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Single decay-lepton angular distributions in polarized $e^+e^- \to t\bar{t}$ and simple angular asymmetries as a measure of CP-violating top dipole couplings

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Abstract. In the presence of an electric dipole coupling of $t\bar{t}$ to a photon, and an analogous 'weak' dipole coupling to the Z, CP violation in the process $e^+e^- \to t\bar{t}$ results in modified polarization of the top and the anti-top. This polarization can be analyzed by studying the angular distributions of decay charged leptons when the top or anti-top decays leptonically. Analytic expressions are presented for these distributions when either t or $\bar{t}$ decays leptonically, including $\mathcal{O}(\alpha_s)$ QCD corrections in the soft-gluon approximation. The angular distributions are insensitive to anomalous interactions in top decay. Two types of simple CP-violating polar-angle asymmetries and two azimuthal asymmetries, which do not need the full reconstruction of the t or $\bar{t}$, are studied. Independent 90% CL limits that may be obtained on the real and imaginary parts of the electric and weak dipole couplings at a linear collider operating at $\sqrt{s} = 500$ GeV with integrated luminosity 500 fb^{-1} and also at $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb^{-1} have been evaluated. The effect of longitudinal electron and/or positron beam polarizations has been included.

Keywords. Top quark; CP violation; e^+e^- collider.

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1. Introduction

An e^+e^- linear collider operating at centre-of-mass (cm) energy of 500 GeV or higher and with an integrated luminosity of several hundred inverse femtobarns should be able to study with precision various properties of the top quark. The possibility of setting up such a collider is under consideration at a number of places at the moment, particularly, the Joint Linear Collider (JLC) in Japan [1], TESLA in Germany [2], and the Next Linear Collider (NLC) in the USA [3].

While the standard model (SM) predicts CP violation outside the K-, D- and B-meson systems to be unobservably small, in some extensions of SM, CP violation might be considerably enhanced, especially in the presence of a heavy top quark. In particular, CP-violating electric dipole form factor of the top quark, and the analogous CP-violating 'weak' dipole form factor in the $t\bar{t}$ coupling to Z could be enhanced. These CP-violating form factors could be determined in a model-independent way at high energy e^+e^- linear colliders, where $e^+e^- \to t\bar{t}$ would proceed through virtual γ and Z exchange.

Since a heavy top quark with a mass of the order of 175 GeV decays before it hadronizes [4], it has been suggested [5] that top polarization asymmetry in $e^+e^- \to t\bar{t}$ can be used to determine the CP-violating dipole form factors, since polarization information would be retained in the decay product distribution. There have been several proposals in which the CP-violating dipole couplings could be measured in decay momentum correlations or asymmetries with or without beam polarization. For a review, see [6].

In this context it is important to note that top polarization can only be studied using top decay. Therefore, for the information from decay distributions to reflect top polarization correctly, the decay amplitudes for various top polarization states have to be known accurately. In particular, if there are any anomalous effects in the decay process, they have to be known accurately. Better still, the decay distributions chosen for the study have to be insensitive to anomalous effects in the decay process. The single-lepton angular distributions that we discuss in this work satisfy the latter condition – they accurately reflect the polarization of the top quark resulting from the production process, while one can continue to use SM in the decay process.

It has been found that one-loop QCD corrections to the process $e^+e^- \to t\bar{t}$ can be as high as 30% for cm energy $\sqrt{s} = 500$ GeV [7]. It is therefore important to examine the effect of these QCD corrections in the decay lepton distributions [8], and their consequent effect on the measurement of CP-violating couplings.

In this paper we re-visit some suggestions made [9–11] for the measurement of top dipole moments in $e^+e^- \to t\bar{t}$ using angular asymmetries of the charged lepton produced in the semi-leptonic decay of one of t and $\bar{t}$, while the other decays hadronically. The purpose is to highlight certain features of the proposal which have become more significant in the light of recent developments, and to update the numerical results. The improvements included in this update are several. Firstly, $\mathcal{O}(\alpha_s)$ QCD corrections in the soft-gluon approximation have now been included. Secondly, a simplification used in earlier work [9–11], that of neglecting CP violation in top decay, has been dispensed with in the light of recent work [12,13]. It turns out that for angular asymmetries of the charged lepton considered here, CP violation in the decay (or for that matter even arbitrary CP-conserving modifications of the tbW vertex) has no effect, if the b-quark mass is neglected. Finally, there is now a better idea of luminosities possible at a future linear collider. Together with updated values of beam polarization now considered feasible, the estimates of possible limits on dipole moments would be more realistic. Thus the estimates in earlier work have been improved upon and put on sounder theoretical footing.

Earlier proposals have considered a variety of CP-violating observables, with varying sensitivities. These include, in addition to angular asymmetries, also vector and tensor correlations [14,15], and expectation values of optimal variables [16]. (For a discussion on relative sensitivities of some variables, see [17].) We have chosen certain angular asymmetries here which have some advantages over others, even though they may not be the most sensitive ones. The advantages are:

(i) Our asymmetries are in the laboratory frame, making them directly observable.
(ii) They depend on final state momenta, rather than on top polarization. Polarization is measured only indirectly through the decay distributions. We, therefore, concentrate only on actual decay-lepton distributions, which are the simplest to observe.
(iii) The observables we choose either do not depend on precise determination of energy

and momentum of top quarks, or, in case of azimuthal asymmetries of the lepton, depend minimally on the top momentum direction for the sake of defining the coordinate axes. This has the advantage of higher accuracy.

(iv) As stated before, leptonic angular distribution is free from background CP violation in top decay, and gives a direct handle on anomalous couplings in top production.

(v) The polar-angle asymmetries we consider can be obtained in analytical form, which is useful for making quick computations. It is possible to get analytical forms for certain azimuthal asymmetries as well, provided no angular cuts are imposed.

(vi) The asymmetries considered here are rather simple conceptually, and hopefully, also from the practical measurement point of view.

Our single-lepton asymmetries have another obvious advantage, that since either t or $\bar{t}$ is allowed to decay hadronically, there is a gain in statistics, as compared to double-lepton asymmetries.

Our results are based on fully analytical calculation of single lepton distributions in the production and subsequent decay of $t\bar{t}$. We present fully differential angular distribution as well as the distribution in the polar angle of the lepton with respect to the beam direction in the centre-of-mass (cm) frame for arbitrary longitudinal beam polarizations. These distributions for SM were first obtained by Arens and Sehgal [18]. Distributions including the effect of CP violation only in production were obtained in [10,11], whereas, with all anomalous effects included in the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices, as well as decay tbW vertex were obtained in [12,13]. Angular distributions in SM with $\mathscr{O}(\alpha_s)$ QCD corrections in the soft-gluon approximation were obtained in [8]. The distributions including anomalous effects in both top production and decay, and including $\mathscr{O}(\alpha_s)$ QCD corrections in the soft-gluon approximation are presented here for the first time. While QCD corrections to $e^+e^- \to t\bar{t}$ are substantial, to the extent of about 30% at $\sqrt{s} = 500$ GeV, their effect on leptonic angular distributions is much smaller [8]. The main effect on the results will be to the sensitivity, through the $1/\sqrt{N}$ factor, where N is the number of events. A part of this work was reported in [19].

The rest of the paper is organized as follows: In §2, we describe the calculation of the decay-lepton angular distribution from a decaying t or $\bar{t}$ in $e^+e^- \to t\bar{t}$. In §3, we describe CP-violating asymmetries. Numerical results are presented in §4, and §5 contains our conclusions. The Appendix contains certain expressions which are too lengthy to be put in the main text.

