

# Fast Neutrino Decay in The Minimal Seesaw Model

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## Abstract

Neutrino decay in the minimal seesaw model containing three right handed neutrinos and a complex  $SU(2) \times U(1)$  singlet Higgs in addition to the standard model fields is considered. A global horizontal symmetry  $U(1)_H$  is imposed, which on spontaneous breaking gives rise to a Goldstone boson. This symmetry is chosen in a way that makes a) the contribution of heavy ( $\leq$  MeV) majorana neutrinos to the neutrinoless double beta decay amplitude vanish and b) allows the heavy neutrino to decay to a lighter neutrino and the Goldstone boson. It is shown that this decay can occur at a rate much faster than in the original Majoron model even if one does not introduce any additional Higgs fields as is done in the literature. Possibility of describing the 17 keV neutrino in this minimal seesaw model is investigated. While most of the cosmological and astrophysical constraints on the 17 keV neutrino can be satisfied in this model, the laboratory limits coming from the neutrino oscillations cannot be easily met. An extension which removes this inadequacy and offers a consistent description of the 17 keV neutrino is discussed.

# 1 Introduction

Apart from playing an important role in cosmology and astrophysics, neutrino masses would provide an unambiguous signature of physics beyond the minimal  $SU(3) \times SU(2) \times U(1)$  model. Various mechanisms for generating these masses in gauge theories [1] have been proposed. Among them, the seesaw mechanism of Gell-Mann, Ramond Slansky and Yanagida (GRSY) [2] explains their smallness compared to other fermions in a natural way and has been extensively studied. The presence of (at least) one right-handed neutrino with a large (Majorana) mass  $\hat{M}$  is an essential ingredient in the GRSY mechanism. This large mass  $\hat{M}$  suppresses the masses of ordinary left-handed neutrinos which are typically given by

$$m_{\nu_i} \sim m_i^2 / \hat{M}, \quad (1)$$

$i=1,2,3$  being the generation indices and  $m_i$  is the Dirac mass connecting the left- and the right-handed neutrinos of the  $i$ th generation.  $\hat{M}$  is related to the breaking of GUT or left-right symmetry in many models, but, in general, it could assume any value from  $O(\text{GeV})$  to  $M_{\text{Planck}}$  in a phenomenologically consistent manner.  $m_i$  could be linked to masses of the charge- $\frac{2}{3}$  quarks  $u_i$  or to those of the charged leptons  $e_i$ . One interesting aspect of the GRSY mechanism is that there exists a range of natural values of parameters  $m_i$  and  $\hat{M}$  for which the neutrino masses are in the observable range and near their experimental limits. For example, if  $\hat{M}$  is identified [3] with the weak scale ( $\approx 100$  GeV) and  $m_i$  with the charged lepton masses, then  $m_{\nu_e} \approx 2.5$  eV,  $m_{\nu_\mu} \approx 100$  keV, and  $m_{\nu_\tau} \approx 30$  MeV. Likewise, if  $m_i$  are identified with the masses of  $u_i$ , then one could have observable mass at least for  $\nu_\tau$ , even for a large  $\hat{M}$ , e.g.,  $\hat{M} \approx 10^7$  GeV and  $m_t \approx 100$  GeV implies  $m_{\nu_\tau} \approx 1$  MeV.

Seesaw models which predict  $m_{\nu_\mu}$  and  $m_{\nu_\tau}$  in the above range have to satisfy two important constraints. The first comes from neutrinoless double beta decay ( $0\nu\beta\beta$ ) which requires the effective mass  $m_{\text{eff}}$  for  $\nu_e$  to be less than a few electron volts. A heavy neutrino mixing with  $\nu_e$  could contribute significantly to this effective mass, e.g., a neutrino with 1 MeV mass contributes  $>O(\text{eV})$  to  $m_{\text{eff}}$  if its mixing  $\beta$  with  $\nu_e$  is  $> 10^{-3}$ . One must avoid large contribution to  $0\nu\beta\beta$  coming in this way from heavy masses. The second constraint comes from the neutrino contribution to the energy density of the universe [4]. In terms of masses, this requires the sum of the masses of the normal neutrinos to be  $\leq 100$  eV. Hence  $\nu_\mu$  and  $\nu_\tau$  have either to decay or to annihilate in the early universe if their masses are large compared to this limit. Both decay as well as annihilation could occur if the theory contains

a massless Goldstone boson, viz., the Majoron. Such a Goldstone boson can be naturally incorporated [5] in the seesaw model by introducing a complex  $SU(2) \times U(1)$  singlet Higgs field whose vacuum expectation value sets the scale  $\hat{M}$  and breaks lepton number spontaneously, generating the Majoron. A minimal model of this type is however shown [6] to be inadequate for generating fast decay of the heavy neutrino. A typical decay rate for the process  $\nu_h \rightarrow \nu_l + J$  is shown [6] to be

$$\Gamma \approx \frac{\beta^2}{16\pi} \frac{m_{\nu_h}^5}{\hat{M}^4}. \quad (2)$$

The life time implied by the above equation exceeds the age of the universe for light ( $m_{\nu_h} \approx 10\text{keV}$ ) neutrino  $\nu_h$  and relatively large ( $\geq \text{TeV}$ )  $\hat{M}$ . The life time can be lowered by lowering the value of  $\hat{M}$ , but it is possible to construct models where the decay  $\nu_h \rightarrow \nu_l + J$  occurs at a much faster rate than given by eq.(2). Models proposed [7, 8] in the literature achieve this by enlarging the Higgs content. They typically require adding more than one singlet to the minimal seesaw model.

