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Top Quarks and CP Violation in Polarized e^+e^- Collisions

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Abstract

Electroweak dipole moments of the top quark are conjectured and their effect in polarized e^+e^- collisions is examined for the expectation values of two different CP-odd observables. One of these observables probes the real part of the dipole moments whereas the other probes their imaginary part. It turns out that varying the polarization of the electron beam substantially enhances the resolving power of the experiment.

Now that substantial evidence for the existence of the top quark has finally been gathered [1], the exploration of its properties becomes an important endeavour. It is not unnatural to expect these properties to differ from those of the other known quarks, since its mass is already substantially heavier. The questions which naturally arise are: How well are its couplings described by the standard model? Is the top quark really elementary or is it a composite of other more elementary objects? The answers to these questions may be found in studying the interactions of the top quark with other particles and comparing them with predictions from the standard model and other extended models, or analyzing them in a model-independent manner. In particular, it would be interesting to investigate whether top couplings can conserve CP, a symmetry so far known to be violated only in the K-meson system.

Possible CP violating couplings of fermions are electric dipole type interactions with the electromagnetic field [2], and the analogous "weak" dipole coupling to the Z^0 field. These can arise at the one-loop level, for example, in certain models of CP violation like the two-Higgs-doublet model or the minimal supersymmetric standard model. They would show up in the production of $t\bar{t}$ in e^+e^- or hadronic colliders. It is our purpose to study the possibility of measuring these dipole type of couplings in e^+e^- colliders, especially with longitudinal beam polarization. We do not restrict ourselves to any particular model, but parametrize the CP violation in terms of effective electric and weak dipole form factors.

At a linear collider of the next generation (CLIC, JLC, NLC, TESLA, VLEPP,...) top quark pairs should be produced in sufficient abundance (a few thousand events) to allow simple studies of its nature. The main production mechanism proceeds at the Born level by the s-channel annihilation of initial electron-positron pairs into virtual photons or neutral weak gauge bosons, and their subsequent splitting into top-antitop pairs

$$e^+e^- \to \gamma^*, Z^{0*} \to t\bar{t} \ . \tag{1}$$

There have been several suggestions to measure possible electroweak dipole moments of the top quark in top pair-production at e^+e^- and hadronic colliders. Various experiments have been suggested to perform these measurements by making use of CP-odd quantities. These include measuring the polarization asymmetry [3, 4, 5] of the $t\bar{t}$ pair through the energy asymmetry of the charged leptons arising in their decays [4, 6], as well as the up-down asymmetry of these leptons with respect to the production plane of $t\bar{t}$ [6],[1] and various CP-odd momentum correlations among the decay products of $t\bar{t}$ [8, 9]. It has also been realized in the context of the measurement of the τ dipole moment that longitudinal polarization of the electron beam (and possibly also the positron beam) in the e^+e^- experiments can enhance certain momentum correlations and can lead to enhanced sensitivity [10]. We investigate here the effect on similar correlations of longitudinal beam polarization in the process (1).

Quite apart from the actual numerical results we obtain, we will see that the main advantage of using polarized beams is that whereas in the absence of polarization a CP-odd correlation gives information on a certain combination of the electric and weak dipole

[1]The effect of longitudinal beam polarization of these asymmetries is studied in Ref. [7]

moments, its polarization dependence can be used to get independent information on a different combination. This helps to measure or obtain limits on both types of dipole moments. In the absence of polarization, obtaining the same information would require measuring at least two independent correlations.

It should be straightforward to obtain high degrees of longitudinal polarization of the initial state electron beams at a linear collider. The same is not true, though, for the positron beams. This poses the problem that in experiments with only the electron beam polarized, the initial state would not be a CP eigenstate, and therefore CP-odd correlations are not necessarily a measure of the CP violation of the interaction. However, if we neglect radiative corrections, only the left-right and right-left combinations of electron and positron helicities couple to the γ and Z^0 in the limit of vanishing electron mass. Hence the corrections, if any, are tiny. Hard collinear emission of a photon from the initial e^- or e^+ can flip helicity, however, even in the limit of vanishing electron mass [11]. This leads to non-zero CP-odd correlations even in the absence of CP-invariant interactions, and this background must be calculated and subtracted. While we postpone a detailed calculation, we will argue that our results are not seriously affected by this background.

