

Predictive Framework with a Pair of Degenerate Neutrinos at a High Scale

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Abstract

Radiative generation of the solar scale Δ_\odot is discussed in the presence of leptonic CP violation. We assume that both the solar scale and U_{e3} are zero at a high scale and the weak radiative corrections generate them. It is shown that all leptonic mass matrices satisfying these requirements lead to a unique prediction $\Delta_\odot \cos 2\theta_\odot \approx 4\delta_\tau \sin^2 \theta_A |m_{ee}|^2$ for the solar scale in terms of the radiative correction parameter δ_τ , the physical solar (atmospheric) mixing angles $\theta_\odot(\theta_A)$ and the Majorana neutrino mass m_{ee} probed in neutrinoless double beta decay. This relation is independent of the mixing matrix and CP-violating phases at the high scale. The presence of CP-violating phases leads to dilution in the solar mixing angle defined at the high scale. Because of this, bi-maximal mixing pattern at the high energy leads to large but non-maximal solar mixing in the low-energy theory. An illustrative model with this feature is discussed.

I. INTRODUCTION:

The neutrino masses and mixing pattern implied [1,2] by the neutrino oscillation experiments requires (i) two large mixing angles θ_\odot and θ_A to explain respectively the results of the solar and the atmospheric neutrino experiments, (ii) corresponding mass scales Δ_\odot and Δ_{atm} satisfying

$$\frac{\Delta_\odot}{\Delta_{\text{atm}}} \approx 3 \times 10^{-2} \quad (1)$$

and (iii) a third mixing angle which is very small $\theta_3 \leq 9^\circ$.

The smallness of θ_3 compared to other two large angles and that of $\frac{\Delta_\odot}{\Delta_{\text{atm}}}$ are two of the major puzzles [3] in neutrino physics requiring theoretical explanation. One possibility is to suppose that some symmetry leads to vanishing of Δ_\odot or θ_3 or both and its breaking is responsible for small values of these parameters. Vanishing of Δ_\odot can be a consequence of lepton like $U(1)$ or more general non-abelian symmetry such as $SU(2)_H$. The vanishing of θ_3 can also be attributed to some symmetry, e.g., $L_e - L_\mu - L_\tau$ invariance [4] studied extensively in the literature. One needs to provide a mechanism for symmetry breaking in order to generate the solar scale. An economical possibility is to suppose that physics at a high scale leads to vanishing solar scale and the standard weak gauge bosons (and their superpartners in supersymmetric theory) are responsible for its generation at a low scale [5,6,7]. This provides a well-defined symmetry breaking pattern which can be deduced [8] by studying the evolution of the neutrino mass matrix through the renormalization group (RG) equations [9]. This evolution has been studied very extensively [10] with a different physical motivation.

Let us suppose that neutrino masses m_{ν_i} and the mixing angle θ_3 generated by physics at a high scale (e.g., seesaw mechanism) are given by

$$m_{\nu_i} = (m, -m, m') \quad ; \quad \sin \theta_3 \equiv |(U_0)_{e3}| = 0 \quad , \quad (2)$$

where U_0 is the neutrino mixing matrix at the high scale. Clearly, a very large class of the neutrino and the charged lepton mass matrices can lead to eq.(2). The RG evolution provides a systematic way to study generation of the solar scale and U_{e3} at a low energy in all these models. It was found in [11] that all models of leptonic mass matrices leading to eq.(2) give a unique prediction for the solar scale when CP is conserved,

$$\Delta_\odot \cos 2\theta_\odot = 4\delta_\tau s_A^2 |m_{ee}|^2 + \mathcal{O}(\delta_\mu, \delta_\tau^2) \quad . \quad (3)$$

Here, m_{ee} is the effective neutrino mass probed in neutrinoless double beta decay ($0\nu\beta\beta$) and δ_τ denotes [8] the size of the radiative correction induced by the Yukawa coupling of the τ :

$$\delta_\tau \approx c \left(\frac{m_\tau}{4\pi v} \right)^2 \ln \frac{M_X}{M_Z} . \quad (4)$$

$c = \frac{3}{2}, -\frac{1}{\cos^2\beta}$ in respective cases of the standard model (SM) and the minimal supersymmetric standard model (MSSM).