One of the motivations of the present works is to look critically at the minimal model with only one singlet Higgs and three right-handed neutrinos added to the standard model. We shall show that if one does not insist on the conservation of total lepton number then it is indeed possible to obtain in this minimal model a heavy neutrino decay rate much faster than given in eq.(2). The Majoron in this scenario gets related to a spontaneously broken horizontal symmetry acting on leptons. It is possible to choose this symmetry in such a way that the contribution of the heavy neutrinos to  $0\nu\beta\beta$  cancels naturally. Thus within this minimal model one could accommodate heavy neutrinos without conflicting with any of the known constraints.

The second aim of the paper is to see if the heavy neutrino in this minimal model can be identified with the reported [9] 17 keV neutrino. Like any other heavy neutrino, the 17 keV neutrino has to satisfy [10] constraints imposed by cosmology (relic density and nucleosynthesis), astrophysics (the supernova SN1987a) and the laboratory experiments ( $0\nu\beta\beta$  and oscillations). The desire to satisfy these constraints has given rise to ingenious but fairly involved [11] models for the 17 keV neutrino. We shall show that the minimal model considered here can meet most of the stringent constraints. We shall give an explicit example where this happens. We however find that the mixing angles predicted in this example do not agree with the laboratory limits on them coming from neutrino oscillations if the mixing of the 17 keV neutrino with  $\nu_e$  is indeed as large as  $\approx 1\%$ . One could avoid this

easily by adding an  $SU(2)$  triplet of Higgs field to the minimal seesaw model. The resulting model provides a much more economical and yet successful description of the 17 keV neutrino than most of the proposed [11] schemes.

In the next section, we discuss the minimal seesaw model and show that it is possible to obtain a decay rate higher than given by eq.(2) for a heavy neutrino in this model. The third section contains a specific example which has fast decay rate and suppressed neutrinoless double beta decay. The fourth section summarizes the constraints on the 17 keV neutrino. In the fifth section we provide a model for the 17 keV neutrino. The last section summarizes our results.

## 2 Heavy Neutrino Decay

We consider in this section an  $SU(2) \times U(1) \times U(1)_H$  model containing three right-handed neutrinos and a complex  $SU(2) \times U(1)$  singlet scalar field  $\eta$  in addition to the standard fields. The right-handed neutrinos are needed to obtain seesaw masses while  $\eta$  as well as a global symmetry  $U(1)_H$  is required to obtain Majoron in the manner suggested [5] by CMP. We shall refer to this model as the minimal seesaw model (MSM).  $U(1)_H$  was identified with the total lepton number in reference [5]. As was shown [6] later, this leads to eq.(2) and hence to a slower rate for the decay of heavy neutrino to a lighter neutrino and Majoron. This decay rate can be increased [7] if  $U(1)_H$  distinguishes between generations. Explicit models where this happens [7, 8] were discussed but this involved adding more than one singlet scalar field. As we discuss now, this is unnecessary and one could obtain a fast decay in the framework of the MSM.

If we do not insist on the conservation of total lepton number then the most general Yukawa interaction invariant under  $SU(2) \times U(1) \times U(1)_H$  is given as follows:

$$-\mathcal{L}_Y = \overline{\nu'_L} \frac{m}{\langle \phi^0 \rangle} \nu'_L \phi^0 + \frac{1}{2} \overline{\nu'_R}^c \left( \frac{M_\eta}{\langle \eta \rangle} \eta + \frac{M_{\eta^*}}{\langle \eta^* \rangle} \eta^* + M_B \right) \nu'_R + H.c. \quad (3)$$

$\phi^0$  is a neutral member of the doublet and  $\nu'_L$ ,  $\nu'_R$  are weak eigenstate neutrino fields.  $m, M_\eta, M_{\eta^*}$  and  $M_B$  are matrices in generation space. The entries in  $m$  are typically of the order of the charged-lepton or the up-quark masses

while those in  $M_\eta, M_{\eta^*}$  and  $M_B$  are assumed to be much larger. Because of the simultaneous presence of  $\eta$  and  $\eta^*$  in eq.(3),  $\mathcal{L}_Y$  cannot conserve total lepton number. Once lepton number conservation is not insisted upon, there is no reason to forbid the bare mass term  $M_B$  which has also been included in the above equation. All the terms in eq.(3) are however required to be invariant under a global  $U(1)_H$  corresponding to some linear combination of the individual lepton numbers of each generation.