At lowest order the polarized cross section for the process (1) is

$$\sigma(e^+e^- \to t\bar{t}) = 4\pi\alpha^2 s \sqrt{1-x} \ \ \Sigma \ , \tag{2}$$

where

$$x = \frac{4m_t^2}{s} \tag{3}$$

and

$$\begin{aligned}
\Sigma &= \frac{1}{s^2}\left(1+\frac{x}{2}\right) v_e^{\gamma 2} v_t^{\gamma 2} \\
&+ \frac{2}{s(s-m_Z^2)}\left(1+\frac{x}{2}\right) v_e^\gamma v_t^\gamma \left(v_e^Z - p a_e^Z\right) v_t^Z \\
&+ \frac{1}{(s-m_Z^2)^2}\left(v_e^{Z2} + a_e^{Z2} - 2p v_e^Z a_e^Z\right)\left[(1+\frac{x}{2}) v_t^{Z2} + (1-x) a_t^{Z2}\right] \ .
\end{aligned} \tag{4}$$

As usual, s is the centre of mass energy squared, θ_w is the weak mixing angle taken in what follows to be given by $\sin^2\theta_w = .22$, $\alpha = 1/128$ is the fine structure constant and $m_Z = 91$ GeV and $m_t = 175$ GeV are the Z^0 and top quark masses. The latter value has been chosen according to recent experimental results [1, 12]. The degree of longitudinal polarization of the initial electron beam is p, where $p = \pm 1$ for right and left helicities respectively. The electron and top vector and axial couplings to the photon and the Z^0 are given by

$$\begin{aligned}
v_e^\gamma &= 1 \quad , \quad v_e^Z = (1-4\sin^2\theta_w)/(4\sin\theta_w\cos\theta_w) \\
a_e^\gamma &= 0 \quad , \quad a_e^Z = 1/(4\sin\theta_w\cos\theta_w) \\
v_t^\gamma &= -\frac{2}{3} \quad , \quad v_t^Z = (-1+\frac{8}{3}\sin^2\theta_w)/(4\sin\theta_w\cos\theta_w) \\
a_t^\gamma &= 0 \quad , \quad a_t^Z = -1/(4\sin\theta_w\cos\theta_w) \ .
\end{aligned} \tag{5}$$

It is a straightforward task to incorporate the width of the Z^0. The numerical effect, though, is minute but has been taken into account in the numerical analysis. The dependence of the top pair-production cross section on the centre of mass energy is depicted in Fig. 1 for a fully left- and right-polarized electron beam and in the absence of polarization.

The two top quarks produced in the reaction (1) subsequently decay into charged weak gauge bosons and bottom quarks with a branching ratio which can for all practical purposes be taken equal to one:

$$t \to W^+ b \qquad\qquad \bar{t} \to W^- \bar{b} \; . \tag{6}$$

Some information about the polarization of the final state top quarks can also be inferred from the angular distributions of its decay products. Indeed, in the centre of mass frame of the (anti-) top quark, the (anti-) bottom quark emerges with an angle θ with respect to the spin of its parent which is distributed according to

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{1}{2}(1 \pm \beta\cos\theta) \; , \tag{7}$$

where the mass of the bottom quark is neglected and

$$\beta = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} \; . \tag{8}$$

In the presence of an electroweak dipole moment of the top quark, the standard model lagrangian has to be supplemented with the following terms:

$$\mathcal{L}_{EWDM} = -\frac{i}{2}\, d_t^V \; [\bar{t}\sigma^{\mu\nu}\gamma_5 t] \; (\partial_\mu V_\nu - \partial_\nu V_\mu) \qquad\qquad (V = \gamma, Z) \; , \tag{9}$$

where d_t^γ and d_t^Z are the magnetic and weak dipole moments of the top quark. It should be noted at this stage that these moments are by no means constants, but rather energy dependent form factors.