This equation implies a strong correlation between the solar scale and m_{ee} . In particular, it requires that m_{ee} should be close to its present limit if the large mixing angle (LMA) solution [1] is to be reproduced. CP conservation was assumed in the analysis presented in [11]. When CP violation is allowed, the vanishing of Δ_\odot at high scale implies that the first two masses are degenerate in eq.(2) only up to a phase. It is well-known [8,10,12] that CP violating phases can alter evolution of the neutrino masses in a non-trivial and drastic manner. It is thus important to include the effects of such phases. We find that the CP violating phases α, β associated with the neutrino masses influence the predicted value of the solar mixing angle θ_\odot significantly but in such a way that the basic prediction eq.(3) obtained in a CP conserving theory remains unaffected. Unlike in eq.(3), the radiatively generated U_{e3} depends on the phases α and β . In the following, we derive these basic results and discuss their consequences.

II. ASSUMPTIONS AND RESULTS

A complex symmetric neutrino mass matrix can always be diagonalized by a unitary matrix U . This U can be identified with the MNS [13] matrix when the neutrino mass matrix is specified in the flavour basis. U is known to be determined by three mixing angles θ_i and three phases δ, α, β and can be parameterized as

$$U = R_{23}(\theta_2)R_{13}(\theta_3 e^{-i\delta})R_{12}(\theta_1)diag.(1, e^{\frac{i\alpha}{2}}, e^{\frac{i\beta}{2}}) \quad (5)$$

The phase δ is analogous to the CKM phases and α, β are the phases associated with the Majorana masses.

We denote the neutrino mass matrix in the flavour basis at a high scale by $\mathcal{M}_{0\nu}$ and the corresponding mixing matrix by U_0 . The U_0 is obtained by choosing $\theta_3 = 0$ in eq.(5). Explicitly,

$$U_0 = \begin{pmatrix} c_1 & s_1 & 0 \\ -c_2 s_1 & c_2 c_1 & s_2 \\ s_1 s_2 & -c_1 s_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{i\alpha}{2}} & 0 \\ 0 & 0 & e^{\frac{i\beta}{2}} \end{pmatrix}. \quad (6)$$

where $c_1 = \cos \theta_1$, $s_1 = \sin \theta_1$ etc.

The solar scale vanishes if

$$U_0^T \mathcal{M}_{0\nu} U_0 = \text{diag.}(m, m, m') \quad (7)$$

Several points are to be noted in connection with the above equation.

- One can choose m and m' real and positive without loss of generality. The non-zero $|m'^2 - m^2|$ can be identified with the atmospheric scale Δ_{atm} .
- Given the mass ordering as in eq.(7), $(U_0)_{e3}$ denotes the mixing element probed at CHOOZ. We assumed it to be zero which amounted to choosing $\theta_3 = 0$ in eq.(5). This then implies that the CKM phase can be assumed zero at a high scale. CP violation is still present through non-vanishing α and β .
- The neutrino mass matrix $\mathcal{M}_{0\nu}$ can be determined by inverting eq.(7):

$$\mathcal{M}_{0\nu} = U_0^* \text{diag.}(m, m, m') U_0^\dagger \quad (8)$$

The matrix U_0 is however not unique. Degeneracy of masses in eq.(7) implies that U_0 is arbitrary up to an orthogonal transformation O_{12} by an angle θ'_1 in the 12 plane. Thus U_0 and $U_0 O_{12}$ imply the same physics. This arbitrariness in U_0 amounts to the following redefinition of the angle θ_1 appearing in eq.(6):

$$\begin{aligned} |c_1| &\rightarrow |c_1 c'_1 - s_1 s'_1 e^{\frac{i\alpha}{2}}|, \\ |s_1| &\rightarrow |c_1 s'_1 + s_1 c'_1 e^{\frac{i\alpha}{2}}|, \end{aligned} \quad (9)$$

This freedom implies that the solar angle corresponding to the 12 mixing cannot be uniquely defined at a high scale. Eq.(9) at the same time does not allow us to completely rotate away the solar angle unless $\alpha = 2n\pi$. The arbitrariness in defining the solar angle will be removed by the radiative corrections. We can thus set θ'_1 to zero without loss of generality.