2. Calculation of lepton angular distributions

We describe in this section the calculation of l^+ (l^-) distribution in $e^+e^- \to t\bar{t}$ and the subsequent decay $t \to bl^+\nu_l$ $(\bar{t} \to \overline{b}l^-\overline{\nu_l})$. We adopt the narrow-width approximation for t and $\bar{t}$, as well as for $W^\pm$ produced in t, $\bar{t}$ decay.

We assume the top quark couplings to γ and Z to be given by the vertex factor $ie\Gamma^j_\mu$, where

$$\Gamma^j_\mu \;=\; c^j_v \gamma_\mu \;+\; c^j_a \gamma_\mu \gamma_5 \;+\; \frac{c^j_d}{2\,m_t}\, i\gamma_5\,(p_t - p_{\bar{t}})_\mu, \;\; j \;=\; \gamma, Z, \tag{1}$$

with

$$c_v^\gamma = \frac{2}{3}, \quad c_a^\gamma = 0,$$
$$c_v^Z = \frac{\left(\frac{1}{4} - \frac{2}{3} x_W\right)}{\sqrt{x_W (1 - x_W)}},$$
$$c_a^Z = -\frac{1}{4\sqrt{x_W (1 - x_W)}}, \tag{2}$$

and $x_W = \sin^2\theta_W$, θ_W being the weak mixing angle. In addition to the SM couplings $c_{v,a}^{\gamma,Z}$ we have introduced the CP-violating electric and weak dipole form factors, ec_d^γ/m_t and ec_d^Z/m_t, which are assumed small. The Dirac equation is used to rewrite the usual dipole coupling $\sigma_{\mu\nu}(p_t + p_{\bar{t}})^\nu \gamma_5$ as $i\gamma_5(p_t - p_{\bar{t}})_\mu$, dropping small corrections to the vector and axial-vector couplings. We will work in the approximation in which we keep only linear terms in c_d^γ and c_d^Z. Addition of other CP-conserving form factors will not change our results in the linear approximation.

To include $\mathscr{O}(\alpha_s)$ corrections in the soft-gluon approximation (SGA), we need to modify the above vertices, as explained in [8]. These modified vertices are given by

$$\Gamma_\mu^\gamma = c_v^\gamma \gamma_\mu + \left[c_M^\gamma + i\gamma_5 c_d^\gamma\right] \frac{(p_t - p_{\bar{t}})_\mu}{2m_t}, \tag{3}$$

$$\Gamma_\mu^Z = c_v^Z \gamma_\mu + c_a^Z \gamma_\mu \gamma_5 + \left[c_M^Z + i\gamma_5 c_d^Z\right] \frac{(p_t - p_{\bar{t}})_\mu}{2m_t}, \tag{4}$$

where

$$c_v^\gamma = \frac{2}{3}(1 + A), \tag{5}$$
$$c_v^Z = \frac{1}{\sin\theta_W \cos\theta_W} \left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)(1 + A), \tag{6}$$
$$c_a^\gamma = 0, \tag{7}$$
$$c_a^Z = \frac{1}{\sin\theta_W \cos\theta_W} \left(-\frac{1}{4}\right)(1 + A + 2B), \tag{8}$$
$$c_M^\gamma = \frac{2}{3}B, \tag{9}$$
$$c_M^Z = \frac{1}{\sin\theta_W \cos\theta_W} \left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right) B. \tag{10}$$

The form factors A and B are given to order α_s in SGA (see, for example, [7,20]) by

$$\begin{aligned} \mathrm{Re}A = \hat{\alpha}_s \Bigg[& \left(\frac{1+\beta^2}{\beta} \log\frac{1+\beta}{1-\beta} - 2\right) \log\frac{4\omega_{\max}^2}{m_t^2} - 4 \\ & + \frac{2+3\beta^2}{\beta} \log\frac{1+\beta}{1-\beta} + \frac{1+\beta^2}{\beta} \Bigg\{ \log\frac{1-\beta}{1+\beta} \left(3\log\frac{2\beta}{1+\beta}\right. \\ & \left. + \log\frac{2\beta}{1-\beta}\right) + 4\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{3}\pi^2 \Bigg\} \Bigg], \end{aligned} \tag{11}$$

$$\mathrm{Re}\, B = \hat{\alpha}_s \frac{1-\beta^2}{\beta} \log \frac{1+\beta}{1-\beta}, \tag{12}$$

$$\mathrm{Im} B = -\hat{\alpha}_s \pi \frac{1-\beta^2}{\beta}, \tag{13}$$

where $\hat{\alpha}_s = \alpha_s/(3\pi)$, $\beta = \sqrt{1-4m_t^2/s}$, and Li_2 is the Spence function. ReA in eq. (11) contains the effective form factor for a cut-off $\omega_{\max}$ on the gluon energy after the infrared singularities have been cancelled between the virtual- and soft-gluon contributions in the on-shell renormalization scheme. Only the real part of the form factor A has been given, because the contribution of the imaginary part is proportional to the Z width, and hence negligibly small [7,21]. The imaginary part of B, however, contributes to the azimuthal distributions.

The helicity amplitudes for $e^+e^- \to \gamma^*, Z^* \to t\bar{t}$ in the cm frame, including $c_d^{\gamma,Z}$ and $c_M^{\gamma,Z}$ couplings, have been given in [22] (see also [5]).

We write the contribution of a general tbW vertex to t and $\bar{t}$ decays as

$$\begin{aligned}\Gamma^\mu_{tbW} = &-\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \Big[\gamma^\mu (f_{1L} P_L + f_{1R} P_R) \\ &- \frac{i}{m_W} \sigma^{\mu\nu} (p_t - p_b)_\nu (f_{2L} P_L + f_{2R} P_R)\Big] u(p_t),\end{aligned} \tag{14}$$

$$\begin{aligned}\bar{\Gamma}^\mu_{tbW} = &-\frac{g}{\sqrt{2}} V^*_{tb} \bar{v}(p_{\bar{t}}) \Big[\gamma^\mu (\bar{f}_{1L} P_L + \bar{f}_{1R} P_R) \\ &- \frac{i}{m_W} \sigma^{\mu\nu} (p_{\bar{t}} - p_{\bar{b}})_\nu (\bar{f}_{2L} P_L + \bar{f}_{2R} P_R)\Big] v(p_{\bar{b}}),\end{aligned} \tag{15}$$

where $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$, and V_{tb} the Cabibbo–Kobayashi–Maskawa matrix element, which we take to be equal to one. If CP is conserved, the form factors f above obey the relations

$$f_{1L} = \bar{f}_{1L}; \quad f_{1R} = \bar{f}_{1R} \tag{16}$$

and

$$f_{2L} = \bar{f}_{2R}; \quad f_{2R} = \bar{f}_{2L}. \tag{17}$$

Like c_d^γ and c_d^Z above, we will also treat $f_{2L,R}$ and $\bar{f}_{2L,R}$ as small, and retain only terms linear in them. For the form factors f_{1L} and $\bar{f}_{1L}$, we retain their SM values, viz., $f_{1L} = \bar{f}_{1L} = 1$. f_{1R} and $\bar{f}_{1R}$ do not contribute in the limit of vanishing b mass, which is used here. Also, f_{2L} and $\bar{f}_{2R}$ drop out in this limit.

