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QCD corrections to decay-lepton polar and azimuthal angular distributions in $e^+e^- \to t\,\bar{t}$ in the soft-gluon approximation

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Abstract. QCD corrections to order α_s in the soft-gluon approximation to angular distributions of decay charged leptons in the process $e^+e^- \to t\bar{t}$, followed by semileptonic decay of t or $\bar{t}$, are obtained in the e^+e^- centre-of-mass frame. As compared to distributions in the top rest frame, these have the advantage that they would allow direct comparison with experiment without the need to reconstruct the top rest frame. The results also do not depend on the choice of a spin quantization axis for t or $\bar{t}$. Analytic expression for the triple distribution in the polar angle of t and polar and azimuthal angles of the lepton is obtained. Analytic expression is also derived for the distribution in the charged-lepton polar angle. Numerical values are discussed for $\sqrt{s} = 400$, 800 and 1500 GeV.

Keywords. Top quark; electron–positron annihilation; QCD corrections.

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1. Introduction

The discovery [1] of a heavy top quark, with a mass of about 174 GeV which is close to the electroweak symmetry breaking scale, raises the interesting possibility that the study of its properties will provide hints to the mechanism of symmetry breaking. While most of the gross properties of the top quark will be investigated at the Tevatron and LHC, more accurate determination of its couplings will have to await the construction of a linear e^+e^- collider. The prospects of the construction of such a linear collider, which will provide detailed information also on the $W^\pm$, Z and Higgs, are currently under intense discussion, and it is very important at the present time to focus on the details of the physics issues ([2] and references therein).

In this context, top polarization is of great interest. There has been a lot of work on production of polarized top quarks in the standard model (SM) in hadron [3] collisions, and e^+e^- collisions in the continuum [4], as well as at the threshold [5]. A comparison of the theoretical predictions for single-top polarization as well as $t\bar{t}$ spin correlations with experiment can provide a verification of SM couplings and QCD corrections, or give clues to possible new physics beyond SM in the couplings of the top quark [6–18] (see [19] for a review of CP violation in top physics).

Undoubtedly, the study of the top polarization is possible because of its large mass, which ensures that the top decays fast enough for spin information not to be lost due to hadronization [20]. Thus, kinematic distributions of top decay products can be analyzed to obtain polarization information. However, most studies on top polarization, particularly on QCD corrections, have presumed an accurate reconstruction of the spin, and are generally not concerned with decay distributions. Some works do consider decay distributions, but in the top rest frame [18,21,22]. In these cases, the onus of accurate determination of the spin quantization axis and that of the top energy-momentum and hence of the top rest frame, is largely left on the experimentalists. The remaining papers which discuss decay distributions in the laboratory frame are restricted to energy distributions of decay products [13], and do not calculate angular distributions.

The choice of spin quantization axis has also been an issue of discussion [21,23], and it has been remarked that a certain 'off-diagonal' spin basis has certain advantages.

In an alternative approach, adopted usually in the context of the study of top polarization arising from new physics, predictions are made directly for decay-lepton [7–12,14–16] or bottom-quark [15,17] distributions in the laboratory frame. Such an approach makes the issue of the choice of spin basis for the top quark superfluous. Moreover, if the study is restricted to energy and polar angle distributions of top decay products, it even obviates the need for accurate determination of the energy or momentum direction of the top quark [10]. Even in case azimuthal distributions are studied, where an additional direction is needed to define the azimuthal angle, the knowledge of the magnitude of the top momentum is not required. It is sufficient if the direction is determined with reasonable accuracy.

In this paper we shall be concerned with the laboratory-frame angular distribution of secondary leptons arising from the decay of the top quarks in $e^+e^- \to t\bar{t}$ in the context of QCD corrections to order α_s. QCD corrections to top polarization in $e^+e^- \to t\bar{t}$ have been calculated earlier by many groups [22,24–30]. In cases where the corrections have been applied to the process including top decay, the discussion of angular distributions is restricted to the top rest frame [22]. QCD corrections to the lepton energy distributions have been treated in the top rest frame in [18] and in the lab. frame in [13]. This paper provides, for the first time, angular distribution in the e^+e^- centre-of-mass (c.m.) frame. As a first approach, this work is restricted, for simplicity, to the soft-gluon approximation (SGA). SGA has been found to give a satisfactory description of top polarization in single-top production [29], and it is hoped that it will suffice to give a reasonable quantitative description.

The study of the laboratory (lab.) frame angular distribution of secondary leptons, besides admitting direct experimental observation, has another advantage. It has been found [15,16] that the angular distribution is not altered, to first-order approximation, by modifications of the tbW decay vertex, provided the b-quark mass is neglected. Thus, our result would hold to a high degree of accuracy even when $\mathcal{O}(\alpha_s)$ soft-gluon QCD corrections to top decay are included, since these can be represented by the same form factors [31] considered in [15,16]. We do not, therefore, need to calculate these explicitly. It is sufficient to include $\mathcal{O}(\alpha_s)$ corrections to the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices. This, of course, assumes that QCD corrections of the nonfactorizable type [32], where a virtual gluon is exchanged gluon between t ($\bar{t}$) and $\bar{b}$ (b) from $\bar{t}$ (t) decay, can be neglected. We have assumed that these are negligible.

The procedure adopted here is as follows: We make use of effective $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices derived in earlier works in the soft-gluon approximation, using an appropriate cut-off on the soft-gluon energy. In principle, these effective vertices are obtained by suitably cancelling the infra-red divergences in the virtual-gluon contribution to the differential cross section for $e^+e^- \to t\bar{t}$ against the real soft-gluon contribution to the differential cross section for $e^+e^- \to t\bar{t}g$. For practical purposes, restricting to SGA, it is sufficient to modify the tree-level $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices suitably to produce the desired result. Thus, assuming $\mathscr{O}(\alpha_s)$ effective SGA vertices, we have obtained helicity amplitudes for $e^+e^- \to t\bar{t}$, and hence spin-density matrices for production. This implies an assumption that these effective vertices provide, in SGA, a correct approximate description of the off-diagonal density matrix elements as well as the diagonal ones entering the differential cross sections. Justification for this would need explicit calculation of hard-gluon effects, and is beyond the scope of this work.