Eq.(3) gives the following mass term for the neutrinos

$$-\mathcal{L}_{mass} = \frac{1}{2} \begin{pmatrix} \overline{\nu'_L} & \overline{\nu'_R^c} \end{pmatrix} \mathcal{M} \begin{pmatrix} \nu'_L^c \\ \nu'_R \end{pmatrix} + H.c., \quad (4)$$

where

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix}, \quad (5)$$

and

$$M = M_\eta + M_{\eta^*} + M_B. \quad (6)$$

for simplicity, we shall assume CP conservation and take all masses and vacuum expectation values real. Moreover, we shall work in the seesaw limit,  $\hat{M} \gg \hat{m}$ ,  $\hat{M}, \hat{m}$  being typical values of the entries in  $m$  and  $M$  respectively. Diagonalization of  $\mathcal{M}$  proceeds in the well-known [6] way in this limit. First, we block diagonalize  $\mathcal{M}$ , i.e.,

$$U \mathcal{M} U^T = \begin{pmatrix} -mM^{-1}m^T & 0 \\ 0 & M + \frac{1}{2}M^{-1}m^Tm + \frac{1}{2}m^TmM^{-1} \end{pmatrix} + O\left(\frac{\hat{m}^3}{\hat{M}^2}\right), \quad (7)$$

where,

$$U = \begin{pmatrix} 1 - \frac{1}{2}\rho\rho^T & -\rho \\ \rho^T & 1 - \frac{1}{2}\rho^T\rho \end{pmatrix} + O(\rho^3) \quad (8)$$

and  $\rho = mM^{-1}$ .  $-mM^{-1}m^T$  defines the effective mass matrix  $m_{eff}$  for the light neutrinos in the seesaw limit. Let  $O$  be  $3 \times 3$  matrix which diagonalizes  $m_{eff}$

$$O m_{eff} O^T = \text{diag.}(\xi_1 m_{\nu_1}, \xi_2 m_{\nu_2}, \xi_3 m_{\nu_3}). \quad (9)$$

$m_{\nu_i}$  ( $i=1,2,3$ ) are positive masses for three light neutrinos.  $\xi_i$  are signature factors which depend upon the structure of the matrix  $m_{eff}$ . They can be removed by a redefinition of the phases of the neutrino fields, i.e.,

$$P O m_{eff} O^T P^T = \text{diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \quad (10)$$

with

$$P_{ij} = \delta_{ij} e^{\frac{i\pi}{4}(1-\xi_i)}. \quad (11)$$

The relation between the weak eigenstates  $\nu'_L, \nu'^c_R$  and the light mass eigenstates follows from eq.(10),

$$\nu'_{iL} = [O^T P^*]_{ij} \nu_{jL} + \dots \quad (12)$$

$$\nu'^c_{iR} = -[M^{-1} m^T O^T P]_{ij} \nu_{jL} + \dots \quad (13)$$

The terms involving  $\dots$  contain fields describing the heavy Majorana neutrinos with masses  $\approx O(\hat{M})$ . We are primarily interested in the light fields  $\nu_{iL}$ .

The vacuum expectation value of  $\eta$  breaks the  $U(1)_H$  symmetry spontaneously giving rise to a Majoron  $J$ . The latter is related to  $\eta$  by

$$J \equiv \sqrt{2} \text{Im} \eta. \quad (14)$$

The couplings of the light neutrinos to the Majoron arise through couplings of the right-handed fields  $\nu_R$  to  $\eta$ . Using eq.(3),

$$-\mathcal{L}_J = \frac{i}{2\sqrt{2} \langle \eta \rangle} \overline{\nu_R^c} (M_\eta - M_{\eta^*}) \nu'_R J + H.c. \quad (15)$$

In terms of the mass eigenstates  $\nu_i$  of the light neutrino fields, eq.(12,13), we have

$$-\mathcal{L}_J = \frac{i}{2\sqrt{2} \langle \eta \rangle} g_{ij} e^{-\frac{i\pi}{4}(2-\xi_i-\xi_j)} \overline{\nu_i} \nu_{jL}^c J + H.c. \quad (16)$$

where,

$$g_{ij} = [O m_J O^T]_{ij}, \quad (17)$$

with the Majoron coupling matrix  $m_J$  defined as

$$m_J = m M^{-1} (M_\eta - M_{\eta^*}) M^{-1} m^T. \quad (18)$$

In terms of the Majorona fields  $\nu_i = \nu_{iL} + \nu_{iL}^c$ , we have for any pair  $i, j (i > j)$ :

$$\begin{aligned} -\mathcal{L}_J &= \frac{g_{ij}}{\sqrt{2} \langle \eta \rangle} \overline{\nu_i} \nu_j J \quad \text{if } \xi_i + \xi_j = 0 \\ &= \pm \frac{g_{ij}}{\sqrt{2} \langle \eta \rangle} \overline{\nu_i} \nu_j J \quad \text{if } \xi_i + \xi_j = \pm 2. \end{aligned} \quad (19)$$

