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Effect of anomalous tbW vertex on decay-lepton distributions in $e^+e^- \to t\bar{t}$ and CP-violating asymmetries

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Abstract. We obtain analytic expressions for the energy and polar-angle double differential distributions of a secondary lepton $l^+(l^-)$ arising from the decay of t ($\bar{t}$) in $e^+e^- \to t\bar{t}$ with an anomalous tbW decay vertex. We also obtain analytic expressions for the various differential cross-sections with the lepton energy integrated over. In this case, we find that the angular distributions of the secondary lepton do not depend on the anomalous coupling in the decay, regardless of possible anomalous couplings occurring in the production amplitude for $e^+e^- \to t\bar{t}$. Our study includes the effect of longitudinal e^- and e^+ beam polarization. We also study the lepton energy and beam polarization dependence of certain CP-violating lepton angular asymmetries arising from an anomalous tbW decay vertex and compare them with the asymmetries arising due to CP-violation in the production process due to the top electric or weak dipole moment.

Keywords. Decay of top quark; anomalous couplings; CP-violation.

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1. Introduction

The standard model (SM) has been found to be in agreement with experiment in many of its aspects. However, the properties of the top quark, in particular, the nature and strength of its couplings, have yet to be studied accurately. A future linear e^+e^- collider, operating at a centre-of-mass (cm) energy above the $t\bar{t}$ threshold, will be able to determine with greater accuracy the couplings of the top quark to the gauge bosons γ, Z, W, and possibly, to the Higgs [1]. A comparison of these with SM expectations will be able to shed light on new physics effects, if any.

The role of a linear e^+e^- collider (LC) in probing $t\bar{t}$ interactions through the polarization of the t and/or $\bar{t}$ has received a fair amount of attention [2–4]. In particular, the polarization effects can be used to probe the anomalous magnetic and electric dipole couplings of t, $\bar{t}$ to γ and Z [5–11], as well as chromoelectric and chromomagnetic dipole couplings to gluons [12]. A common underlying idea behind these studies is, of course, that the top quark being heavy decays before it can hadronize [13]. Hence its decay distributions would retain information about the spin of the quark and would be useful in analyzing its polarization.

The top quark polarization can thus be observed using its dominant decay into a bottom quark and a W, the latter decaying into a quark pair or a charged lepton and a neutrino. The accuracy of polarization measurements is therefore dependent on an accurate knowledge of the tbW vertex. It is of utmost importance, then, to study possible anomalous contributions to the tbW vertex, which might arise from the same sources as anomalous contributions to the production process.

In this paper we look at the process $e^+e^- \to t\bar{t}$, with either of t or $\bar{t}$ decaying hadronically, while the other decays into a b quark, a charged lepton and a neutrino. We will be interested only in the kinematic distribution of the charged lepton as a probe of anomalous interactions either in the production process ($e^+e^- \to t\bar{t}$), or in the tbW decay vertex. Our aim would be to examine features of various distributions which might isolate or emphasize anomalous effects in production over those in decay, or vice versa.

To this aim we have obtained analytic expressions for a full angular distribution, and also combined energy and polar-angle distribution of the secondary decay lepton in the cm frame, in the presence of non-standard contributions to production as well as decay. We have also included the effect of e^+ and e^- beam longitudinal polarization.

The energy distribution of a single lepton in a semileptonic process described above, as well as energy correlations between leptons arising from t and $\bar{t}$ decay, have been studied before in [5,9,14]. Angular distribution of leptons, as well as angular correlations have been studied in the context of CP-violating (electric and weak dipole) moments of the top quark alone [5-11] and/or of CP-violation in decay [15]. Many of these studies have been done in the context of specific models. An analytic expression for combined lepton energy and polar-angle distribution in the context of anomalous couplings present in the top production as well as decay, which has been obtained here, did not exist in the literature until very recently [16,17]. It should be noted, however, that our approach is considerably different from that of [16]. For example, we have used helicity amplitudes rather than the method of Kawasaki, Shirafuji and Tsai [18] used by them to get decay lepton distributions. We thus provide an independent cross check and confirmation of their expressions.

As it turns out, without the observation of lepton energy, it is impossible to see the effect of anomalous tbW vertex in the lepton angular distribution. It is thus important to look at combined energy-angle distribution.

We have made use of our expressions to study CP-violation in the production and decay of $t\bar{t}$. We have calculated the contribution of the different sources of CP-violation, viz., top electric and weak dipole couplings and a CP-violating tbW vertex, to the dependence on lepton energy of certain simple angular asymmetries. We have also examined the role of longitudinal e^+ and e^- beam polarization in discriminating among these different sources of CP-violation.

One of the results of this work is that so long as the lepton energy is integrated over, the distributions in terms of lepton and/or top angles are independent of non-standard effects in top decay, regardless of whether there are any non-standard effects in production or not. This has two consequences. Firstly, this implies that lepton angular distributions can be safely used for analyzing top polarization without fear of an error coming from possible non-standard effects in the tbW coupling. Thus, angular distributions can be used to study non-standard effects in top production without contamination from non-standard effects in decay. On the other hand, it is also evident that for studying non-standard effects in top decay, one has necessarily to look at the energy dependence. We have found that crucial changes of sign of certain asymmetries with polarization, and with lepton energy, can be used to enhance the relative importance of the contributions of CP-violation coming from the dipole moments of the top quark, and from the decay.

The plan of the rest of the paper is as follows. In the next section we set up our formalism and list helicity amplitudes for t and $\bar{t}$ decay in the leptonic channel. In §3 we obtain energy integrated charged lepton angular distributions. In §4 we obtain the distribution for the charged leptons in terms of energy and polar angle. Section 5 contains the results and discussion.

2. Formalism and helicity amplitudes

We describe in this section the calculation of helicity amplitudes in $e^+e^- \to t\bar{t}$ and the subsequent decay $t \to bl^+\nu_l$ ($\bar{t} \to \bar{b}l^- \bar{\nu}_l$). We adopt the narrow-width approximation for t and $\bar{t}$, as well as for $W^\pm$ produced in $t,\ \bar{t}$ decay.

We assume the top quark couplings to γ and Z to be given by the vertex factor $ie\Gamma^j_\mu$, where

$$\Gamma^j_\mu \ = \ c^j_v\,\gamma_\mu \ + \ c^j_a\,\gamma_\mu\,\gamma_5 \ + \ \frac{c^j_d}{2\,m_t}\,i\gamma_5\,(p_t \ - \ p_{\bar{t}})_\mu, \ \ j \ = \ \gamma, Z, \tag{1}$$

with

$$\begin{aligned} c^\gamma_v &= \frac{2}{3}, \quad c^\gamma_a \ = \ 0, \\ c^Z_v &= \frac{\left(\frac{1}{4} - \frac{2}{3}\,x_w\right)}{\sqrt{x_w\,(1 - x_w)}}, \\ c^Z_a &= -\frac{1}{4\sqrt{x_w\,(1 - x_w)}}, \end{aligned} \tag{2}$$

and $x_w \ = \ \sin^2\theta_w$, θ_w being the weak mixing angle. We have assumed in (1) that the only addition to the SM couplings $c^{\gamma,Z}_{v,a}$ are the CP-violating electric and weak dipole form factors, ec^γ_d/m_t and ec^Z_d/m_t, which are assumed small. We will call these electric dipole moment (edm) and weak dipole moment (wdm) for short. Including additional anomalous couplings, viz., vector, axial vector and magnetic dipole couplings is not a problem, and our deriviation of distributions would go through in that case. However, numerical calculations in this paper are restricted to only CP-violating effects and hence we do not include other form factors in eq. (1). Use has also been made of the Dirac equation in rewriting the usual dipole coupling $\sigma_{\mu\nu}(p_t + p_{\bar{t}})^\nu\gamma_5$ as $i\gamma_5(p_t - p_{\bar{t}})_\mu$, dropping small corrections to the vector and axial-vector couplings.