The matrix $\mathcal{M}_{0\nu}$ determined by eq.(8) is modified by radiative corrections. The radiatively corrected form of $\mathcal{M}_{0\nu}$ follows from the relevant RG equations. We assume RG equations corresponding to the SM or the MSSM. The modified neutrino mass matrix is given [8] in this case by

$$\mathcal{M}_{0\nu} \rightarrow \mathcal{M}_\nu \approx I_g I_t (I U_0^* \text{diag.}(m, m, m_3) U_0^\dagger I) \quad (10)$$

where $I_{g,t}$ are calculable numbers depending on the gauge and top quark Yukawa couplings. I is a flavour dependent matrix given by

$$I \approx \text{diag.}(1 + \delta_e, 1 + \delta_\mu, 1 + \delta_\tau) .$$

$\delta_{e,\mu}$ are obtained from eq.(4) by replacing the tau mass by the electron and the muon masses respectively. The physical neutrino masses and mixing are obtained by diagonalizing the above matrix. We do this approximately by retaining the contribution of the tau Yukawa coupling and by working to the lowest order in δ_τ . The effect of radiative corrections is best seen by going to the basis in which the neutrino mass matrix $\mathcal{M}_{0\nu}$ defined at high scale is diagonal. This is done through a rotation by the original U_0 on eq.(10)

$$\begin{aligned} \tilde{\mathcal{M}}_\nu &\equiv U_0^T \mathcal{M}_\nu U_0 \\ &\approx m I_g I_t \begin{pmatrix} 1 + 2\delta_\tau s_1^2 s_2^2 & -2\delta_\tau \cos \frac{\alpha}{2} c_1 s_1 s_2^2 & \delta_\tau c_2 s_2 s_1 (e^{\frac{i\beta}{2}} + r e^{-\frac{i\beta}{2}}) \\ -2\delta_\tau \cos \frac{\alpha}{2} c_1 s_1 s_2^2 & (1 + 2\delta_\tau c_1^2 s_2^2) & -\delta_\tau c_2 s_2 c_1 (e^{-i\chi} + r e^{i\chi}) \\ \delta_\tau c_2 s_2 s_1 (e^{\frac{i\beta}{2}} + r e^{-\frac{i\beta}{2}}) & -\delta_\tau c_2 s_2 c_1 (e^{-i\chi} + r e^{i\chi}) & r(1 + 2\delta_\tau c_2^2) \end{pmatrix} \end{aligned} \quad (11)$$

where $r \equiv \frac{m'}{m}$ and $\chi \equiv \frac{(\alpha - \beta)}{2}$.

Approximate diagonalization of the above matrix is straightforward. We omit the details of this diagonalization but mention the salient points.

- The structure of the upper 2×2 block in eq.(11) implies that the correction to the solar mixing angle arising from its diagonalization is not $\mathcal{O}(\delta_\tau)$ but $\mathcal{O}(1)$ unless $\alpha = \pi$. This is a well-studied phenomenon [8] which is sometimes referred to as radiative instability of the mixing angle. This feature is related here to the arbitrariness (eq.(9)) in defining the mixing angle at high scale. Perturbation present in eq.(11) helps in removing this ambiguity.