We have considered three possibilities, corresponding to the electron beam being unpolarized ($P=0$), fully left-handed polarized ($P=-1$), and fully right-handed polarized ($P=+1$). Since we give explicit analytical expressions, suitable modification to more realistic polarizations would be straightforward.

The results for the case of polar distributions of the decay lepton, restricted to $\sqrt{s}$ values of 400 and 800 GeV, were reported earlier in a short paper [33]. We present here more details, as well as azimuthal distributions. We also consider the possibility of a higher energy option of the linear collider, with $\sqrt{s}=1500$ GeV.

Our main result may be summarized as follows: By and large the distribution in the polar angle θ_l of the secondary lepton w.r.t. the e^- beam direction is unchanged in shape on inclusion of QCD corrections in SGA. The θ_l distribution for $\sqrt{s}=400$ GeV is very accurately described by overall multiplication by a K factor ($K \equiv 1+\kappa > 1$), except for extreme values of θ_l, and that too for the case of $P=+1$. For the other energies considered, κ continues to be a slowly varying function of θ_l. This has the important consequence that earlier results on the sensitivity of lepton angular distributions or asymmetries to anomalous top couplings, obtained for $\sqrt{s}$ values around 400 GeV without QCD corrections being taken into account, would go through by a simple modification by a factor of $1/\sqrt{K}$ [34]. The triple distributions in θ_t, θ_l and ϕ_l show an asymmetry around $\phi_l = 180^\circ$, which is not present at Born level.

2. Derivation of angular distributions to $\mathscr{O}(\alpha_s)$

We first obtain expressions for helicity amplitudes for

$$e^-(p_{e^-}) + e^+(p_{e^+}) \to t(p_t) + \bar{t}(p_{\bar{t}}) \tag{1}$$

going through virtual γ and Z in the e^+e^- c.m. frame, including QCD corrections in SGA. The starting point is the QCD-modified $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices obtained earlier (see, for example, [29,35]). We can write them [29] in the limit of vanishing electron mass as

$$\Gamma_\mu^\gamma = e\left[c_v^\gamma \gamma_\mu + c_M^\gamma \frac{(p_t - p_{\bar{t}})_\mu}{2m_t}\right], \tag{2}$$

$$\Gamma_\mu^Z = e\left[c_v^Z \gamma_\mu + c_a^Z \gamma_\mu \gamma_5 + c_M^Z \frac{(p_t - p_{\bar{t}})_\mu}{2m_t}\right], \tag{3}$$

where

$$c_v^\gamma = \frac{2}{3}(1+A), \tag{4}$$

$$c_v^Z = \frac{1}{\sin\theta_W \cos\theta_W}\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)(1+A), \tag{5}$$

$$c_a^\gamma = 0, \tag{6}$$

$$c_a^Z = \frac{1}{\sin\theta_W \cos\theta_W}\left(-\frac{1}{4}\right)(1+A+2B), \tag{7}$$

$$c_M^\gamma = \frac{2}{3}B, \tag{8}$$

$$c_M^Z = \frac{1}{\sin\theta_W \cos\theta_W}\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)B. \tag{9}$$

The form factors A and B are given to order α_s in SGA by

$$\begin{aligned} \mathrm{Re}A = \hat{\alpha}_s \Bigg[& \left(\frac{1+\beta^2}{\beta}\log\frac{1+\beta}{1-\beta} - 2\right)\log\frac{4\omega_{\max}^2}{m_t^2} - 4 \\ & + \frac{2+3\beta^2}{\beta}\log\frac{1+\beta}{1-\beta} + \frac{1+\beta^2}{\beta}\left\{\log\frac{1-\beta}{1+\beta}\left(3\log\frac{2\beta}{1+\beta}\right.\right. \\ & \left.\left. + \log\frac{2\beta}{1-\beta}\right) + 4\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{3}\pi^2\right\}\Bigg], \end{aligned} \tag{10}$$

$$\mathrm{Re}B = \hat{\alpha}_s \frac{1-\beta^2}{\beta}\log\frac{1+\beta}{1-\beta}, \tag{11}$$

$$\mathrm{Im}B = -\hat{\alpha}_s \pi \frac{1-\beta^2}{\beta}, \tag{12}$$

where $\hat{\alpha}_s = \alpha_s/(3\pi)$, $\beta = \sqrt{1-4m_t^2/s}$, and Li_2 is the Spence function. ReA in eq. (10) contains the effective form factor for a cut-off $\omega_{\max}$ on the gluon energy after the infrared singularities have been cancelled between the virtual- and soft-gluon contributions in the on-shell renormalization scheme. Only the real part of the form factor A has been given, because the contribution of the imaginary part is proportional to the Z width, and hence negligibly small [26,29]. The imaginary part of B, however, contributes to the triple angular distribution.

The vertices in eqs (2) and (3) can be used to obtain helicity amplitudes for $e^+e^- \to t\bar{t}$, including the contribution of s-channel γ and Z exchanges. The result is, in a notation where the subscripts of M denote the signs of the helicities of e^-, e^+, t and $\bar{t}$, in that order,

$$M_{+-\pm\pm} = \pm\frac{4e^2}{s}\sin\theta_t\frac{1}{\gamma}\left[\left(c_V^\gamma + r_\mathrm{R}c_V^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_\mathrm{R}c_M^Z\right)\right], \tag{13}$$

$$M_{-+\pm\pm} = \pm\frac{4e^2}{s}\sin\theta_t\frac{1}{\gamma}\left[\left(c_V^\gamma + r_\mathrm{L}c_V^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_\mathrm{L}c_M^Z\right)\right], \tag{14}$$

