PRAMANA
— journal of
physics

Supplement to Vol. 45
October 1995
pp. 263-276

CP violation at colliders

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Abstract. The prospects of experimental detection of CP violation at e^+e^- and $pp/p\bar{p}$ colliders are reviewed. After a general discussion on the quantities which can measure CP violation and on the implications of the CPT theorem, various possibilities of measuring CP violation arising outside the standard model are taken up. CP violation in leptonic processes, especially polarization effects in $e^+e^- \to l^+l^-$ are discussed next. CP violation in $t\bar{t}$ and W^+W^- production and decay is also described.

1. Introduction

1.1 CP violation in the standard model

CP violation in the standard model (SM), as is well known, arises due to complex Yukawa couplings, and finally shows up through quark mixing in the Cabibbo-Kobayashi-Maskawa matrix as a single phase. The reparametrization invariant quantity

$$J = \sin^2\theta_1 \sin\theta_2 \sin\theta_3 \cos\theta_1 \cos\theta_2 \cos\theta_3 \sin\delta \tag{1}$$

is a measure of CP violation in SM, and though small, produces measurable effects in the K meson system, and hopefully the effects in the b-quark states will be measurable. The effects in other sectors (as for example the neutron electric dipole moment) and in high-energy processes is generally predicted to be too small to be observed.

In the leptonic sector of SM the prospects of CP violation being observable are worse, since there is no analogue of the CKM phase in the absence of neutrino masses. CP violation in leptonic systems has to feed in from the hadronic sector through loops. For example, electric dipole moments (EDM) of charged leptons are induced due to the W EDM, and are found to vanish up to three-loop order. The electron EDM is then estimated to be around 10^{-41} e cm. [1]

Thus, any new observable CP violation would be a signal of non-standard physics. It might be mentioned that perhaps there is already a hint towards non-standard CP violation in current ideas on electroweak baryogenesis.

1.2 Scenarios for CP violation beyond SM

CP violation beyond SM can arise due to almost any extra Yukawa couplings which can be complex, and possibly also due to new Higgs self-interactions and complex scalar vacuum expectation values. Thus, introduction of extra fermions or scalars could give rise to new sources of CP violation.

Retaining the gauge group to be $SU(2)_L \times U(1)$, CP violation can arise due to extra fermion or Higgs doublets or singlets. Since the SM measure of CP violation J (eq.(1)) is small owing to the small mixing angles among quark generations as experimentally observed, larger CP violating effects would arise if there are extra generations of quarks, whose mixing angles may be less constrained. If there are new fermions (quarks or leptons) in exotic representations (left-handed singlets and/or right-handed doublets of $SU(2)_L$) there are further complex flavour-changing couplings to Z which violate CP.

Supersymmetry requires the addition of extra scalars and fermions, whose couplings violate CP. In left-right symmetric models, again, there further sources of CP violation.

1.3 Use of effective Lagrangians for model-independent analysis

There is a large variety of sources of CP violation beyond SM, and rather than discuss predictions of each model for each observable quantity, it is more economical to analyze CP-violating quantities in terms of the parameters of an effective Lagrangian. Examples of CP-violating terms in an effective Lagrangian with which we will be concerned here are given below:

$$\begin{aligned}\mathcal{L}_{eff} &= -\frac{i}{2}\sum_i d_{\psi_i}\overline{\psi}_i\sigma^{\mu\nu}\gamma_5\psi_i F_{\mu\nu} - \frac{i}{2}\sum_i \tilde{d}_{\psi_i}\overline{\psi}_i\sigma^{\mu\nu}\gamma_5\psi_i(\partial_\mu Z_\nu - \partial_\nu Z_\mu) \\ &+ \sum_{V=A,Z} ig_V\left[\tilde{\kappa}^V W^\dagger_\mu W_\nu \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}^V}{m_W^2} W^\dagger_{\lambda\mu} W^\mu_\nu \tilde{V}^{\nu\lambda}\right. \\ &\left. + g_4^V W^\dagger_\mu W_\nu(\partial^\mu V^\nu + \partial^\nu V^\mu)\right]. \end{aligned} \tag{2}$$

($\tilde{V}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}V_{\alpha\beta}$; $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$; $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$). These terms are of dimension ≤ 6. In eq.(2), ψ_i refers to various fermionic fields (quarks and leptons), whose electric and "weak" dipole moment is given by d_{ψ_i} and $\tilde{d}_{\psi_i}$, respectively. It should be noted that all the parameters are in reality scale dependent "form factors", and can be complex.

2. Some general considerations

2.1 Observable quantities which measure CP violation

There are basically two types of observables which can be used to characterize CP violation: asymmetries and correlations.