We write the contribution of a general tbW vertex to t and $\bar{t}$ decays as

$$\begin{aligned} \Gamma^\mu_{tbW} = -\frac{g}{\sqrt{2}}V_{tb}\bar{u}(p_b)\Big[&\gamma^\mu(f_{1L}P_L + f_{1R}P_R) \\ &-\frac{i}{m_W}\sigma^{\mu\nu}(p_t - p_b)_\nu(f_{2L}P_L + f_{2R}P_R)\Big]u(p_t), \end{aligned} \tag{3}$$

$$\bar{\Gamma}^{\mu}_{tbW} = -\frac{g}{\sqrt{2}} V^{*}_{tb} \bar{v}(p_{\bar{t}}) \left[\gamma^{\mu} (\bar{f}_{1L} P_L + \bar{f}_{1R} P_R) \right. \\ \left. - \frac{i}{m_W} \sigma^{\mu\nu} (p_{\bar{t}} - p_{\bar{b}})_{\nu} (\bar{f}_{2L} P_L + \bar{f}_{2R} P_R) \right] v(p_{\bar{b}}), \tag{4}$$

where $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$, and V_{tb} is the Cabibbo–Kobayashi–Maskawa matrix element, which we take to be equal to one. If CP is conserved, the form factors f above obey the relations

$$f_{1L} = \bar{f}_{1L}; \quad f_{1R} = \bar{f}_{1R}, \tag{5}$$

and

$$f_{2L} = \bar{f}_{2R}; \quad f_{2R} = \bar{f}_{2L}. \tag{6}$$

Like c_d^{γ} and c_d^{Z} above, we will also treat $f_{2L,R}$ and $\bar{f}_{2L,R}$ as small, and retain only terms linear in them. For the form factors f_{1L} and $\bar{f}_{1L}$, we retain their SM values, viz., $f_{1L} = \bar{f}_{1L} = 1$. f_{1R} and $\bar{f}_{1R}$ do not contribute in the limit of vanishing b mass, which is used here. Also, f_{2L} and $\bar{f}_{2R}$ drop out in this limit.

The helicity amplitudes for $e^+e^- \to \gamma^*, Z^* \to t\bar{t}$ in the centre-of-mass (cm) frame, including $c_d^{\gamma,Z}$ couplings, have been given in [5] (see also Kane *et al*, ref. [2]), so we do not repeat them here.

To calculate the decay helicity amplitudes, we use the standard Dirac gamma matrix representations, and the following forms for Dirac spinors with definite helicity.

For the spinors for l, b and ν, and their antiparticles, all of which are assumed massless, we use the representations

$$u_-(P) = \sqrt{P^0} \begin{pmatrix} -\sin\left(\frac{\Theta}{2}\right) \mathrm{e}^{-i\Phi} \\ \cos\left(\frac{\Theta}{2}\right) \\ \sin\left(\frac{\Theta}{2}\right) \mathrm{e}^{-i\Phi} \\ -\cos\left(\frac{\Theta}{2}\right) \end{pmatrix}; \; v_+(P) = \sqrt{P^0} \begin{pmatrix} \sin\left(\frac{\Theta}{2}\right) \mathrm{e}^{-i\Phi} \\ -\cos\left(\frac{\Theta}{2}\right) \\ -\sin\left(\frac{\Theta}{2}\right) \mathrm{e}^{-i\Phi} \\ \cos\left(\frac{\Theta}{2}\right) \end{pmatrix}, \tag{7}$$

where the subscript denotes the sign of the helicity and Θ and Φ are the polar and azimuthal angles of the momentum P of the respective particle or antiparticle.

For the spinors corresponding to t and $\bar{t}$ in their respective rest frames, we use

$$u_{t+} = \sqrt{2m_t} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; u_{t-} = \sqrt{2m_t} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \tag{8}$$

$$v_{t+} = \sqrt{2m_t} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; v_{t-} = \sqrt{2m_t} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{9}$$

The non-vanishing helicity amplitudes, respectively M and $\overline{M}$, for

$$t \to bW^+, \;\; W^+ \to l^+\nu_l$$

and

$$\bar{t} \to \bar{b}W^-, \;\; W^- \to l^-\bar{\nu}_l$$

in the respective rest frames of t, $\bar{t}$, are given below (we neglect all fermion masses except m_t, the top mass):

$$\begin{aligned} M_{+-+-} = & -2\sqrt{2}g^2\Delta_W(q^2) \\ & \times \left\{ \left(1+\frac{f_{2R}}{\sqrt{r}}\right)\cos\frac{\theta_{l+}}{2}\left[\cos\frac{\theta_{\nu_l}}{2}\sin\frac{\theta_b}{2}\mathrm{e}^{i\phi_b} - \sin\frac{\theta_{\nu_l}}{2}\cos\frac{\theta_b}{2}\mathrm{e}^{i\phi_{\nu_l}}\right] \right. \\ & \left. - \frac{f_{2R}}{\sqrt{2}}\sin\frac{\theta_b}{2}\mathrm{e}^{i\phi_b}\left[\sin\frac{\theta_{\nu_l}}{2}\sin\frac{\theta_{l+}}{2}\mathrm{e}^{i(\phi_{\nu_l}-\phi_{l+})} + \cos\frac{\theta_{\nu_l}}{2}\cos\frac{\theta_{l+}}{2}\right]\right\}, \end{aligned} \tag{10}$$

$$\begin{aligned} M_{--+-} = & -2\sqrt{2}g^2\Delta_W(q^2) \\ & \times \left\{ \left(1+\frac{f_{2R}}{\sqrt{r}}\right)\sin\frac{\theta_{l+}}{2}\mathrm{e}^{-i\phi_{l+}}\left[\cos\frac{\theta_{\nu_l}}{2}\sin\frac{\theta_b}{2}\mathrm{e}^{i\phi_b} - \sin\frac{\theta_{\nu_l}}{2}\cos\frac{\theta_b}{2}e^{i\phi_{\nu_l}}\right] \right. \\ & \left. + \frac{f_{2R}}{\sqrt{r}}\cos\frac{\theta_b}{2}\left[\sin\frac{\theta_{\nu_l}}{2}\sin\frac{\theta_{l+}}{2}\mathrm{e}^{i(\phi_{\nu_l}-\phi_{l+})} + \cos\frac{\theta_{\nu_l}}{2}\cos\frac{\theta_{l+}}{2}\right]\right\}, \end{aligned} \tag{11}$$

$$\begin{aligned} \bar{M}_{++-+} = & -2\sqrt{2}g^2\Delta_W(q^2) \\ & \times \left\{ \left(1+\frac{\bar{f}_{2L}}{\sqrt{r}}\right)\cos\frac{\theta_{l-}}{2}\left[\cos\frac{\theta_{\bar{\nu}_l}}{2}\sin\frac{\theta_{\bar{b}}}{2}\mathrm{e}^{-i\phi_{\bar{b}}} - \sin\frac{\theta_{\bar{\nu}_l}}{2}\cos\frac{\theta_{\bar{b}}}{2}\mathrm{e}^{-i\phi_{\bar{\nu}_l}}\right] \right. \\ & \left. - \frac{\bar{f}_{2L}}{\sqrt{r}}\sin\frac{\theta_{\bar{b}}}{2}\mathrm{e}^{-i\phi_{\bar{b}}}\left[\sin\frac{\theta_{\bar{\nu}_l}}{2}\sin\frac{\theta_{l-}}{2}\mathrm{e}^{i(\phi_{l-}-\phi_{\bar{\nu}_l})} + \cos\frac{\theta_{\bar{\nu}_l}}{2}\cos\frac{\theta_{l-}}{2}\right]\right\}, \end{aligned}$$

$$\begin{aligned} \bar{M}_{-+-+} = & -2\sqrt{2}g^2\Delta_W(q^2) \\ & \times \left\{ \left(1+\frac{\bar{f}_{2L}}{\sqrt{r}}\right)\sin\frac{\theta_{l-}}{2}\mathrm{e}^{i\phi_{l-}}\left[\cos\frac{\theta_{\bar{\nu}_l}}{2}\sin\frac{\theta_{\bar{b}}}{2}\mathrm{e}^{-i\phi_{\bar{b}}} - \sin\frac{\theta_{\bar{\nu}_l}}{2}\cos\frac{\theta_{\bar{b}}}{2}\mathrm{e}^{-i\phi_{\bar{\nu}_l}}\right] \right. \\ & \left. + \frac{\bar{f}_{2L}}{\sqrt{r}}\cos\frac{\theta_{\bar{b}}}{2}\left[\sin\frac{\theta_{\bar{\nu}_l}}{2}\sin\frac{\theta_{l-}}{2}\mathrm{e}^{i(\phi_{l-}-\phi_{\bar{\nu}_l})} + \cos\frac{\theta_{\bar{\nu}_l}}{2}\cos\frac{\theta_{l-}}{2}\right]\right\}, \end{aligned} \tag{12}$$

where

$$\Delta_W(q^2) = \frac{1}{q^2 - m_W^2 + i\Gamma_W m_W} \tag{13}$$

is the W propagator, q its momentum, and

$$r = \frac{m_W^2}{m_t^2}. \tag{14}$$

The subscripts $\pm$ refer to signs of the helicities, the order of the helicities being t, b, l^+, ν_l ($\bar{t}$, $\bar{b}$, l^-, $\bar{\nu}_l$). The various θ's are polar angles of the particles (in the t rest frame) and of the antiparticles (in the $\bar{t}$ rest frame) labelled by the suffixes with respect to a z axis which is the direction in which the top momentum is boosted to go from its rest frame to the cm frame. ϕ's are the azimuthal angles with respect to an x axis chosen in the plane containing the e^- momentum and positive z directions. We could have chosen here the polar and azimuthal angles of the lepton to be zero in the top rest frame. However, we maintain nonzero values for future use, when we boost to the cm frame.