The helicity amplitudes for

$$t \to bW^+, \; W^+ \to l^+ \nu_l$$

and

$$\bar{t} \to \bar{b} W^-, \; W^- \to l^- \bar{\nu}_l$$

in the respective rest frames of t, $\bar{t}$, in the limit that all masses except the top mass are neglected, are given in ref. [13].

Combining the production and decay amplitudes in the narrow-width approximation for $t, \bar{t}, W^+, W^-$, and using appropriate Lorentz boosts to calculate everything in the e^+e^- cm frame, we get the l^+ and l^- angular distributions for the case of e^-, e^+ with polarization P_e, $P_{\bar{e}}$ to be:

$$\begin{aligned}\frac{\mathrm{d}^3\sigma^\pm}{\mathrm{d}\cos\theta_t \,\mathrm{d}\cos\theta_l \,\mathrm{d}\phi_l} = {} & \frac{3\alpha^2\beta m_t^2}{8s^2} B_t B_{\bar{t}} \frac{1}{(1-\beta\cos\theta_{tl})^3} \\ & \times \left[\mathscr{A}^\pm(1-\beta\cos\theta_{tl}) + \mathscr{B}^\pm(\cos\theta_{tl}-\beta)\right. \\ & + \mathscr{C}^\pm(1-\beta^2)\sin\theta_t\sin\theta_l(\cos\theta_t\cos\phi_l - \sin\theta_t\cot\theta_l) \\ & \left. + \mathscr{D}^\pm(1-\beta^2)\sin\theta_t\sin\theta_l\sin\phi_l\right], \end{aligned} \tag{18}$$

where σ^+ and σ^- refer respectively to l^+ and l^- distributions, with the same notation for the kinematic variables of particles and antiparticles. Thus, θ_t is the polar angle of t (or $\bar{t}$), and E_l, θ_l, ϕ_l are the energy, polar angle and azimuthal angle of l^+ (or l^-). All the angles are now in the cm frame, with the z-axis chosen along the e^- momentum, and the x-axis chosen in the plane containing the e^- and t directions. θ_{tl} is the angle between the t and l^+ directions (or $\bar{t}$ and l^- directions). β is the t (or $\bar{t}$) velocity: $\beta = \sqrt{1-4m_t^2/s}$, and $\gamma = 1/\sqrt{1-\beta^2}$. B_t and $B_{\bar{t}}$ are respectively the branching ratios of t and $\bar{t}$ into the final states being considered.

The coefficients $\mathscr{A}^\pm$, $\mathscr{B}^\pm$, $\mathscr{C}^\pm$ and $\mathscr{D}^\pm$ are given by

$$\mathscr{A}^\pm = A_0 \pm A_1\cos\theta_t + A_2\cos^2\theta_t, \tag{19}$$

$$\mathscr{B}^\pm = B_0^\pm \pm B_1\cos\theta_t + B_2^\pm\cos^2\theta_t, \tag{20}$$

$$\mathscr{C}^\pm = \pm C_0^\pm + C_1^\pm\cos\theta_t, \tag{21}$$

$$\mathscr{D}^\pm = \pm D_0^\pm + D_1^\pm\cos\theta_t. \tag{22}$$

The quantities A_i, $B_i^\pm$, $C_i^\pm$ and $D_i^\pm$ occurring in the above equations are functions of the masses, s, the degrees of e and $\bar{e}$ polarization (P_e and $P_{\bar{e}}$), and the coupling constants. They are listed in the Appendix.

It should be emphasized that, as shown in [12,13], the distribution in (18) does not depend on anomalous effects in the tbW vertices (14) and (15). In the limit of small b-quark mass, and in the linear approximation, the effect of all possible form factors in the angular distributions is the same overall factor which appears in the total width. Consequently, this factor cancels out with another appearing in the denominator [12,13]. As a consequence, the angular distributions are totally independent of any anomalous effects, CP-violating or CP-conserving, in the decay vertex. Thus even $\mathscr{O}(\alpha_s)$ QCD corrections to the tbW vertices would not be felt in (18).

To obtain the single-differential polar-angle distribution, we integrate over ϕ from 0 to 2π, and finally over $\cos\theta_t$ from -1 to $+1$. The final result is

$$\frac{\mathrm{d}\sigma^\pm}{\mathrm{d}\cos\theta_l} = \frac{3\pi\alpha^2}{32s} B_t B_{\bar{t}} \beta \left\{ 4A_0 \mp 2A_1 \left(\frac{1-\beta^2}{\beta^2}\log\frac{1+\beta}{1-\beta} - \frac{2}{\beta}\right)\cos\theta_l \right.$$

$$
\begin{aligned}
&+2A_2\left(\frac{1-\beta^2}{\beta^3}\log\frac{1+\beta}{1-\beta}(1-3\cos^2\theta_l)\right.\\
&\left.-\frac{2}{\beta^2}(1-3\cos^2\theta_l-\beta^2+2\beta^2\cos^2\theta_l)\right)\\
&\pm 2B_1\frac{1-\beta^2}{\beta^2}\left(\frac{1}{\beta}\log\frac{1+\beta}{1-\beta}-2\right)\cos\theta_l\\
&+B_2^{\pm}\frac{1-\beta^2}{\beta^3}\left(\frac{\beta^2-3}{\beta}\log\frac{1+\beta}{1-\beta}+6\right)(1-3\cos^2\theta_l)\\
&\pm 2C_0^{\pm}\frac{1-\beta^2}{\beta^2}\left(\frac{1-\beta^2}{\beta}\log\frac{1+\beta}{1-\beta}-2\right)\cos\theta_l\\
&\left.-C_1^{\pm}\frac{1-\beta^2}{\beta^3}\left(\frac{3(1-\beta^2)}{\beta}\log\frac{1+\beta}{1-\beta}-2(3-2\beta^2)\right)(1-3\cos^2\theta_l)\right\}. \qquad (23)
\end{aligned}
$$

This is the same expression as in [10] and [13]. However, the significance of the functions A_i, B_i, C_i and D_i is different in each case.

We now proceed to a discussion of CP-odd asymmetries resulting from the use of the above distributions.

3. CP-violating angular asymmetries

We will work with two different types of asymmetries, one which does not depend on the azimuthal angles of the decay lepton, so that the azimuthal angle is fully integrated over, and the other dependent on the azimuthal angle. In all cases, we assume a cut-off of θ_0 on the forward and backward directions of the charged lepton. Some cut-off on the forward and backward angles is certainly needed from an experimental point of view; we furthermore exploit the cut-off to optimize the sensitivity.