The couplings  $g_{ij}$  of neutrinos to Majoron are generically of  $O(\frac{\hat{m}^2}{\hat{M}^2})$ . However, in specific cases, the matrix  $g$  could be diagonal and the neutrino decay amplitude may be suppressed. This happens, for example, if  $U(1)_H$  is

identified with the total lepton number [6].  $M_B$  is zero and only one of  $M_\eta$  and  $M_{\eta^*}$  is allowed to be present in eq.(3). As a result, the Majoron coupling matrix  $m_J$  coincides with  $m_{eff}$  and  $g$  is diagonal. The off-diagonal Majoron couplings arise [6] at  $O(\frac{\hat{m}^4}{M^4})$  in this case and the decay rate for  $\nu_j \rightarrow \nu_i + J$  is enormously suppressed as in eq.(2)

In general, the off-diagonal couplings of neutrinos to Majoron arise if any two of the  $M_B$ ,  $M_\eta$  and  $M_{\eta^*}$  are nonzero. The matrix  $m_J$  is different from  $m_{eff}$  in this case. As long as the  $m_J$  does not commute with  $m_{eff}$ , the coupling matrix  $g$  contains off-diagonal entries at  $O(\frac{\hat{m}^2}{M^2})$  leading to a fast decay rate for  $\nu_j \rightarrow \nu_i + J$ . In this situation, one could have heavy neutrinos without any conflict with cosmology. We shall present a model where this happens in the next section.

### 3 A Specific Model

Following the analysis presented in the earlier section, we now present an explicit model. We shall make a specific choice of  $U(1)_H$  which not only leads to a fast decay for the heavy neutrino but also implies vanishing of the neutrinoless double beta decay amplitude in a natural manner. As already discussed, a fast decay rate can result if  $U(1)_H$  allows at least two of the three possible mass terms  $M_\eta, M_{\eta^*}$  and  $M_B$ . This can be done by an appropriate choice of  $U(1)_H$ . The requirement of a vanishing neutrinoless double beta decay amplitude also constrains the choice of  $U(1)_H$ . As discussed by Wolfenstein [12] the neutrinoless double beta decay amplitude is proportional to the 11 element of the light ( $\leq O(\text{MeV})$ ) neutrino mass matrix in the basis which makes the charged lepton mass matrix diagonal.  $U(1)_H$  must be chosen to ensure this.

We assume that the ordinary doublet field  $\phi$  is neutral under  $U(1)_H$ . The Majoron does not have tree-level couplings to fields other than those of neutrinos in this case. As a result, the scale of the  $U(1)_H$  breaking is not required to be very high as in some models [13] with a non-trivial  $\phi$ . We shall also require  $U(1)_H$  to be vectorial and assume that no two generations transform identically under  $U(1)_H$ . These requirements simplify the task of making the neutrinoless double beta decay amplitude vanish. With these requirements imposed, the charged lepton mass matrix as well as the Dirac mass term  $m$  in the neutrino mass matrix automatically become diagonal.

The neutrinoless double beta decay amplitude then vanishes if the 11 element of  $m_{eff} \equiv -mM^{-1}m^T$  and hence of  $M^{-1}$  is zero. In the absence of any non-abelian symmetry which relates various Yukawa couplings,  $(M^{-1})_{11}$  can be zero only if  $M_{23}$  as well as  $M_{22}$  and/or  $M_{33}$  vanish. We shall try to be most general and allow the maximum number of entries in  $M^{-1}$  to be nonzero. Then, a little consideration shows that only two choices are possible for  $U(1)_H$ . These correspond to  $L_e - 3L_\mu - L_\tau$  and  $L_e - 3L_\tau - L_\mu$ . We explicitly discuss the former choice. Analogous considerations are valid for the latter. With this choice for  $U(1)_H$ ,  $M_\eta$ ,  $M_{\eta^*}$  and  $M_B$  defined in eq.(3) are given by

$$M_\eta = \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix} \quad (20)$$

$$M_{\eta^*} = \begin{pmatrix} M_3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (21)$$

$$M_B = \begin{pmatrix} 0 & 0 & M_0 \\ 0 & 0 & 0 \\ M_0 & 0 & 0 \end{pmatrix} \quad (22)$$

If we parametrize the elements of the diagonal matrix  $m$  by  $m_i$  then it follows using eq.(6) that

$$m_{eff} \equiv -mM^{-1}m^T = \begin{pmatrix} 0 & X & 0 \\ X & Y & Z \\ 0 & Z & W \end{pmatrix} \quad (23)$$

with

$$X = -\frac{m_1 m_2}{M_1} \quad (24)$$

$$Y = \frac{m_2^2}{M_1^2 M_2} (M_2 M_3 - M_0^2) \quad (25)$$

$$Z = \frac{m_2 m_3 M_0}{M_1 M_2} \quad (26)$$

$$W = -\frac{m_3^2}{M_2}. \quad (27)$$

$m_{eff}$  can be diagonalized by an orthogonal matrix. The zero entries in  $m_{eff}$  imply two independent relations among three mixing angles and masses. We shall extract these relations under the physically relevant (see next section) assumption of all the mixing angles being small. In this case,