An example of an asymmetry is the partial-width asymmetry for decay of particles i and $\bar{i}$ which are CP conjugates of each other:

$$A = \frac{\Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f})}{\Gamma(i \to f) + \Gamma(\bar{i} \to \bar{f})}. \tag{3}$$

If CP is a symmetry of the theory, $A = 0$. Non-vanishing A implies violation of CP. A is a convenient parameter because it is dimensionless and lies between -1

and 1. In particular, if i is an eigenstate of CP, $\bar{i} = i$, and A in (3) simplifies and measures the fractional difference in the decay rates of i to two CP-conjugate states f and $\bar{f}$. As we shall see later, the CPT theorem implies that A is zero even if CP is violated unless the amplitude has an absorptive part which can arise because of final-state interactions or loop effects in perturbation theory.

Another type of asymmetry is an asymmetry in a final-state variable like energy or angle. It is defined in general (for $i = \bar{i}$) as

$$A = \frac{\sum_{f \in S} d\sigma(i \to f) - \sum_{f \in \bar{S}} d\sigma(i \to f)}{\sum_{f \in S} d\sigma(i \to f) + \sum_{f \in \bar{S}} d\sigma(i \to f)}. \tag{4}$$

Here S and $\bar{S}$ are sets of states with CP-conjugate kinematic ranges, and f is a final state assumed to have particles conjugate to one another in pairs. An example is energy asymmetry between the energies E_+ and E_- of CP-conjugate particles in f:

$$A = \frac{\int_{E_+ > E_-} d\sigma(i \to f) - \int_{E_+ < E_-} d\sigma(i \to f)}{\int_{E_+, E_-} d\sigma(i \to f)}. \tag{5}$$

The other category of quantities consists of CP-odd correlations which are expectations values of CP-odd operators in a process with initial as well final states described by CP-even density matrices. Thus for an observable $\mathcal{O}(\{\mathbf{p}_{A_i}, \mathbf{s}_{A_i}\})$ which is a function of momenta $\mathbf{p}_{A_i}$ and spins $\mathbf{s}_{A_i}$ of particles A_i, and which transforms under CP as

$$\mathcal{O}(\{\mathbf{p}_{A_i}, \mathbf{s}_{A_i}\}) \to \mathcal{O}(\{-\mathbf{p}_{A_i}, \mathbf{s}_{A_i}\}) = -\mathcal{O}(\{\mathbf{p}_{A_i}, \mathbf{s}_{A_i}\}), \tag{6}$$

the CP-odd correlation is

$$\langle \mathcal{O} \rangle = \frac{\int d\sigma \mathcal{O}(\{\mathbf{p}_{A_i}, \mathbf{s}_{A_i}\})}{\int d\sigma}. \tag{7}$$

A non-zero value of such a correlation signals CP violation.

It may be noted that an asymmetry in the variable $\mathcal{O}$may be described as a special case of a correlation of $\epsilon(\mathcal{O})$, where ϵ is the antisymmetric step function:

$$\langle \epsilon(\mathcal{O}) \rangle = \frac{\int_{\mathcal{O}>0} d\sigma - \int_{\mathcal{O}<0} d\sigma}{\int_{\mathcal{O}} d\sigma}. \tag{8}$$

2.2 Statistical significance

Whether or not a measured asymmetry or correlation can really signal CP violation naturally depends on its statistical significance decided by the statistical fluctuation expected in the event sample.

For a rate asymmetry A, the number of asymmetric events ΔN is

$$\Delta N = AN, \tag{9}$$

where N is the total number of events in the channel considered. The statistical fluctuation in these N events is $\sqrt{N}$. Hence for discrimination of the signal, at the one standard deviation level, we require

$$\Delta N > \sqrt{N}, \tag{10}$$

or

$$A > \frac{1}{\sqrt{N}}. \tag{11}$$

Thus, it would be possible to measure an asymmetry if its predicted value is larger than $1/\sqrt{N}$. To put it differently, the number of events should be larger than $1/A^2$.

In the case of a CP-odd correlation $\langle \mathcal{O} \rangle$, CP-invariant interactions can give individual events with $\mathcal{O} \neq 0$, averaging out to zero. Thus the mean square deviation

$$\Delta\mathcal{O} = \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2} \tag{12}$$

is a measure of the background coming from CP-invariant interactions. For N events in the channel, the CP-even events give rise to a fluctuation $\Delta\mathcal{O}/\sqrt{N}$. The signal $\langle \mathcal{O} \rangle$ should be larger than this to be measurable at the one standard deviation level:

$$\langle \mathcal{O} \rangle > \frac{1}{\sqrt{N}} \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}. \tag{13}$$

There is a further experimental requirement for measuring CP violation. All experimental cuts must respect CP invariance. If not, they would introduce artificial asymmetries, diluting or obliterating the genuine signal of CP violation.