The density matrix elements calculated from the above helicity amplitudes in the rest frame of the top are given by

$$\Gamma_{\pm\pm} = g^4|\Delta(q^2)|^2(m_t^2 - 2p_t \cdot p_{l^+})(1 \pm \cos\theta_{l^+}) \left(1 + \frac{\mathrm{Re} f_{2R}}{\sqrt{r}} \frac{m_W^2}{p_t \cdot p_{l^+}}\right), \tag{15}$$

$$\Gamma_{\pm\mp} = g^4|\Delta(q^2)|^2(m_t^2 - 2p_t \cdot p_{l^+}) \sin\theta_{l^+} \mathrm{e}^{\pm i\phi_{l^+}} \left(1 + \frac{\mathrm{Re} f_{2R}}{\sqrt{r}} \frac{m_W^2}{p_t \cdot p_{l^+}}\right), \tag{16}$$

$$\bar{\Gamma}_{\pm\pm} = g^4|\Delta(q^2)|^2(m_t^2 - 2p_{\bar{t}} \cdot p_{l^-})(1 \pm \cos\theta_{l^-}) \left(1 + \frac{\mathrm{Re} \bar{f}_{2L}}{\sqrt{r}} \frac{m_W^2}{p_{\bar{t}} \cdot p_{l^-}}\right), \tag{17}$$

$$\bar{\Gamma}_{\pm\mp} = g^4|\Delta(q^2)|^2(m_t^2 - 2p_{\bar{t}} \cdot p_{l^-}) \sin\theta_{l^-} \mathrm{e}^{\mp i\phi_{l^-}} \left(1 + \frac{\mathrm{Re} \bar{f}_{2L}}{\sqrt{r}} \frac{m_W^2}{p_{\bar{t}} \cdot p_{l^-}}\right), \tag{18}$$

where an averaging over the the azimuthal angles ϕ_b and $\phi_{\bar{b}}$ has been done. Notice that the only change in the decay density matrix relative to the expression in SM is the overall factor $1 + (\mathrm{Re} f_{2R}/\sqrt{r})(m_W^2/p_t \cdot p_{l^+})$ or $1 + ((\mathrm{Re} \bar{f}_{2L}/\sqrt{r})(m_W^2/p_{\bar{t}} \cdot p_{l^-}))$. This has the important consequence that regardless of any anomalous contributions to the production process, the decay-lepton double differential distribution (calculated below) gets modified by anomalous tbW couplings in the narrow-width approximation by the same overall factor. It is also interesting to note that only the real part of the anomalous couplings appear in the overall factor. This actually corresponds to the absorptive part of the decay amplitude.

We now make a Lorentz transformation to the laboratory frame using the fact that Γ_{ij} and $\bar{\Gamma}_{ij}$ are invariant, and the following transformations for the angles:

$$1 \pm \cos\theta_{l^+} = \frac{(1 \mp \beta)(1 \pm \cos\theta_{tl^+}^{\mathrm{c.m.}})}{1 - \beta\cos\theta_{tl^+}^{\mathrm{c.m.}}}, \tag{19}$$

$$\begin{aligned}\sin\theta_{l^+}\mathrm{e}^{i\phi_{l^+}} = \frac{\sqrt{1-\beta^2}}{1 - \beta\cos\theta_{tl^+}^{\mathrm{c.m.}}} (&\sin\theta_{l^+}^{\mathrm{c.m.}} \cos\theta_t^{\mathrm{c.m.}} \cos\phi_{l^+}^{\mathrm{c.m.}} \\ &- \cos\theta_{l^+}^{\mathrm{c.m.}} \sin\theta_t^{\mathrm{c.m.}} + i\sin\theta_{l^+}^{\mathrm{c.m.}} \sin\phi_{l^+}^{\mathrm{c.m.}}),\end{aligned} \tag{20}$$

where $\theta_{tl^+}^{\mathrm{c.m.}}$ is the angle between t and l^+ directions in the c.m. frame. There are similar expressions for the angles of $\bar{t}$, and its decay products, which we do not give explicitly. We will henceforth calculate everything in the cm frame. We will therefore drop the label 'c.m.' over the angles, it being understood that all variables are in the e^+e^- c.m. frame.

Combining the production and decay density matrices in the narrow-width approximation for $t, \bar{t}, W^+, W^-$, we get the l^+ and l^- distributions for the case of e^-, e^+ with polarization P_e, $P_{\bar{e}}$ to be

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^\pm}{\mathrm{d}\cos\theta_t \mathrm{d}E_l \mathrm{d}\cos\theta_l \mathrm{d}\phi_l} &= \frac{3\alpha^4\beta}{16x_w^2\sqrt{s}} \frac{E_l}{\Gamma_t\Gamma_W m_W} \\
&\times \left(1 + \frac{\mathrm{Re} f^\pm}{\sqrt{r}} \frac{2m_W^2}{E_l\sqrt{s}(1-\beta\cos\theta_{tl})}\right) \left(\frac{1}{1-\beta\cos\theta_{tl}} - \frac{4E_l}{\sqrt{s}(1-\beta^2)}\right) \\
&\times \left\{A(1-\beta\cos\theta_{tl}) + B^\pm(\cos\theta_{tl} - \beta)\right. \\
&\quad + C^\pm(1-\beta^2)\sin\theta_t\sin\theta_l(\cos\theta_t\cos\phi_l - \sin\theta_t\cot\theta_l) \\
&\quad \left. + D^\pm(1-\beta^2)\sin\theta_t\sin\theta_l\sin\phi_l\right\}, \qquad (21)
\end{aligned}
$$

where A, B, C and D are quantities related to the production density matrices ρ_{ij} and $\bar{\rho}_{ij}$ for t and $\bar{t}$ respectively, by

$$A = \rho_{++} + \rho_{--} = \bar{\rho}_{++} + \bar{\rho}_{--}, \qquad (22)$$

$$B^+ = \rho_{++} - \rho_{--}, \qquad (23)$$

$$B^- = \bar{\rho}_{++} - \bar{\rho}_{--}, \qquad (24)$$

$$C^+ = \mathrm{Re}\rho_{+-} \frac{\sqrt{s}}{m_t\sin\theta_t}, \qquad (25)$$

$$C^- = \mathrm{Re}\bar{\rho}_{+-} \frac{\sqrt{s}}{m_t\sin\theta_t}, \qquad (26)$$

$$D^+ = \mathrm{Im}\rho_{+-} \frac{\sqrt{s}}{m_t\sin\theta_t}, \qquad (27)$$

$$D^- = \mathrm{Im}\bar{\rho}_{+-} \frac{\sqrt{s}}{m_t\sin\theta_t}. \qquad (28)$$

The quantities A_i, B_i, C_i and D_i occurring in the above equations are functions of the masses, s, the degrees of e^- and e^+ polarization (P_{e^-} and P_{e^+}), and the coupling constants. They are listed in the appendix of [10].

In eq. (21), σ^+ and σ^- refer respectively to l^+ and l^- distributions, with the same notation for the kinematic variables of particles and antiparticles. Thus, θ_t is the polar angle of t (or $\bar{t}$), and E_l, θ_l, ϕ_l are the energy, polar angle and azimuthal angle of l^+

(or l^-). All the angles are now in the cm frame, with the z axis chosen along the e^- momentum, and the x axis chosen in the plane containing the e^- and t directions. θ_{tl} is the angle between the t and l^+ directions (or $\bar{t}$ and l^- directions). β is the t (or $\bar{t}$) velocity: $\beta = \sqrt{1 - 4m_t^2/s}$. Also, we use f^+ and f^- to denote f_{2R} and $\bar{f}_{2L}$, respectively. We note that the only effect of the anomalous decay vertex is to multiply the differential cross-section by an overall factor, which is dependent on the lepton energy and the decay angle of the lepton with respect to the top direction.

We can now proceed in either of the two ways. If we are not interested in energy dependence, we can integrate eq. (21) over E_l, and then over ϕ_l, and finally over $\cos\theta_t$, as done in ref. [10] to get the $\mathrm{d}\sigma/\mathrm{d}\cos\theta_l$. In the intermediate steps, we would get angular distributions which contain dependence on the top-quark polar angle. We follow this procedure in the next section. Alternatively, if we are interested in the double differential cross-section $\mathrm{d}^2\sigma/(\mathrm{d}E_l\,\mathrm{d}\cos\theta_l)$, we have to proceed somewhat differently. That procedure is outlined in §4.