The upper 2×2 block of eq.(11) is diagonalized by

$$U_{12} = \begin{pmatrix} \tilde{c}_1 & \tilde{s}_1 & 0 \\ -\tilde{s}_1 & \tilde{c}_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (12)$$

with

$$\tan 2\tilde{\theta}_1 = -\cos \frac{\alpha}{2} \tan 2\theta_1, \quad (13)$$

- The (13) and (23) elements of the matrix $U_{12}^T \tilde{\mathcal{M}}_\nu U_{12}$ resulting after rotation through U_{12} are $\mathcal{O}(m\delta_\tau)$ and the 33 element is approximately m' . The effect of these off-diagonal elements is to induce additional mixing of the $\mathcal{O}(\epsilon)$ where $\epsilon \approx \frac{m\delta_\tau}{m'-m} \approx \frac{2m^2\delta_\tau}{\Delta_{\text{atm}}}$. This mixing is quite small for physically interesting mass range $m \leq \mathcal{O}(\text{eV})$. Thus a complete diagonalization of $U_{12}^T \tilde{\mathcal{M}}_\nu U_{12}$ is performed by two additional rotations with mixing angles which are $\mathcal{O}(\epsilon)$. Explicitly,

$$U_{23}^T U_{13}^T U_{12}^T U_0^T \mathcal{M}_\nu U_0 U_{12} U_{13} U_{23} \approx \text{diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \quad (14)$$

The U_{13} is responsible for generation of the CHOOZ angle while U_{23} provides a small radiative correction to the atmospheric mixing angle θ_2 . We neglect correction to θ_2 in the following. U_{13} is given by

$$U_{13} = \begin{pmatrix} \tilde{c}_3 & 0 & \tilde{s}_3 e^{i\Psi} \\ 0 & 1 & 0 \\ -\tilde{s}_3 e^{-i\Psi} & 0 & \tilde{c}_3 \end{pmatrix}, \quad (15)$$

Eq.(14) gives us the complete mixing matrix which can be approximately written as

$$U \approx U_0 U_{12} U_{13} \approx \begin{pmatrix} c_\odot \tilde{c}_3 e^{-i\eta} & i s_\odot e^{-i\eta} & c_\odot \tilde{s}_3 e^{i(\Psi-\eta)} \\ i c_A s_\odot \tilde{c}_3 e^{i(\eta+\alpha/2)} - s_A \tilde{s}_3 e^{i(\beta/2-\psi)} & c_A c_\odot e^{i(\eta+\alpha/2)} & i c_A s_\odot \tilde{s}_3 e^{i(\psi+\eta+\alpha/2)} + s_A \tilde{c}_3 e^{i\beta/2} \\ -i s_A s_\odot \tilde{c}_3 e^{i(\eta+\alpha/2)} - c_A \tilde{s}_3 e^{i(\beta/2-\psi)} & -s_A c_\odot e^{i(\eta+\alpha/2)} & -i s_A s_\odot \tilde{s}_3 e^{i\psi} + c_A \tilde{c}_3 e^{i\beta/2} \end{pmatrix}. \quad (16)$$

where,

$$\begin{aligned}
c_\odot e^{i\eta} &= (c_1 \tilde{c}_1 - s_1 \tilde{s}_1 e^{-i\frac{\alpha}{2}}) , \\
s_\odot e^{i\eta} &= i(c_1 \tilde{s}_1 + s_1 \tilde{c}_1 e^{-i\frac{\alpha}{2}}) , \\
s_A &= s_2 + \mathcal{O}(\delta_\tau) .
\end{aligned} \tag{17}$$

θ_3 is radiatively generated and is given by

$$\begin{aligned}
|U_{e3}| = |s_3| = |\tilde{s}_3 c_\odot| &= |\delta_\tau s_A c_A c_\odot s_\odot \frac{1+r}{1-r} \cos \delta| \\
&\approx |\delta_\tau \sin 2\theta_A \sin 2\theta_\odot \frac{m^2}{2\Delta_{\text{atm}}} \cos \delta| .
\end{aligned} \tag{18}$$

where $r \equiv \frac{m'}{m}$ can be expressed as:

$$|r| = (1 \pm \frac{\Delta_{\text{atm}}}{m^2})^{1/2} \approx 1 \pm \frac{\Delta_{\text{atm}}}{2m^2} \tag{19}$$