2.3 CPT theorem and all that

Since a combined CPT transformation is a good symmetry according to the CPT theorem, CP invariance (or violation) implies T invariance (or violation), and *vice versa*. However, it should be borne in mind that observation of a T-odd asymmetry or correlation is not necessarily an indication of CP (or even T) violation. The reason for this is the anti-unitary nature of the time-reversal operator in quantum mechanics. As a consequence of this, a T operation not only reverses spin and three-momenta of all particles, but also interchanges initial and final states. This last interchange is difficult to meet with in practice, and one usually has a situation where only momenta and spins are reversed, with the initial and final states kept as such. In that case, non-zero T-odd observables do not necessarily signal genuine T violation.

There is, however, a case when T-odd observables imply T violation, and that is when final-state interactions and loop effects can be neglected. In that case the transition operator $\mathcal{T}$ obeys $\mathcal{T} = \mathcal{T}^\dagger$, since the right-hand side in the unitarity relation

$$\mathcal{T} - \mathcal{T}^\dagger = i\mathcal{T}^\dagger\mathcal{T} \tag{14}$$

can be neglected. Then

$$\langle f|\mathcal{T}|i\rangle \approx \langle f|\mathcal{T}^\dagger|i\rangle = \langle i|\mathcal{T}|f\rangle^*. \tag{15}$$

Now if T invariance holds, then

$$\langle f|\mathcal{T}|i\rangle = \langle i_T|\mathcal{T}|f_T\rangle, \tag{16}$$

where $|i_T\rangle$, $|f_T\rangle$ represent states with all momenta and spins inverted in sign. Combining eq.(16) with (15) for time-reversed states, we have

$$|\langle f|\mathcal{T}|i\rangle| = |\langle f_T|\mathcal{T}|i_T\rangle|. \tag{17}$$

In this case, if a T-odd observable is non-zero, it implies T violation. Thus, T invariance (and CP invariance through the CPT theorem) implies equality of amplitudes with all momenta and spins reversed if and only if final-state interaction (absorptive part for the amplitude) vanishes.

Put differently, this means that final-state interactions can mimic T violation, but not genuine CP violation. One should therefore use, as far as possible, CP-odd observables to test CP invariance, not T-odd observables (unless they are also CP odd).

For a genuine CP-odd quantity, there are two possibilities,

A. it is T odd, and therefore CPT even, or

B. it is T even, and therefore CPT odd.

In case B, there is no violation of the CPT theorem provided the amplitude has an absorptive part. (This is again due to the fact the CPT operator is antiunitary, and interchanges initial and final states). Thus non-vanishing of CPT-odd operators necessarily requires an absorptive part of the amplitude to exist.

The absorptive part of CP-odd CPT-odd quantities in perturbation theory usually comes from loop contributions where the intermediate state can be on shell. An interesting way of realizing this possibility in the case of an intermediate state of an unstable particle is through the Breit-Wigner form of its propagator. In this case the absorptive part is proportional to its width. This trick has been used in the case of the top, Z and Higgs propagators [2, 3, 4]. One must however be careful to subtract out the part of the width corresponding to decay into the initial or final state for consistency with the CPT theorem [5]. It has also been pointed out recently [6] that off-diagonal contributions to the self-energy of the virtual particles are also needed for consistency with the CPT theorem.

3. CP violation in the leptonic sector

3.1 Scenarios for leptonic CP violation

In the standard model, no right-handed neutrinos are introduced. As a result, there is no mass matrix to diagonalize for the neutrinos. Hence the CKM matrix is the unit matrix, and no CP-violating phases can arise. However, in extensions of the standard model, CP violation can arise either because of the presence of neutrino masses or because of extra leptons introduced (even though neutrinos may be massless), or both.

A. Massive neutrinos. Neutrinos can have Dirac or Majorana masses. CP violation in the Dirac case is exactly analogous to that in the quark sector of the standard model. In case of Majorana masses, the freedom of phase redefinition of the neutral lepton fields is reduced because Majorana mass terms are not invariant under phase transformations. As a result there are more CP-violating phases in the CKM matrix than the corresponding Dirac case. It is thus possible to have CP violation with even two generations of Majorana neutrinos.

B. Massless neutrinos with exotic leptons. It is possible to have CP violation because of either charged or neutral leptons in exotic representations of $SU(2) \times$

$U(1)$. The leptons then have flavour-violating couplings to Z or Higgses, which can be complex and hence CP violating.

We consider below some leptonic CP-violating processes at high-energy colliders which make use of the above mechanisms of CP violation. The importance of leptonic processes stems from the fact that they are relatively clean from the experimental point of view.