In the following we concentrate on the two CP-odd observables [10]

$$O_1 \;=\; (\vec{p}_b \times \vec{p}_{\bar{b}}) \cdot \vec{1}_z \tag{10}$$

$$O_2 \;=\; (\vec{p}_b + \vec{p}_{\bar{b}}) \cdot \vec{1}_z \; , \tag{11}$$

where $\vec{1}_z$ is the unit vector aligned with the incoming positron beam. In the presence of electroweak dipole moments (9), these observables aquire non-vanishing expectation values. These are not sensitive to possible CP violation in top decay, and we have therefore ignored it. Neglecting the higher order terms in the dipole moments $d_t^{\gamma,Z}$, we find

$$
\begin{aligned}
\langle O_1 \rangle &= \frac{s m_t}{12}\ (1-x)\ \epsilon^2\ \beta\ \Sigma^{-1} \\
& \Big\{\ \frac{1}{s^2}\ C^{\gamma\gamma}\ v_e^{\gamma 2}\ v_t^{\gamma}\ \mathcal{R}e\ d_t^{\gamma} \\
& + \frac{1}{s(s-m_Z^2)}\ C^{Z\gamma}\ v_e^{\gamma} v_e^{Z}\ (v_t^Z - \frac{\beta}{3} a_t^Z)\ \mathcal{R}e\ d_t^{\gamma} \\
& + \frac{1}{s(s-m_Z^2)}\ C^{Z\gamma}\ v_e^{\gamma} v_e^{Z}\ v_t^{\gamma}\ \mathcal{R}e\ d_t^{Z} \\
& + \frac{1}{(s-m_Z^2)^2}\ C^{ZZ}\ (v_e^{Z2} + a_e^{Z2})\ (v_t^Z - \frac{\beta}{3} a_t^Z)\ \mathcal{R}e\ d_t^{Z}\ \Big\} \qquad (12) \\
\langle O_2 \rangle &= \frac{\sqrt{s} m_t}{3}\ (1-x)\ \epsilon\ \beta\ \Sigma^{-1} \\
& \Big\{\ \frac{1}{s(s-m_Z^2)}\ C^{Z\gamma}\ v_e^{\gamma} v_e^{Z}\ a_t^{Z}\ \mathcal{I}m\ d_t^{\gamma} \\
& + \frac{1}{(s-m_Z^2)^2}\ C^{ZZ}\ (v_e^{Z2} + a_e^{Z2})\ a_t^{Z}\ \mathcal{I}m\ d_t^{Z}\ \Big\}\ , \qquad (13)
\end{aligned}
$$

where

$$
\epsilon = 1 - \frac{m_W^2}{m_t^2} \qquad (14)
$$

and

$$
\begin{aligned}
C^{\gamma\gamma} &= -p \\
C^{\gamma Z} &= \frac{a_e^Z}{v_e^Z} - p \\
C^{ZZ} &= \frac{2 v_e^Z a_e^Z}{v_e^{Z2} + a_e^{Z2}} - p\ . \qquad (15)
\end{aligned}
$$

Note that O_1 being CPT-even it is sensitive to the real parts of the dipole moments. In contrast, O_2 is CPT-odd and provides thus a measure of the imaginary parts of the moments.

If a non-vanishing average values $\langle O \rangle$ is observed, it has a statistical significance only as far as it is compared with the expected variance $\langle O^2 \rangle$. For instance, to observe a deviation from the standard model expectation with better than 3 standard deviations (*i.e.* 99.7% confidence) one needs

$$
\langle O \rangle \ \geq\ 3 \sqrt{\frac{\langle O^2 \rangle}{n_{t\bar{t}}}}\ , \qquad (16)
$$

where $n_{t\bar{t}} = \mathcal{L}\sigma(e^+e^- \to t\bar{t})$ is the number of events and $\mathcal{L}$ is the collider luminosity.