$O$  can be parametrized by

$$O \approx \begin{pmatrix} 1 - \frac{1}{2}(\alpha^2 + \beta^2) & \alpha & \beta \\ -\alpha & 1 - \frac{1}{2}(\alpha^2 + \gamma^2) & \gamma \\ -\beta & -\gamma & 1 - \frac{1}{2}(\beta^2 + \gamma^2) \end{pmatrix}. \quad (28)$$

This choice of  $O$  as well as eq.(9) lead to the following relations corresponding to  $(m_{eff})_{11}$  and  $(m_{eff})_{13}$  being zero respectively:

$$\xi_1 m_{\nu_1} (1 - \alpha^2 - \beta^2) + \xi_2 m_{\nu_2} \alpha^2 + \xi_3 m_{\nu_3} \beta^2 \approx 0 \quad (29)$$

$$\xi_1 m_{\nu_1} \beta - \gamma \alpha \xi_2 m_{\nu_2} - \xi_3 m_{\nu_3} \beta \approx 0. \quad (30)$$

Neglecting contribution of  $m_{\nu_1}$  in the above equations, we obtain

$$\frac{\alpha^2}{\beta^2} \approx -\frac{\xi_3 m_{\nu_3}}{\xi_2 m_{\nu_2}} \quad (31)$$

$$\frac{\gamma \alpha}{\beta} \approx -\frac{\xi_3 m_{\nu_3}}{\xi_2 m_{\nu_2}}. \quad (32)$$

Eq.(31) requires  $\frac{\xi_3}{\xi_2}$  to be negative. Choosing  $\xi_3 = -1$ ,  $\xi_2 = \xi_1 = 1$ , the elements of  $m_{eff}$  can be expressed in terms of masses and mixing angles:

$$\begin{aligned} X &\approx -\alpha m_{\nu_2} - \beta \gamma m_{\nu_3} + \alpha m_{\nu_1} \\ Y &\approx \alpha^2 m_{\nu_1} - \gamma^2 m_{\nu_3} + m_{\nu_2} (1 - \alpha^2 - \gamma^2) \\ Z &\approx \alpha \beta m_{\nu_1} + \gamma (m_{\nu_3} + m_{\nu_2}) \\ W &\approx \gamma^2 m_{\nu_2} + \beta^2 m_{\nu_1} - m_{\nu_3} (1 - \beta^2 - \gamma^2). \end{aligned} \quad (33)$$

Eq. (33) allows us to fix the Majoron couplings completely in terms of mixing angles and masses. With  $M_\eta$ ,  $M_{\eta^*}$  and  $M_B$  explicitly given by eqs.(20-22), the couplings  $g_{12}$  and  $g_{13}$  responsible for the decay of  $\nu_{2,3}$  to  $\nu_1$  and a Majoron can be worked out from eq.(17). These are given by

$$\begin{aligned} g_{12} &\approx -2\alpha Y - \alpha \gamma Z - \beta Z \\ &\approx -2\alpha m_{\nu_2} \\ g_{13} &\approx -\alpha Z + 2\alpha \gamma Y + \beta \gamma Z \\ &\approx \beta m_{\nu_3} \end{aligned} \quad (34)$$

where we have used eqs.(31-32) and retained only the leading contributions assuming all the mixing angles to be of similar magnitudes. We shall neglect the contribution of  $m_{\nu_1}$ . The rates for the decay of  $\nu_{2,3}$  are given by

$$\begin{aligned} \Gamma(\nu_3 \rightarrow \nu_1 + J) &= \frac{(g_{13})^2}{32\pi(m_{\nu_3})^3} \frac{(m_{\nu_3} + m_{\nu_1})^2 (m_{\nu_3}^2 - m_{\nu_1}^2)}{<\eta>^2} \\ &\approx \frac{\beta^2}{32\pi} \frac{m_{\nu_3}^3}{<\eta>^2}, \end{aligned} \quad (35)$$

$$\begin{aligned}
\Gamma(\nu_2 \rightarrow \nu_1 + J) &= \frac{(g_{12})^2}{32\pi(m_{\nu_2})^3} \frac{(m_{\nu_2} + m_{\nu_1})^2(m_{\nu_2}^2 - m_{\nu_1}^2)}{<\eta>^2} \\
&\approx \frac{\alpha^2}{8\pi} \frac{m_{\nu_2}^3}{<\eta>^2}.
\end{aligned} \tag{36}$$

Analogous result also holds for the decay rate of the  $\nu_3$  going to  $\nu_2$ . The considerations based on the relic density of neutrinos imply stringent constraints on the lifetime of neutrinos. This is discussed in [4] as a function of the heavy neutrino mass. Typically, for a neutrino with mass around 1 MeV, one has  $\tau \leq 10^9$  sec. Considerations based on the structure formation in the universe imply more stringent bound [14] on  $\tau$ . If the neutrino lifetime is very long then the decay products of the heavy neutrinos could make the universe radiation dominated after the recombination epoch. If this happens, then the density perturbations at the time of recombination cannot grow adequately preventing the formation of structures. Requiring that the life time is short enough for the universe to remain matter dominated till the present epoch [15] one gets

$$\tau \leq 1.7 \times 10^3 \text{ sec} \left( \frac{m_\nu}{1 \text{ MeV}} \right)^{-2} \tag{37}$$