3.2 Leptonic flavour violating Z decays

Leptons can have flavour-violating couplings to Z giving rise to flavour violating Z decays into charged leptons either at tree level or at one-loop level:

$$Z \to l_i \overline{l_j} \quad (i \neq j). \tag{18}$$

The corresponding CP-violating asymmetry is

$$A = \frac{\Gamma(Z \to l_i \overline{l_j}) - \Gamma(Z \to \overline{l_i} l_j)}{\Gamma(Z \to l_i \overline{l_j}) + \Gamma(Z \to \overline{l_i} l_j)}. \tag{19}$$

This is T even and therefore CPT odd. It therefore needs an absorptive part to be present.

Flavour-violating tree-level couplings of charged leptons to Z arise in models with exotic charged leptons transforming as either left-handed singlets and/or right-handed doublets. In such a case the Glashow-Weinberg condition for flavour-diagonal couplings is not sătisfied, and (18) occurs at tree level. For A of (19) to be non-zero, one-loop correction to (18) is also needed, and only the absorptive part of that amplitude contributes. The asymmetry is then $\mathcal{O}(\alpha)$[7].

On the other hand, models with exotic neutral leptons have flavour-violating couplings of neutral leptons at the tree-level giving rise to (18) at the loop level [8]. The absorptive part now comes from one of these loop diagrams. A is now $\mathcal{O}(1)$. However, unlike in the previous case, the rate of the flavour-violating process (18) is $\mathcal{O}(\alpha^3)$. Thus, the minimum total number N_Z of Z events for an observable asymmetry is in both cases $\mathcal{O}(1/\alpha^3)$. However, constraints on leptonic mixing angles and masses of exotic leptons make this process too rare to observe at LEP.

3.3 CP violation in $e^+e^- \to \gamma^*, Z^* \to l^+l^-$

In case of CP violation in $e^+e^- \to \gamma, Z \to l^+l^-$ there are two general results [9]:
(i) No CP violation can be seen without measuring initial or final spins. This follows basically because no CP-odd observable can be constructed without spins.
(ii) The only CP-violating couplings for the on-shell process are the dipole moment type couplings of e or l (electric or "weak" dipole moments). Since there are strong experimental limits on the electron and muon electric dipole moments ($d_e \lesssim 10^{-27}e$ cm, $d_\mu \lesssim 10^{-19}e$ cm), τ may be a good candidate for l. In fact, the weak moment of τ has been constrained using τ polarization in this reaction (see below).

Since it is clear from (i) that either initial or final spins have to be observed to detect CP violation, we consider below both these cases for $e^+e^- \to Z \to \tau^+\tau^-$.

3.3.1 CP tests using τ polarization in $e^+e^- \to \tau^+\tau^-$: This possibility has been discussed by several authors [2,10–12]. For the process

$$e^-(p_-) + e^+(p_+) \to \tau^-(k_-, s_-) + \tau^+(k_+, s_+), \tag{20}$$

possible CP-odd quantities that can be constructed out of the momenta and spins in the centre-of-mass (cm) frame are $(\mathbf{p}_- - \mathbf{p}_+)\cdot(\mathbf{s}_- - \mathbf{s}_+)$ and $(\mathbf{p}_- - \mathbf{p}_+)\cdot\mathbf{s}_- \times \mathbf{s}_+$.

To measure these quantities, one must, of course, be able to measure $\mathbf{s}_\pm$. This can be done by looking at decay distributions of $\tau_\pm$. In the rest frame of τ, the angular distribution of an observed decay particle is

$$\frac{d\Gamma}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha \mathbf{s}\cdot\widehat{\mathbf{q}}^*\right), \tag{21}$$

where $\widehat{\mathbf{q}}^*$ is the unit vector along the momentum of the observed particle, and α is a constant called the analyzing power of the channel. For example,

$$\begin{aligned}\alpha &= \mp 1 \quad \text{for} \quad \tau^\pm \to \pi^\pm \nu_\tau, \\ &= \pm 1/3 \quad \text{for} \quad \tau^\pm \to l^\pm \nu_l \nu_\tau,\end{aligned}$$

as deduced from the theory of weak τ decay. Using (21), spin correlations can be translated to momentum correlations.