3. Angular distributions

We now proceed to obtain an expression for angular distributions as outlined earlier. We start with eq. (21) and carry out the E_l integration. The limits of integration are

$$\frac{m_W^2}{\sqrt{s}}\frac{1}{1-\beta\cos\theta_{tl}} \le E_l \le \frac{m_t^2}{\sqrt{s}}\frac{1}{1-\beta\cos\theta_{tl}}. \tag{29}$$

The integration is simple to carry out, and we get the result

$$\begin{aligned}
\frac{\mathrm{d}\sigma^\pm}{\mathrm{d}\cos\theta_t\,\mathrm{d}\cos\theta_l\,\mathrm{d}\phi_l} &= \frac{3\alpha^4\beta}{16x_w^2\sqrt{s}}\frac{1}{\Gamma_t\Gamma_W m_W} \\
&\times \frac{m_t^4}{6s}(1-r)^2\left(1+2r-6\mathrm{Re}f^\pm\sqrt{r}\right)\frac{1}{(1-\beta\cos\theta_{tl})^3} \\
&\times \left\{A(1-\beta\cos\theta_{tl}) + B^\pm(\cos\theta_{tl}-\beta)\right. \\
&\quad + C^\pm(1-\beta^2)\sin\theta_t\sin\theta_l(\cos\theta_t\cos\phi_l - \sin\theta_t\cot\theta_l) \\
&\quad \left. + D^\pm(1-\beta^2)\sin\theta_t\sin\theta_l\sin\phi_l\right\}.
\end{aligned} \tag{30}$$

As this equation shows, the effect of anomalous tbW couplings $f^\pm$ on the distribution is through an overall constant factor $1+2r-6\mathrm{Re}f^\pm\sqrt{r}$. Any further integration will not affect this factor. In fact, this is precisely the factor which enters the t and $\bar{t}$ total widths to first order in $f^\pm$:

$$\Gamma_t = \frac{\alpha^2}{24x_w^2}\frac{m_t^3}{\Gamma_W m_W}(1-r)^2[(1+2r) - 6\mathrm{Re}f_{2R}\sqrt{r}], \tag{31}$$

with a similar expression for $\Gamma_{\bar{t}}$, with f_{2R} replaced by $\bar{f}_{2L}$. Using the expression for Γ_t from eq. (31) in (30), we get an expression which is identical to the one obtained with SM couplings for t and $\bar{t}$ decay.

Thus, at least in the linear approximation scheme for anomalous decay couplings which we are employing, anomalous couplings (CP conserving as well as CP violating) in the decay have no effect on the angular distributions. This holds for arbitrary longitudinal polarizations of e^+ and e^- beams and arbitrary anomalous $\gamma t\bar{t}$ and $Zt\bar{t}$ couplings, whether CP violating or CP conserving. It should be clarified that this result does not depend on a linear approximation in the anomalous $\gamma t\bar{t}$ and $Zt\bar{t}$ couplings, for then the result would be trivial. The result holds for arbitrary values of the quantities A, B, C, D parametrizing the production density matrix. Thus these could include higher orders of dipole couplings, or other anomalous couplings without affecting our result.

We can further do an integration over ϕ_l and $\cos\theta_t$ to obtain for the single differential cross-section the same expression as with SM couplings for the tbW vertex. This expression was given in [10], and has subsequently been found to agree with the result of Grzadkowski and Hioki by them [16]. However, in view of typographical errors in the expression in [10], we give the correct form here:

$$\begin{aligned}\frac{\mathrm{d}\sigma^\pm}{\mathrm{d}\cos\theta_l} = \frac{3\pi\alpha^2}{32s}\beta\Bigg\{ & 4A_0 \mp 2A_1\left(\frac{1-\beta^2}{\beta^2}\log\frac{1+\beta}{1-\beta} - \frac{2}{\beta}\right)\cos\theta_l \\ & +2A_2\left(\frac{1-\beta^2}{\beta^3}\log\frac{1+\beta}{1-\beta}(1-3\cos^2\theta_l)\right. \\ & \left.-\frac{2}{\beta^2}(1-3\cos^2\theta_l-\beta^2+2\beta^2\cos^2\theta_l)\right) \\ & \pm 2B_1\frac{1-\beta^2}{\beta^2}\left(\frac{1}{\beta}\log\frac{1+\beta}{1-\beta}-2\right)\cos\theta_l \\ & +B_2^\pm\frac{1-\beta^2}{\beta^3}\left(\frac{\beta^2-3}{\beta}\log\frac{1+\beta}{1-\beta}+6\right)(1-3\cos^2\theta_l) \\ & \pm 2C_0^\pm\frac{1-\beta^2}{\beta^2}\left(\frac{1-\beta^2}{\beta}\log\frac{1+\beta}{1-\beta}-2\right)\cos\theta_l \\ & -C_1^\pm\frac{1-\beta^2}{\beta^3}\left(\frac{3(1-\beta^2)}{\beta}\log\frac{1+\beta}{1-\beta}-2(3-2\beta^2)\right)(1-3\cos^2\theta_l)\Bigg\}. \quad (32)\end{aligned}$$

The quantities A_i, B_i and C_i in the above equation are coefficients of powers of $\cos\theta_t$, as defined below (D_i do not appear in the above equation, but we define them here for completeness):

$$A = A_0 + A_1\cos\theta_t + A_2\cos^2\theta_t, \quad (33)$$

$$B^\pm = B_0^\pm + B_1\cos\theta_t + B_2^\pm\cos^2\theta_t, \quad (34)$$

$$C^\pm = C_0^\pm + C_1^\pm\cos\theta_t, \quad (35)$$

$$D^\pm = D_0^\pm + D_1^\pm\cos\theta_t. \quad (36)$$

The values of the coefficients are given in the appendix of [10], in the presence of CP-violating electric and weak dipole moments. We only note here that contribution of CP-even magnetic and weak magnetic dipole moments may easily be included in those expressions using the helicity amplitudes given, e.g., by Ladinsky and Yuan in [2]. However, we will not make use of these in this paper.

4. Energy and angle double differential distributions

In order to obtain a distribution only in terms of lepton variables, we need to integrate the expression in (21) for the differential cross-section over $\cos\theta_t$. However, if we are interested only in the lepton energy and polar angle distributions, and not in the azimuthal angle of the the lepton, it is more convenient to proceed as follows. We first make a change of variables from $\cos\theta_t$ and ϕ_l to $\cos\theta_{tl}$ and α, where α is defined by

$$\cos\theta_t = \cos\theta_{tl}\cos\theta_l + \sin\theta_{tl}\sin\theta_l\cos\alpha. \tag{37}$$

We then have

$$\mathrm{d}\cos\theta_t\,\mathrm{d}\phi_l = \mathrm{d}\cos\theta_{tl}\,\mathrm{d}\alpha, \tag{38}$$

and the relations for the sine of the angle θ_t,

$$\sin\theta_t\sin\phi_l = \sin\theta_{tl}\sin\alpha,\ \sin\theta_t\cos\phi_l = \cos\theta_{tl}\sin\theta_l - \sin\theta_{tl}\cos\theta_l\cos\alpha. \tag{39}$$

We can now integrate (21) over α over the range 0 to 2π to get

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{\pm}}{\mathrm{d}E_l\,\mathrm{d}c_l\,\mathrm{d}c_{tl}} &= \frac{3\pi\alpha^4\beta}{32\sqrt{s}\sin\theta_W^4}\frac{E_l}{\Gamma_t\Gamma_W m_W}\\
&\times\left\{\left[\left(A_0-\frac{B_0^{\pm}}{\beta}\right)\pm\left(A_1-\frac{B_1}{\beta}\right)c_l c_{tl}+\left(A_2-\frac{B_2^{\pm}}{\beta}\right)(c_l^2c_{tl}^2+\frac{1}{2}s_l^2s_{tl}^2)\right]\right.\\
&+\left[B_0^{\pm}\pm B_1 c_l c_{tl}+B_2^{\pm}(c_l^2c_{tl}^2+\frac{1}{2}s_l^2s_{tl}^2)\right]\frac{(1-\beta^2)}{\beta}\frac{1}{(1-\beta c_{tl})}\\
&+\left.\left[\mp C_0^{\pm}c_l s_{tl}^2+\frac{1}{2}C_1^{\pm}(1-3c_l^2)c_{tl}s_{tl}^2\right]\right\}\\
&\times\left[1-\frac{E_l\sqrt{s}}{m_t^2}(1-\beta c_{tl})\right]\left[1+\frac{2\mathrm{Re}f^{\pm}}{\sqrt{r}}\frac{m_W^2}{E_l\sqrt{s}(1-\beta c_{tl})}\right].
\end{aligned} \tag{40}$$

In the above equation we have used the notation

$$c_l\equiv\cos\theta_l,\ s_l\equiv\sin\theta_l;\quad c_{tl}\equiv\cos\theta_{tl},\ s_{tl}\equiv\sin\theta_{tl}. \tag{41}$$

Note that in the above equation, as well as in the following equations, Γ_t is given by eq. (31) above, which includes the correction from the anomalous decay vertex.