$$M_{+-\pm\mp} = \frac{4e^2}{s}(1\pm\cos\theta_t)\left[\pm\left(c_V^\gamma + r_\mathrm{R}c_V^Z\right) + \beta\left(c_a^\gamma + r_\mathrm{R}c_a^Z\right)\right], \tag{15}$$

$$M_{-+\pm\mp} = \frac{4e^2}{s}(1\mp\cos\theta_t)\left[\mp\left(c_V^\gamma + r_\mathrm{L}c_V^Z\right) - \beta\left(c_a^\gamma + r_\mathrm{L}c_a^Z\right)\right], \tag{16}$$

where θ_t is the angle the top-quark momentum makes with the e^- momentum, $\gamma = 1/\sqrt{1-\beta^2}$, and $r_\mathrm{L,R}$ are related to the left- and right-handed $Ze\bar{e}$ couplings, and are given by

$$r_\mathrm{L} = \left(\frac{s}{s-m_Z^2}\right)\frac{1}{\sin\theta_W\cos\theta_W}, \tag{17}$$

$$r_\mathrm{R} = -\left(\frac{s}{s-m_Z^2}\right)\tan\theta_W. \tag{18}$$

Since we are interested in lepton distributions arising from top decay, we also evaluate the helicity amplitudes for $t \to bl^+\nu_l$ (or $\bar{t} \to \bar{b}l^-\bar{\nu}_l$), which will be combined with the production amplitudes in the narrow-width approximation for t ($\bar{t}$) and W^+ (W^-). In principle, QCD corrections also should be included in the decay process. However, in SGA, these could be written in terms of effective form factors [31]. As found earlier [15,16], in the linear approximation, these form factors do not affect the charged-lepton angular distribution. Hence we need not calculate these form factors. Nevertheless, to illustrate this point we will discuss an arbitrary $\bar{t}bW$ vertex, in the limit of vanishing bottom mass.

The most general $\bar{t}bW$ vertex may be written as

$$\begin{aligned}\Gamma^\mu_{t\to bW} = -\frac{g}{\sqrt{2}}V_{tb}\bar{u}(p_b)\Big[&\gamma^\mu(f_{1\mathrm{L}}P_\mathrm{L} + f_{1\mathrm{R}}P_\mathrm{R}) \\ &-\frac{i}{m_W}\sigma^{\mu\nu}(p_t-p_b)_\nu(f_{2\mathrm{L}}P_\mathrm{L} + f_{2\mathrm{R}}P_\mathrm{R})\Big]u(p_t),\end{aligned} \tag{19}$$

where $P_\mathrm{L,R} = \frac{1}{2}(1\mp\gamma_5)$ and V_{tb} is the Cabibbo–Kobayashi–Maskawa (CKM) matrix element, which we will take to be equal to one. $f_{1\mathrm{R}}$ and $f_{2\mathrm{L}}$ do not contribute in the limit of vanishing b mass, and can be omitted. Moreover, we will set $f_{1\mathrm{L}} = 1$, the value at tree level.

The result does not depend on the value of $f_{1\mathrm{L}}$, as we will see soon. The decay helicity amplitudes in the t rest frame can be found in [16], and we do not repeat them here.

It was found in [16] that, in this case, the spin density matrix for $t \to bl^+\nu_l$, on integration over the b-quark azimuthal angle, is the same as that for the Born vertex, except for an overall factor $\left(1 + \mathrm{Re} f_{2\mathrm{R}}(m_W m_t / p_t \cdot p_{l^+})\right)$ multiplying all the matrix elements. Moreover, after performing a boost to the lab. frame, and integrating over the charged-lepton energy, the $f_{2\mathrm{R}}$ dependent factor cancelled against the corresponding factor in the correction to the top width. If we had not set $f_{1\mathrm{L}} = 1$, the $f_{1\mathrm{L}}$ dependence would also have likewise cancelled.

The final result for the angular distribution in the lab. frame can be written as

$$\begin{aligned}\frac{\mathrm{d}^3\sigma}{\mathrm{d}\cos\theta_t \mathrm{d}\cos\theta_l \mathrm{d}\phi_l} = \frac{3\alpha^2\beta m_t^2}{8s^2} B_l \frac{1}{(1-\beta\cos\theta_{tl})^3} \big[&\mathscr{A}(1-\beta\cos\theta_{tl}) + \mathscr{B}(\cos\theta_{tl} - \beta) \\ &+ \mathscr{C}(1-\beta^2)\sin\theta_t \sin\theta_l (\cos\theta_t \cos\phi_l - \sin\theta_t \cot\theta_l) \\ &+ \mathscr{D}(1-\beta^2)\sin\theta_t \sin\theta_l \sin\phi_l \big], \end{aligned} \tag{20}$$

where θ_t and θ_l are polar angles respectively of the t and l^+ momenta with respect to the e^- beam direction chosen as the z-axis, and ϕ_l is the azimuthal angle of the l^+ momentum relative to an axis chosen in the $t\bar{t}$ production plane. B_l is the leptonic branching ratio of the top. θ_{tl} is the angle between the t and l^+ directions, given by

$$\cos\theta_{tl} = \cos\theta_t \cos\theta_l + \sin\theta_t \sin\theta_l \cos\phi_l, \tag{21}$$

and the coefficients $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$ and $\mathscr{D}$ are given by

$$\mathscr{A} = A_0 + A_1 \cos\theta_t + A_2 \cos^2\theta_t, \tag{22}$$

$$\mathscr{B} = B_0 + B_1 \cos\theta_t + B_2 \cos^2\theta_t, \tag{23}$$

$$\mathscr{C} = C_0 + C_1 \cos\theta_t, \tag{24}$$

$$\mathscr{D} = D_0 + D_1 \cos\theta_t, \tag{25}$$

with [36]