In terms of the observed momenta, possible CP-odd variables are $\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ \times \mathbf{q}_-)$ $(CPT = +1)$, $\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ + \mathbf{q}_-)$ $(CPT = -1)$, $\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ \times \mathbf{q}_-)\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ - \mathbf{q}_-)$ $(CPT = +1)$, $\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ + \mathbf{q}_-)\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ - \mathbf{q}_-)$ $(CPT = -1)$. Expectation values of the last two were suggested by Bernreuther *et al.* [11, 12] for measuring respectively the real and imaginary parts of the τ weak dipole form factor $\tilde{d}_\tau(m_Z^2)$. The suggestion has been carried out at LEP for $\mathrm{Re}\,\tilde{d}_\tau(m_Z^2)$ by the OPAL [13] and ALEPH [14] groups. OPAL looked at inclusive leptonic and hadronic decays of τ, whereas ALEPH analyzed all channels exclusively. The results obtained are the 95% confidence-level upper limits $\mathrm{Re}\,\tilde{d}_\tau(m_Z^2) < 7.0 \times 10^{-17} e$ cm (OPAL [13]) and $\mathrm{Re}\,\tilde{d}_\tau(m_Z^2) < 3.7 \times 10^{-17} e$ cm (ALEPH [14]).

The theoretical prediction for the 1 s.d. limit obtainable in the measurement of $\mathrm{Im}\,\tilde{d}_\tau(m_Z^2)$ is 10^{-16} using the correlation $\langle \widehat{\mathbf{p}}\cdot(\mathbf{q}_+ + \mathbf{q}_-)\widehat{\mathbf{p}}\cdot(\mathbf{q}_+ - \mathbf{q}_-)\rangle$ and a sample of 10^7 Z's [12].

3.3.2 Longitudinal beam polarization: The Stanford Linear Collider (SLC), operating presently at the Z resonance, has an e^- polarization of about 62%, and is likely to reach 75% in the future. The present sample collected is of 50,000 Z's, and the hope is to reach 5×10^5, or even 10^6 Z's.

Can this longitudinal e^- polarization help in measuring the τ weak dipole moment? The answer is "yes" [15]. In fact, as we shall see, SLC can do better than LEP so far as $\mathrm{Im}\tilde{d}_\tau$ is concerned.

The essential point is that the vector polarization of Z gets enhanced in the presence of e^+e^- longitudinal polarization. For vanishing beam polarization, $P_{e^-} = P_{e^+} = 0$, the Z vector polarization is

$$P_Z^{(0)} = \frac{2 g_{Ve} g_{Ae}}{g_{Ae}^2 + g_{Ve}^2} \approx 0.16, \tag{22}$$

where g_{Ve}, g_{Ae} are vector and axial-vector couplings of e^+e^- to Z. For non-zero polarization,

$$P_Z = \frac{P_Z^{(0)} - P_{e^+e^-}}{1 - P_Z^{(0)} P_{e^+e^-}}, \tag{23}$$

where

$$P_{e^+e^-} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-}p_{e^+}}. \tag{24}$$

Thus, $P_Z \approx 0.71$ for $P_{e^-} = -.62$ and $P_{e^+} = 0$, which is an enhancement by a factor of about 4.5.

It is therefore profitable to look for CP-odd observables involving the Z spin $\mathbf{s}_Z = P_Z\widehat{\mathbf{p}}$, where $\widehat{\mathbf{p}}$ is the unit vector along $\mathbf{p}_+ = -\mathbf{p}_-$ in the c.m. frame. Examples of such observables are $\widehat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$ and $\widehat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)$. While both are CP odd, the former is CPT even and the latter is CPT odd.

The above is not entirely correct in principle. CP-odd correlations give a measure of underlying CP violation only if the initial state is CP even. Otherwise there may be contributions to correlations which arise from CP-invariant interactions due to the CP-odd part of the initial state. In the case when only the electron beam is polarized, the initial state is not CP even. In practice, however, this CP-even background is small because for $m_e \to 0$, only the CP-even helicity combinations $e_L^- e_R^+$ and $e_R^- e_L^+$ survive, making the corrections proportional to $m_e/m_Z \approx 5\times 10^{-6}$. If one includes order α collinear photon emission from the initial state, which could flip the helicity of the $e^\pm$, then like-helicity e^+e^- states could also survive for vanishing electron mass [16]. However, it turns out that this being a non-resonant effect, the corresponding cross section at the Z peak is small. It is therefore expected that the correlations coming from CP-invariant SM interactions in such a case will be negligible.

The correlations $\langle O_1 \rangle \sim \langle \widehat{\mathbf{p}} \cdot (\mathbf{q}_+ \times \mathbf{q}_-)\rangle$ and $\langle O_2 \rangle \sim \langle \widehat{\mathbf{p}} \cdot (\mathbf{q}_+ + \mathbf{q}_-)\rangle$ have been calculated analytically for the single-pion and ρ decay mode of each τ. Also calculated analytically are $\langle O_1^2 \rangle$ and $\langle O_2^2 \rangle$ needed for obtaining the 1 s.d. limit on the measurability of $\tilde{d}_\tau$ obtained using eq.(13) [15].

