

Angular distributions as a probe of anomalous ZZH and γZH interactions at a linear collider with polarized beams

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Abstract

We examine the contribution of general Z^*ZH and γ^*ZH three-point interactions arising from new physics to the Higgs production process $e^+e^- \rightarrow HZ$. From Lorentz covariance, each of these vertices may be written in terms of three (complex) form factors, whose real and imaginary parts together make six independent couplings. We take into account possible longitudinal or transverse beam polarization likely to be available at a linear collider. We show how partial cross sections and angular asymmetries in suitable combinations with appropriate beam polarizations, can be used to disentangle various couplings from one another. A striking result is that using transverse polarization, one of the γZH couplings, not otherwise accessible, can be determined independently of all other couplings. Transverse polarization also helps in the independent determination of a combination of two other couplings, in contrast to a combination of four accessible with unpolarized or longitudinally polarized beams. We also obtain the sensitivity of the various observables in constraining the new-physics interactions at a linear collider operating at a centre-of-mass energy of 500 GeV with longitudinal or transverse polarization.

1 Introduction

Despite the dramatic success of the standard model (SM), an essential component of SM responsible for generating masses in the theory, viz., the Higgs mechanism, remains untested. The SM Higgs boson, signalling symmetry breaking in SM by means of one scalar doublet of $SU(2)$, is yet to be discovered. A scalar boson with the properties of the SM Higgs boson is likely to be discovered at the Large Hadron Collider (LHC). However, there are a number of scenarios beyond the standard model for spontaneous symmetry breaking, and ascertaining the mass and other properties of the scalar boson or bosons is an important task. This task would prove extremely difficult for LHC. However, scenarios beyond SM, with more than just one Higgs doublet, as in the case of the minimal supersymmetric standard model (MSSM), would be more amenable to discovery at a linear e^+e^- collider operating at a centre-of-mass (cm) energy of 500 GeV. We are at a stage when the International Linear Collider (ILC) seems poised to become a reality [1].

Scenarios going beyond the SM mechanism of symmetry breaking, and incorporating new mechanisms of CP violation, have also become a necessity in order to understand baryogenesis which resulted in the present-day baryon-antibaryon asymmetry in the universe. In a theory with an extended Higgs sector and new mechanisms of CP violation, the physical Higgs bosons are not necessarily eigenstates of CP. In such a case, the production of a physical Higgs can proceed through more than one channel, and the interference between two channels can give rise to a CP-violating signal in the production.

Here we consider in a general model-independent way the production of a Higgs mass eigenstate H in a possible extension of SM through the process $e^+e^- \rightarrow HZ$ mediated by s -channel virtual γ and Z . This is an important mechanism for the production of the Higgs, the other important mechanisms being $e^+e^- \rightarrow e^+e^-H$ and $e^+e^- \rightarrow \nu\bar{\nu}H$ proceeding via vector-boson fusion. $e^+e^- \rightarrow HZ$ is generally assumed to get a contribution from a diagram with an s -channel exchange of Z . At the lowest order, the ZZH vertex in this diagram would be simply a point-like coupling (Fig. 1). Interactions beyond SM can modify this point-like vertex by means of a momentum-dependent form factor, as well as by adding more complicated momentum-dependent forms of anomalous interactions considered in [2]-[10]. The corresponding diagram is shown in Fig. 2, where the anomalous ZZH vertex is denoted by a blob. There is also a diagram with a photon propagator and an anomalous

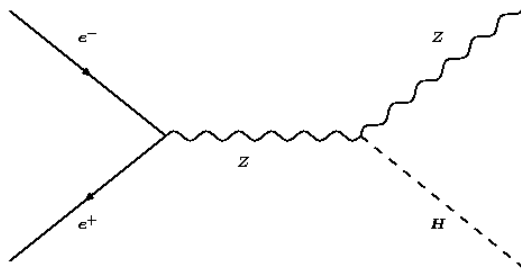


Figure 1: Higgs production diagram with an s -channel exchange of Z with point-like ZZH coupling.

γZH vertex, which does not occur in SM at tree level. This is shown in Fig. 3 by a blob. This coupling vanishes in SM at tree level, but can get contributions at higher order in SM or in extensions of SM. Such anomalous γZH couplings were considered earlier in [3, 5, 9, 10].

Refs. [11, 12] considered a beyond-SM contribution represented by a four-point e^+e^-HZ coupling general enough to include the effects of the diagrams of Figs. 2 and 3, as well as additional couplings going beyond s -channel exchanges. By considering appropriate relations between those form factors and momentum dependencies, we can derive expressions we consider here. While the four-point coupling is most general, the dominant contributions are likely to arise from the three-point couplings considered here.

We write the most general Z^*ZH and γ^*ZH couplings consistent with Lorentz invariance. We do not assume CP conservation. We then obtain angular distributions for the Z arising from the square of amplitude M_1 for the diagram in Fig. 1 with a point-like ZZH coupling, together with the cross terms between M_1 and the amplitude M_2 for the diagram in Fig. 2, and the amplitude M_3 for the diagram in Fig. 3. We neglect terms quadratic in M_2 and M_3 , assuming that the new-physics contribution is small compared to the dominant SM contribution. We include the possibility that the beams have polarization, either longitudinal or transverse. While we have restricted the actual calculation to SM couplings in calculating M_1 , it should be borne in mind that in models with more than one Higgs doublet this amplitude would differ by an overall factor depending on the mixing among the Higgs

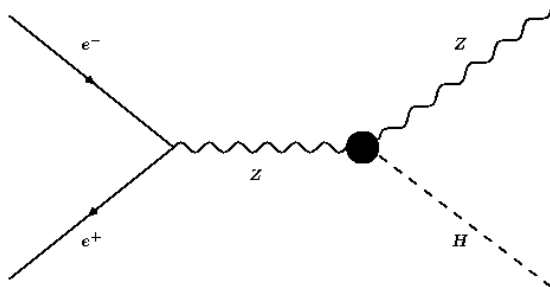


Figure 2: Higgs production diagram with an s -channel exchange of Z with anomalous ZZH coupling.

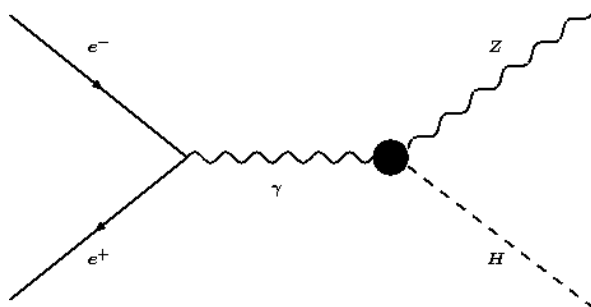


Figure 3: Higgs production diagram with an s -channel exchange of Z with anomalous γZH coupling.

doublets. Thus our results are trivially applicable to such extensions of SM, by an appropriate rescaling of the coupling.