The positive (negative) sign in the above equation applies to the case $m' > m$ ($m' < m$). All the phases appearing in eq.(23) are expressible in terms of the original phases α and β . Explicitly,

$$\sin \eta = \frac{s_1}{\sqrt{2}c_\odot} \sin \frac{\alpha}{2} (1 - \frac{\cos 2\theta_1}{\cos 2\theta_\odot})^{1/2} , \tag{20}$$

$$\beta_L = \pi + \beta - \alpha - 2\eta , \tag{21}$$

$$\delta = -\frac{\beta_L}{2} + \mathcal{O}\left(\frac{\Delta_{\text{atm}}}{2m^2}\right)^2 . \tag{22}$$

The mixing matrix in eq.(16) assumes the following simple form once non-leading terms of $\mathcal{O}(\tilde{s}_3)$ are neglected ¹:

$$U \equiv U_0 U_{12} U_{13} \approx \begin{pmatrix} c_\odot & s_\odot & s_3 e^{-i\delta} \\ -c_A s_\odot & c_A c_\odot & s_A \\ s_A s_\odot & -s_A c_\odot & c_A \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & e^{\frac{i\beta_L}{2}} \end{pmatrix} . \tag{23}$$

It is clear from eqs.(13,17) that the radiative corrections have removed the ambiguity in the choice of the solar angle by fixing the θ'_1 in eq.(9) to $\tilde{\theta}_1$ given in eq.(13). The same equation also determines the phase η as given in eq.(20). Interestingly enough, the phase of

¹In writing this form, we have made a phase rotation on eq.(16) from the left and right by two appropriate diagonal phase matrices $P_{L,R}$. The P_L is absorbed in redefining the charged lepton fields giving us the final eq.(23).

the second mass state (which was α at the high scale) is now determined to be π in eq.(23). This corresponds to (almost) equal and opposite neutrino masses at the low scale.

The major physical effects of the radiative corrections are generation of the solar scale, mixing angle θ_3 and the CKM phase δ which was absent at the high scale. The solar scale follows from the eigenvalues of eq.(11):

$$\begin{aligned} m_{\nu_1} &\approx I_g I_t m (1 + 2\delta_\tau s_A^2 s_\odot^2) + \mathcal{O}(\delta_\tau^2) , \\ m_{\nu_2} &\approx I_g I_t m (1 + 2\delta_\tau s_A^2 c_\odot^2) + \mathcal{O}(\delta_\tau^2) , \\ m_{\nu_3} &\approx I_g I_t m' (1 + 2\delta_\tau c_A^2) + \mathcal{O}(\delta_\tau^2) . \end{aligned} \quad (24)$$

which lead to

$$\Delta_\odot \equiv m_{\nu_2}^2 - m_{\nu_1}^2 = 4I_g^2 I_t^2 m^2 \delta_\tau s_A^2 \cos 2\theta_\odot + \mathcal{O}(\delta_\tau^2) . \quad (25)$$

The Majorana mass m_{ee} is obtained using eqs.(23,24):

$$|m_{ee}|^2 \equiv |U_{ei}^2 m_{\nu_i}|^2 \approx I_g^2 I_t^2 m^2 \cos^2 2\theta_\odot + \mathcal{O}(\delta_\tau) , \quad (26)$$

The initial phases α, β appear in the above equation only implicitly through the solar angle θ_\odot . This is a consequence of the fact that physical neutrinos resulting after the radiative corrections consist of a pseudo-Dirac pair with (almost) equal and opposite masses.

Eqs.(18,25) are predictions of the scheme. The common mass $m I_g I_t$ of the degenerate pair can be identified with the electron neutrino mass m_{ν_e} probed through the direct neutrino mass search, e.g., in tritium beta decay [14]. It is also probed through measurement of the Majorana mass parameter m_{ee} [15]. One can in fact eliminate m from eq.(25) using eq.(26). This leads to the prediction (3) already mentioned in the introduction. This prediction involves only low energy measurable parameters and is independent of the CP violating phase.