The results for only the single-pion channel are summarized in tables 1a and 1b, which give, respectively for O_1 and O_2 and for electron polarizations $P_{e^-} = 0, \pm 0.62$, the correlations in units of $\tilde{d}_\tau m_Z/e$, the square root of the variance, and the 1σ limit on $\mathrm{Re}\,\tilde{d}_\tau$ and $\mathrm{Im}\,\tilde{d}_\tau$ obtainable with 10^6 Z's. The enhancement of $\langle O_{1,2} \rangle$ and hence the sensitivity of $\tilde{d}_\tau$ measurement with polarization is evident from the tables.

Table 1a Correlation of O_1, the standard deviation from SM interactions, and the 1 s.d. limit on the real and imaginary parts of the weak dipole moment, for different polarizations P_e.

P_e	$\langle O_1 \rangle$ (GeV^2) for $\mathrm{Re}\,\tilde{d}_\tau = e/m_Z$	$\sqrt{\langle O_1^2 \rangle}$ (GeV^2)	1 s.d. limit on $\mathrm{Re}\,\tilde{d}_\tau$ for 10^6 Z's (in e cm)
0	0.90	12.86	1.5×10^{-16}
+0.62	−2.89	12.86	4.6×10^{-17}
−0.62	4.01	12.86	3.3×10^{-17}

Table 1b Quantities as in table 1a, but for O_2

P_e	$\langle O_2 \rangle$ (GeV) for $\mathrm{Im}\,\tilde{d}_\tau = e/m_Z$	$\sqrt{\langle O_2^2 \rangle}$ (GeV)	1 s.d. limit on $\mathrm{Im}\,\tilde{d}_\tau$ for 10^6 Z's (in e cm)
0	−0.16	9.57	6.2×10^{-16}
+0.62	0.51	9.57	1.9×10^{-16}
−0.62	−0.70	9.57	1.4×10^{-16}

The sensitivity can be further improved by looking at only the P_{e^-} dependent part of the cross section. This can be done by looking at a sample corresponding to the difference in the number of events for a polarization P_{e^-} and polarization $-P_{e^-}$. The correlations are evaluated with respect to $d\sigma(P_{e^-}) - d\sigma(-P_{e^-})$. This reduces the number of events. However, correlations are enhanced by a larger factor giving a net gain in the sensitivity. The result for $P_{e^-} = .62$ and 10^6 Z's is given in table 2.

Table 2 Quantities as in table 1a, but for a distribution asymmetrized between polarizations +0.62 and −0.62.

Observable	$\langle O \rangle$ for $\lvert\tilde{d}_\tau\rvert = e/m_Z$	$\sqrt{\langle O^2 \rangle}$	1 s.d. limit on $\lvert d_\tau \rvert$ for 10^6 Z's (in e cm)
O_1	35.55 GeV^2	12.86 GeV^2	1.2×10^{-17}
O_2	−6.22 GeV	9.57 GeV	5.0×10^{-17}

The 1 s.d. limit for $\mathrm{Im}\tilde{d}_\tau$ should be compared with the LEP expectation of 10^{-16} e cm for a larger sample of 10^7 Z's [12].

The use of correlations for measuring the τ edm at the proposed τ-charm factories employing longitudinal polarization of electron and positron beams has also been studied in detail in [17].

3.3.3 Transverse beam polarization: Use of transversely polarized e^+e^- beams for the study of CP violation has been studied by several people (see for example [2, 18]). For a reaction

$$e^-(p_-, s_-) + e^+(p_+, s_+) \to f(k_+) + \bar{f}(k_-) \tag{25}$$

in the c.m. frame, where f denotes a fermion, possible T-odd triple products are $\mathbf{s}_- \cdot (\mathbf{p} \times \mathbf{k})$, $\mathbf{s}_+ \cdot (\mathbf{p} \times \mathbf{k})$, $\mathbf{p} \cdot (\mathbf{s}_- \times \mathbf{s}_+)$, $\mathbf{k} \cdot (\mathbf{s}_- \times \mathbf{s}_+)$, where $\mathbf{p}_+ = -\mathbf{p}_- = \mathbf{p}$ and $\mathbf{k}_+ = -\mathbf{k}_- = \mathbf{k}$. Of these triple products, the last two are purely CP odd, whereas only the difference of the first two is CP odd. Burgess and Robinson [18] have done an analysis of $e^+e^- \to \tau^+\tau^-, c\bar{c}, b\bar{b}$ in terms of operators

$$\mathcal{L}_W = \lambda_W \left[\bar{L} P_R D^\mu E D_\mu \phi\right] + H.c., \tag{26}$$

$$\mathcal{L}_Y = \frac{i}{2}\lambda_Y \left[g_1 B_{\mu\nu} \left(\overline{L}\gamma^{\mu\nu} P_R E\right) \phi\right] + H.c. \tag{27}$$