We are thus addressing the question of how well the form factors for the anomalous ZZH and γZH couplings in $e^+e^- \rightarrow HZ$ can be determined from the observation of Z angular distributions in the presence of unpolarized beams or beams with either longitudinal or transverse polarizations. This question taking into account a new-physics contribution which merely modifies the form of the ZZH vertex has been addressed before in several works [2, 4, 6, 7, 8]. This amounts to assuming that the γZH couplings are zero or negligible. Refs. [3, 5, 9, 10] do take into account both γZH and ZZH couplings. However, they relate both to coefficients of terms of higher dimensions in an effective Lagrangian, whereas we treat all couplings

as independent of one another. Moreover, [5] does not discuss effects of beam polarization. On the other hand, we attempt to seek ways to determine the couplings completely independent of one another. Refs. [9, 10] does have a similar approach to ours. They make use of optimal observables and consider only longitudinal electron polarization, whereas we seek to use simpler observables and asymmetries constructed out of the Z angular variables, and consider the effects of longitudinal and transverse polarization of both e^- and e^+ beams. The authors of [9] also include τ polarization and b -jet charge identification which we do not require.

One specific practical aspect in which our approach differs from that of the effective Lagrangians is that while the couplings are all taken to be real in the latter approach, we allow the couplings to be complex, and in principle, momentum-dependent form factors.

Polarized beams are likely to be available at a linear collider, and several studies have shown the importance of longitudinal polarization in reducing backgrounds and improving the sensitivity to new effects [13]. The question of whether transverse beam polarization, which could be obtained with the use of spin rotators, would be useful in probing new physics, has been addressed in recent times in the context of the ILC [11]-[19]. In earlier work, it has been observed that polarization does not give any new information about the anomalous ZZH couplings when they are assumed real [3]. However, the sensitivity can be improved by suitable choice of polarization. Moreover, polarization can indeed give information about the imaginary parts of the couplings. A model-independent approach on kinematic observables in one- and two-particle final states when longitudinal or transverse beam polarization is present, which covers our present process, can be found in [20].

In this work, our emphasis has been on simultaneous independent determination of couplings, to the extent possible, making use of a combination of asymmetries and/or polarizations. We have also tried to consider rather simple observables, conceptually, as well as from an experimental point of view. With this objective in mind, we use only Z angular distributions without including the polarization or the decay of the Z . This amounts to using the sum of the momenta of the Z decay products. Since we do not require charge determination, this has the advantage that one can include both leptonic and hadronic decays of the Z . On the other hand, if a measurement on the Higgs-boson decay products is made, we can also use the two-neutrino decay channels of Z since the missing energy-momentum would be fully determined.

When all couplings are assumed to be independent and nonzero, we find that angular asymmetries are linear combinations of a certain number of anomalous couplings (in our approximation of neglecting terms quadratic in anomalous couplings). By using that many number of observables, for example, different asymmetries, or the same asymmetry measured for different beam polarizations, one can solve simultaneous linear equations to determine the couplings involved. This is the approach we follow here. A similar technique of considering combinations of different polarizations was made use of, for example, in [21].

We find that longitudinal polarization is particularly useful in achieving our purpose of determining a different combination of couplings compared to the unpolarized case. As it turns out, the cross section with transverse polarization generally provides combinations of the same couplings as longitudinal polarization. A marked exception is the angular dependence associated with the coupling $\text{Im } a_\gamma$ (the couplings are defined in the next section) – it is possible to use an azimuthal asymmetry which depends entirely on this coupling when the beams are transversely polarized, and its measurement would determine this coupling directly. Unpolarized or longitudinally polarized beams provide no access to $\text{Im } a_\gamma$. Another azimuthal asymmetry in the presence of transverse polarization helps to isolate a combination of two couplings $\text{Re } a_\gamma$ and $\text{Re } \Delta a_Z$ out of the four which contribute to the differential cross section with longitudinal polarization.

In the next section we write down the possible model-independent ZZH and γZH couplings. In Section 3, we obtain the angular distributions arising from these couplings in the presence of beam polarization. Section 4 deals with asymmetries which can be used for separating various form factors and Section 5 describes the numerical results. Section 6 contains our conclusions and a discussion.

2 Form factors for the process $e^+e^- \rightarrow HZ$

Assuming Lorentz invariance, the general structure for the vertex corresponding to the process $V_\mu^*(k_1) \rightarrow Z_\nu(k_2)H$, where $V \equiv \gamma$ or Z , can be written as [4, 6, 7, 9]

$$\Gamma_{\mu\nu} = g_V m_Z \left[a_V g_{\mu\nu} + \frac{b_V}{m_Z^2} (k_{1\nu} k_{2\mu} - g_{\mu\nu} k_1 \cdot k_2) + \frac{\tilde{b}_V}{m_Z^2} \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right], \quad (1)$$

where a_V , b_V and \tilde{b}_V , are form factors, which are in general complex. We have omitted terms proportional to $k_{1\mu}$ and $k_{2\nu}$, which do not contribute to the process $e^+e^- \rightarrow HZ$ in the limit of vanishing electron mass. The constant g_Z is chosen to be $g/\cos\theta_W$, so that $a_Z = 1$ for SM. g_γ is chosen to be e . Of the interactions in (1), the terms with \tilde{b}_Z and \tilde{b}_γ are CP violating, whereas the others are CP conserving. Henceforth we will write $a_Z = 1 + \Delta a_Z$, Δa_Z being the deviation of a_Z from its tree-level SM value. The other form factors are vanishing in SM at tree level. Thus the above ‘‘couplings,’’ which are deviations from the tree-level SM values, could arise from loops in SM or from new physics beyond SM. We could of course work with a set of modified couplings where the anomalous couplings denote deviations from the tree-level values in a specific extension of the SM model, like a concrete two-Higgs doublet model. The corresponding modifications are trivial to incorporate.

The expression for the amplitude for the process

$$e^-(p_1) + e^+(p_2) \rightarrow Z^\alpha(q) + H(k), \quad (2)$$

arising from the SM diagram of Fig. 1 with a point-like ZZH vertex, is

$$M_{\text{SM}} = -\frac{e^2}{4\sin^2\theta_W\cos^2\theta_W} \frac{m_Z}{s - m_Z^2} \bar{v}(p_2)\gamma^\alpha(g_V - \gamma_5 g_A)u(p_1), \quad (3)$$

where the vector and axial-vector couplings of the Z to electrons are given by

$$g_V^e = -1 + 4\sin^2\theta_W, \quad g_A^e = -1, \quad (4)$$

and θ_W is the weak mixing angle.

3 Differential cross sections

We now obtain the differential cross section for the process (2) keeping the pure SM contribution, and the interference between the SM amplitude of Fig. 1 and the amplitudes with anomalous γZH and ZZH couplings of Figs. 2 and 3, respectively. We ignore terms bilinear in the anomalous couplings, assuming that the new-physics contribution is small. We treat the two cases of longitudinal and transverse polarizations for the electron and positron beams separately. We neglect the mass of the electron.

Helicity amplitudes for the process were obtained earlier in the context of an effective Lagrangian approach [5, 3, 9, 10], and could be made use of for obtaining the differential cross section for the case of longitudinal polarization, and, with less ease, for the case of transverse polarization. We have used instead trace techniques employing the symbolic manipulation program ‘FORM’ [22].

Note that though we have used SM couplings for the leading contribution, it is trivial to modify these by overall factors for cases of other models (like two-Higgs-doublet models). Our expressions are not, however, applicable for the case when the Higgs is a pure pseudoscalar in models conserving CP, since in that case, the SM-like lowest-order couplings are absent.

We choose the z axis to be the direction of the e^- momentum, and the xz plane to coincide with the HZ production plane in the case when the initial beams are unpolarized or longitudinally polarized. The positive x axis is chosen, in the case of transverse polarization, to be along the direction of the e^- polarization. We then define θ and ϕ to be the polar and azimuthal angles of the momentum \vec{q} of the Z . We use the convention $\epsilon^{0123} = +1$ for the Levi-Civita tensor.