The c_{tl} integrals can be carried out analytically, the limits of integration depending on the lepton energy. These may be written as

$$L < c_{tl} < U, \tag{42}$$

where L and U take different values in different energy ranges. They are given by

$$L = \max\left(-1,\frac{1}{\beta}\left(1-\frac{m_t^2}{E_l\sqrt{s}}\right)\right),\ U = \min\left(1,\frac{1}{\beta}\left(1-\frac{m_W^2}{E_l\sqrt{s}}\right)\right). \tag{43}$$

For a given value of $\sqrt{s}$, this gives three possible ranges of energy, with distinct sets of limits on c_{tl}. These ranges of energy are described in detail in [3].

The result of integration over c_{tl} from L to U is given by

$$\begin{aligned}
\frac{\mathrm{d}\sigma^\pm}{\mathrm{d}E_l\,\mathrm{d}c_l} &= \frac{3\pi\alpha^4\beta}{32\sqrt{s}\sin\theta_W^4}\frac{E_l}{\Gamma_t\Gamma_W m_W} \\
&\times \Bigg[\left(A_0 - \frac{B_0^\pm}{\beta}\right)X_0 \pm c_l\left(A_1 - \frac{B_1}{\beta}\right)X_1 \\
&\quad + \frac{1}{2}(3c_l^2-1)\left(A_2 - \frac{B_2^\pm}{\beta}\right)X_2 + \frac{1}{2}s_l^2\left(A_2 - \frac{B_2^\pm}{\beta}\right)X_2' \\
&\quad + \frac{1-\beta^2}{\beta}\left[\left(B_0^\pm + \frac{1}{2}s_l^2 B_2^\pm\right)Y_0 \pm B_1 c_l Y_1 + \frac{1}{2}B_2^\pm(3c_l^2-1)Y_2\right] \\
&\quad \mp (1-\beta^2)C_0^\pm c_l Z_0 - \frac{1}{2}(1-\beta^2)C_1^\pm(3c_l^2-1)Z_1\Bigg]. \qquad (44)
\end{aligned}$$

Here we have used

$$X_0 = P(U-L) + \frac{1}{2}Q(U^2-L^2) + \frac{R}{\beta}\mathcal{L}, \qquad (45)$$

$$X_1 = \frac{1}{2}P(U^2-L^2) + \frac{1}{3}Q(U^3-L^3) - \frac{R}{\beta}F, \qquad (46)$$

$$X_2 = \frac{1}{3}P(U^3-L^3) + \frac{1}{4}Q(U^4-L^4) - \frac{R}{\beta^2}\left(F + \frac{1}{2}\beta(U^2-L^2)\right), \qquad (47)$$

$$X_2' = P(U-L) + \frac{1}{2}Q(U^2-L^2) + \frac{R}{\beta}\mathcal{L}, \qquad (48)$$

$$Y_0 = \frac{1}{\beta}P'\mathcal{L} - \frac{1}{\beta}Q(U-L) + R\frac{U-L}{(1-L\beta)(1-U\beta)}, \qquad (49)$$

$$Y_1 = -\frac{1}{\beta}P'F - \frac{1}{2}\frac{Q}{\beta}(U^2-L^2) + \frac{R}{\beta}\left(\frac{U-L}{(1-L\beta)(1-U\beta)} - \frac{1}{\beta}\mathcal{L}\right), \qquad (50)$$

$$\begin{aligned}
Y_2 = &-\frac{1}{\beta^2}P'\left(F + \frac{1}{2}\beta(U^2-L^2)\right) - \frac{Q}{3\beta}(U^3-L^3) \\
&+\frac{R}{\beta^2}\left(U - L - \frac{2}{\beta}\mathcal{L} + \frac{U-L}{(1-L\beta)(1-U\beta)}\right), \qquad (51)
\end{aligned}$$

$$Z_0 = \frac{1}{\beta^2} P' \left(U - L + \frac{1}{2}\beta(U^2 - L^2) - \frac{1-\beta^2}{\beta}\mathcal{L} \right) + \frac{Q}{\beta}\left(-(U-L) + \frac{1}{3}(U^3 - L^3) \right) + \frac{R}{\beta^2}\left(-2F + (U-L) - \frac{(1-\beta^2)(U-L)}{(1-L\beta)(1-U\beta)} \right), \tag{52}$$

$$Z_1 = \frac{1}{\beta^3} P' \left((1-\beta^2)F + \frac{1}{2}\beta(U^2 - L^2) + \frac{\beta^2}{3}(U^3 - L^3) \right) - \frac{Q}{2\beta}\left((U^2 - L^2) - \frac{1}{2}(U^4 - L^4) \right) + \frac{R}{\beta^3}\left(-2F - \frac{\beta}{2}(U^2 - L^2) + \frac{1-\beta^2}{\beta}\mathcal{L} - \frac{(1-\beta^2)(U-L)}{(1-L\beta)(1-U\beta)} \right). \tag{53}$$

Also,

$$P = \left(1 - \frac{E_l\sqrt{s}}{m_t^2} \right) - 2f^{\pm}\sqrt{r}, \tag{54}$$

$$Q = \frac{E_l\sqrt{s}}{m_t^2}\beta, \tag{55}$$

$$R = \frac{m_W^2}{E_l\sqrt{s}} \frac{2f^{\pm}}{\sqrt{r}}, \tag{56}$$

$$P' = P + Q/\beta = 1 - 2f^{\pm}\sqrt{r}, \tag{57}$$

$$F = U - L - \frac{1}{\beta}\log\frac{1-L\beta}{1-U\beta}, \tag{58}$$

$$\mathcal{L} = \log\frac{1-L\beta}{1-U\beta}. \tag{59}$$

We can use the above expression for the double differential cross-section to obtain CP-violating asymmetries. For example, the difference in the differential cross-sections for l^+ and l^- for a given value of E_l but for values of $\cos\theta_l$ differing in sign is a CP-odd quantity. The double differential cross-sections can easily be integrated to obtain analytic expressions for certain angular asymmetries as a function of the lepton energy. We had earlier proposed two asymmetries in the case where the lepton energy was completely integrated over [10,11], viz. the charge asymmetry (with a cut-off θ_0 in forward and backward angles), and the sum of the forward-backward asymmetries (again with an angular cut-off) for l^+ and for l^-.

We redefine these for the case when energy is not integrated over:

$$\mathcal{A}_{\rm ch}(E_l,\theta_0) = \frac{\displaystyle\int_{\theta_0}^{\pi-\theta_0} \mathrm{d}\theta_l \left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}E_l\,\mathrm{d}\theta_l} - \frac{\mathrm{d}\sigma^-}{\mathrm{d}E_l\,\mathrm{d}\theta_l} \right)}{\displaystyle\int_{\theta_0}^{\pi-\theta_0} \mathrm{d}\theta_l \left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}E_l\,\mathrm{d}\theta_l} + \frac{\mathrm{d}\sigma^-}{\mathrm{d}E_l\,\mathrm{d}\theta_l} \right)}, \tag{60}$$

$$\mathcal{A}_{\rm FB}(E_l,\theta_0) = \frac{\displaystyle\int_{\theta_0}^{\frac{\pi}{2}} \mathrm{d}\theta_l \left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}E_l\,\mathrm{d}\theta_l} + \frac{\mathrm{d}\sigma^-}{\mathrm{d}E_l\,\mathrm{d}\theta_l} \right) - \int_{\frac{\pi}{2}}^{\pi-\theta_0} \mathrm{d}\theta_l \left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}E_l\,\mathrm{d}\theta_l} + \frac{\mathrm{d}\sigma^-}{\mathrm{d}E_l\,\mathrm{d}\theta_l} \right)}{\displaystyle\int_{\theta_0}^{\pi-\theta_0} \mathrm{d}\theta_l \left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}E_l\,\mathrm{d}\theta_l} + \frac{\mathrm{d}\sigma^-}{\mathrm{d}E_l\,\mathrm{d}\theta_l} \right)}. \tag{61}$$

The first of these, for the case when there is no angular cut-off, and with only CP-violation in top production included, was discussed in [5,19] for unpolarized beams, and in [9,20] for longitudinally polarized beams.

We will calculate these asymmetries as functions of the lepton energy.

We do not display the expressions for these asymmetries. The corresponding angular integration is trivial, and the expressions lengthy.

In the next section we apply the above expressions to obtain numerical values for distributions and asymmetries.