Their results for $\lambda_W = (400\,\mathrm{GeV})^{-2}$ and $L = 4.8 \times 10^5\,\mathrm{pb}^{-1}$ at LEP are given in table 3, where A is the asymmetry for $\mathbf{s}_- \cdot (\mathbf{p} \times \mathbf{k})$ given by

$$\mathcal{A} = \left[\int d\sigma(\mathbf{p}_i, \mathbf{s}_i) - \int d\sigma(-\mathbf{p}_i, -\mathbf{s}_i)\right] \mathbf{s}_- \cdot (\mathbf{p} \times \mathbf{k}). \tag{28}$$

Though this is an interesting effect, a theoretical estimate for λ_W is needed before concluding whether the effect would be observable. Assuming a systematic error of 0.1%, the 2-σ limits possible on λ_W are estimated to be $(570\,\mathrm{GeV})^{-2}$ and $(660\,\mathrm{GeV})^{-2}$ respectively for up- and down-type quarks.

Table 3 The transverse polarization asymmetry $\mathcal{A}$ (defined in the text) compared with the standard model events for $e^+e^- \to \tau^+\tau^-, c\bar{c}, b\bar{b}$.

	$\tau^+\tau^-$	$c\bar{c}$	$b\bar{b}$
N (SM events)$\times 10^{-8}$	6.5	24	31
$\mathcal{A} \times 10^{-5}$	−6.1	97	−170
$\mathcal{A}/N \times 10^3$	0.9	4.0	5.5

4. CP violation in top pair production

Evidence for a top quark of mass of about 174 GeV at the Tevatron has been reported by the CDF collaboration [19]. Even though the data is not conclusive, it is generally believed that the top quark will eventually be found with a mass of a similar magnitude. Top-antitop pairs can then be produced at large rates at future colliders and used for various studies. In particular, since a heavy top ($m_t > 120$ GeV) decays before it hadronizes [20], information about its polarization is preserved in its decay products. Schmidt and Peskin [21] have suggested (elaborating on an old suggestion of Donoghue and Valencia [22]) looking for the asymmetry between $t_L\bar{t}_L$ and $t_R\bar{t}_R$ as a signal for CP violation (see also [23]). Note that this is possible only for a heavy particle like the top quark because for a light particle, the dominant helicity combination would be $t_L\bar{t}_R$ or $t_R\bar{t}_L$, each being self-conjugate.

The asymmetry $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ can be probed through the energy spectra of prompt leptons from $t \to Wb$; $W \to l\nu$. This is understood as follows.

For a heavy top, the dominant W helicity in $t \to Wb$ is 0. Now, due to $V - A$ interaction, b is produced with left-handed helicity (neglecting the b mass). Hence in the t rest frame, W^+ momentum is dominantly along the t spin direction. It follows that l^+ in W^+ decay is produced preferentially in the direction of the t spin. In fact, the distribution is $1 + \cos\psi$, where ψ is the angle between the l^+ momentum direction and the t spin direction. In going to the laboratory frame, the t gets boosted. Thus l^+ from t_R is more energetic than l^+ from t_L, and l^- from $\bar{t}_L$

has more energy than l^- from $\bar{t}_R$. Therefore, in the decay of $t_L\bar{t}_L$, l^- from $\bar{t}_L$ has higher energy than l^+ from t_L, and the reverse is true for $t_R\bar{t}_R$. Thus the energy asymmetry of leptons measures $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$.

Schmidt and Peskin [21] looked at this asymmetry in hadron collisions in a CP-violating multi-Higgs model where the CP violation is described by the Lagrangian terms

$$\delta\mathcal{L} = -\frac{m_t}{v}\phi\bar{t}\,[AP_L + A^* P_R]\,t, \tag{29}$$

where only the effect of the dominant lightest Higgs field ϕ is kept. v is the SM Higgs vacuum expectation value, and A is a complex combination of mixing angles and phases.

Since $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ is CP odd and T even, the CPT theorem requires the existence of an absorptive part for it to be non-zero. Looking at the tree and one-loop diagrams for $q\bar{q} \leftrightarrow t\bar{t}$, they conclude that

$$\mathcal{A} = \frac{N(t_L\bar{t}_L) - N(t_R\bar{t}_R)}{N(t_L\bar{t}_L) + N(t_R\bar{t}_R)} \approx 10^{-3} \tag{30}$$

for $m_t = 150$ GeV, $m_\phi = 100$–400 GeV and $\mathrm{Im}[A^2] = \sqrt{2}$. Thus the asymmetry $\mathcal{A}$ would be observable with 10^8 $t\bar{t}$ a year.

For pp collisions, since the pp state is not a CP eigenstate, there is also a CP-invariant contribution present, but this is shown to be small [21].