In the first case, namely polar asymmetries, we define two independent CP-violating asymmetries, which depend on different linear combinations of $\mathrm{Im}c_d^\gamma$ and $\mathrm{Im}c_d^Z$. (It is not possible to define CP-odd quantities which determine $\mathrm{Re}c_d^{\gamma,Z}$ using single-lepton polar distributions, as can be seen from the expression for the CP-odd combination $((\mathrm{d}\sigma^+/\mathrm{d}\cos\theta_l)(\theta_l))-((\mathrm{d}\sigma^-/\mathrm{d}\cos\theta_l)(\pi-\theta_l)))$. One is simply the total lepton-charge asymmetry, with a cut-off of θ_0 on the forward and backward directions:

$$
A_{\mathrm{ch}}(\theta_0)=\frac{\displaystyle\int_{\theta_0}^{\pi-\theta_0}\mathrm{d}\theta_l\left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}\theta_l}-\frac{\mathrm{d}\sigma^-}{\mathrm{d}\theta_l}\right)}{\displaystyle\int_{\theta_0}^{\pi-\theta_0}\mathrm{d}\theta_l\left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}\theta_l}+\frac{\mathrm{d}\sigma^-}{\mathrm{d}\theta_l}\right)}. \qquad (24)
$$

The other is the leptonic forward–backward asymmetry combined with charge asymmetry, again with the angles within θ_0 of the forward and backward directions excluded:

$$
A_{\mathrm{fb}}(\theta_0)=\frac{\displaystyle\int_{\theta_0}^{\pi/2}\mathrm{d}\theta_l\left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}\theta_l}+\frac{\mathrm{d}\sigma^-}{\mathrm{d}\theta_l}\right)-\int_{\pi/2}^{\pi-\theta_0}\mathrm{d}\theta_l\left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}\theta_l}+\frac{\mathrm{d}\sigma^-}{\mathrm{d}\theta_l}\right)}{\displaystyle\int_{\theta_0}^{\pi-\theta_0}\mathrm{d}\theta_l\left(\frac{\mathrm{d}\sigma^+}{\mathrm{d}\theta_l}+\frac{\mathrm{d}\sigma^-}{\mathrm{d}\theta_l}\right)}. \qquad (25)
$$

Analytic expressions for both these aymmetries may be easily obtained using (23), and are not displayed here explicitly.

We note the fact that $\mathscr{A}_{\rm ch}(\theta_0)$ vanishes for $\theta_0 = 0$, since then it is simply the asymmetry between the total rates of production of l^+ and l^-. It then vanishes so long as CP violation in decay does not contribute. $\mathscr{A}_{\rm fb}(\theta_0)$, however, is non-zero for $\theta_0 = 0$. This implies that the CP-violating charge asymmetry does not exist unless a cut-off is imposed on the lepton production angle. $\mathscr{A}_{\rm fb}(\theta_0)$, however, is non-zero for $\theta_0 = 0$.

We now define angular asymmetries of the second type, which depend on the range of the azimuthal angle ϕ_l of the charged lepton. These are called the up–down and left–right asymmetries, and depend respectively on the real and imaginary parts of the dipole couplings.

The up–down asymmetry is defined by

$$A_{\rm ud}(\theta_0) = \frac{1}{2\,\sigma(\theta_0)} \int_{\theta_0}^{\pi-\theta_0} \left[\frac{{\rm d}\sigma^+_{\rm up}}{{\rm d}\theta_l} - \frac{{\rm d}\sigma^+_{\rm down}}{{\rm d}\theta_l} + \frac{{\rm d}\sigma^-_{\rm up}}{{\rm d}\theta_l} - \frac{{\rm d}\sigma^-_{\rm down}}{{\rm d}\theta_l} \right] {\rm d}\theta_l, \tag{26}$$

where

$$\sigma(\theta_0) = \int_{\theta_0}^{\pi-\theta_0} \frac{{\rm d}\sigma}{{\rm d}\theta_l} {\rm d}\theta_l \tag{27}$$

is the SM cross-section for the semi-leptonic final state, with a forward and backward cut-off of θ_0 on θ_l. Here up/down refers to $(p_{l^\pm})_y \gtrless 0,\ (p_{l^\pm})_y$ being the y component of $\vec{p}_{l^\pm}$ with respect to a coordinate system chosen in the e^+e^- center-of-mass (cm) frame so that the z-axis is along $\vec{p}_e$, and the y-axis is along $\vec{p}_e \times \vec{p}_t$. The $t\bar{t}$ production plane is thus the xz plane. Thus, 'up' refers to the range $0 < \phi_l < \pi$, and 'down' refers to the range $\pi < \phi_l < 2\pi$.

The left–right asymmetry is defined by

$$A_{\rm lr}(\theta_0) = \frac{1}{2\,\sigma(\theta_0)} \int_{\theta_0}^{\pi-\theta_0} \left[\frac{{\rm d}\sigma^+_{\rm left}}{{\rm d}\theta_l} - \frac{{\rm d}\sigma^+_{\rm right}}{{\rm d}\theta_l} + \frac{{\rm d}\sigma^-_{\rm left}}{{\rm d}\theta_l} - \frac{{\rm d}\sigma^-_{\rm right}}{{\rm d}\theta_l} \right] {\rm d}\theta_l. \tag{28}$$

Here left/right refers to $(p_{l^\pm})_x \gtrless 0,\ (p_{l^\pm})_x$ being the x component of $\vec{p}_{l^\pm}$ with respect to the coordinate system defined above. Thus, 'left' refers to the range $-\pi/2 < \phi_l < \pi/2$, and 'right' refers to the range $\pi/2 < \phi_l < 3\pi/2$.

Analytic expressions for the up–down and left–right symmetry are not available for non-zero cut-off in θ_l. Hence, the angular integrations have been done numerically in what follows.

These azimuthal asymmetries with a different choice of axes were discussed in [22,9], without a cut-off θ_0. Two other asymmetries were defined in [9], which helped to disentangle the two dipole couplings from each other. However, we do not discuss these here. Instead, we will assume that the electron beam polarization can be made to change sign to give additional observable quantities to enable this disentanglement.

All these asymmetries are a measure of CP violation in the unpolarized case and in the case when polarization is present, but $P_e = -P_{\bar{e}}$. When $P_e \neq -P_{\bar{e}}$, the initial state is not invariant under CP, and therefore CP-invariant interactions can contribute to the asymmetries. However, to leading order in α, these CP-invariant contributions vanish in the limit

$m_e = 0$. Order-α collinear helicity-flip photon emission can give a CP-even contribution. However, this background has been estimated in [23], and found to be negligible for certain CP-odd correlations for the kind of luminosities under consideration. It has also been estimated for $A_{\rm fb}$ and $A_{\rm ch}$, and again found negligible [24]. The background is zero in the case of $A_{\rm ud}$ [24]. It is expected that the background will also be negligible for $A_{\rm lr}$ though it has not been calculated explicitly.

4. Numerical results

In this section we describe results for the calculation of 90% confidence level (CL) limits that could be put on $\mathrm{Re}c_d^{\gamma,Z}$ and $\mathrm{Im}c_d^{\gamma,Z}$ using the asymmetries described in the previous sections.

We look at only semi-leptonic final states. That is to say, when t decays leptonically, we assume $\bar{t}$ decays hadronically, and vice versa. We sum over the electron and muon decay channels. Thus, $B_t B_{\bar{t}}$ is taken to be $2/3 \times 2/9$.

We have considered unpolarized beams, as well as the case when the electron beam has a longitudinal polarization of 80%, either left-handed or right-handed. We have also considered the possibility of two runs for identical time-spans with the polarization reversed in the second run. The positron beam is assumed to be unpolarized. Later on, we discuss the results in the case when the positron beam is also polarized.

We assume an integrated luminosity of 500 fb^{-1} for a cm energy of 500 GeV, and an integrated luminosity of 1000 fb^{-1} for a cm energy of 1000 GeV. The limits for different integrated luminosities can easily be obtained by scaling appropriately the limits presented here by the inverse square root of the factor by which the luminosity is scaled. We comment later on the results for a cm energy of 800 GeV with an integrated luminosity of 800 GeV.

We use the parameters $\alpha = 1/128$, $\alpha_s(m_Z^2) = 0.118$, $m_Z = 91.187$ GeV, $m_W = 80.41$ GeV, $m_t = 175$ GeV and $\sin^2\theta_W = 0.2315$. We have used, following [7], a gluon energy cut-off of $\omega_{\rm max} = (\sqrt{s} - 2m_t)/5$. While qualitative results would be insensitive, exact quantitative results would, of course, depend on the choice of cut-off.