5. Results and discussion

We will use expressions obtained in the previous sections to look at CP-violating asymmetries arising from top edm, wdm, and from tbW vertex. The asymmetries would get a contribution simultaneously from all these three sources. However, we have treated these sources one at a time, with the understanding that contributions from these would get added linearly in the asymmetries. In a given model, these contributions would occur in a fixed linear combination. However, in a model-independent approach, methods have to be devised to obtain the parameters simultaneously, independent of one another.

In our calculations in this section, we shall assume an LC operating at a cm energy of 500 GeV. We will assume a top-quark mass of 174 GeV and $x_w = 0.23$. We assume that while one of t and $\bar{t}$ decays into a leptonic channel containing either e or μ, the other decays hadronically into b and two jets. This means that all the earlier expressions for differential cross-sections have to be multiplied by the product of the hadronic branching ratio 2/3 and the leptonic branching ratio 2/9.

We have assumed that electron and positron beams are unpolarized, or fully longitudinally polarized, with the electron and positron beam polarizations equal and opposite. This may clearly not be possible in practice, since polarization of positron beams is more difficult. However, this strong assumption is not necessary, since what appears in the expressions for asymmetries is only the effective polarization

$$P = (P_{e^-} - P_{e^+})/(1 - P_{e^-}P_{e^+}). \tag{62}$$

Thus, for example, it is sufficient if $P_{e^-} = -1$ and $P_{e^+} = 0$ for P to be -1 [21].

The expressions for absolute (differential) cross-sections would depend on P as well as on $1 - P_{e^-}P_{e^+}$. In what follows, we will label polarizations by P.

In figures 1, 2 and 3 we give three dimensional plots of the double differential cross-section $\mathrm{d}\sigma/(\mathrm{d}E_l \mathrm{d}c_l)$ as a function of E_l and c_l in SM, for the beam polarizations of $P_{e^-} = P_{e^+} = 0$, $P_{e^-} = -P_{e^+} = -1$ and $P_{e^-} = -P_{e^+} = +1$, respectively. The plots shown are for l^+. The corresponding ones for l^- are obtained simply by reflecting about $\theta_l = \pi/2$, since CP is conserved in SM. In figures 4, 5 and 6 we plot, respectively for $P = 0$, $P = -1$ and $P = +1$, the ratio of the CP-violating contribution to the differential cross-section from only CP-violation in the decay to the SM value of the differential cross-section

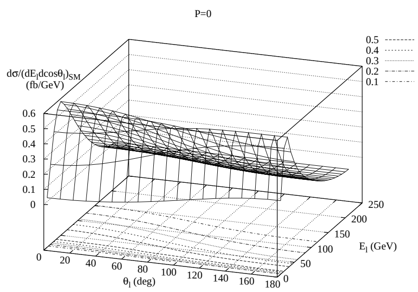

Figure 1. The standard model double differential cross-section as a function of energy E_l and polar angle θ_l of the charged lepton l^+ (either μ^+ or e^+) for unpolarized beams. The bottom plane shows a few contours of constant cross-section.

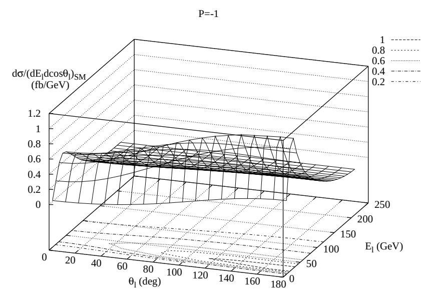

Figure 2. The standard model double differential cross-section as in figure 1, but for beam polarizations $P_{e^-} = -P_{e^+} = -1$. The bottom plane shows a few contours of constant cross-section.

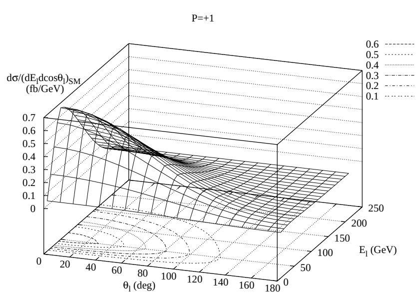

Figure 3. The standard model double differential cross-section as in figure 1, but for effective beam polarization $P_{e^-} = -P_{e^+} = +1$. The bottom plane shows a few contours of constant cross-section.

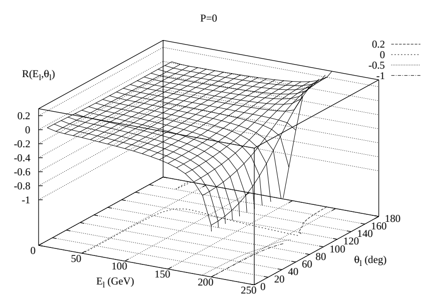

Figure 4. The ratio $R(E_l, \theta_l)$ of the CP-violating part of the cross-section to the SM cross-section defined in the text for $\mathrm{Re} f_{2R} = -\mathrm{Re}\bar{f}_{2L} = 10^{-2}$ with unpolarized beams. A few contours of constant R values are shown on the bottom plane.

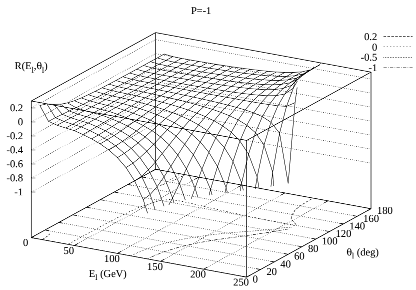

Figure 5. The ratio $R(E_l, \theta_l)$ of the CP-violating part of the cross-section to the SM cross-section defined in the text for $\mathrm{Re}f_{2R} = -\mathrm{Re}\bar{f}_{2L} = 10^{-2}$ with effective beam polarization $P = -1$. A few contours of constant values of R are shown on the bottom plane.

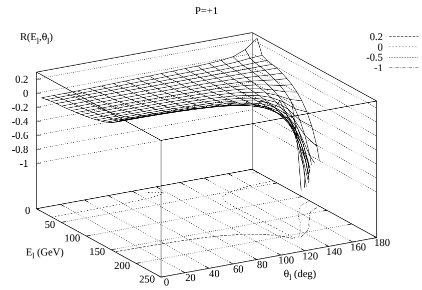

Figure 6. The ratio $R(E_l, \theta_l)$ of the CP-violating part of the cross-section to the SM cross-section defined in the text for $\mathrm{Re}f_{2R} = -\mathrm{Re}\bar{f}_{2L} = 10^{-2}$ with effective beam polarization $P = +1$. A few contours of constant values of R are shown on the bottom plane.

$$R(E_l, \theta_l) = \frac{\dfrac{\mathrm{d}\sigma^+}{\mathrm{d}E_l \mathrm{d}c_l}(E_l, \theta_l) - \dfrac{\mathrm{d}\sigma^-}{\mathrm{d}E_l \mathrm{d}c_l}(E_l, \pi - \theta_l)}{2\dfrac{\mathrm{d}\sigma_{\mathrm{SM}}}{\mathrm{d}E_l \mathrm{d}c_l}(E_l, \theta_l)} \tag{63}$$

for $\mathrm{Re}f_{2R} = -\mathrm{Re}\bar{f}_{2L} = 10^{-2}$. In all the three-dimensional plots, figures 1–6, some selected contours are shown on the bottom plane.

As noted earlier, in practice there has to be a cut-off in θ_l in the forward and backward directions. We shall mainly use a cut-off $\theta_0 = 30°$, though we have examined the variation of our asymmetries with cut-off angle in what follows. To get an idea of the effect of cut-off on event rates, we have plotted the energy dependence of the SM differential cross-section for cut-off values of $\theta_0 = 30°$ and $\theta_0 = 60°$ in the forward and backward directions in figure 7, for the polarized and unpolarized cases.

In figure 8 we plot the lepton charge asymmetry A_{ch} for two values of cut-off angles, $\theta_0 = 30°$ and $\theta_0 = 60°$, for the case of unpolarized e^+ and e^- beams. The dependence of the asymmetry on lepton energy is shown assuming one source of CP-violation at a time – the top electric dipole moment (edm) $d_t^\gamma = (0.1)e/m_t \approx 10^{-17}\, e$ cm, a top weak dipole moment (wdm) of $d_t^Z = e/m_t$, and top decay CP-violation (CPV) corresponding to $\mathrm{Re}f_{2R} = -\mathrm{Re}\bar{f}_{2L} = 0.1$. For the last case, there is no significant change in going from $\theta_0 = 30°$ to $\theta_0 = 60°$. It is interesting to note that all asymmetries change sign for a lepton energy of 50–60 GeV. As is evident, a value of 0.1 for $\mathrm{Re}f_{2R} = -\mathrm{Re}\bar{f}_{2L}$ can produce the same order of asymmetry as top edm of $10^{-17}\, e$ cm, or top wdm of $10^{-16}\, e$ cm.