We can use eqs(35,37) to derive the bound on the Majorana mass scale  $<\eta>$

$$<\eta> \leq (1.5 \times 10^7 \text{ GeV}) \left( \frac{\beta}{.1} \right) \left( \frac{m_{\nu_3}}{\text{MeV}} \right)^{1/2} \tag{38}$$

Thus a fairly large value is allowed for the scale of the singlet vacuum expectation value. The situation here is to be contrasted with the original singlet Majoron model. In this model the decay rate is typically given by eq.(2) and the cosmological limit requires

$$<\eta> \leq (1.5 \times 10^2 \text{ GeV}) \left( \frac{\beta}{.1} \right)^{1/2} \left( \frac{m_{\nu_3}}{\text{MeV}} \right)^{3/4} \tag{39}$$

Before we close this section, we would like to point out that the structure similar to eq.(23) for the neutrino mass matrix was obtained by Valle [8]. He however did not include both the terms  $\eta$  and  $\eta^*$  in the Yukawa couplings but had two different Higgs scalars to obtain essentially the same structure. This extension is not necessary and one could stay with the minimal structure. His main motivation was to understand the 17 keV neutrino using the structure(23). This is not possible as we shall discuss in the next section, unless one does a delicate fine tuning of the parameters.

## 4 17 keV neutrino: Constraints

The existence of a 17 keV neutrino  $\nu_{17}$  mixing significantly with  $\nu_e$  was first reported in 1985. Subsequent experiments failed to observe it. Last year, there were further evidences for and against the existence of  $\nu_{17}$ . This has generated a considerable theoretical interest and various constraints to be satisfied in any model incorporating  $\nu_{17}$  have been worked out in detail [10]. The severe constraints mainly come from a) the near absence of the neutrinoless double beta decay, b) the observed neutrino signals from the supernova SN1987a, c) the bound on the relic density of the universe and d) nucleosynthesis. The need for satisfying all these constraints simultaneously has given rise to models [11] for  $\nu_{17}$  which invoke new physics. As we now discuss, the minimal model considered above can in fact meet all the above mentioned requirements. The detailed prediction on mixing angles made in the model however disagrees with the known limits coming from the neutrino oscillation experiments.

One way to incorporate the absence of neutrinoless double beta decay amplitude is to make  $\nu_{17}$  a Dirac particle. The right-handed component of this Dirac  $\nu_{17}$  can either be sterile or it could be one of the known antineutrinos. In the former case, the observed length of the neutrino pulse from the supernova SN1987a puts strong constraints [16] on the neutrino mass. This case is marginally allowed, the latest [17] limit being around 25 keV.

The alternative in which  $\nu_{17}$  is a Dirac particle with a non-sterile right-handed component can be realized by imposing [18] an unbroken  $L_e - L_\mu + L_\tau$  symmetry. The supernova does not imply any restriction in this case. This scenario is however constrained by cosmological arguments. The  $\nu_{17}$  remains stable in the minimal model with  $L_e - L_\mu + L_\tau$  symmetry. The relic density of  $\nu_{17}$  in this model can exceed the cosmological limit unless annihilation of  $\nu_{17}$  into Majorons occurs very rapidly. But if this annihilation is strong enough, the Majoron stays in equilibrium till the nucleosynthesis era [10]. This again conflicts with the known bound [4] on the contribution of the additional species to nucleosynthesis.

In the light of the above arguments, we are forced to consider a majorana  $\nu_{17}$  if we wish to understand it within the conventional seesaw framework. The MSM has not been seriously considered as a model for  $\nu_{17}$  in the literature because of difficulty in satisfying the constraint coming from the neutrinoless double decay and because of the expected [6] slow rate for the

decay of neutrino. The example presented in the earlier section shows that this is not the case. This example in fact satisfies all the above constraints. By virtue of being a non-sterile Majorana particle, there is no constraint on  $\nu_{17}$  from the supernova. Fast decay avoids conflict with cosmology. Neutrinoless double beta decay amplitude nearly vanish in this model and the Majoron does not stay in thermal equilibrium at the nucleosynthesis era if the scale  $\hat{M}$  exceeds [19] about few GeV. Despite this, the model cannot describe  $\nu_{17}$  in a consistent manner as we now show.