Figure 1 shows the SM cross-section $\sigma(\theta_0)$, defined in eq. (27), for t or $\bar{t}$ production, followed by its semi-leptonic decay, with a cut-off θ_0 on the lepton polar angle, plotted against θ_0 for the two choices of $\sqrt{s}$ and for different electron beam polarizations.

Figure 2 shows the asymmetry $A_{\rm ch}$ defined in eq. (24) arising when either of the (imaginary parts of) electric and weak dipole couplings takes the value 1, the other taking the value 0, plotted as a function of the cut-off θ_0, for the polarized and unpolarized cases, for two different cm energies. Figure 3 is the corresponding figure for $A_{\rm fb}$ defined in eq. (25).

Similarly, the asymmetries $A_{\rm ud}$ from eq. (26) and $A_{\rm lr}$ from eq. (28), which depend respectively on the real and imaginary parts of $c_d^{\gamma,Z}$, are shown in figures 4 and 5. Again, only one of the couplings takes a non-zero value, in this case 0.1, while the others are vanishing.

Tables 1–5 show the results on the limits obtainable for each of these possibilities. In all cases, the value of the cut-off θ_0 has been chosen to get the best sensitivity for that specific item. In case of $A_{\rm fb}$, the sensitivity is maximum for $\theta_0 = 0$. In that case, the cut-off has been arbitrarily chosen to be 10°.

In table 1, we give the 90% confidence level (CL) limits that can be obtained on $\mathrm{Im}c_d^{\gamma}$ and $\mathrm{Im}c_d^{Z}$, assuming one of them to be non-zero, the other taken to be vanishing. The limit

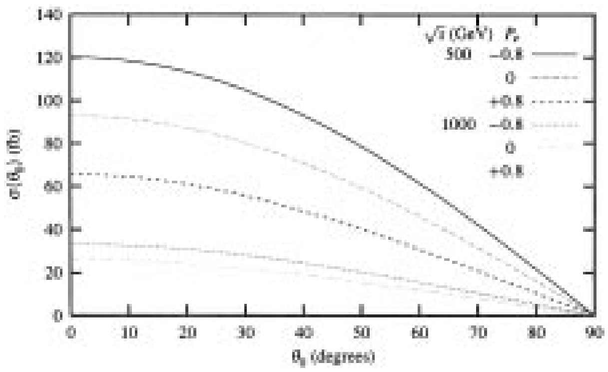

Figure 1. The SM cross-section for decay leptons in the process $e^+e^- \to t\bar{t}$ plotted as a function of the cut-off θ_0 on the lepton polar angle in the forward and backward directions for e^- beam longitudinal polarizations $P_e = -0.8, 0, +0.8$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

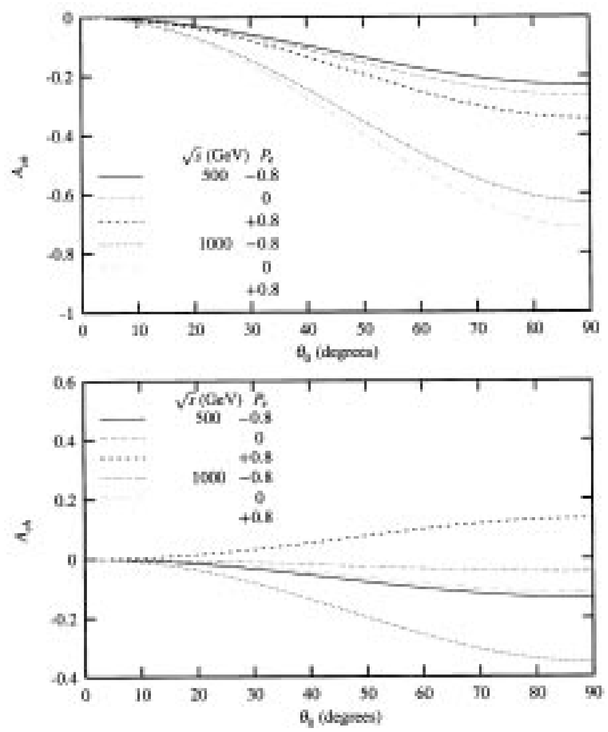

Figure 2. The asymmetry $A_{\rm ch}$ defined in the text, for $\mathrm{Im}c_d^\gamma = 1$, $\mathrm{Im}c_d^Z = 0$ (top), and for $\mathrm{Im}c_d^\gamma = 0$, $\mathrm{Im}c_d^Z = 1$ (bottom), plotted as a function of the cut-off θ_0 on the lepton polar angle in the forward and backward directions for e^- beam longitudinal polarizations $P_e = -0.8, 0, +0.8$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

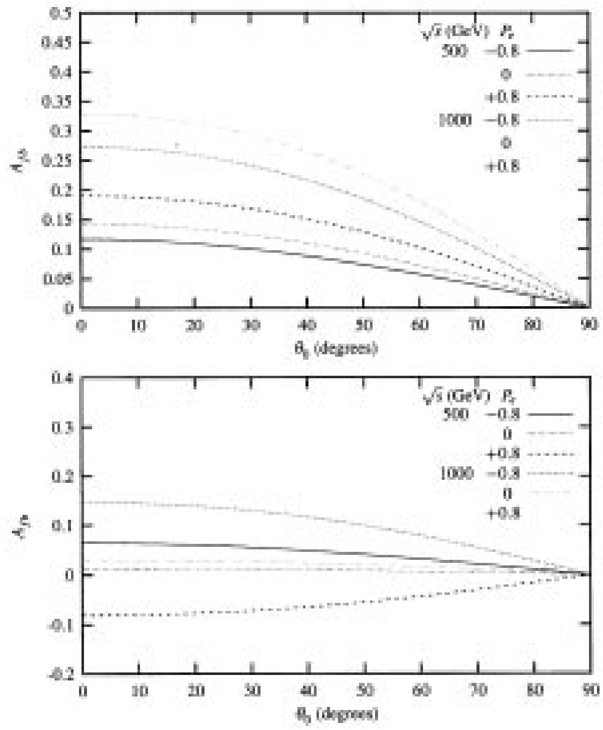

Figure 3. The asymmetry $A_{\rm fb}$ defined in the text, for $\text{Im}c_d^\gamma = 1$, $\text{Im}c_d^Z = 0$ (top), and for $\text{Im}c_d^\gamma = 0$, $\text{Im}c_d^Z = 1$ (bottom), plotted as a function of the cut-off θ_0 on the lepton polar angle in the forward and backward directions for e^- beam longitudinal polarizations $P_e = -0.8, 0, +0.8$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

is defined as the value of $\text{Im}c_d^\gamma$ or $\text{Im}c_d^Z$ for which the corresponding asymmetry $A_{\rm ch}$ or $A_{\rm fb}$ becomes equal to $1.64/\sqrt{N}$, where N is the total number of events.

Table 2 shows possible 90% CL limits for the unpolarized case, when results from $A_{\rm ch}$ and $A_{\rm fb}$ are combined. The idea is that each asymmetry measures a different linear combination of $\text{Im}c_d^\gamma$ and $\text{Im}c_d^Z$. So a null result for the two asymmetries will correspond to two different bands of regions allowed at 90% CL in the space of $\text{Im}c_d^\gamma$ and $\text{Im}c_d^Z$. The overlapping region of the two bands leads to the limits given in table 2. In this case, for 90% CL, the asymmetry is required to be $2.15/\sqrt{N}$, corresponding to two degrees of freedom. Incidentally, the same procedure followed for $P_e = \pm 0.8$ gives much worse limits.