3.1 Angular distributions for longitudinal polarization

The angular distribution for the process (2) with longitudinal polarizations P_L and \bar{P}_L respectively of the e^- and e^+ beams may be written as

$$\frac{d\sigma_L}{d\Omega} = (1 - P_L \bar{P}_L) [A_L + B_L \sin^2 \theta + C_L \cos \theta], \quad (5)$$

where A_L, B_L, C_L are further written in terms of contributions from SM alone (superscript “SM”), interference between SM and ZZH terms (superscript Z), and interference between SM and γZH (superscript γ):

$$A_L = A_L^{\text{SM}} + A_L^Z + A_L^\gamma, \quad (6)$$

$$B_L = B_L^{\text{SM}} + B_L^Z + B_L^\gamma, \quad (7)$$

$$C_L = C_L^Z + C_L^\gamma. \quad (8)$$

In case of C_L , which is the coefficient of a CP-odd term, there is no contribution from SM. The expressions for the various terms used above are as follows.

$$A_L^{\text{SM}} = B_L^{\text{SM}} \frac{2m_Z^2}{|\vec{q}|^2} = (g_V^{e2} + g_A^{e2} - 2g_V^e g_A^e P_L^{\text{eff}}) K^{\text{SM}}, \quad C_L^{\text{SM}} = 0, \quad (9)$$

where

$$K^{\text{SM}} = \frac{\alpha^2 |\vec{q}|}{2\sqrt{s} \sin^4 2\theta_W} \frac{m_Z^2}{(s - m_Z^2)^2}. \quad (10)$$

We also have

$$A_L^Z = 2 \left(\text{Re } \Delta a_Z + \text{Re } b_Z \frac{\sqrt{s} q^0}{m_Z^2} \right) (g_V^{e2} + g_A^{e2} - 2g_V^e g_A^e P_L^{\text{eff}}) K^{\text{SM}}, \quad (11)$$

$$B_L^Z = 2 \text{Re } \Delta a_Z \frac{|\vec{q}|^2}{2m_Z^2} (g_V^{e2} + g_A^{e2} - 2g_V^e g_A^e P_L^{\text{eff}}) K^{\text{SM}}, \quad (12)$$

$$C_L^Z = 2 \text{Im } \tilde{b}_Z \frac{\sqrt{s} |\vec{q}|}{m_Z^2} ((g_V^{e2} + g_A^{e2}) P_L^{\text{eff}} - 2g_V^e g_A^e) K^{\text{SM}}. \quad (13)$$

Next,

$$A_L^\gamma = \left(\text{Re } a_\gamma + \text{Re } b_\gamma \frac{\sqrt{s} q^0}{m_Z^2} \right) (g_V^e - g_A^e P_L^{\text{eff}}) K^\gamma, \quad (14)$$

$$B_L^\gamma = \text{Re } a_\gamma \frac{|\vec{q}|^2}{2m_Z^2} (g_V^e - g_A^e P_L^{\text{eff}}) K^\gamma, \quad C_L^\gamma = \frac{\sqrt{s} |\vec{q}|}{m_Z^2} \text{Im } \tilde{b}_\gamma (g_A^e - g_V^e P_L^{\text{eff}}) K^\gamma, \quad (15)$$

where

$$K^\gamma = \frac{\alpha^2 |\vec{q}|}{\sqrt{s} \sin^2 2\theta_W} \frac{m_Z^2}{s(s - m_Z^2)}. \quad (16)$$

In the above, we have used the effective polarization

$$P_L^{\text{eff}} = \frac{P_L - \overline{P}_L}{1 - P_L \overline{P}_L}. \quad (17)$$

The expressions for the Z energy q^0 and the magnitude of its three-momentum $|\vec{q}|$ are

$$q^0 = \frac{s + m_Z^2 - m_H^2}{2\sqrt{s}}, \quad |\vec{q}| = \frac{\sqrt{s^2 + (m_Z^2 - m_H^2)^2 - 2s(m_Z^2 + m_H^2)}}{2\sqrt{s}}. \quad (18)$$

Immediate inferences from these expressions are: (i) If the six coefficients $A_L^{\gamma,Z}$, $B_L^{\gamma,Z}$ and $C_L^{\gamma,Z}$ could be determined independently using angular distributions and polarization, it would be possible to determine the six anomalous couplings $\text{Re } a_\gamma$, $\text{Re } \Delta a_Z$, $\text{Re } b_\gamma$, $\text{Re } b_Z$, $\text{Im } \tilde{b}_\gamma$ and $\text{Im } \tilde{b}_Z$. (ii) Imaginary parts of a_γ , Δa_Z , b_γ , b_Z , and real parts of \tilde{b}_γ , \tilde{b}_Z do not contribute to the angular distributions at this order, and hence remain undetermined. (iii)

Numerically g_V^e is small (about -0.12 for $\sin^2 \theta_W = 0.22$), while $g_A^e = -1$. Hence, in the absence of polarization, from among the anomalous contributions, the terms A_L^Z , B_L^Z and C_L^γ dominate over the others. If these coefficients are determined from angular distributions, it would be possible to measure $\text{Re } \Delta a_Z$, $\text{Re } b_Z$ and $\text{Im } \tilde{b}_\gamma$ with greater sensitivity. On the other hand, there would be very low sensitivity to the remaining couplings, viz., $\text{Re } a_\gamma$, $\text{Re } b_\gamma$ and $\text{Im } \tilde{b}_Z$. (iv) Within the combinations of couplings which appear in A_L^γ and A_L^Z , the contributions of $\text{Re } b_\gamma$ and $\text{Re } b_Z$ are enhanced because of the factor $\sqrt{s}q^0/m_Z^2$ multiplying them. This improves their sensitivity. (v) With longitudinal polarization turned on, with a reasonably large value of P_L^{eff} , the coefficients C_L^Z , A_L^γ and B_L^γ would become significant. In that case, the sensitivity to $\text{Re } a_\gamma$, $\text{Re } b_\gamma$ and $\text{Im } \tilde{b}_Z$ would be improved. (vi) In view of (v), it is clear that a combination of angular distributions for the polarized and unpolarized cases will help in disentangling the different couplings.

3.2 Angular distributions for transverse polarization

For the transverse case, we take the e^- polarization to be along the x axis and that of the e^+ in the xy plane, making an angle of δ with the x axis, so that $\delta = 0$ corresponds to parallel e^- and e^+ transverse polarizations. The expression for the cross section with transverse polarization P_T for the e^- beam and \bar{P}_T for the e^+ beam is

$$\begin{aligned} \frac{d\sigma_T}{d\Omega} = & [A_T + B_T \sin^2 \theta + C_T \cos \theta \\ & + P_T \bar{P}_T \sin^2 \theta \{D_T \cos(2\phi - \delta) + E_T \sin(2\phi - \delta)\}], \end{aligned} \quad (19)$$

where A_T , B_T , C_T , D_T and E_T are further written in terms of contributions from SM alone (superscript ‘‘SM’’), interference between SM and ZZH terms (superscript Z), and interference between SM and γZH (superscript γ), in exact analogy with expressions for A_L , B_L and C_L given earlier for the longitudinal polarization case. The expressions for the separate contributions for these coefficients are as follows.