It should be noted that, as seen from figure 7, the SM lepton energy distribution peaks at an energy of about 40–50 GeV, and falls for low as well as high values of energy. Since it is the SM differential cross-section which occurs in the denominator of the charge asymmetry (and the forward–backward asymmetry discussed below), large values of the asymmetry for low and high values of the lepton energy must be treated with caution. They can be of order 1 simply because the SM cross-section in the denominator is small for those values of energy.

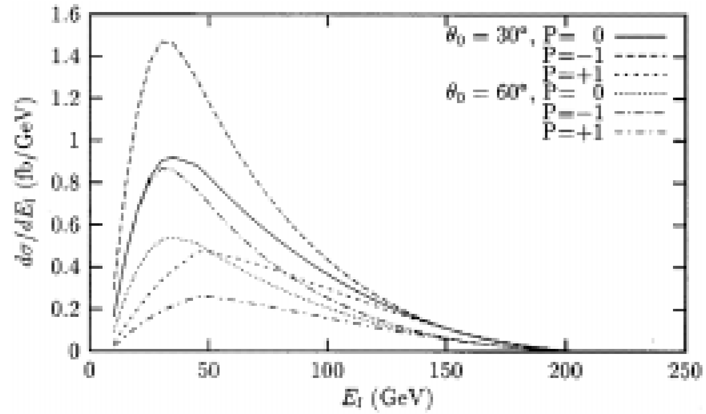

Figure 7. SM differential cross-section integrated over θ_l with a cut-off θ_0 in the forward and backward directions. The curves are shown for $\theta_0 = 30°$ and $\theta_0 = 60°$ and for effective beam polarizations $P = 0, -1, +1$.

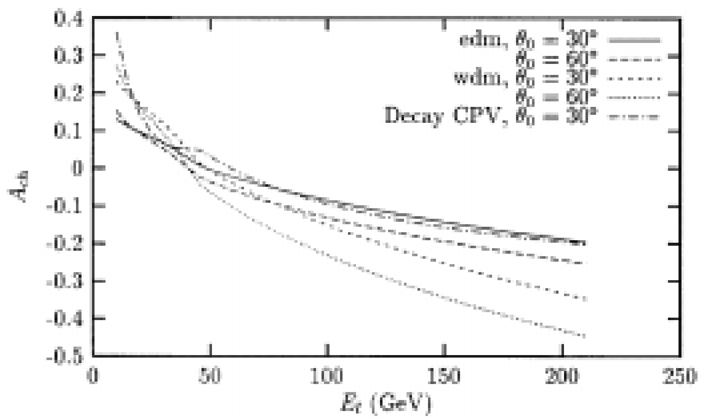

Figure 8. The charge asymmetry $A_{\rm ch}$ as function of E_l with two cut-off angles, for the different sources of CP-violation, and unpolarized beams. The top edm is taken to be $(0.1)e/m_t$, the wdm to be e/m_t, and $\mathrm{Re}f_{2R} = -\mathrm{Re}\bar{f}_{2L} = 0.1$.

Figures 9, 10 and 11 show the charge asymmetry $A_{\rm ch}$ for a cut-off of $\theta_0 = 30°$ for the cases of nonzero edm, wdm, and decay CPV, respectively, each for effective beam polarization of P of 0, -1 and $+1$. It is clear that while charge asymmetries arising from both edm and wdm are sensitive to polarization for low and high values of lepton energy, $A_{\rm ch}$ due to decay CPV is not very sensitive to P. $A_{\rm ch}$ due to wdm even has opposite signs for $P = +1$ and -1. This shows that the wdm contribution to $A_{\rm ch}$ may be easily separated by using data for positive and negative values of P. On the other hand, $A_{\rm ch}$ due to decay CPV may be separated by concentrating on $E_l \approx 50$ GeV, where the other two asymmetries are close to zero and change sign.

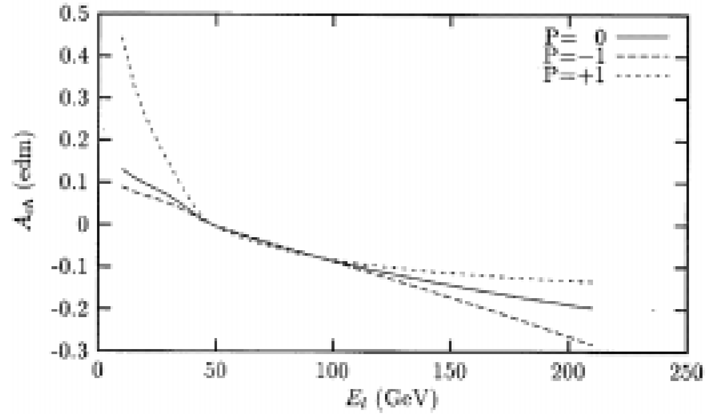

Figure 9. The charge asymmetry $A_{\rm ch}$ arising from the top edm as a function of E_l for $\theta_0 = 30^\circ$ for three values of the effective e^+e^- polarization P. The top edm assumed is $(0.1)e/m_t$.

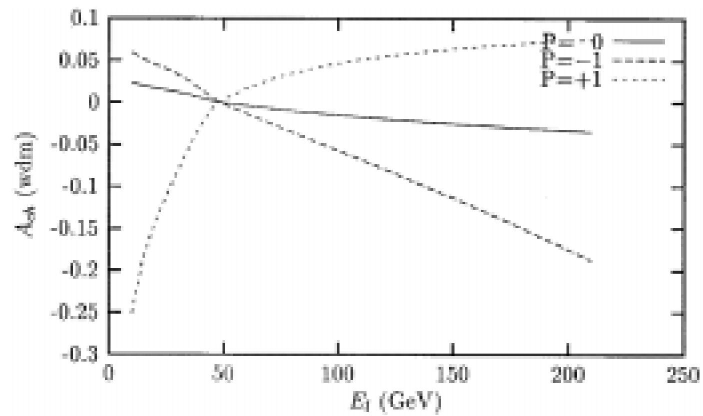

Figure 10. The charge asymmetry $A_{\rm ch}$ arising from the top weak dipole moment as a function of E_l for $\theta_0 = 30^\circ$ for three values of the effective e^+e^- polarization P. The top wdm assumed is $(0.1)e/m_t$.

Figure 12, which is the analogue of figure 8, shows the forward–backward asymmetry (symmetrized over lepton charges) $A_{\rm FB}$. It is seen from the figure that the edm contribution dominates over the wdm contribution for equal values of d_t^γ and d_t^Z, and in fact, it is very nearly 10 times the wdm contribution, by accident. Unlike $A_{\rm ch}$, there is no change of sign. $A_{\rm FB}$ due to decay CPV, on the other hand, changes sign twice. Now the asymmetries are larger for a cut-off $\theta = 30^\circ$ rather than for $\theta = 60^\circ$, unlike in the case of $A_{\rm ch}$.

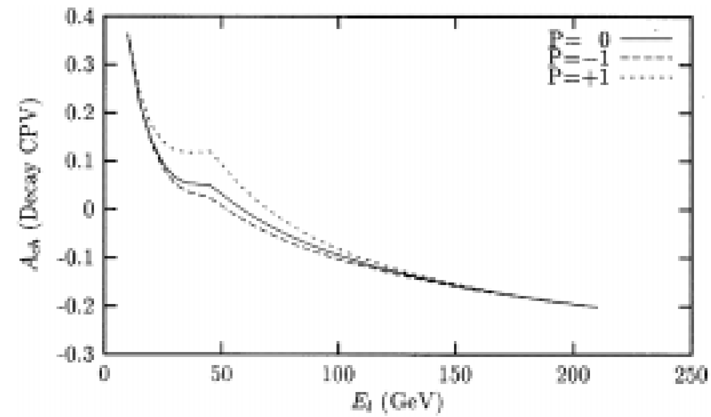

Figure 11. The charge asymmetry $A_{\rm ch}$ arising from the CP-violating tbW vertex as a function of E_l for $\theta_0 = 30^\circ$ for three values of the effective e^+e^- polarization P. ${\rm Re} f_{2R} = -{\rm Re}\bar{f}_{2L} = 0.1$ is assumed.