We find for the variances

$$\begin{aligned}
\langle O_1^2\rangle &= \frac{sm_t^2}{2880}\ \epsilon^4\ \Sigma^{-1} \\
&\quad \Big\{\ \frac{1}{s^2}\ v_e^{\gamma 2}\ \Big[v_t^{\gamma 2}\ \Big(24+2x-11x^2+4\beta^2(1-x)^2\Big)\Big] \\
&\quad + \frac{2}{s(s-m_Z^2)}\ v_e^\gamma v_e^Z\ \Big[v_t^\gamma v_t^Z\ \Big(24+2x-11x^2+4\beta^2(1-x)^2\Big) \\
&\qquad\qquad -v_t^\gamma a_t^Z\ 2(1-x)(6-x)\beta\Big] \\
&\quad + \frac{1}{(s-m_Z^2)^2}\ (v_e^{Z^2}+a_e^{Z^2})\ \Big[v_t^{Z^2}\ \Big(24+2x-11x^2+4\beta^2(1-x)^2\Big) \\
&\qquad\qquad a_t^{Z^2}\ \Big(24-14x-4\beta^2(1-x)\Big)(1-x) \\
&\qquad\qquad -v_t^Z a_t^Z\ 4(1-x)(6-x)\beta\Big]\ \ \Big\} \qquad (17) \\
\langle O_2^2\rangle &= \frac{s}{720}\ \epsilon^2\ \Sigma^{-1} \\
&\quad \Big\{\ \frac{1}{s^2}\ v_e^{\gamma 2}\ \Big[v_t^{\gamma 2}\ (4+7x+4x^2)(3-\beta^2)\Big] \\
&\quad + \frac{2}{s(s-m_Z^2)}\ v_e^\gamma v_e^Z\ \Big[v_t^\gamma v_t^Z\ (4+7x+4x^2)(3-\beta^2)\Big] \\
&\quad + \frac{1}{(s-m_Z^2)^2}\ (v_e^{Z^2}+a_e^{Z^2})\ \Big[v_t^{Z^2}\ (4+7x+4x^2)(3-\beta^2) \\
&\qquad\qquad +a_t^{Z^2}\ \Big(12+18x-4\beta^2(1-x)\Big)(1-x)\Big]\ \ \Big\}\,. \qquad (18)
\end{aligned}$$

Implementing Eq. (16) one obtains the areas in the (d_t^γ, d_t^Z) plane which cannot be explored with sufficient confidence. Clearly, because of the linear dependence of the expectation values (12,13) on the dipole moments, these areas are delimited by straight lines which are equidistant from the standard model expectation $d_t^\gamma = d_t^Z = 0$. The slope of these straight lines varies with the polarization of the initial electron beam, as is shown in Figs 2 and 3. For fully polarized beams the bands are narrowest. As the degree of polarization is decreased, the bands rotate around fixed points and become wider. The results of Figs 2 and 3 have been obtained assuming a centre of mass energy of 500 GeV, a luminosity of 10 fb^{-1} and an overall conservative b- and W-tagging efficiency of 10%. Note that if the tagging efficiencies were to be improved without loss of purity from 10% to 50% or 90%, the bounds on the dipole moments in Figs 2 and 3 would shrink by a factor of .45 or .33.

It is clear from Figs 2 and 3 that in the absence of polarization, the correlations $\langle O_1\rangle$ and $\langle O_2\rangle$ are extremely insensitive to $\mathcal{R}e\ d_t^\gamma$ and $\mathcal{I}m\ d_t^{Z^0}$ respectively. However, the inclusion of polarization makes the correlations sensitive to real as well as imaginary parts of both type of dipole moments, due to the rotation of the bands described above. Note also that a single measurement (with or without polarization) cannot exclude large dipole moments: in some unfortunate situations, the electric and weak dipoles can assume large values, but their effects cancel out so that no CP violation is apparent. However, if the information from two measurements with opposite electron polarization is combined, only small values (in the vicinity of the origin) of the electroweak dipole moments can escape detection.