$$A_T^{\text{SM}} = B_T^{\text{SM}} \frac{2m_Z^2}{|\vec{q}|^2} = (g_V^{e2} + g_A^{e2}) K^{\text{SM}}, \quad (20)$$

$$C_T^{\text{SM}} = 0, \quad D_T^{\text{SM}} = \frac{|\vec{q}|^2}{2m_Z^2} (g_V^{e2} - g_A^{e2}) K^{\text{SM}}, \quad E_T^{\text{SM}} = 0, \quad (21)$$

$$A_T^Z = 2(g_V^{e2} + g_A^{e2}) \left(\text{Re } \Delta a_Z + \text{Re } b_Z \frac{\sqrt{s} q^0}{m_Z^2} \right) K^{SM}, \quad (22)$$

$$B_T^Z = 2 \frac{|\vec{q}|^2}{2m_Z^2} \text{Re } \Delta a_Z (g_V^{e2} + g_A^{e2}) K^{SM}, \quad C_T^Z = 2 \text{Im } \tilde{b}_Z \frac{\sqrt{s} |\vec{q}|}{m_Z^2} 2g_V^e g_A^e K^{SM}, \quad (23)$$

$$D_T^Z = 2 \frac{|\vec{q}|^2}{2m_Z^2} (-\text{Re } \Delta a_Z) (g_V^{e2} - g_A^{e2}) K^{SM}, \quad E_T^Z = 0, \quad (24)$$

$$A_T^\gamma = \left(\text{Re } a_\gamma + \text{Re } b_\gamma \frac{\sqrt{s} q^0}{m_Z^2} \right) (g_V^e) K^\gamma, \quad (25)$$

$$B_T^\gamma = \frac{|\vec{q}|^2}{2m_Z^2} \text{Re } a_\gamma (g_V^e) K^\gamma, \quad C_T^\gamma = \frac{\sqrt{s} |\vec{q}|}{m_Z^2} \text{Im } \tilde{b}_\gamma (g_A^e) K^\gamma, \quad (26)$$

$$D_T^\gamma = \frac{|\vec{q}|^2}{2m_Z^2} \text{Re } a_\gamma (-g_V^e) K^\gamma, \quad E_T^\gamma = \frac{|\vec{q}|^2}{2m_Z^2} \text{Im } a_\gamma (g_A^e) K^\gamma. \quad (27)$$

Taking a look at the above equations, one can infer the following: (i) For studying any effects dependent on transverse polarization, and therefore, of the azimuthal distribution of the Z , both electron and positron beams have to be polarized. (ii) If the azimuthal angle ϕ of Z is integrated over, there is no difference between the transversely polarized and unpolarized cross sections [23]. Thus the usefulness of transverse polarization comes from the study of nontrivial ϕ dependence. (iii) A glaring advantage of using transverse polarization would be to determine $\text{Im } a_\gamma$ from the $\sin(2\phi - \delta)$ dependence of the angular distribution. It can be seen that E_T receives contribution only from E_T^γ , which determines $\text{Im } a_\gamma$ independently of any other coupling. Moreover, $\text{Im } a_\gamma$ does not contribute to unpolarized or longitudinally polarized cases. (iv) The $\cos(2\phi - \delta)$ dependence of the angular distribution (the D_T term) determines a combination only of the couplings $\text{Re } \Delta a_Z$ and $\text{Re } a_\gamma$. On the other hand, in the case of unpolarized or longitudinally polarized beams the coefficient B_L does depend only on $\text{Re } a_\gamma$ and $\text{Re } \Delta a_Z$, and if measured, can give information on $\text{Re } a_\gamma$ and $\text{Re } \Delta a_Z$ independently of $\text{Re } b_\gamma$ and $\text{Re } b_Z$. However, there is no simple asymmetry which allows B_L to be measured separately from A_L , which depends on a combination of all four of $\text{Re } a_\gamma$, $\text{Re } \Delta a_Z$, $\text{Re } b_\gamma$ and $\text{Re } b_Z$. Of course, it is in principle possible to separate A_L and B_L using either a fit or using cross sections integrated over different ranges. The latter approach has been used in Sec. 5.1 to obtain simultaneous limits on all of $\text{Re } a_\gamma$, $\text{Re } \Delta a_Z$, $\text{Re } b_\gamma$ and $\text{Re } b_Z$. (v) The real parts of the

CP-violating couplings \tilde{b}_Z and \tilde{b}_γ remain undetermined with either longitudinal or transverse polarization. (vi) $Im \Delta a_Z$ also remains undetermined.

We now examine how the angular distributions in the presence of polarizations may be used to determine the various form factors.

4 Polarization and Angular asymmetries

In this section we discuss observables like partial cross sections and angular asymmetries which can be used to determine the anomalous couplings.

One of the simplest observables is a partial cross section, i.e., the differential cross section integrated over all azimuthal angles, but over a limited range in θ . A cut-off in the forward and backward directions is natural for avoiding the beam pipe, and so this would be an obvious cut on θ . Such a partial cross section would get contribution from the SM terms as well as a linear combination of the real parts of anomalous couplings $Re \Delta a_Z$, $Re a_\gamma$, $Re b_Z$ and $Re b_\gamma$, provided the range in θ is forward-backward symmetric. The result for unpolarized and transversely polarized beams would be identical [23].

On the other hand, with longitudinal polarization, the partial cross section depends on a different linear combination of the real parts of the anomalous couplings. Thus, combining results of the measurement with unpolarized beams with those of the measurement with longitudinally polarized beams with e^- and e^+ polarizations of the same sign or opposite signs would give three relations with which to constrain the four couplings.

The expression for the partial cross sections in the longitudinal polarization case, in terms of the coefficients A_L and B_L used in the differential cross section is

$$\sigma_L(\theta_0) = (1 - P_L \bar{P}_L) 4\pi \cos \theta_0 \left[A_L + \left(1 - \frac{1}{3} \cos^2 \theta_0 \right) B_L \right], \quad (28)$$

where θ_0 is the cut-off angle. As mentioned earlier, the partial cross section in the case of transverse polarization is the same as that for unpolarized beams.

The terms proportional to $\cos \theta$ can be determined using a simple forward-backward asymmetry:

$$A_{\text{FB}}(\theta_0) = \frac{1}{\sigma(\theta_0)} \left[\int_{\theta_0}^{\pi/2} \frac{d\sigma}{d\theta} d\theta - \int_{\pi/2}^{\pi-\theta_0} \frac{d\sigma}{d\theta} d\theta \right], \quad (29)$$

where

$$\sigma(\theta_0) = \int_{\theta_0}^{\pi-\theta_0} \frac{d\sigma}{d\theta} d\theta, \quad (30)$$

and θ_0 is a cut-off in the forward and backward directions which could be chosen to optimize the sensitivity.

The expression for $A_{\text{FB}}^L(\theta_0)$ for longitudinal polarization is

$$A_{\text{FB}}^L(\theta_0) = \frac{C_L \cos \theta_0}{2 [A_L^{\text{SM}} + B_L^{\text{SM}} (1 - \frac{1}{3} \cos^2 \theta_0)]}, \quad (31)$$

where we have used only the SM cross section in the denominator because we work to linear order in the anomalous couplings. This asymmetry is odd under CP and is proportional to C and therefore to a combination of $\text{Im } \tilde{b}_Z$ and $\text{Im } \tilde{b}_\gamma$. This combination is dependent on the degree of (longitudinal) polarization, and is therefore sensitive to polarization. The asymmetry for transverse polarization is the same as that for zero polarization.