III. PHENOMENOLOGICAL CONSEQUENCES

The five observables, namely θ_\odot , Δ_\odot , $|m_{ee}|$, $m_{\nu_e} = m I_g I_t$ and U_{e3} , are correlated through eqs.(3,18,25). We now study the consequences of this correlation. δ_τ is negative in case of the MSSM. This implies a negative $\Delta_\odot \cos 2\theta_\odot$ and hence only much less preferred dark region of the solar parameter space. This excludes the LMA and LOW solutions in case of the MSSM but the SM can easily allow them. These solutions are realized only for the specific range in $|m_{ee}|$. This is displayed in Fig.(1) where we show contours of the Δ_\odot and the

electron neutrino mass m_{ν_e} in the $\tan^2 \theta_\odot - |m_{ee}|$ plane. The values of $\tan^2 \theta_\odot$ are restricted to the typical range $\sim 0.2 - 0.8$ allowed by the LMA or LOW solution. For these values, one obtains a Δ_\odot in the required range $10^{-4} - 10^{-5} \text{ eV}^2$ provided $m_{ee} \sim 0.1 - 1 \text{ eV}$. This value is close to the experimental limit [15]. We also show the contours corresponding to the electron neutrino mass m_{ν_e} equal to 0.5 eV and 2.0 eV in the same plot. It is seen that m_{ν_e} is restricted to lie in the range $0.5 - 2 \text{ eV}$ in case of the LMA solution.

The predicted values of the $|U_{e3}|$ are also shown in the figure. $|U_{e3}|$ is seen to be restricted to a typical range $\sim 0.001 - 0.02$ in the region of the $m_{ee} - \tan^2 \theta_\odot$ plane allowed by the LMA solution.

The results presented above are completely in terms of the low energy variables and do not need any knowledge of the parameters at the high scale. Let us now comment on possible choices of the high scale parameter. Eqs.(13,17) are equivalent to the relation

$$\sin^2 2\theta_\odot = \sin^2 2\theta_1 \sin^2 \frac{\alpha}{2}, \quad (27)$$

The initial value of the solar mixing angle θ_1 is subject to the arbitrariness noted in eq.(9). We see that irrespective of this, any choice of θ_1 and α must satisfy

$$\left(\sin^2 2\theta_1, \sin^2 \frac{\alpha}{2}\right) \geq \sin^2 2\theta_\odot \sim (0.6 - 0.9). \quad (28)$$

It is seen that the mixing angle is reduced compared to its value at a high scale. This is phenomenologically interesting. One of the preferred phenomenological schemes corresponds to bi-maximal mixing [16] which can arise from symmetry considerations. However, the present solar data do not favour strictly maximal mixing [1]. Eq.(27) shows that one can start with bi-maximal mixing at a high scale and radiative corrections would lead to the desired reduction provided the CP-violating phase α is chosen non-zero and different from π at a high scale. The presence of α also plays another important role. $\alpha = \pi$ and maximal solar mixing corresponds to vanishing m_{ee} with the consequence that the solar scale arise only at $\mathcal{O}(\delta_\tau^2, \delta_\mu)$, see eq.(3). The natural value for the solar scale lies in the vacuum region in this case [17,18]. An α different from π alters this and allows strictly bi-maximal mixing and a non-zero m_{ee} at a high scale.

The radiative reduction in the solar angle found here is to be contrasted with a similar analysis presented recently in [19]. This analysis assumed vanishing U_{e3} but a non-zero Δ_\odot at the high scale itself. It was then found [19] that radiative corrections tend to increase the $\sin^2 2\theta_\odot$ compared to its value at the high scale. This does not allow bi-maximal mixing at the high scale in contrast to what is found here.