As in the case of pp or $p\bar{p}$ collisions, lepton energy asymmetry can be used to measure CP violation in $e^+e^- \to t\bar{t}$ [24]. The authors of ref.[24] also consider another asymmetry, viz., the up-down asymmetry of the charged leptons with respect to the $t\bar{t}$ production plane in the laboratory frame. It is also possible to construct a "left-right" asymmetry of leptons with respect to a plane perpendicular to the $t\bar{t}$ production plane, but containing the $t\bar{t}$ momentum direction [25]. Certain combinations of up-down and left-right symmetry with forward-backward asymmetry can also be considered. All these probe different combinations of CP-violating couplings [25].

The result of ref.[24] is that asymmetry is at the few per cent level for the top-quark electric and weak dipole moments $d_t, \tilde{d}_t \sim e/m_t$. For $\sqrt{s} = 300$ GeV and $m_t = 120$ GeV, $\sigma \approx 1400$ fb, which corresponds to 20,000 prompt leptons a year for a luminosity of 10^{33} cm^{-2}s^{-1}. In such a case, $d_t, \tilde{d}_t$ can be determined to a few percent level. This should be compared with the prediction of $10^{-2} - 10^{-3}$ from the Higgs model for CP violation.

Apart from the above asymmetries, CP-odd correlations could provide a measure of CP violation in $t\bar{t}$ production [26]. Certain correlations are more sensitive to CP violation in t decay, rather than production [26].

Another process which has been suggested is $e^+e^- \to t\bar{t}\nu\bar{\nu}$ through $W^+W^- \to t\bar{t}$ [4]. Two diagrams contribute to $W^+W^- \to t\bar{t}$, one corresponding to t-channel b exchange and the other with an s-channel heavy Higgs ϕ, which can be on shell for $m_\phi > 2m_t$. Then, the absorptive part needed for a CP-odd CPT-odd asymmetry is provided by the width of the Higgs. A sizable asymmetry can be obtained thus [4].

At linear colliders, there is a possibility of producing electron beams with longitudinal polarization. This may be exploited to enhance the sensitivity of measurememnt of the top dipole moments as well as to measure the electric and weak dipole moments independently [25, 27]

5. CP violation in other processes

The process $e^+e^- \to W^+W^-$, which will be studied in the near future at LEP200, will be the first one to be able to test the SM couplings of the electroweak gauge bosons. The process is expected to put bounds on non-standard γ and Z couplings to W^+W^-. The non-standard couplings could be CP violating ones, as in (2). These can be studied in a way similar to the one used for probing CP violation in $e^+e^- \to \tau^+\tau^-$ and $e^+e^- \to t\bar{t}$ considered earlier. As in the case of $e^+e^- \to t\bar{t}$ with leptonic $t, \bar{t}$ decays, energy or up-down asymmetry of leptons may be used [28, 29]. Whereas Chang *et al.* [28] consider asymmetries, Mani *et al* [29] gave estimated the energy correlation ratio

$$A = \frac{\langle E_- \rangle - \langle E_+ \rangle}{\langle E_- \rangle + \langle E_+ \rangle},$$

and suggest the angular correlation ratio

$$\delta = \frac{\langle \theta_- \rangle - \langle \theta_+ \rangle}{\langle \theta_- \rangle + \langle \theta_+ \rangle},$$

where $E_\pm$ are the energies of the leptons $l^\pm$ produced in the decay of $W^\pm$, and $\theta_\pm$ are the angles of $l^\pm$ momenta with respect to the e^+ beam direction. The general expectation is $A \approx 10^{-3}$ for $\tilde{\kappa}$ or $\tilde{\lambda} \approx 0.1$.

Some other processes considered in the literature are $t, \bar{t}$ decay asymmetries in $\phi \to t\bar{t}$, where ϕ is a heavy Higgs [30], decay lepton asymmetries in $e^+e^- \to \tilde{\chi}\tilde{\chi}$, where $\tilde{\chi}$ is a neutralino in the minimal supersymmetric standard model [31], decay correlations in $\gamma\gamma \to W^+W^-$ [32], and forward-backward asymmetry in $e^+e^- \to \gamma Z$ [33].

6. Conclusions

We have seen above the various points to be kept in mind when selecting processes and variables for detecting and measuring CP violation, a number of processes and scenarios of CP-violating signatures which could be looked for. The above discussion is mainly aimed at arriving at an idea of the sensitivities possible in different measurements. In general, the results in most popular models of CP violation beyond SM indicate that CP violation in the most optimistic theoretical scenario would be measurable only with some difficulty in the existing or presently envisaged experiments. Nevertheless, it would be good to keep one's eyes open to these possibilities.

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