It should be noted that only imaginary parts of couplings enter. This is related to the fact that the CP-violating asymmetry $A_{\text{FB}}(\theta_0)$ is odd under naive CPT. It follows that for it to have a non-zero value, the amplitude should have an absorptive part [24].

We now treat the cases of longitudinally and transversely polarized beams separately.

Case (a) Longitudinal polarization:

The forward-backward asymmetry of eq. (29) in the presence of longitudinal polarization, which we denote by $A_{\text{FB}}^L(\theta_0)$, determines a different combination of the same couplings $\text{Im } \tilde{b}_Z$ and $\text{Im } \tilde{b}_\gamma$. Thus observing asymmetries with and without polarization, the two imaginary parts can be determined independently.

In the same way, a combination of the cross section for the unpolarized and longitudinally polarized beams can be used to determine two different combinations of the remaining couplings which appear in (5). However, one can get information only on the real parts of $\Delta a_Z, b_Z, a_\gamma, b_\gamma$, not their imaginary parts.

Case (b) Transverse polarization:

In the case of the angular distribution with transversely polarized beams, there is a dependence on the azimuthal angle ϕ of the Z . Thus, in addition to ϕ -independent terms which are the same as those in the unpolarized case,

there are terms with factors $\sin^2 \theta \cos 2\phi$ and $\sin^2 \theta \sin 2\phi$. The ϕ -dependent terms occur with the factor of $P_T \bar{P}_T$. Thus, both beams need to have transverse polarization for a nontrivial azimuthal dependence.

We can define an azimuthal asymmetry which can be used to separate out $Im(a_\gamma)$:

$$A^T(\theta_0) = \frac{1}{\sigma_T^{SM}(\theta_0)} \left[\int_{\theta_0}^{\pi-\theta_0} d\theta \left(\int_0^{\pi/2} d\phi - \int_{\pi/2}^{\pi} d\phi + \int_{\pi}^{3\pi/2} d\phi - \int_{3\pi/2}^{2\pi} d\phi \right) \frac{d\sigma_T}{d\theta d\phi} \right], \quad (32)$$

Next, we define an asymmetry which separates out a linear combination of $Re \Delta a_Z$ and $Re a_\gamma$ as follows:

$$A'^T(\theta_0) = \frac{1}{\sigma_T^{SM}(\theta_0)} \left[\int_{\theta_0}^{\pi-\theta_0} d\theta \left(\int_{-\pi/4}^{\pi/4} d\phi - \int_{\pi/4}^{3\pi/4} d\phi + \int_{3\pi/4}^{5\pi/4} d\phi - \int_{5\pi/4}^{7\pi/4} d\phi \right) \frac{d\sigma_T}{d\theta d\phi} \right], \quad (33)$$

The integrals in the above may be evaluated to yield

$$A^T(\theta_0) = \frac{2}{\pi} P_T \bar{P}_T \frac{(D_T \sin \delta + E_T \cos \delta) (1 - \frac{1}{3} \cos^2 \theta_0)}{A_T^{SM} + B_T^{SM} (1 - \frac{1}{3} \cos^2 \theta_0)}, \quad (34)$$

and

$$A'^T(\theta_0) = \frac{2}{\pi} P_T \bar{P}_T \frac{(D_T \cos \delta - E_T \sin \delta) (1 - \frac{1}{3} \cos^2 \theta_0)}{A_T^{SM} + B_T^{SM} (1 - \frac{1}{3} \cos^2 \theta_0)}, \quad (35)$$

where we use only the SM cross section in the denominators, since we work to first order in anomalous couplings. The simplest scenario is when $\delta = 0$ or π . In that case, we see that the two asymmetries A^T and A'^T can measure, respectively, $Im a_\gamma$ and a combination of $Re \Delta a_Z$ and $Re a_\gamma$. The former is odd under naive time reversal, whereas the latter is even. The CPT theorem then implies that these would be respectively dependent on real and imaginary parts of form factors [24]. We thus have the important result that a measurement of $A^T(\theta_0)$ when the electron and positron polarizations are parallel to each other directly gives us a measurement of $Im a_\gamma$, which cannot

be measured without the use of transverse polarization. This, in the present context, is the most important use of transverse polarization¹.

In the next section we discuss numerical evaluation of the cross sections and asymmetries, and demonstrate how information using more than one observable, or one observable, but different polarization choices can be used to disentangle the different anomalous couplings. We will also study the numerical limits that can be put on the couplings at a linear collider.

5 Numerical Calculations

We now evaluate various observables and their sensitivities for a linear collider operating at $\sqrt{s} = 500$ GeV. We assume that longitudinal beam polarizations of $P_L = \pm 0.8$ and $\bar{P}_L = \pm 0.6$ can be reached, and that rotating the spins to point in the transverse direction will not entail any loss of polarization. With this choice of individual polarizations, the factor $1 - P_L \bar{P}_L$, occurring in the expression for the cross section, is 0.52 or 1.48 depending on whether the electron and positron have like-sign or unlike-sign polarizations. (We take the sign of polarization to be positive for right-handed polarization). The quantity P_L^{eff} , defined in eq. (17), which appears in various expressions is then 0.385 or 0.946 in the two cases of like-sign and unlike-sign polarizations.

In case of transverse polarization, we assume $\delta = 0$ corresponding to the simplest configuration of the electron and positron spins.

We have chosen $m_H = 120$ GeV for the main part of our calculations. We comment later on the results for larger Higgs masses.

We have made use of the following values of other parameters: $M_Z = 91.19$ GeV, $\alpha(m_Z) = 1/128$, $\sin^2 \theta = 0.22$. For studying the sensitivity of the linear collider, we have assumed an integrated luminosity of $L \equiv \int \mathcal{L} dt = 500 \text{ fb}^{-1}$.

5.1 Cross section

The simplest observable is the total rate that can be used to determine some combination of anomalous couplings. If we integrate the differential

¹It may be mentioned that in [11], where e^+e^-HZ contact interactions were used, another set of azimuthal asymmetries in combination with forward-backward asymmetries were defined, which are not present in the present case, and would therefore signal the presence of four-point interactions.

cross section with respect to polar and azimuthal angle over the full ranges, we would get a combination of the couplings $\text{Re } \Delta a_Z$, $\text{Re } b_Z$, $\text{Re } a_\gamma$ and $\text{Re } b_\gamma$. Different combinations of these same couplings enter the unpolarized cross section and cross sections with same-sign or opposite-sign polarizations of the beams. Transverse polarization, on the other hand, gives the same combination of the couplings as in the unpolarized case.