The analysis presented so far holds for all models satisfying eq.(2) at a high scale. While many possibilities exist, let us give an illustrative example which corresponds to bi-maximal mixing. This is specified by the following neutrino mass matrix in the flavour basis.

$$\mathcal{M}_{0\nu} = R_{23}(\theta_2) \begin{pmatrix} a & ib & 0 \\ ib & a & 0 \\ 0 & 0 & m'e^{-i\beta} \end{pmatrix} R_{23}^T(\theta_2) , \quad (29)$$

The parameters a , b , m' and θ_2 are assumed real.

Define the mixing matrix \tilde{U}_0 as:

$$\tilde{U}_0 = e^{-\frac{i\alpha}{4}} R_{23}(\theta_2) \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{\frac{i\alpha}{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{\frac{i\alpha}{2}} & 0 \\ 0 & 0 & e^{i\frac{\beta}{2}} \end{pmatrix} . \quad (30)$$

This matrix diagonalizes eq.(29):

$$\tilde{U}_0^T \mathcal{M}_{0\nu} \tilde{U}_0 = \text{Diag.}(m, m, m')$$

, with

$$\begin{aligned} \tan \frac{\alpha}{2} &= -\frac{b}{a} , \\ m^2 &= (a^2 + b^2) . \end{aligned} \quad (31)$$

We thus have maximally mixed degenerate neutrinos at a high scale. The maximal θ_1 implies maximal $\tilde{\theta}_1$ through eq.(13). The solar angle θ_\odot and the low scale CP violating phase follows respectively from eq.(17) and eq.(20):

$$\tan \theta_\odot = \cot \frac{\alpha}{4} \quad ; \quad \delta = \frac{\pi + \alpha - 2\beta}{4}$$

It is seen that $\alpha \sim \pi$ leads to large solar mixing. Specifically, $b \sim \sqrt{3}a$ leads to $\theta_\odot \sim 30^\circ$. The radiatively generated solar scale can naturally fall in the LMA region as already discussed before in the general context. Eq.(29) in this way leads to the required pattern of neutrino masses and mixing.

The texture presented here is quite similar to the one studied in ref [17] which assumed zero a and a real ib . For the reasons already mentioned, this model leads only to the vacuum

solution after radiative corrections are included. The presence of a and additional phase in b alters this and allows one to obtain the LMA solution.

It is possible [18] to obtain the above texture in the context of seesaw model by invoking additional horizontal ² symmetry. One way [3] of realizing the above texture is to assume a charged lepton mixing matrix with only $\mu - \tau$ mixing. This would generate R_{23} in eq.(29). The neutrino mass matrix in the weak basis is then given by the block diagonal form explicitly displayed in eq.(29). An explicit model was presented with these features in [18] in a CP conserving situation. We do not elaborate on it here since a trivial modification ³ of this model incorporating CP violation leads to the mass matrix presented in eq.(29).

IV. SUMMARY

The presently available information on neutrino oscillations can be nicely understood if two of the neutrinos pair up to form a pseudo-Dirac state. This can be obtained from a degenerate neutrino pair by means of standard radiative corrections. We discussed basic predictions of this picture including the important effects of the CP-violating phases. The scheme presented here has testable predictions: The LMA solution requires $m_{ee} \sim 0.1 - 1$ eV close to its present limit, relatively small $U_{e3} \sim 0.001 - .01$ and observable $\sim 0.5 - 2$ eV neutrino mass in beta decay.

The CP violating phases α and β associated with the Majorana masses do not effect the basic prediction (3) of the scheme but play an important role in diluting the solar mixing angle defined at a high scale. This allows bi-maximal mixing at the high scale and large but non-maximal solar angle at the low scale in accordance with the demand of the current solar neutrino results.

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² $SU(2)_H$ symmetry has also been recently used in [20] to generate bimaximal mixing pattern.

³This modification amounts to assuming a non-zero vacuum expectation value (vev) for the CP-odd field T^2 and vanishing vev for the CP even field T^1 in the notation of [18].