$$\begin{aligned} A_0 = 2\big\{&(2-\beta^2)\left[2|c_v^\gamma|^2 + 2(r_\mathrm{L}+r_\mathrm{R})\,\mathrm{Re}\left(c_v^\gamma c_v^{Z*}\right) + (r_\mathrm{L}^2+r_\mathrm{R}^2)|c_v^Z|^2\right] \\ &+\beta^2(r_\mathrm{L}^2+r_\mathrm{R}^2)|c_a^Z|^2 - 2\beta^2\,\mathrm{Re}\left[2c_v^\gamma c_M^{\gamma*} + (r_\mathrm{L}+r_\mathrm{R})\left(c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*}\right)\right. \\ &\left.+(r_\mathrm{L}^2+r_\mathrm{R}^2)c_v^Z c_M^{Z*}\right]\big\}(1-P_eP_{\bar{e}}) + 2\big\{\beta^2(r_\mathrm{L}^2-r_\mathrm{R}^2)|c_a^Z|^2 \\ &+(2-\beta^2)\left[2(r_\mathrm{L}-r_\mathrm{R})\,\mathrm{Re}\left(c_v^\gamma c_v^{Z*}\right) + (r_\mathrm{L}^2-r_\mathrm{R}^2)|c_v^Z|^2\right] \\ &-2\beta^2\,\mathrm{Re}\left[(r_\mathrm{L}-r_\mathrm{R})\left(c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*}\right) + (r_\mathrm{L}^2-r_\mathrm{R}^2)c_v^Z c_M^{Z*}\right]\big\}(P_{\bar{e}}-P_e), \\ A_1 = -8\beta\,\mathrm{Re}\big(&c_a^{Z*}\big\{\left[(r_\mathrm{L}-r_\mathrm{R})c_v^\gamma + (r_\mathrm{L}^2-r_\mathrm{R}^2)c_v^Z\right](1-P_eP_{\bar{e}}) \\ &+\left[(r_\mathrm{L}+r_\mathrm{R})c_v^\gamma + (r_\mathrm{L}^2+r_\mathrm{R}^2)c_v^Z\right](P_{\bar{e}}-P_e)\big\}\big), \\ A_2 = 2\beta^2\big\{&\left[2|c_v^\gamma|^2 + 4\,\mathrm{Re}\left(c_v^\gamma c_M^{\gamma*}\right) + 2(r_\mathrm{L}+r_\mathrm{R})\,\mathrm{Re}\left(c_v^\gamma c_v^{Z*} + c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*}\right)\right. \\ &\left.+(r_\mathrm{L}^2+r_\mathrm{R}^2)\left(|c_v^Z|^2 + |c_a^Z|^2 + 2\,\mathrm{Re}\left(c_v^Z c_M^{Z*}\right)\right)\right](1-P_eP_{\bar{e}}) \\ &+\left[2(r_\mathrm{L}-r_\mathrm{R})\,\mathrm{Re}\left(c_v^\gamma c_v^{Z*} + c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*}\right)\right. \\ &\left.+(r_\mathrm{L}^2-r_\mathrm{R}^2)\left(|c_v^Z|^2 + |c_a^Z|^2 + 2\,\mathrm{Re}\left(c_v^Z c_M^{Z*}\right)\right)\right](P_{\bar{e}}-P_e)\big\}, \end{aligned}$$

$$
\begin{aligned}
B_0 &= 4\beta \left\{ r_{\rm L}\, {\rm Re}\left[\left(c_v^\gamma + r_{\rm L} c_v^Z\right) c_a^{Z*}\right](1-P_e)(1+P_{\overline{e}}) \right.\\
&\quad \left. + r_{\rm R}\, {\rm Re}\left[\left(c_v^\gamma + r_{\rm R} c_v^Z\right) c_a^{Z*}\right](1+P_e)(1-P_{\overline{e}}) \right\},\\
B_1 &= -4\left\{ \left[|c_v^\gamma + r_{\rm L} c_v^Z|^2 + \beta^2 r_{\rm L}^2 |c_a^Z|^2\right](1-P_e)(1+P_{\overline{e}}) \right.\\
&\quad \left. - \left[|c_v^\gamma + r_{\rm R} c_v^Z|^2 + \beta^2 r_{\rm R}^2 |c_a^Z|^2\right](1+P_e)(1-P_{\overline{e}}) \right\},\\
B_2 &= 4\beta \left\{ r_{\rm L}\, {\rm Re}\left[\left(c_v^\gamma + r_{\rm L} c_v^Z\right) c_a^{Z*}\right](1-P_e)(1+P_{\overline{e}}) \right.\\
&\quad \left. + r_{\rm R}\, {\rm Re}\left[\left(c_v^\gamma + r_{\rm R} c_v^Z\right) c_a^{Z*}\right](1+P_e)(1-P_{\overline{e}}) \right\},\\
C_0 &= 4\left\{ |c_v^\gamma + r_{\rm L} c_v^Z|^2 - \beta^2\gamma^2 {\rm Re}\left[\left(c_v^\gamma + r_{\rm L} c_v^Z\right)\left(c_M^{\gamma*} + r_{\rm L} c_M^{Z*}\right)\right]\right\}(1-P_e)(1+P_{\overline{e}})\\
&\quad -4\left\{ |c_v^\gamma + r_{\rm R} c_v^Z|^2 - \beta^2\gamma^2 {\rm Re}\left[\left(c_v^\gamma + r_{\rm R} c_v^Z\right)\left(c_M^{\gamma*} + r_{\rm R} c_M^{Z*}\right)\right]\right\}(1+P_e)(1-P_{\overline{e}}),\\
C_1 &= -4\beta\, {\rm Re}\left\{ \left[\left(c_v^\gamma + r_{\rm L} c_v^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_{\rm L} c_M^Z\right)\right] r_{\rm L} c_a^{Z*}(1-P_e)(1+P_{\overline{e}}) \right.\\
&\quad \left. + \left[\left(c_v^\gamma + r_{\rm R} c_v^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_{\rm R} c_M^Z\right)\right] r_{\rm R} c_a^{Z*}(1+P_e)(1-P_{\overline{e}}) \right\},\\
D_0 &= 4\beta\, {\rm Im}\left[\left\{ \left(c_v^\gamma + r_{\rm L} c_v^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_{\rm L} c_M^Z\right)\right\} r_{\rm L} c_a^{Z*}(1-P_e)(1+P_{\overline{e}}) \right.\\
&\quad \left. - \left\{ \left(c_v^\gamma + r_{\rm R} c_v^Z\right) - \beta^2\gamma^2\left(c_M^\gamma + r_{\rm R} c_M^Z\right)\right\} r_{\rm R} c_a^{Z*}(1+P_e)(1-P_{\overline{e}}) \right] \sin\theta_t,\\
D_1 &= 4\beta^2\gamma^2 {\rm Im}\left[\left(c_M^\gamma + r_{\rm L} c_M^Z\right)\left(c_v^{\gamma*} + r_{\rm L} c_v^{Z*}\right)(1-P_e)(1+P_{\overline{e}}) \right.\\
&\quad \left. + \left(c_M^\gamma + r_{\rm R} c_M^Z\right)\left(c_v^{\gamma*} + r_{\rm R} c_v^{Z*}\right)(1+P_e)(1-P_{\overline{e}}) \right] \sin\theta_t.
\end{aligned}
$$