Although the top pair-production cross section drops rapidly beyond 420 GeV, the effect of the electroweak dipole moments keeps augmenting with increasing energies. This can easily be traced back to the higher dimension of the operators (9). As can easily be inferred from the asymptotic behaviours of $\langle O \rangle$, $\langle O^2 \rangle$ and $n_{t\bar{t}}$, the sensitivity of O_1 saturates while the sensitivity of O_2 is bound to decrease beyond a certain energy, which turns out to be around 750 GeV. At this energy the sensitivities of O_1 and O_2 improve by 60% and 30% respectively. At higher energies the sensitivity of O_1 almost doubles. It should, however, be kept in mind that the dipole moments being actually energy dependent form factors, their magnitude is expected to decrease rapidly at energies exceeding the scale of "new physics".

We now examine to what extent collinear helicity-flip photon emission from the initial state affects our analysis. First of all, there can be no CP-conserving background at order α from such a process for $\langle O_1 \rangle$, since O_1 is T-odd. Any CP-conserving process can only contribute to it if the amplitude has an absorptive part, and to order α, initial-state photon emission occurs only at tree-level. In contrast, $\langle O_2 \rangle$ can get such a contribution, since O_2 is T even. In principle this standard model contribution to the signal can be computed and subtracted from the values of $\langle O_2 \rangle$ (13). However, since the helicity-flip photon spectrum is hard [11], a cut on the final-state energy requiring it to be larger than $(1-\delta)\sqrt{s}$ would suppress this background correlation by a factor of at least $\alpha/(2\pi)\delta^3$. Even an easily implementable cut with $\delta = 0.05$ would thus suffice to render this background harmless. Of course, this would mean restricting the analysis to hadronic decay modes of the $W^\pm$. However, the loss in efficiency is not dramatic and can be considered as being included in the aforementioned 10% overall b- and W-tagging efficiency.

We have compared our results with the sensitivities obtained by studying energy and up-down asymmetries of leptons arising in top decay [6] or correlations in the absence of polarization [8]. We find that under similar assumptions the polarized experiment we suggest can improve the observability limits on the electroweak dipole moments by more than an order of magnitude.

To conclude, we have studied the role of longitudinal polarization of electron beams in high energy e^+e^- collisions for measuring the real and imaginary parts of the top quark electric and weak dipole moments by determining correlations of certain CP-odd observables. We have calculated the correlations as well as the variances of the observables using analytical expressions, and obtained the sensitivities of these observables to the dipole moments at a linear collider operating at $\sqrt{s} = 500$ GeV and with an integrated luminosity of 10 fb^{-1}. We find that the sensitivities for the electroweak dipole moments are greatly enhanced in the presence of polarized electron beams. The real and imaginary parts of the dipole moments can be probed at the 3-σ level down to values below 1 e atto-m.