The anomalous part of the cross section in eq. (28) can be written as

$$\begin{aligned} \sigma_L(\theta_0) - \sigma_L^{\text{SM}}(\theta_0) = \sigma_L^{\text{SM}}(\theta_0) & \left[2 \left(\text{Re } \Delta a_Z + \frac{2\sqrt{s}q^0}{2m_Z^2 + (1 - \frac{1}{3} \cos^2 \theta_0) |\vec{q}|^2} \text{Re } b_Z \right) \right. \\ & \left. + \frac{(g_V^e - g_A^e P_L^{\text{eff}})}{(g_V^e + g_A^e - 2g_V^e g_A^e P_L^{\text{eff}})} \frac{K_\gamma}{K_{SM}} \left(\text{Re } a_\gamma + \frac{2\sqrt{s}q^0}{2m_Z^2 + (1 - \frac{1}{3} \cos^2 \theta_0) |\vec{q}|^2} \text{Re } b_\gamma \right) \right] \end{aligned} \quad (36)$$

It can be seen that for fixed cut-off, measuring the cross section for two different polarization combinations can determine the two combinations of two anomalous couplings each. We define the following two combinations:

$$c_Z = 2 \left(\text{Re } \Delta a_Z + \frac{2\sqrt{s}q^0}{2m_Z^2 + (1 - \frac{1}{3} \cos^2 \theta_0) |\vec{q}|^2} \text{Re } b_Z \right) \quad (37)$$

and

$$c_\gamma = \frac{2g_V^e \sin^2 2\theta_W s - m_Z^2}{g_V^e + g_A^e} \frac{s - m_Z^2}{s} \left(\text{Re } a_\gamma + \frac{2\sqrt{s}q^0}{2m_Z^2 + (1 - \frac{1}{3} \cos^2 \theta_0) |\vec{q}|^2} \text{Re } b_\gamma \right). \quad (38)$$

Further, using the same combinations of polarizations, c_Z and c_γ can again be determined for a different value of cut-off θ_0 . This would give two equations for each of c_Z and c_γ . It would then be possible to determine all four of $\text{Re } \Delta a_Z$, $\text{Re } b_Z$, $\text{Re } a_\gamma$ and $\text{Re } b_\gamma$ independent of one another.

Fig. 4 shows the 95% CL constraints in the $c_\gamma - c_Z$ plane from polarization combinations (P_L, \bar{P}_L) of $(0, 0)$, $(0.8, +0.6)$ and $(0.8, -0.6)$, using a cut-off $\theta_0 = \pi/16$. The lines correspond to the solutions of the equation

$$|\sigma_L(\theta_0) - \sigma_L^{\text{SM}}(\theta_0)| = 2.45 \sqrt{\sigma_L^{\text{SM}}(\theta_0)/L} \quad (39)$$

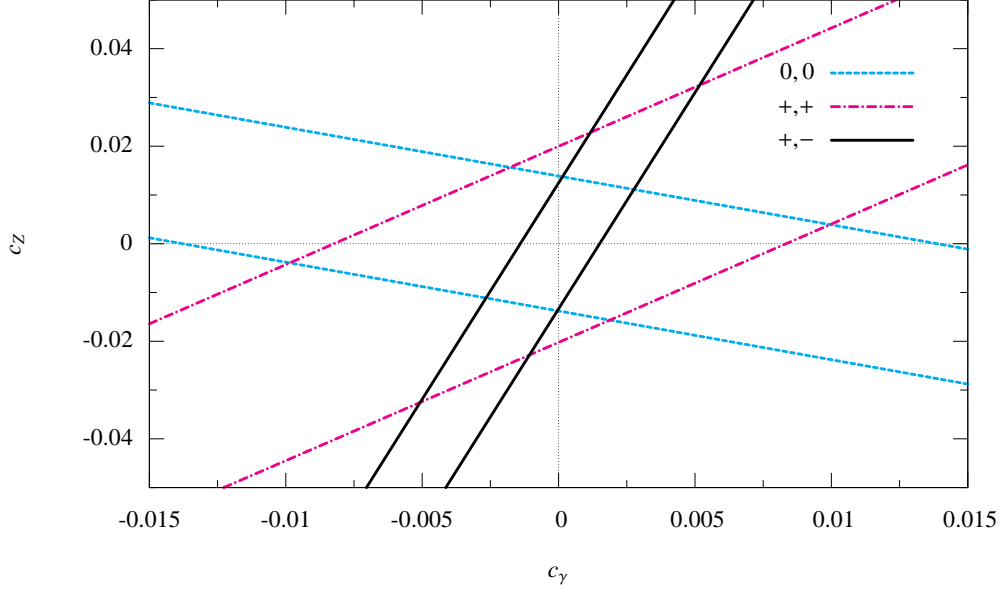


Figure 4: The region in the $c_\gamma - c_Z$ plane accessible at the 95% CL with cross sections with different beam polarization configurations for integrated luminosity $L = 500 \text{ fb}^{-1}$. $0,0$, $+,+$ and $+,-$ stand for the cases of zero, like-sign and opposite-sign e^- and e^+ polarizations. The cut-off θ_0 is taken to be $\pi/16$.

for the three polarization combinations. The best simultaneous limits on c_γ and c_Z are obtained using a combination of unpolarized beams and longitudinally polarized beams with opposite signs, viz.,

$$|\text{Re } c_\gamma| \leq 0.00271, \quad |\text{Re } c_Z| \leq 0.0137. \quad (40)$$

The individual limits that can be obtained keeping one coupling to be nonzero at a time and setting the rest to be zero are shown in Table 1.

A direct procedure would of course be to determine all four couplings by solving four simultaneous equations obtained by using two combinations of polarization, each for two values of cut-off. Applying this approach for polarization combinations $P_L = \overline{P}_L = 0$ and $(P_L, \overline{P}_L) = (0.8, -0.6)$, and the

	$ \text{Re } a_\gamma $	$ \text{Re } \Delta a_Z $	$ \text{Re } b_\gamma $	$ \text{Re } b_Z $
Unpolarized	0.0705	0.00553	0.0149	0.00117
$P_L = 0.8, \overline{P}_L = +0.6$	0.0423	0.00805;	0.00890	0.00169
$P_L = 0.8, \overline{P}_L = -0.6$	0.00741	0.00516	0.00156	0.00109

Table 1: Individual 95% CL limits on the couplings $\text{Re } a_\gamma$, $\text{Re } \Delta a_Z$, $\text{Re } b_\gamma$, $\text{Re } b_Z$ obtained from the cross section for a cut-off $\theta_0 = \pi/16$ for different beam polarization combinations.

cut-off values $\theta_0 = \pi/16$ and $\theta_0 = \pi/4$, we find the 95% CL limits of

$$|\text{Re } a_\gamma| \leq 0.320; \quad |\text{Re } \Delta a_Z| \leq 0.128; \quad |\text{Re } b_\gamma| \leq 0.0721; \quad |\text{Re } b_Z| \leq 0.0287. \quad (41)$$

The two other polarization combinations, viz., $P_L = \overline{P}_L = 0$ or $(P_L, \overline{P}_L) = (0.8, +0.6)$ used with $(P_L, \overline{P}_L) = (0.8, -0.6)$, give worse limits than these.

5.2 Forward-backward Asymmetry

As can be seen from eq. (31), the forward-backward asymmetry defined in eq. (29) can be a probe of the combination of imaginary part of the couplings $\text{Im } \tilde{b}_Z$ and $\text{Im } \tilde{b}_\gamma$. We examine the accuracy to which this combination can be determined. The limits which can be placed at the 95% CL on the two parameters contributing to the asymmetry is given by equating the asymmetry to $2.45/\sqrt{N_{SM}}$, where N_{SM} is the number of SM events. This leads to the relation

$$|A_{FB}| = \frac{2.45}{\sqrt{L\sigma_L^{SM}}}, \quad (42)$$

where L is the integrated luminosity.

We show in Fig. 5 a plot of the relation eq. (42) in the space of the couplings involved for unpolarized beams, and for the two combinations of longitudinal polarizations $(P_L, \overline{P}_L) \equiv (0.8, +0.6)$, denoted by $(+, +)$ and $(P_L, \overline{P}_L) \equiv (0.8, -0.6)$, denoted by $(+, -)$. The intersection of the lines corresponding to any two combinations gives a closed region which is the allowed region at 95% CL.