Integrating over ϕ_l and θ_{tl} we get the θ_l distribution:

$$
\begin{aligned}
\frac{{\rm d}\sigma}{{\rm d}\cos\theta_l} = \frac{3\pi\alpha^2}{32s}\beta B_l \Bigg\{ & \left(4A_0 + \frac{4}{3}A_2\right) + \Bigg[-2A_1\left(\frac{1-\beta^2}{\beta^2}\log\frac{1+\beta}{1-\beta} - \frac{2}{\beta}\right)\\
& + 2B_1 \frac{1-\beta^2}{\beta^2}\left(\frac{1}{\beta}\log\frac{1+\beta}{1-\beta} - 2\right)\\
& + 2C_0 \frac{1-\beta^2}{\beta^2}\left(\frac{1-\beta^2}{\beta}\log\frac{1+\beta}{1-\beta} - 2\right)\Bigg] \cos\theta_l\\
& + \Bigg[2A_2\left(\frac{1-\beta^2}{\beta^3}\log\frac{1+\beta}{1-\beta} - \frac{2}{3\beta^2}\left(3-2\beta^2\right)\right)\\
& + \frac{1-\beta^2}{\beta^3}\Bigg\{B_2\left(\frac{\beta^2-3}{\beta}\log\frac{1+\beta}{1-\beta} + 6\right)\\
& - C_1\left(\frac{3(1-\beta^2)}{\beta}\log\frac{1+\beta}{1-\beta} - 2(3-2\beta^2)\right)\Bigg\}\Bigg] \left(1 - 3\cos^2\theta_l\right)\Bigg\}. \qquad (26)
\end{aligned}
$$

3. Numerical results

After having obtained analytic expressions for angular distributions, we now examine the numerical values of the QCD corrections.

We use the parameters $\alpha = 1/128$, $\alpha_s(m_Z^2) = 0.118$, $m_Z = 91.187$ GeV, $m_W = 80.41$ GeV, $m_t = 175$ GeV and $\sin^2\theta_W = 0.2315$. We consider leptonic decays into one specific channel (electrons or muons or tau leptons), corresponding to a branching ratio of 1/9. We have used, following [29], a gluon energy cut-off of $\omega_{\rm max} = (\sqrt{s} - 2m_t)/5$. While qualitative results would be insensitive, exact quantitative results would of course depend on the choice of cut-off.

In figures 1a–c we show the single differential cross section $d\sigma/d\cos\theta_l$ in picobarns with and without QCD corrections, for three values of $\sqrt{s}$, viz., (a) 400 GeV, (b) 800 GeV and (c) 1500 GeV, and for e^- beam polarizations $P = 0, -1, +1$. It can be seen that the distribution with QCD corrections follows, in general, the shape of the lowest order distribution.

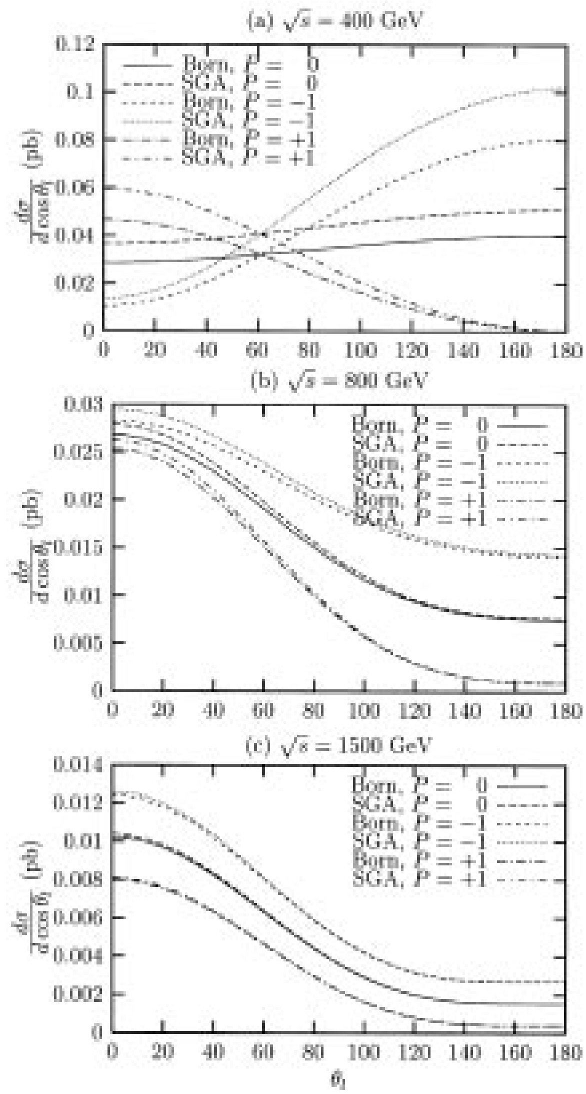

Figure 1. The distribution in θ_l with and without QCD corrections for (**a**) $\sqrt{s} = 400$ GeV, (**b**) $\sqrt{s} = 800$ GeV and (**c**) $\sqrt{s} = 1500$ GeV plotted against θ_l, for e^- beam polarizations $P = 0, -1, +1$ in each case.