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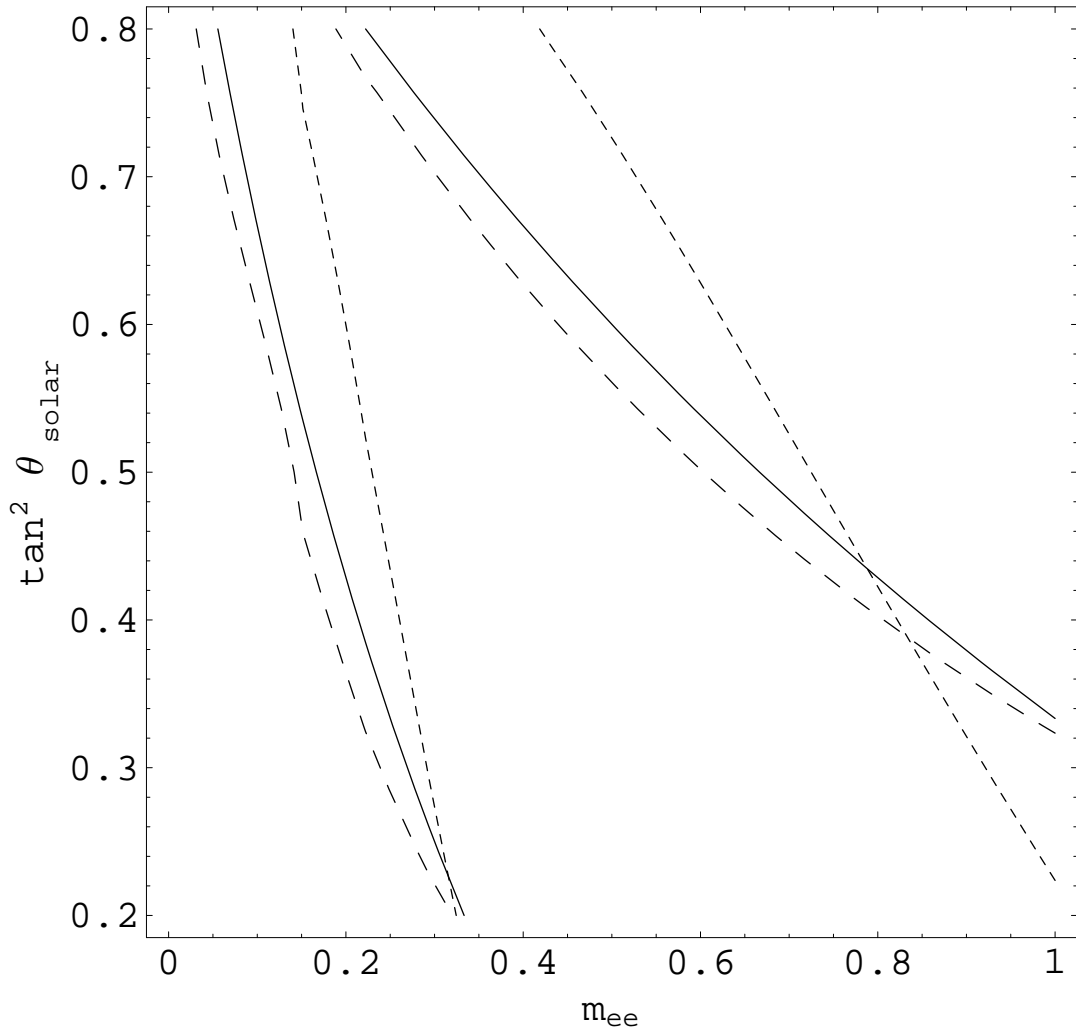


FIG. 1. Contours of Δ_{\odot} (dotted), $|U_{e3}|$ (dashed) and the electron neutrino mass m_{ν_e} (solid) as a function of $\tan^2 \theta_{\odot}$ and m_{ee} (in eV). The upper (lower) curves correspond to $\Delta_{\odot} = 10^{-4}$ (10^{-5}) eV^2 , $|U_{e3}| = 0.02$ (0.01) and the $m_{\nu_e} = 2.0$ (0.5) eV respectively.