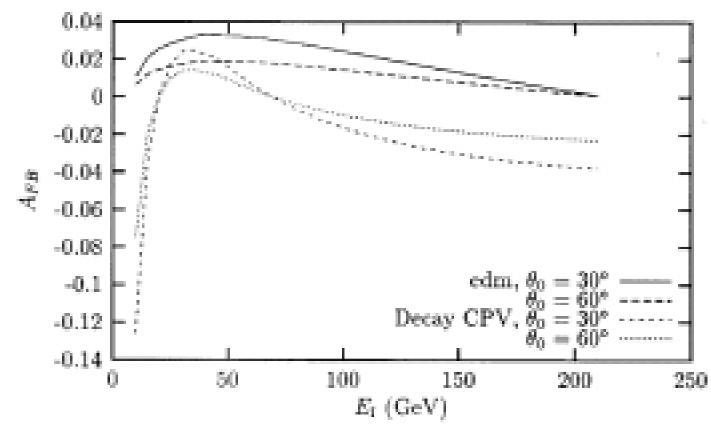

Figure 12. The asymmetry $A_{\rm FB}$ as function of E_l with two cut-off angles, for the different sources of CP-violation, for unpolarized beams. The top edm is taken as $(0.1)e/m_t$, and ${\rm Re} f_{2R} = -{\rm Re}\bar{f}_{2L} = 0.1$. The curve for a top wdm of e/m_t approximately coincides with the curve for edm, and is not shown separately.

Looking at the polarization dependences of $A_{\rm FB}$ in figures 13, 14 and 15 it is clear that in all cases there is strong sensitivity to P. Again $A_{\rm FB}$ due to wdm changes sign with P, whereas that due to edm does not. However, $A_{\rm FB}$ due to decay CPV also changes sign with P for low as well as high values of E_l.

It is useful to get an idea of the sensitivity of a linear collider with an integrated luminosity of 100 fb^{-1} to the CP-violating parameters. The number of events expected in an energy bin of 10 GeV centred around $E_l \approx 40$ GeV in the unpolarized case would be $N \approx 900$, corresponding to $\sqrt{N} = 30$. For a 2σ effect, the asymmetry then

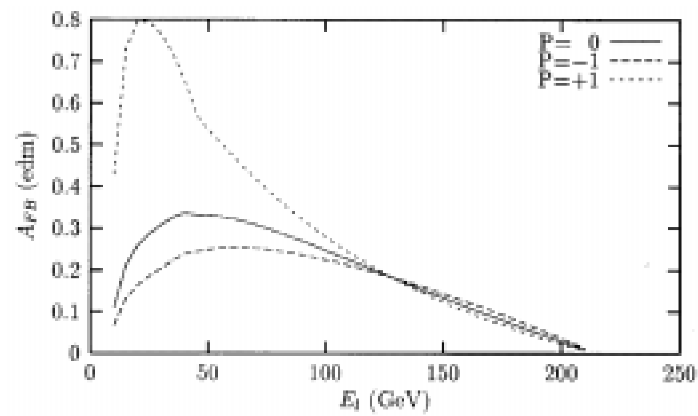

Figure 13. The asymmetry $A_{\rm FB}$ arising from the top electric dipole moment as a function of E_l for $\theta_0 = 30°$ for three values of the effective e^+e^- polarization P. The top edm assumed is e/m_t.

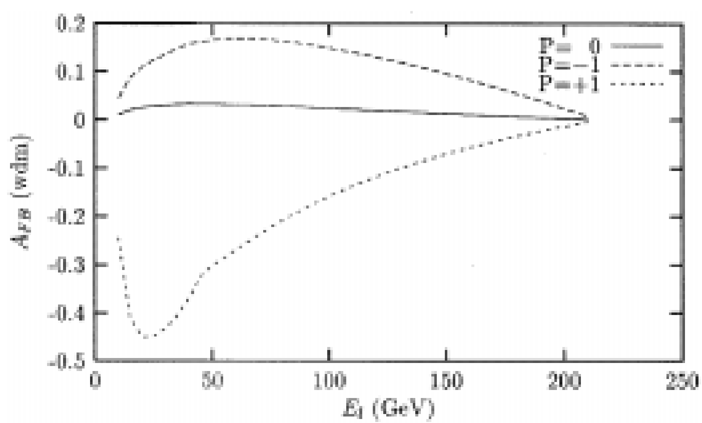

Figure 14. The asymmetry $A_{\rm FB}$ arising from the top weak dipole moment as a function of E_l for $\theta_0 = 30°$ for three values of the effective e^+e^- polarization P. The top edm assumed is e/m_t.

should be at least 0.06. A charge asymmetry of this magnitude would correspond to edm of about 10^{-18} e cm, or wdm of about 10^{-18} e cm, or $f^{\pm} \approx 0.06$. Combining data from several bins and doing a likelihood analysis could easily improve this sensitivity to $f^{\pm} \approx 10^{-3}$. Since the expectation from popular models like the minimal supersymmetric standard model for $f^{\pm}$ is in this range, it would be interesting to look for the asymmetries we have discussed. A more detailed study of the statistical significance and possible backgrounds would be worthwhile.

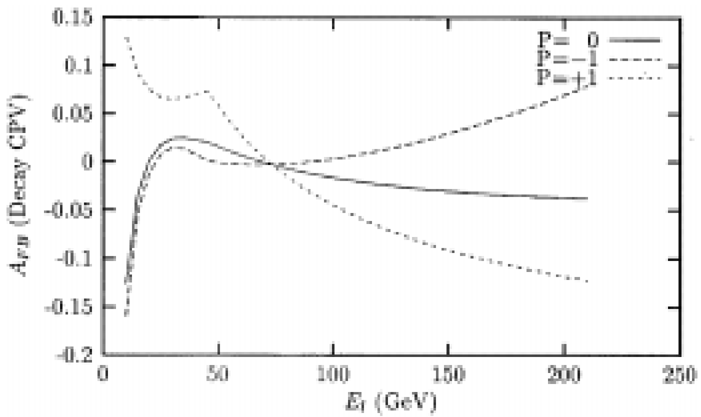

Figure 15. The asymmetry $A_{\rm FB}$ arising from the CP-violating tbW vertex as a function of E_l for $\theta_0 = 30^\circ$ for three values of the effective e^+e^- polarization P. Re$f_{2R} = -{\rm Re}\bar{f}_{2L} = 0.1$ is assumed.

To summarize, after obtaining analytic expressions for angular distributions and energy-angle double distributions including anomalous effects in production as well as decay, we have studied the CP-violating asymmetries $A_{\rm ch}$ and $A_{\rm FB}$ as functions of decay-lepton energy and the initial beam polarization. Since anomalous effects in the decay do not appear in the angular distributions where energy has been integrated over, these are not useful for the study of CP-violation in decay. On the other hand, these are most useful for the study of CP-violation in production, as discussed in [11].

To be able to get a handle on CP-violation in the tbW vertex, we have compared the E_l and polarization dependence of $A_{\rm ch}$ and $A_{\rm FB}$ arising from the top-quark edm, wdm, and decay CPV separately. We find interesting features like zeros and sign changes in the asymmetries as a function of energy, which are different for the different sources of CP-violation. In general there is strong dependence on effective beam polarization P, which not only enhances the asymmetry in most cases, but might also help in discrimination amongst the various sources of CP-violation. We have not studied in detail the procedure for discriminating, but have indicated significant features which might be used. A detailed study of various sensitivities would be useful.

We end with a few comments.

The CP-violating asymmetries we have considered are simple in principle as also straightfoward to implement from the experimental point of view, as they do not require the determination of the top-quark direction or momentum. It is possible to use correlations of optimal observables, which would maximize the statistical sensitivity [7], at the expense of simplicity.

Note that the asymmetries we have chosen are odd under CP, but even under ‘naive’ time reversal T_N, i.e., sign reversal of spins and momenta, without an interchange of initial and final states. The CPT theorem therefore implies that they should necessarily come from the absorptive part of the amplitude [23]. As noted earlier, only the absorptive part of the tbW vertex contributes to the differential cross-sections. It follows that had a CP-odd asymmetry which was odd under $T_{\rm N}$ been chosen, it could not have depended on CP-violation in decay.

In the above, we have taken into account the practical requirement of imposing a cut on the angle of the detected lepton with respect to the beam axis. A similar practical constraint might be necessary for the lepton energy. The detection of a charged lepton will require it to have a minimum energy. However, for a cm energy of 500 GeV, the minimum lepton energy allowed kinematically is about 7.5 GeV. Thus, if the cut needed for detection is not required to be below about 7.5 GeV, our results can be used as such. If, however, a cut is to be larger, this cut may itself introduce a dependence on $f^{\pm}$ of the polar angle distribution, which was found to be absent when the full range of energy is integrated over, as in eq. (32). This would need further study. However, it can be seen that this question can be easily handled by doing an analytic integration over the appropriate energy region using our expressions.

After the completion of this work, the paper of Boos *et al* [24] was listed in the Los Alamos archive, which discusses asymmetries due to anomalous tbW vertex. They do not discuss explicitly CP-violating asymmetries which have been described in this paper.

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