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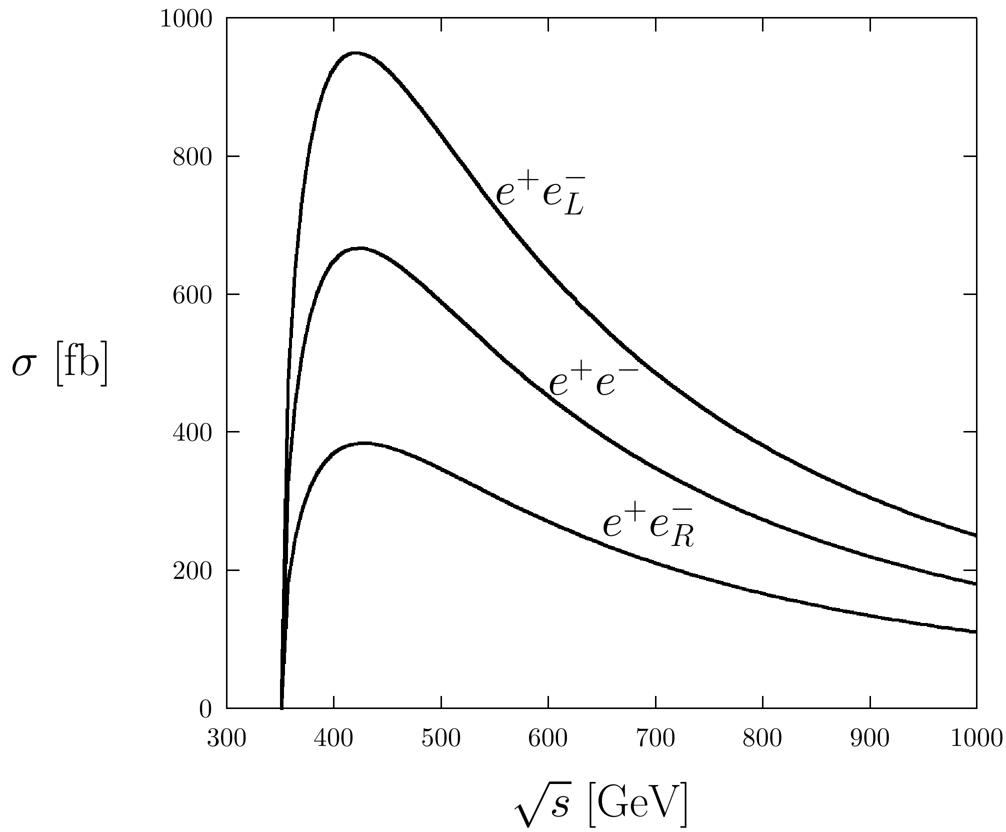

Figure 1: Energy dependence of the top quark pair-production cross section in the presence and absence of polarization. The top quark mass is taken to be 175 GeV.

