definition of the couplings (form factors) are dependent on the value of the scale parameter M which is chosen. Thus changing the value of M will change the limits on the form factors.

6 Conclusions and Discussion

We have parametrized the amplitude for the process $e^+e^- \rightarrow HZ$ using only Lorentz invariance by means of form factors, treating separately the chirality-conserving and chirality-violating cases. We then calculated the differential cross section for the process $e^+e^- \rightarrow HZ$ in terms of these form factors for polarized beams. The motivation was to determine the extent to which longitudinal and transverse polarizations can help in an independent determination of the various form factors.

We found that in the presence of transverse polarization, there is a CP-odd and T-odd contribution to the angular distribution. The coupling combinations this term depends on cannot be determined using longitudinally

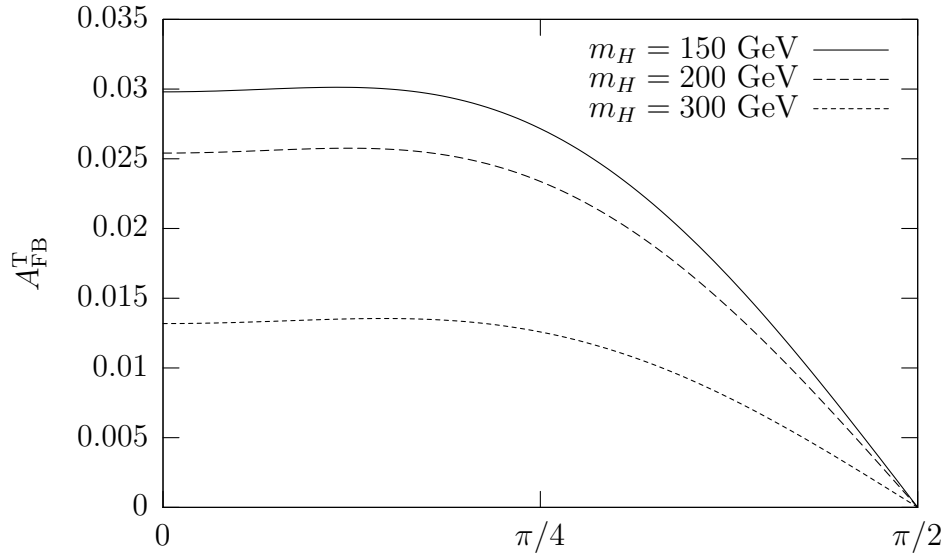


Figure 8: The asymmetry A_{FB}^T for transverse polarizations $P_T = 0.8$ and $\overline{P}_T = 0.6$ plotted against θ_0 for $\text{Re } V_3 = 0.1$. All other couplings are taken to be zero.

polarized beams. Moreover, this transverse-polarization dependent contribution does not arise when only VVH type of couplings are considered. Hence such a term, if observed, would be a unique signal of CP-violating four-point interaction.

It should be emphasized that our results and conclusions are dependent on the assumption that the form factors are independent of t and u . In particular, the CP property of a given term in the distribution would change if the corresponding form factor is an odd function of $\cos\theta$. The reason is that $\cos\theta \equiv q \cdot (p_2 - p_1)/(|\vec{q}|s^{1/2})$ is odd under CP.

We have discussed limits on the couplings that would be expected from a definite configuration of the linear collider. As for the CP-conserving couplings, limits may be obtained even from the existing LEP data, which has excluded SM Higgs up to mass of about 114 GeV. However, we have concentrated only on the limits on the CP-violating couplings. It should be borne in mind that the limits on these depend on the choice of M , the arbitrary parameter of dimension of mass that we introduced.

Though we have used SM couplings for the leading contribution of Fig.

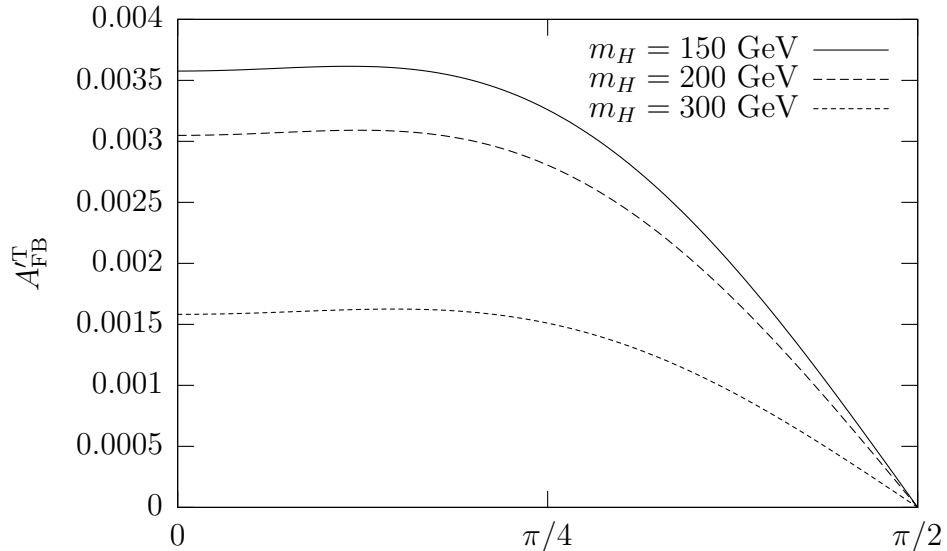


Figure 9: The asymmetry A_{FB}^{T} for transversely polarizations $P_T = 0.8$ and $\overline{P}_T = 0.6$ plotted against θ_0 for $\text{Im } V_3 = 0.1$. All other couplings are taken to be zero.

1, as mentioned earlier, the analysis needs only trivial modification when applied to a model like MSSM or a multi-Higgs-doublet model, and will be useful in such extensions of SM. It is likely that such models will give rise to four-point contributions through box diagrams or loop diagrams with a t -channel exchange of particles. However, to our knowledge, such calculations are not available for CP-violating models. The interesting effects we have discussed would make it useful to carry out such calculations.

We have discussed the angular distribution of the Z in the process $e^+e^- \rightarrow HZ$. Clearly, for the discussion to be of practical use, one has to include the means of detection of Z and H . Thus, it is important to include decays of Z and H and to see what our analysis implies for the decay products. In particular, one has to answer the question as to how leptons or jets from the decay of the Z can be used to measure the asymmetries we discuss, and with what efficiency. To the extent that the sum of the four-momenta of the charged lepton pair or the jet pair can be a measure of the Z four-momentum, it should be possible to reconstruct the asymmetries discussed here with reasonable accuracy. One should also investigate the effect experimental

cuts would have on the accuracy of the determination of the couplings. One should keep in mind the possibility that radiative corrections can lead to quantitative changes in the above results (see, for example, [22]). While these practical questions are not addressed in this work, we feel that the interesting new features we found would make it worthwhile to address them in future.

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