The best simultaneous limits are obtained by considering the region enclosed by the intersections of the lines corresponding to $P_L = \overline{P}_L = 0$ and

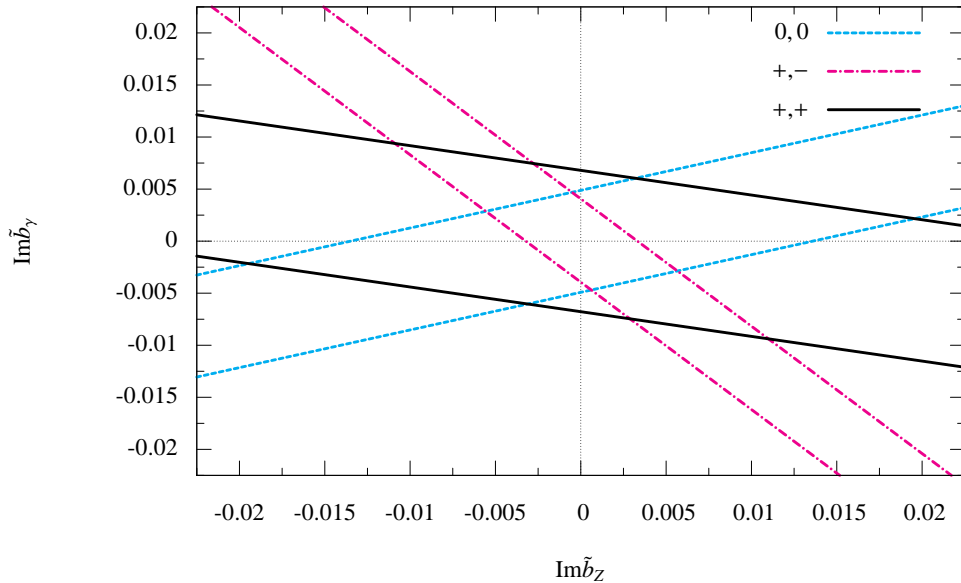


Figure 5: The region in the $\text{Im } \tilde{b}_Z - \text{Im } \tilde{b}_\gamma$ plane accessible at the 95% CL with forward-backward asymmetry with different beam polarization configurations for integrated luminosity $L = 500 \text{ fb}^{-1}$. 0, 0, +, + and +, - stand for the cases of zero, like-sign and opposite-sign e^- and e^+ polarizations.

$(P_L, \overline{P}_L) = (0.8, -0.6)$. These limits are

$$|\text{Im } \tilde{b}_\gamma| \leq 4.69 \cdot 10^{-3}; |\text{Im } \tilde{b}_Z| \leq 5.61 \cdot 10^{-3}. \quad (43)$$

Individual limits on the two couplings obtained from the forward-backward asymmetry by setting one coupling to zero at a time for the three polarization combinations are shown in Table 2.

It can be seen that the limit is improved considerably in the case of opposite-sign polarizations as compared to unpolarized beams for $\text{Im } \tilde{b}_Z$, but only marginally in case of $\text{Im } \tilde{b}_\gamma$. Like-sign polarizations make the limits worse.

5.3 Azimuthal asymmetries

Transversely polarized beams can in principle provide more information through the azimuthal angular distribution which has terms dependent on $\sin^2 \theta$

	$ \text{Im } \tilde{b}_\gamma $	$ \text{Im } \tilde{b}_Z $
Unpolarized	0.00392	0.0108
$P_L = 0.8, \bar{P}_L = +0.6$	0.00543	0.0229
$P_L = 0.8, \bar{P}_L = -0.6$	0.00320	0.00262

Table 2: Individual 95% CL limits on the couplings $\text{Im } \tilde{b}_\gamma$, $\text{Im } \tilde{b}_Z$, obtained from the forward-backward asymmetry for a cut-off $\theta_0 = \pi/16$ for different beam polarization combinations.

$\sin 2\phi$ and $\sin^2 \theta \cos 2\phi$. We can construct observables which isolate these terms.

(a) The $\sin^2 \theta \sin 2\phi$ term

This term, which has a coefficient denoted by E in the expression of eq. (19), can be isolated using the asymmetry $A_T(\theta_0)$ defined in eq. (32), when $\delta = 0$. Since E_T^{SM} and E_T^Z are vanishing, this asymmetry uniquely determines E_T^γ , and hence the coupling $\text{Im } a_\gamma$. This coupling cannot be determined without transverse polarization.

We can also choose to evaluate the expectation value of any operators which are odd functions of $\sin 2\phi$. We have chosen the three operators $\text{sign}(\sin 2\phi)$ whose expectation value corresponds to the asymmetry A_T , $\sin 2\phi$ and $\sin^3 2\phi$. The 95% CL limit that can be placed on $\text{Im } a_\gamma$ was determined for each operator O using

$$|\text{Im } a_\gamma| \leq 1.96 \frac{\sqrt{\langle O^2 \rangle}}{\langle O \rangle_1 \sqrt{L} \sigma_T^{SM}}, \quad (44)$$

where $\langle O \rangle_1$ is expectation value for unit value of the coupling. Table 3 shows limits on the $|\text{Im } a_\gamma|$ at the 95% confidence level for various Higgs masses. It is seen that the best limits are obtained using the operator $\sin 2\phi$.

Operators	$M_H = 120 \text{ GeV}$	$M_H = 200 \text{ GeV}$	$M_H = 300 \text{ GeV}$
$\text{sign}(\sin 2\phi)$	0.0409	0.0522	0.101
$\sin 2\phi$	0.0368	0.0470	0.0913
$\sin^3 2\phi$	0.0388	0.0495	0.0963

Table 3: Limits of $\text{Im } a_\gamma$ for the various Higgs masses

(b) The $\sin^2 \theta \cos 2\phi$ term

The coefficient of the $\cos 2\phi$ term, viz., D_T , is associated with $\text{Re } \Delta a_Z$ and $\text{Re } a_\gamma$. So, by suitably defining an asymmetry corresponding to $\cos 2\phi$, we can probe a linear combination of $\text{Re } \Delta a_Z$ and $\text{Re } a_\gamma$. The asymmetry A'^T defined in eq. (33) serves the purpose. This asymmetry does not vanish for the SM. Hence we use the following expression for determining the limit on the linear combination of couplings at 95% CL.

$$|A'^T - A'^{TSM}| \leq 2.45 \frac{\sqrt{1 - (A'^{TSM})^2}}{\sqrt{L\sigma^{SM}}}, \quad (45)$$

where A'^{SM} is the value of the asymmetry in SM. Since this asymmetry is proportional to the product $P_T \overline{P}_T$, changing the sign of polarization will only give a change of sign of the asymmetry. It is thus not possible to obtain two different combinations of the couplings $\text{Re } a_\gamma$ and $\text{Re } \Delta a_Z$ as in the earlier case of longitudinal polarization. However, it would be possible to obtain simultaneous limits on these couplings by choosing two different cut-offs on the azimuthal angle ϕ , which would give two equations. We have not attempted this in the present work.