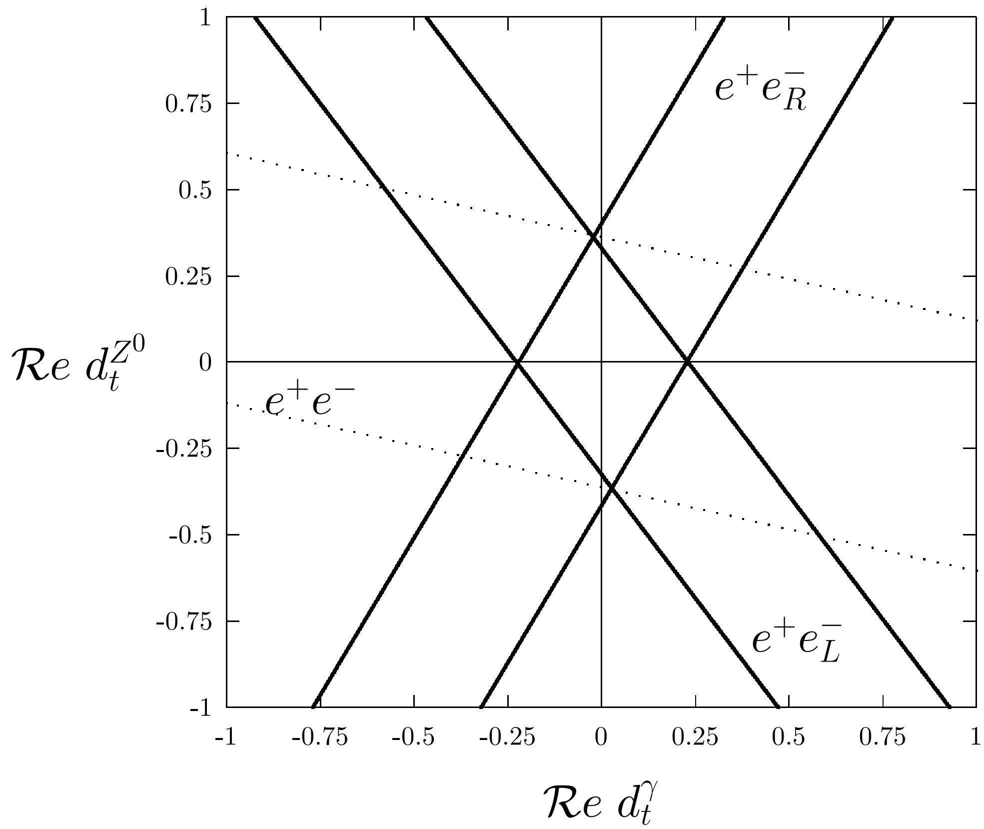

Figure 2: Bounds from $\langle O_1 \rangle$ on $\mathcal{R}e\ d_t^{\gamma,Z}$ [e am] according to Eq. (16).

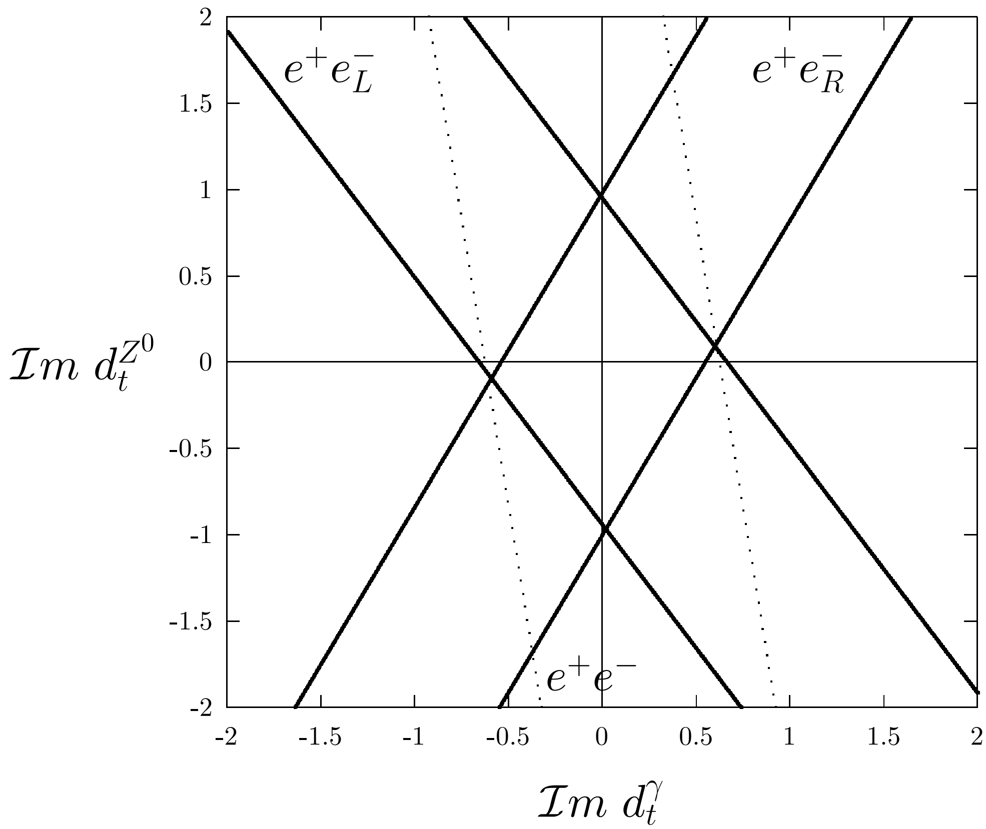

Figure 3: Bounds from $\langle O_2 \rangle$ on $\mathcal{I}m\ d_t^{\gamma,Z}$ [e am] according to Eq. (16).