Similarly, using one of the two asymmetries, but two different polarizations of the electron beam, one can get two bands in the parameter plane, which give simultaneous limits on the dipole couplings. The results for electron polarizations $P_e = \pm 0.8$ are given in table 3 for each of the asymmetries $A_{\rm ch}$ and $A_{\rm fb}$.

Table 4 lists the 90% CL limits which may be obtained on the real and imaginary parts of the dipole couplings using $A_{\rm ud}$ and $A_{\rm lr}$, assuming one of the couplings to be non-zero at a time.

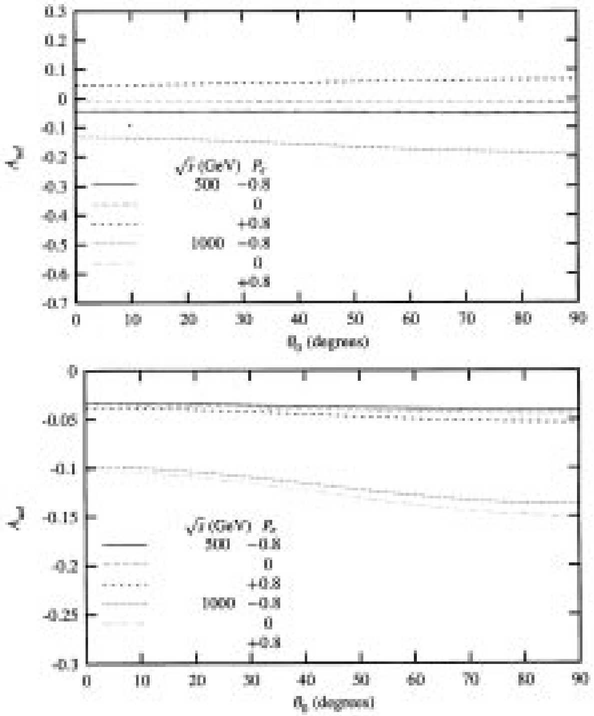

Figure 4. The asymmetry A_{ud} defined in the text, for $\mathrm{Re}c_d^{\gamma} = 0.1$, $\mathrm{Re}c_d^{Z} = 0$ (top), and for $\mathrm{Re}c_d^{\gamma} = 0$, $\mathrm{Re}c_d^{Z} = 0.1$ (bottom), plotted as a function of the cut-off θ_0 on the lepton polar angle in the forward and backward directions for e^- beam longitudinal polarizations $P_e = -0.8, 0, +0.8$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

Table 5 shows simultaneous limits on $\mathrm{Re}c_d^{\gamma}$ and $\mathrm{Re}c_d^{Z}$ obtainable from combining the data on A_{ud} for $P_e = +0.8$ and $P_e = -0.8$, and similarly, limits on $\mathrm{Im}c_d^{\gamma}$ and $\mathrm{Im}c_d^{Z}$ from data on A_{lr} for the two polarizations.

5. Conclusions and discussion

We have presented in analytic form the single-lepton angular distribution in the production and subsequent decay of $t\bar{t}$ in the presence of electric and weak dipole form factors of the top quark, including $\mathcal{O}(\alpha_s)$ QCD corrections in SGA. Anomalous contributions to the tbW decay vertex do not affect these distributions. We have also included effects of longitudinal electron and positron beam polarizations. We have then obtained analytic expressions for certain simple CP-violating polar-angle asymmetries, specially chosen so that they do not require the reconstruction of the t or $\bar{t}$ directions or energies. We have also evaluated numercially azimuthal asymmetries which need minimal information on the t or $\bar{t}$ momentum direction alone. We have analyzed these asymmetries to obtain simultaneous

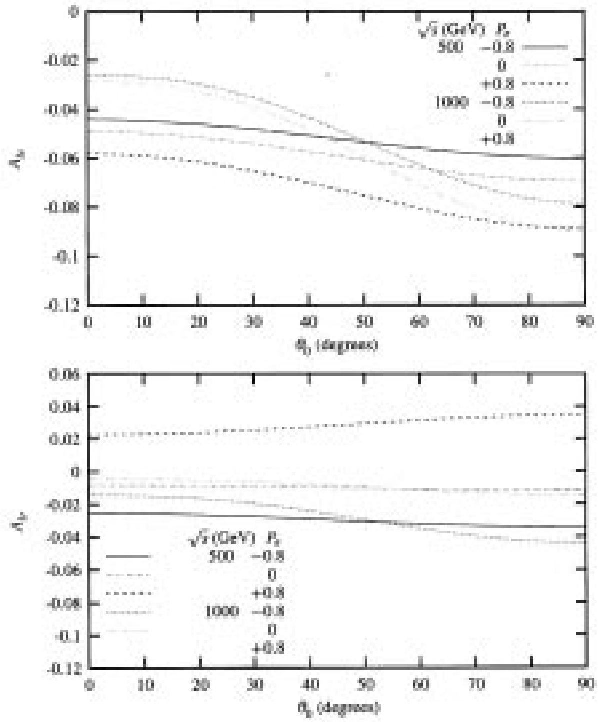

Figure 5. The asymmetry $A_{\rm lr}$ defined in the text, for Im$c_d^\gamma = 0.1$, Im$c_d^Z = 0$ (top), and for Im$c_d^\gamma = 0$, Im$c_d^Z = 0.1$ (bottom), plotted as a function of the cut-off θ_0 on the lepton polar angle in the forward and backward directions for e^- beam longitudinal polarizations $P_e = -0.8, 0, +0.8$ and for values of total cm energy $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

90% CL limits on the electric and weak dipole couplings which would be possible at future linear e^+e^- collider operating at $\sqrt{s} = 500$ GeV with an integrated luminosity of 500 fb^{-1}, and at $\sqrt{s} = 1000$ GeV with an integrated luminosity of 1000 fb^{-1}. We assume electron beam polarization of ±80%, while the positron beam is unpolarized. The results are presented in figures 1–5 and tables 1–5.

In general, simultaneous 90% CL limits on c_d^γ and c_d^Z which can be obtained with the polarized 500 GeV option are of the order of 0.1–0.2, corresponding to dipole moments of about (1–2) $\times 10^{-17} e$ cm, if the asymmetries $A_{\rm ch}$ or $A_{\rm fb}$ are used. The limits improve by a factor of 4 to 6 if the azimuthal asymmetries $A_{\rm ud}$ or $A_{\rm lr}$ are used. However, putting in a top detection efficiency factor of 10% in the case of azimuthal asymmetries, where top direction needs to be determined, would bring down these limts to the same level of (1–2) $\times 10^{-17} e$ cm.

For $\sqrt{s} = 1000$ GeV and an integrated luminosity of 1000 fb^{-1}, the limits obtainable would be better by a factor of 3 or 4 in each case, bringing them to the level of (2–3) $\times 10^{-18} e$ cm in the best cases.

Table 1. Individual 90% CL limits on dipole couplings obtainable from A_{ch} and A_{fb} for $\sqrt{s} = 500$ GeV with integrated luminosity 500 fb^{-1} and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb^{-1} for different electron beam polarizations P_e. Cut-off θ_0 is chosen to optimize the sensitivity.