With all three neutrinos nondegenerate and two quite heavy, the neutrino oscillation experiments require all the three mixing angles to be small. Adopting the parametrization of Caldwell and Langacker [10], the limits on three mixing angles coming from the oscillation experiments are given [10] by

$$\begin{aligned} |\alpha| &< .029 & \text{from} & \nu_e \leftrightarrow \nu_\mu & \text{oscillation} \\ |\beta| &< 0.18 & \text{from} & \nu_e \leftrightarrow \nu_\tau & \text{oscillation} \\ |\gamma| &< .032 & \text{from} & \nu_\mu \leftrightarrow \nu_\tau & \text{oscillation} \end{aligned} \quad (40)$$

Only  $\beta$  can be as large as reported for the 17 keV neutrino. Hence, the  $\nu_{17}$  has to be identified with  $\nu_\tau$ . The absence of the neutrinoless double beta decay also constraints the masses and mixings. This is already evident in eq.(31) and implies

$$\alpha^2 \approx \beta^2 \frac{m_{\nu_3}}{m_{\nu_2}} \quad (41)$$

With  $\beta \approx 0.1$ ,  $m_{\nu_3} \approx 17\text{keV}$  and  $m_{\nu_2}$  corresponding to the experimental limit on  $\nu_\mu$  mass, we have

$$|\alpha| > 0.025 \quad (42)$$

Thus one has both the upper and lower limit on  $\alpha$ . The model of section 3 contains a nontrivial relation among the mixing angles. Combining eq.(31) and (32), this is given by

$$|\gamma| \approx \left| \frac{\alpha}{\beta} \right| \quad (43)$$

The limits expresssed in eq.(40) and (42) are seen to be grossly incompatible with each other if  $\beta$  is around 0.1. As a result the model fails to describe the observed  $\nu_{17}$ .

We had used the  $L_e - 3L_\mu - L_\tau$  symmetry in arriving at the model of section 3. There exists equivalant model with the symmetry  $L_e - 3L_\tau - L_\mu$ . In this model, instead of the 13 element the 12 element of the effective neutrino

mass matrix is zero. As a consequence, the relation (43) gets replaced by

$$|\gamma| \approx \left| \frac{\beta}{\alpha} \right| \quad (44)$$

Unfortunately, this relation is also inconsistent with eq.(40) in view of the upper bound on  $\alpha$ . Thus the other choice of symmetry also does not provide a viable model for  $\nu_{17}$ . As already mentioned, these are the only choices of  $U(1)_H$  which lead to the absence of neutrinoless double beta decay amplitude in rather natural manner. Hence, the MSM of the type described in the earlier section cannot be used to describe  $\nu_{17}$ . This conclusion can be evaded if one fine tunes the parameters [7] of the model.

The bounds on mixing angles expressed by eq.(40) are derived assuming all the neutrinos to be non-degenerate. In case of two of the neutrinos being almost degenerate, the mixing between them is not required to be small. In the present case, if  $\nu_\mu$  and  $\nu_\tau$  are degenerate the problematic bound on  $|\gamma|$  no longer holds. The mass matrix in eq.(23) does not admit this solution unless one fine tunes the parameters. Specifically, one needs to assume  $Y$  and  $W$  in eq.(23) to be much smaller than the off-diagonal elements.  $m_{eff}$  then displays an approximate  $L_e - L_\mu + L_\tau$  symmetry and one gets a Dirac  $\nu_{17}$ . Moreover, unlike the model with a triplet Majoron [18], the heavy neutrino does have off-diagonal couplings to the Majoron even in the limit of  $W$  and  $Y$  becoming zero. Unfortunately, it is not possible to choose a  $U(1)_H$  which automatically ensures vanishing of  $W$  and  $Y$ . Therefore these elements have to be fine tuned. The required fine tuning is quite delicate [10]. In the event of large  $\gamma$  the  $\nu_\mu$  disappearance [20] experiments constrain the  $|m_{\nu_3}^2 - m_{\nu_2}^2|$  to be  $\ll 0.23$  (eV)<sup>2</sup>. If  $W$  and  $Y$  are  $\ll X, Z$  then  $|m_{\nu_2} - m_{\nu_3}| \approx Y + W$ . Thus the ratio,  $|\frac{Y+W}{Z}|$  is required to be  $\leq 10^{-9}$ . In the absence of such fine tuning among the parameters, one needs to enlarge the model of section 3 to accommodate the  $\nu_{17}$ . This we do in the next section.

## 5 17 keV neutrino: A model

The basic difficulty in describing  $\nu_{17}$  within the scheme is the relation (43) which is a consequence of  $(m_{eff})_{13}$  being zero. We can easily extend the MSM to avoid this. The extension amounts to adding an  $SU(2) \times U(1)$  triplet carrying hypercharge  $-2$  and transforming trivially under  $U(1)_H$ . When the neutral member of the triplet acquires a vev, the following  $6 \times 6$  mass matrix

results for the neutrinos

$$\mathcal{M} = \begin{pmatrix} \delta m & m \\ m^T & M \end{pmatrix}, \quad (45)$$

where the matrices  $m$  and  $M$  remain the same as in section (3).  $\delta m$  is now given by

$$\delta m = \begin{pmatrix} 0 & 0 & t \\ 0 & 0 & 0 \\ t & 0 & 0 \end{pmatrix}. \quad (46)$$

$t$  refers to the contribution coming from the triplet, which is assumed to be  $\leq O\left(\frac{\hat{m}^2}{M}\right)$ . One could block-diagonalize the  $\mathcal{M}$  in the seesaw limit with the same  $U$  as in eq.(8).