Figures 2a–c display the fractional deviation of the QCD-corrected distribution from the lowest order distribution

$$\kappa(\theta_l) = \left(\frac{\mathrm{d}\sigma_{\mathrm{Born}}}{\mathrm{d}\cos\theta_l}\right)^{-1} \left(\frac{\mathrm{d}\sigma_{\mathrm{SGA}}}{\mathrm{d}\cos\theta_l} - \frac{\mathrm{d}\sigma_{\mathrm{Born}}}{\mathrm{d}\cos\theta_l}\right). \tag{27}$$

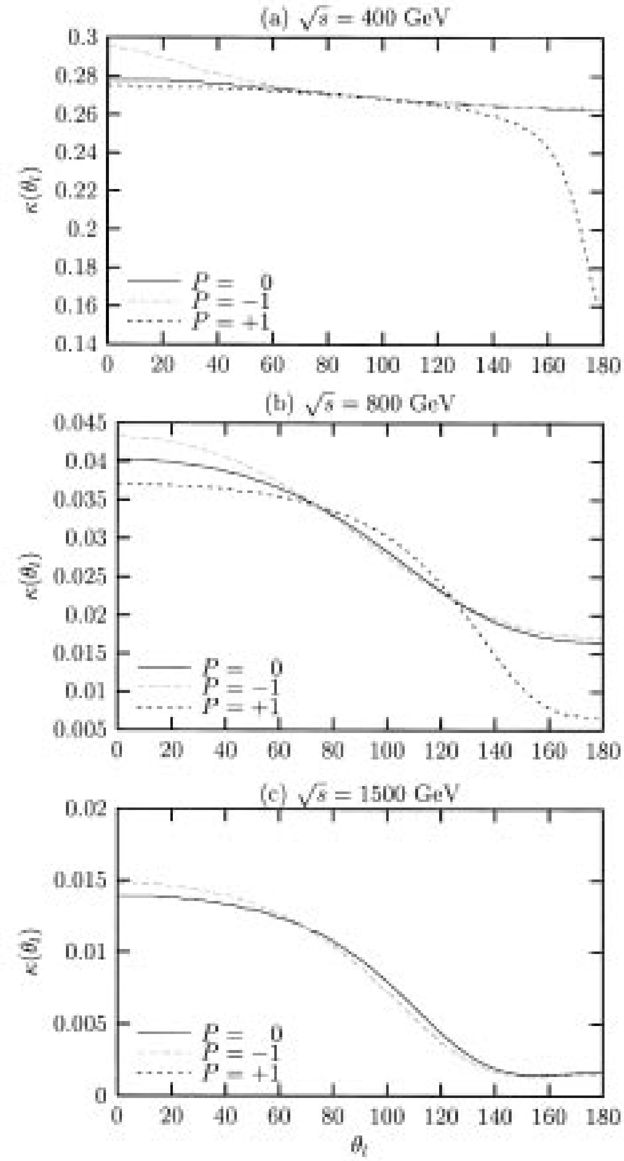

Figure 2. The fractional QCD contribution $\kappa(\theta_l)$ defined in the text for **(a)** $\sqrt{s} = 400$ GeV, **(b)** $\sqrt{s} = 800$ GeV and **(c)** $\sqrt{s} = 1500$ GeV plotted as a function of θ_l, for $P = 0, -1, +1$.

It can be seen that $\kappa(\theta_l)$ is independent of θ_l to a fair degree of accuracy for $\sqrt{s} = 400$ GeV.

In figures 3a–c we show the fractional QCD contributions $(F_{\rm SGA} - F_{\rm Born})/F_{\rm Born}$ where $F(\theta_l)$, is the normalized distribution:

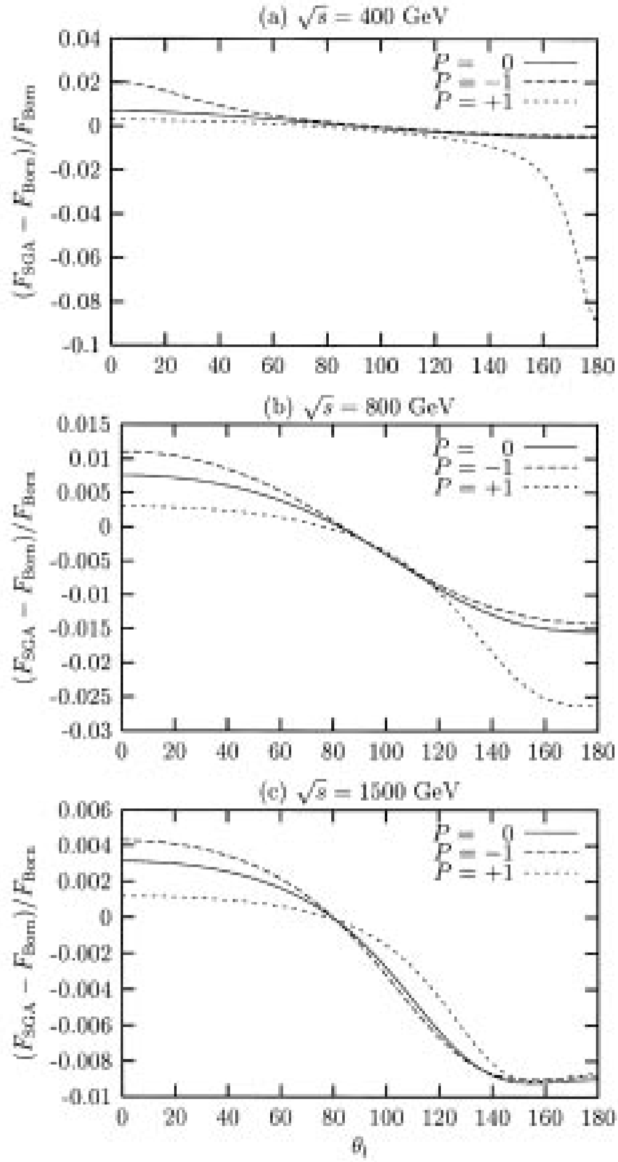

Figure 3. The fractional QCD contribution in normalized angular distributions, $F(\theta_l)$ defined in the text, for (**a**) $\sqrt{s} = 400$ GeV, (**b**) $\sqrt{s} = 800$ GeV and (**c**) $\sqrt{s} = 1500$ GeV plotted as a function of θ_l, for $P = 0, -1, +1$.

$$F(\theta_l) = \frac{1}{\sigma}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_l}\right). \tag{28}$$

It can be seen that the fractional change in the normalized distributions for $\sqrt{s} = 400$ GeV is at most of the order of 1 or 2% (except in the case of $P = +1$, for $\theta_l \geq 160°$). For the other values of $\sqrt{s}$, it is even smaller. This implies that QCD corrected angular distribution is well approximated, at the per cent level, by a constant rescaling by a K factor.

We now discuss the triple differential cross section, which corresponds to a distribution in θ_t, θ_l and ϕ_l, for which we have obtained an analytic expression, eq. (20). This distribution, unlike the θ_l distribution, requires the knowledge of the direction of t. In fact, without knowing the direction of the top momentum, it is not even possible to define ϕ_l. However, the magnitude of the momentum need not be determined precisely.

We have plotted in figure 4a $\mathrm{d}^3\sigma/(\mathrm{d}\cos\theta_t\ \mathrm{d}\cos\theta_l\ \mathrm{d}\phi_l)$, including QCD correction, against ϕ_l for different values of θ_l, for a fixed value of $\theta_t = 30°$, for $\sqrt{s} = 400$ GeV, and for $P = 0$. Figure 4b gives the corresponding fractional deviations from the lowest-order values

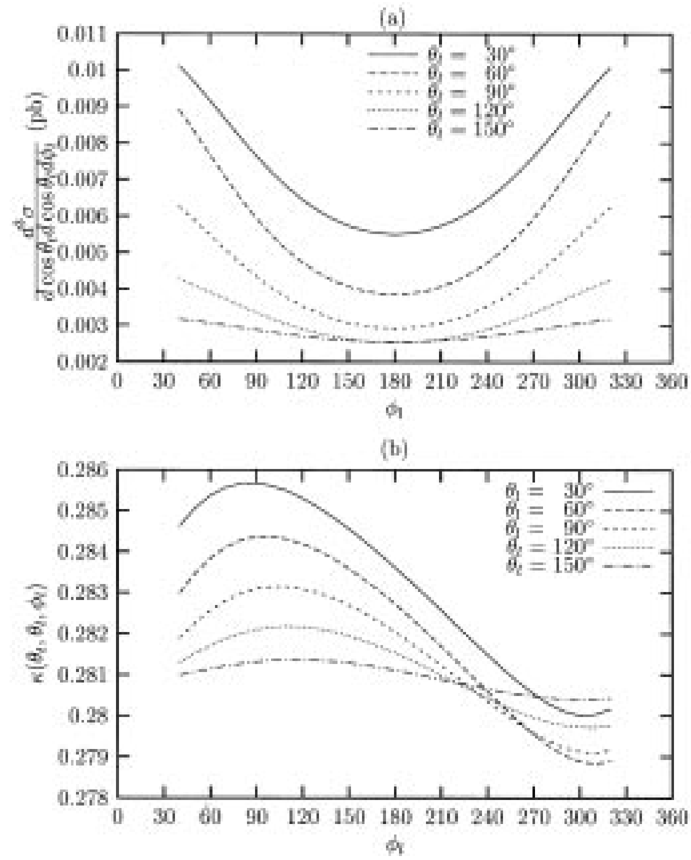

Figure 4. (**a**) The triple differential distribution including the QCD contribution for $\sqrt{s} = 400$ GeV and $\theta_t = 30°$. (**b**) The fractional QCD contribution $\kappa(\theta_t, \theta_l, \phi_l)$ to the triple differential distribution for $\sqrt{s} = 400$ GeV and $\theta_t = 30°$.

$$\kappa(\theta_t, \theta_l, \phi_l) = \left[\frac{\mathrm{d}^3\sigma_{\mathrm{Born}}}{\mathrm{d}\cos\theta_t\,\mathrm{d}\cos\theta_l\,\mathrm{d}\phi_l}\right]^{-1} \times \left[\frac{\mathrm{d}^3\sigma_{\mathrm{SGA}}}{\mathrm{d}\cos\theta_t \mathrm{d}\cos\theta_l \mathrm{d}\phi_l} - \frac{\mathrm{d}^3\sigma_{\mathrm{Born}}}{\mathrm{d}\cos\theta_t\,\mathrm{d}\cos\theta_l\,\mathrm{d}\phi_l}\right]. \tag{29}$$

It can be seen that the QCD contribution remains around 28% for various values of θ_l and ϕ_l. The triple distributions show an asymmetry around $\phi_l = 180°$, which is not present at Born level. This asymmetry is not visible in figure 4a, but is very clear in figure 4b, where fractional QCD contributions to the normalized distributions are plotted.

The same plots as in figure 4a, b are repeated for $\sqrt{s} = 800$ GeV in figures 5a, b and for $\sqrt{s} = 1500$ GeV in figures 6a, b.