		A_{ch}			A_{fb}		
$\sqrt{s}$ (GeV)	P_e	θ_0	$\mathrm{Im}c_d^\gamma$	$\mathrm{Im}c_d^Z$	θ_0	$\mathrm{Im}c_d^\gamma$	$\mathrm{Im}c_d^Z$
	0	64°	0.053	0.31	10°	0.054	0.60
500	+0.8	64°	0.052	0.13	10°	0.049	0.11
	−0.8	63°	0.053	0.092	10°	0.059	0.11
	0	64°	0.029	0.18	10°	0.032	0.36
1000	+0.8	64°	0.028	0.074	10°	0.029	0.069
	−0.8	64°	0.028	0.051	10°	0.034	0.063

Table 2. Simultaneous 90% CL limits on dipole couplings obtainable from A_{ch} and A_{fb} for $\sqrt{s} = 500$ GeV with integrated luminosity 500 fb^{-1} and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb^{-1} for unpolarized beams. Cut-off θ_0 is chosen to optimize the sensitivity.

$\sqrt{s}$ (GeV)	θ_0	$\mathrm{Im}c_d^\gamma$	$\mathrm{Im}c_d^Z$
500	40°	0.37	2.6
1000	40°	0.20	1.5

Table 3. Simultaneous limits on dipole couplings combining data from polarizations $P_e = 0.8$ and $P_e = -0.8$, using separately A_{ch} and A_{fb} for $\sqrt{s} = 500$ GeV with integrated luminosity 500 fb^{-1} and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb^{-1}. Cut-off θ_0 is chosen to optimize the sensitivity.

	A_{ch}			A_{fb}		
$\sqrt{s}$ (GeV)	θ_0	$\mathrm{Im}c_d^\gamma$	$\mathrm{Im}c_d^Z$	θ_0	$\mathrm{Im}c_d^\gamma$	$\mathrm{Im}c_d^Z$
500	64°	0.090	0.19	10°	0.091	0.19
1000	64°	0.049	0.10	10°	0.053	0.11

Though we have not presented detailed results here, a numerical evaluation of possible limits has been carried out for other possibilities, like (i) a slightly higher electron beam polarization of 0.9, (ii) positron beam polarized to the extent of 0.6, in addition to polarized electron beam, a possibility envisaged in the context of TESLA, (iii) a beam energy of 800 GeV, with an integrated luminosity of 800 fb^{-1}. The conclusions are as follows:

An increase in the electron polarization from 0.8 to 0.9 (with the positrons unpolarized) improves the sensitivity by about 30 to 50% in case of polar-angle asymmetries A_{ch} and A_{fb}, and to a lesser extent, 10 to 15% in the case of the measurement of $\mathrm{Re}c_d^\gamma$ and $\mathrm{Im}c_d^Z$ by azimuthal asymmetries.

Table 4. Individual 90% CL limits on dipole couplings obtainable from $A_{\rm ud}$ and $A_{\rm lr}$ for $\sqrt{s} = 500$ GeV with integrated luminosity 500 fb^{-1} and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb^{-1} for different electron beam polarizations P_e. Cut-off θ_0 is chosen to optimize the sensitivity.

		$A_{\rm ud}$			$A_{\rm lr}$		
$\sqrt{s}$ (GeV)	P_e	θ_0	Rec_d^γ	Rec_d^Z	θ_0	Imc_d^γ	Imc_d^Z
	0	25°	0.066	0.022	30°	0.015	0.088
500	+0.8	30°	0.019	0.023	35°	0.015	0.038
	−0.8	25°	0.015	0.020	30°	0.015	0.026
	0	30°	0.029	0.0096	60°	0.021	0.13
1000	+0.8	35°	0.0082	0.010	60°	0.021	0.055
	−0.8	30°	0.0066	0.0089	60°	0.021	0.038

Table 5. Simultaneous limits on dipole couplings combining data from polarizations $P_e = 0.8$ and $P_e = -0.8$, using separately $A_{\rm ud}$ and $A_{\rm lr}$ for $\sqrt{s} = 500$ GeV with integrated luminosity 500 fb^{-1} and for $\sqrt{s} = 1000$ GeV with integrated luminosity 1000 fb^{-1}. Cut-off θ_0 is chosen to optimize the sensitivity.

	$A_{\rm ud}$			$A_{\rm lr}$		
$\sqrt{s}$ (GeV)	θ_0	Rec_d^γ	Rec_d^Z	θ_0	Imc_d^γ	Imc_d^Z
500	25°	0.022	0.029	35°	0.020	0.041
1000	30°	0.0097	0.013	60°	0.028	0.059

Including longitudinal positron polarization of 0.6 (always opposite in sign to the polarization of the electron) improves the sensitivity in all cases by about 20 to 30%.

We conclude that it is probably worthwhile from the top dipole coupling point of view to improve the electron polarization by a small amount rather than to invest in a new or difficult technology to achieve a high positron polarization.

The improvement in sensitivity in going from cm energy of 800 GeV to 1000 GeV, with a simultaneous increase in integrated luminosity from 800 fb^{-1} to 1000 fb^{-1}, is about 5 to 10% in the case of polar-angle asymmetries, and 20 to 25% in the case of $A_{\rm ud}$. However, the sensitivity *worsens* in the case of measurement using $A_{\rm lr}$, by about 10% or so.

Our general conclusion is that the sensitivity to the measurement of individual dipole couplings Rec_d^γ and Imc_d^Z is improved considerably if the electron beam is polarized, a situation which might easily be obtained at linear colliders. As a consequence, simultaneous limits on all the couplings are improved by beam polarization.

The theoretical predictions for $c_d^{\gamma,Z}$ are at the level of 10^{-2}–10^{-3}, as for example, in the neutral-Higgs-exchange and supersymmetric models of CP violation [6,14,22,25]. In other models, like the charged-Higgs-exchange [6] or third-generation leptoquark models [26], the prediction are even lower. Hence the measurements suggested here at the 500 GeV option cannot exclude these modes at the 90% CL. It will be necessary to use the 1000 GeV option with a suitable luminosity to test at least some of the models.

It is necessary to repeat this study including experimental detection efficiencies. Given an overall efficiency, we could still get an idea of the limits on the dipole couplings by scaling them as the inverse square root of the efficiency.

We have not included a cut-off on decay-lepton energies which may be required from a practical point of view. However, our results are perfectly valid if the cut-off is reasonably small. For example, for $\sqrt{s} = 500$ GeV, the minimum lepton energy allowed kinematically is about 7.5 GeV. So a cut-off below that would need no modification of the results.

Contact $e^+e^-t\bar{t}$ interactions violating CP have been ignored in this work. They should be taken into account for a complete treatment of CP violation in $e^+e^- \to t\bar{t}$.

We have restricted ourselves to energies in the $t\bar{t}$ continuum. Studies in the threshold region are also interesting and have been investigated upon [27].