The individual limits using A'^T on $\text{Re } a_\gamma$ and $\text{Re } \Delta a_Z$, each taken nonzero by turns, are

$$|\text{Re } a_\gamma| \leq 0.334, \quad |\text{Re } \Delta a_Z| \leq 0.0270 \quad (46)$$

It is seen that the limit on $\text{Re } a_\gamma$ is not an improvement over the one shown in eq. (41), obtained using the partial cross sections with longitudinal polarization. The limit on $\text{Re } \Delta a_Z$ is a considerable improvement over the limit in eq. (41), though it is much worse than the individual limit obtained using even unpolarized beams (Table 1).

6 Conclusions and discussion

We have obtained angular distributions for the process $e^+e^- \rightarrow ZH$ in the presence of anomalous γZH and ZZH couplings to linear order in these couplings in the presence of longitudinal and transverse beam polarizations. We have then looked at observables and asymmetries which can be used in combinations to disentangle the various couplings to the extent possible. We have also obtained the sensitivities of these observables and asymmetries to the various couplings for a definite configuration of the linear collider.

In certain cases where the contribution of a coupling is suppressed due to the fact that the vector coupling of the Z to e^+e^- is numerically small,

longitudinal polarization helps to enhance the contribution of this coupling. As a result, longitudinal polarization improves the sensitivity. The main advantage of transverse polarization is that it helps to determine $Im a_\gamma$ independent of all other couplings through the $\sin^2 \theta \sin 2\phi$ term. It is not possible to constrain $Im a_\gamma$ without transverse polarization. Another advantage that transverse polarization offers, though not as compelling, is the determination and a combination of the couplings $Re a_\gamma$ and $Re \Delta a_Z$ independently of the couplings $Re b_\gamma$ and $Re b_Z$ through the $\sin^2 \theta \cos 2\phi$ term. It is of course possible to measure the couplings $Re a_\gamma$ and $Re \Delta a_Z$ independently of the couplings $Re b_\gamma$ and $Re b_Z$ using unpolarized or longitudinally polarized beams. However, transverse polarization enables this to be done using a convenient azimuthal asymmetry. In the case of $Re \Delta a_Z$, this procedure proved to be more sensitive than the one determining simultaneous limits employing cross sections for two combinations of longitudinal polarization and two different cut-offs in the polar angle.

We find that with a linear collider operating at a c.m. energy of 500 GeV with the capability of 80% electron polarization and 60% positron polarization with an integrated luminosity of 500 fb^{-1} , using the simple cross section and asymmetry measurements described above it would be possible to place 95% CL individual limits of the order of few times 10^{-3} or better on all couplings taken nonzero one at a time with use of an appropriate combination (P_L and \overline{P}_L of opposite signs) of longitudinal beam polarizations. Polarization gives an improvement in sensitivity by a factor of 5 to 10 as compared to the unpolarized case for the real parts of γZH couplings, and the imaginary parts of ZZH couplings. The use of polarization also enables simultaneous determination (without any coupling being assumed zero) of all couplings which appear in the differential cross section, viz., $Re \Delta a_Z$, $Re a_\gamma$, $Im a_\gamma$, $Re b_Z$, $Re b_\gamma$, $Im \tilde{b}_\gamma$ and $Im \tilde{b}_Z$. The simultaneous limits are, as expected, less stringent, of the order of $0.1 - 0.3$ for $Re a_\gamma$ and $Re \Delta a_Z$, and of the order of $0.03 - 0.07$ on $Re b_\gamma$ and $Re b_Z$. The simultaneous limits on the CP-violating couplings $Im \tilde{b}_\gamma$ and $Im \tilde{b}_Z$ are a little better, being respectively $5 \cdot 10^{-3}$ and $6 \cdot 10^{-3}$. Transverse polarization enables the determination of $Im a_\gamma$ independent of all other couplings, with a possible 95% CL limit of the order of 10^{-2} . With transverse polarization a combination of $Re a_\gamma$ and $Re \Delta a_Z$ can be determined independent of all other couplings with the help of an azimuthal asymmetry. From this combination, individual limits possible on them are respectively 0.33 and $2.7 \cdot 10^{-2}$. While the former is comparable to the simultaneous limit obtained using longitudinal polarization, the latter

is an improvement by an order of magnitude.

It is appropriate to compare our results with those in works using the same parameterization as ours for the anomalous coupling and with an approach similar to ours. Ref. [6] deals with CP-violating ZZH couplings, and it is possible to compare the 95% CL limits obtained in Sec. 5.2 using the forward-backward asymmetry of the Z with the corresponding limits in [6]. With identical values of \sqrt{s} and integrated luminosity, ref. [6] quotes limits of 0.019 and 0.0028 for $Im \tilde{b}_Z$, respectively for unpolarized and longitudinally polarized beams with opposite-sign e^+ and e^- polarizations. The corresponding numbers we have are 0.011 and 0.0026. The agreement is thus good, considering that ref. [6] employs additional experimental cuts, which could reduce the nominal sensitivity. The papers in [7] also deal only with anomalous ZZH couplings, and quote 3σ limits on the couplings. However, the second paper in [7] considers different luminosity options for different polarization combinations. The 3σ limit they quote for $Im \tilde{b}_Z$ is 0.064 for unpolarized beams, and 0.0089 for polarized beams. After correcting for the CL limit of 1.96σ which we use, their limits are still worse by a factor of 2 to 4. This could be attributed to the stringent kinematic cuts imposed by them, and to the different luminosity choice in the case of polarized beams. Similarly, the limits quoted in [7] for $Re \Delta a_Z$ and $Re b_Z$ are worse by a factor of about 2 or 3 in the unpolarized as well as polarized cases. As for the case of γZH couplings, comparison with earlier work is not easy because of the different approach to parameterization of couplings. Also, there is no work dealing in transverse polarization with which we could make a comparison.

In the above, we have assumed a Higgs mass of 120 GeV. For larger values of m_H , for larger Higgs masses, we find decreased sensitivities. We have not studied the sensitivities at higher centre-of-mass energies. However, it is conceivable that the contribution of the anomalous couplings $b_{\gamma,Z}$ and $\tilde{b}_{\gamma,Z}$ which come with momentum-dependent tensors will increase with energy and improve the sensitivity.

Though we have used SM couplings for the leading contribution of Fig. 1, as mentioned earlier, the analysis needs only trivial modification when applied to a model like MSSM or a multi-Higgs-doublet model, and will be useful in such extensions of SM.

We have not included the decay of the Z and the Higgs boson in our analysis. For now, one could simply divide our limits by the square root of the branching ratios and detection efficiencies. Including these decays will entail some loss of efficiency. On the other hand, making use of the

decay products of the Z would also give access to more variables, which might help one to disentangle further the couplings which could not be easily disentangled in our analysis. In particular, $Im \Delta a_Z$, $Im b_\gamma$, $Im b_Z$, $Re \tilde{b}_\gamma$ and $Re \tilde{b}_Z$, which do not appear in the Z angular distributions, will most likely be accessible in the distributions of Z decay products. Work on this is under progress.

We have not considered scenarios with an extra neutral gauge boson Z' . While it is straightforward to include a Z' in our analysis, the number of couplings would be much larger and difficult to disentangle without studying the s dependence of the cross section or asymmetries.

One should also investigate the effect experimental cuts would have on the accuracy of the determination of the couplings. One should keep in mind the possibility that radiative corrections can lead to quantitative changes in the above results (see, for example, [25]). While these practical questions are not addressed in this work, we feel that the interesting new features we found would make it worthwhile to address them in future.

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