$$U\mathcal{M}U^T = \begin{pmatrix} m_{eff} & 0 \\ 0 & M \end{pmatrix}, \quad (47)$$

where

$$\begin{aligned} m_{eff} &= \delta m - mM^{-1}m^T \\ &= \begin{pmatrix} 0 & X & t \\ X & Y & Z \\ t & Z & W \end{pmatrix}. \end{aligned} \quad (48)$$

This  $m_{eff}$  differs from eq.(23) only by the (13) entry. The former can be diagonalized by an orthogonal matrix  $O$  as in eq.(9) and elements of  $m_{eff}$  can be related to mixing angles and masses. In addition to eq.(33), we now have

$$t = \beta(m_{\nu_1} + m_{\nu_3}) - \alpha\gamma m_{\nu_2}. \quad (49)$$

The vanishing of  $t$  in the earlier model led to the problematic relation (43). This is now avoided. The mixing angle and masses now satisfy only one relation corresponding to the vanishing of neutrinoless double beta decay amplitude.

The triplet field is neutral under  $U(1)_H$ . As a result, the Majoron is still given in terms of  $\text{Im}\eta$ . Hence the Majoron couplings  $g_{ij}$  are still given by the basic expression (eq.(17)) derived in section 2. The couplings  $g_{12}$  and  $g_{13}$  are then given as follows

$$\begin{aligned} g_{12} &\approx \gamma t - 2\alpha Y - \alpha\gamma Z - \alpha\beta t - \beta Z \\ &\approx -2\alpha m_{\nu_2} \end{aligned} \quad (50)$$

$$\begin{aligned} g_{13} &\approx t\left(1 - \frac{1}{2}(\alpha^2 + \gamma^2 + 2\beta^2)\right) - \alpha Z + 2\alpha\gamma Y - \beta^2 Z + \beta\gamma Z \\ &\approx \beta m_{\nu_3} \end{aligned} \quad (51)$$

Using the upper limit on  $\alpha$  as given in eq.(40) and  $\beta \approx 0.1$ , we get the following decay rates

$$\begin{aligned}
\Gamma(\nu_3 \rightarrow \nu_1 + J) &= \frac{(g_{13})^2}{32\pi(m_{\nu_3})^3} \frac{(m_{\nu_3} + m_{\nu_1})^2(m_{\nu_3}^2 - m_{\nu_1}^2)}{<\eta>^2} \\
&\approx (1.5 \times 10^{-5} \text{sec}^{-1}) \left(\frac{\beta}{0.1}\right)^2 \left(\frac{m_{\nu_3}}{10 \text{keV}}\right)^3 \left(\frac{10^5 \text{GeV}}{<\eta>}\right)^2 \\
\Gamma(\nu_2 \rightarrow \nu_1 + J) &= \frac{(g_{12})^2}{32\pi(m_{\nu_2})^3} \frac{(m_{\nu_2} + m_{\nu_1})^2(m_{\nu_2}^2 - m_{\nu_1}^2)}{<\eta>^2} \\
&\approx (5.4 \times 10^{-3} \text{sec}^{-1}) \left(\frac{\alpha}{0.03}\right)^2 \left(\frac{m_{\nu_2}}{100 \text{keV}}\right)^3 \left(\frac{10^5 \text{GeV}}{<\eta>}\right)^2
\end{aligned} \tag{52}$$

Analogous result holds for the decay of  $\nu_3$  to  $\nu_1$  and the Majoron. We have chosen  $m_{\nu_3}$  to be 10 keV and a typical value of 100 keV for the muon neutrino mass allowed by the constraint, eq.(31), coming from the vanishing of neutrinoless double beta decay amplitude. The scale  $<\eta>$  is arbitrary but as follows from the above equation both the neutrinos can decay very fast for a large range in this scale.

## 6 Summary

We have considered in this paper, a possibility of describing heavy neutrinos in a phenomenologically consistent manner within the MSM. This is made possible by imposing a global  $U(1)_H$  which is chosen in a way that simultaneously ensures the absence of neutrinoless double beta decay amplitude and also leads to a fast decay rate for the heavy neutrinos. It is widely believed that decay of a heavy neutrino to a lighter neutrino and Majoron is suppressed in the MSM with only one singlet of Higgs field. We have shown this not to be the case.

The MSM comes very close to providing the description of the recently reported 17 keV neutrino. All the astrophysical and cosmological constraints as well the requirement of the vanishing neutrinoless double beta decay amplitude are met in the model. As shown in section (3), the relation among mixing angles predicted in the model do not however seem to be satisfied in case of the 17 keV neutrino. This can be easily avoided in an extension, which also includes an SU(2) triplet Higgs field. The major shortcoming of the model is its inability to solve the solar neutrino problem and to describe

17 keV neutrino at the same time. The mass difference between the neutrinos in this case are much larger than required for solving the solar neutrino problem either through the Mikheyev Smirnov Wolfenstein [21] mechanism or through the magnetic moment [22] of the neutrino. This would certainly require going beyond the conventional seesaw mechanism.

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