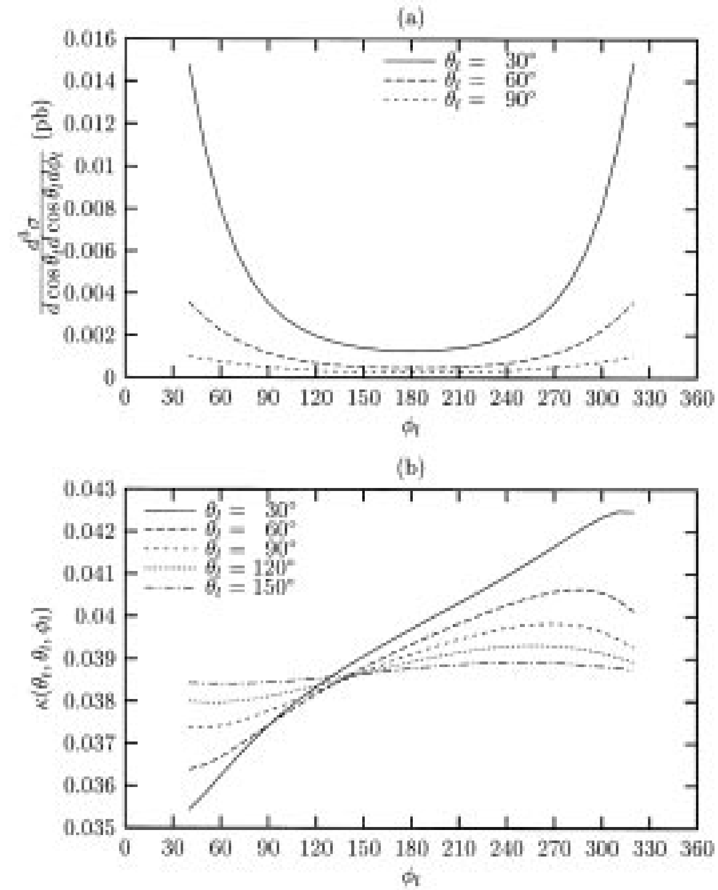

Figure 5. (**a**) The triple differential distribution including the QCD contribution for $\sqrt{s} = 800$ GeV and $\theta_t = 30°$. (**b**) The fractional QCD contribution $\kappa(\theta_t, \theta_l, \phi_l)$ to the triple differential distribution for $\sqrt{s} = 800$ GeV and $\theta_t = 30°$.

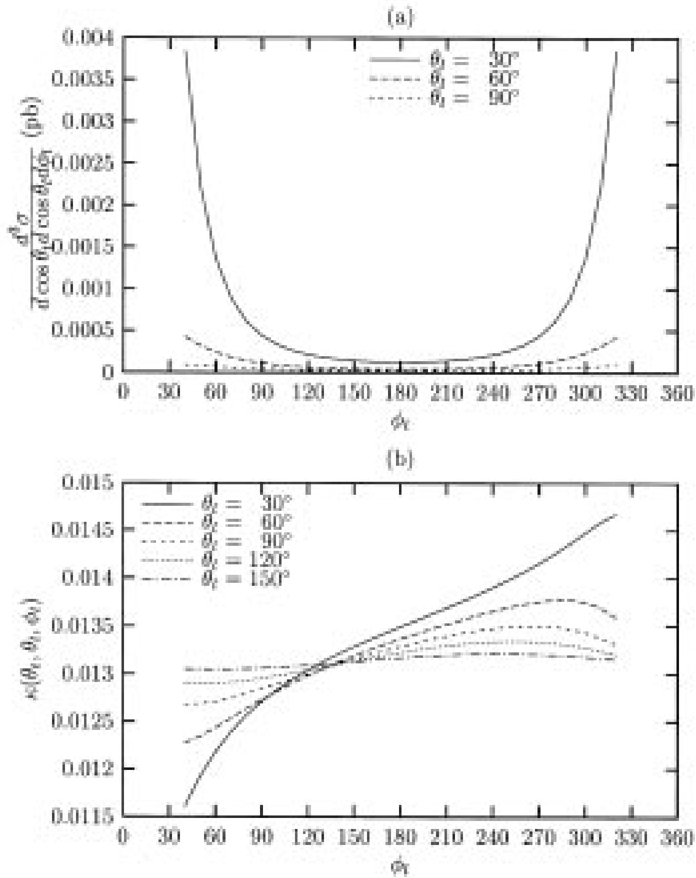

Figure 6. (**a**) The triple differential distribution including the QCD contribution for $\sqrt{s} = 1500$ GeV and $\theta_t = 30°$. (**b**) The fractional QCD contribution $\kappa(\theta_t, \theta_l, \phi_l)$ to the triple differential distribution for $\sqrt{s} = 1500$ GeV and $\theta_t = 30°$.

We have not shown the plots for other values of θ_t because they do not illustrate anything new. We have also not discussed triple distributions for the case of polarized beams for the sake of brevity.

4. Conclusions

To conclude, we have obtained in this paper analytic expressions for angular distributions of leptons from top decay in $e^+e^- \to t\bar{t}$, in the e^+e^- c.m. frame, including QCD corrections to order α_s in the soft-gluon approximation. The distributions are in a form which can be compared directly with experiment. In particular, the single differential θ_l distribution needs neither the reconstruction of the top momentum direction nor the top rest frame. The triple differential distribution does need the top direction to be reconstructed for the definition of the angles. However, the magnitude of the top momentum need not be determined. In either case the results do not depend on the choice of spin quantization axis.

We find that the θ_l distributions are well described by rescaling the zeroth order distributions by a factor K which for $\sqrt{s} = 400$ GeV is roughly independent of θ_l, except for extreme values of θ_l, for the case of right-handed polarized electron beam. For other values of $\sqrt{s}$, it is a slowly varying function of θ_l.

The triple distributions in θ_t, θ_l and ϕ_l show an asymmetry about $\phi_l = 180°$, which is absent at tree level, and would be a good test of the QCD corrections.

It would be useful to carry out the hard-gluon corrections explicitly and check if the soft-gluon approximation used here gives correct quantitative results.

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