Appendix

The expressions for A_i, B_i, C_i and D_i occurring in eq. (8) are listed below. They include to first-order the form factors c_d^γ and c_d^Z, as well as c_M^γ and c_M^Z. Terms containing products of $c_d^{\gamma,Z}$ with $c_M^{\gamma,Z}$ have been dropped. It is also understood that terms proportional to products of A or B (which are of order α_s) and c_d^γ or c_d^Z have to be omitted in the calculations.

$$
\begin{aligned}
A_0 = {} & 2\left\{(2-\beta^2)\left[2|c_v^\gamma|^2 + 2(r_L+r_R)\mathrm{Re}(c_v^\gamma c_v^{Z*}) + (r_L^2+r_R^2)|c_v^Z|^2\right]\right.\\
& +\beta^2(r_L^2+r_R^2)|c_a^Z|^2 - 2\beta^2\left[2\mathrm{Re}(c_v^\gamma c_M^{\gamma*})\right.\\
& \left.+(r_L+r_R)\mathrm{Re}(c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*}) + (r_L^2+r_R^2)\mathrm{Re}(c_v^Z c_M^{Z*})\right]\,(1-P_eP_{\overline{e}})\\
& +\,(2-\beta^2)\left[2(r_L-r_R)\mathrm{Re}(c_v^\gamma c_v^{Z*}) + (r_L^2-r_R^2)|c_v^Z|^2\right]\\
& +\beta^2(r_L^2-r_R^2)|c_a^Z|^2 - 2\beta^2\left[(r_L-r_R)\mathrm{Re}(c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*})\right.\\
& \left.\left.+(r_L^2-r_R^2)\mathrm{Re}(c_v^Z c_M^{Z*})\right]\right\}(P_{\overline{e}}-P_e),\\
A_1 = {} & -8\beta\mathrm{Re}\left(c_a^{Z*}\left\{\left[(r_L-r_R)c_v^\gamma + (r_L^2-r_R^2)c_v^Z\right](1-P_eP_{\overline{e}})\right.\right.\\
& \left.\left.+\left[(r_L+r_R)c_v^\gamma + (r_L^2+r_R^2)c_v^Z\right](P_{\overline{e}}-P_e)\right\}\right),\\
A_2 = {} & 2\beta^2\left\{\left[2|c_v^\gamma|^2 + 4\mathrm{Re}(c_v^\gamma c_M^{\gamma*}) + 2(r_L+r_R)\mathrm{Re}(c_v^\gamma c_v^{Z*} + c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*})\right.\right.\\
& \left.+(r_L^2+r_R^2)\left(|c_v^Z|^2 + |c_a^Z|^2 + 2\mathrm{Re}(c_v^Z c_M^{Z*})\right)\right](1-P_eP_{\overline{e}})\\
& +\left[2(r_L-r_R)\mathrm{Re}(c_v^\gamma c_v^{Z*} + c_v^\gamma c_M^{Z*} + c_v^Z c_M^{\gamma*})\right.\\
& \left.\left.+(r_L^2-r_R^2)\left(|c_v^Z|^2 + |c_a^Z|^2 + 2\mathrm{Re}(c_v^Z c_M^{Z*})\right)\right](P_{\overline{e}}-P_e)\right\},\\
B_0^\pm = {} & 4\beta\left\{\left(\mathrm{Re}c_v^\gamma + r_L\mathrm{Re}c_v^Z\right)\left(r_L\mathrm{Re}c_a^Z \mp \mathrm{Im}c_d^\gamma \mp r_L\mathrm{Im}c_d^Z\right)(1-P_e)(1+P_{\overline{e}})\right.\\
& \left.+\left(\mathrm{Re}c_v^\gamma + r_R\mathrm{Re}c_v^Z\right)\left(r_R\mathrm{Re}c_a^Z \mp \mathrm{Im}c_d^\gamma \mp r_R\mathrm{Im}c_d^Z\right)(1+P_e)(1-P_{\overline{e}})\right\},\\
B_1 = {} & -4\left\{\left[|c_v^\gamma + r_L c_v^Z|^2 + \beta^2 r_L^2|c_a^Z|^2\right](1-P_e)(1+P_{\overline{e}})\right.\\
& \left.-\left[|c_v^\gamma + r_R c_v^Z|^2 + \beta^2 r_R^2|c_a^Z|^2\right](1+P_e)(1-P_{\overline{e}})\right\},\\
B_2^\pm = {} & 4\beta\left\{\left(\mathrm{Re}c_v^\gamma + r_L\mathrm{Re}c_v^Z\right)\left(r_L\mathrm{Re}c_a^Z \pm \mathrm{Im}c_d^\gamma \pm r_L\mathrm{Im}c_d^Z\right)(1-P_e)(1+P_{\overline{e}})\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\mathrm{Re}c_v^\gamma+r_R\mathrm{Re}c_v^Z\right)\left(r_R\mathrm{Re}c_a^Z\pm\mathrm{Im}c_d^\gamma\pm r_R\mathrm{Im}c_d^Z\right)(1+P_e)(1-P_{\overline{e}})\Big\}\,,\\
C_0^\pm &= 4\Big\{\Big[|c_v^\gamma+r_Lc_v^Z|^2-\beta^2\gamma^2\left(\mathrm{Re}c_v^\gamma+r_L\mathrm{Re}c_v^Z\right)\left(\mathrm{Re}c_M^\gamma+r_L\mathrm{Re}c_M^Z\right)\\
&\pm\beta^2\gamma^2 r_L\mathrm{Re}c_a^Z\left(\mathrm{Im}c_d^\gamma+\mathrm{Im}c_d^Zr_L\right)\Big](1-P_e)(1+P_{\overline{e}})\\
&-\Big[|c_v^\gamma+r_Rc_v^Z|^2-\beta^2\gamma^2\left(\mathrm{Re}c_v^\gamma+r_R\mathrm{Re}c_v^Z\right)\left(\mathrm{Re}c_M^\gamma+r_R\mathrm{Re}c_M^Z\right)\\
&\pm\beta^2\gamma^2 r_R\mathrm{Re}c_a^Z\left(\mathrm{Im}c_d^\gamma+\mathrm{Im}c_d^Zr_R\right)\Big](1+P_e)(1-P_{\overline{e}})\Big\}\,,\\
C_1^\pm &= -4\beta\Big\{\Big[\left(\mathrm{Re}c_v^\gamma+r_L\mathrm{Re}c_v^Z\right)\left(r_L\mathrm{Re}c_a^Z\pm\gamma^2\mathrm{Im}c_d^\gamma\pm r_L\gamma^2\mathrm{Im}c_d^Z\right)\\
&-\beta^2\gamma^2r_L\mathrm{Re}c_a^Z\left(\mathrm{Re}c_M^\gamma+r_L\mathrm{Re}c_M^Z)\right)\Big](1-P_e)(1+P_{\overline{e}})\\
&+\Big[\left(\mathrm{Re}c_v^\gamma+r_R\mathrm{Re}c_v^Z\right)\left(r_R\mathrm{Re}c_a^Z\pm\gamma^2\mathrm{Im}c_d^\gamma\pm r_R\gamma^2\mathrm{Im}c_d^Z\right)\\
&-\beta^2\gamma^2r_R\mathrm{Re}c_a^Z\left(\mathrm{Re}c_M^\gamma+r_R\mathrm{Re}c_M^Z)\right)\Big](1+P_e)(1-P_{\overline{e}})\Big\}\,,\\
D_0^\pm &= 4\beta\Big\{\Big[\mathrm{Im}\left[\left((c_v^\gamma+r_Lc_v^Z)-\beta^2\gamma^2(c_M^\gamma+r_Lc_M^Z)\right)r_Lc_a^{Z*}\right]\\
&\mp\gamma^2\left(\mathrm{Re}c_v^\gamma+r_L\mathrm{Re}c_v^Z\right)\left(\mathrm{Re}c_d^\gamma+r_L\mathrm{Re}c_d^Z\right)\Big](1-P_e)(1+P_{\overline{e}})\\
&-\Big[\mathrm{Im}\left[\left((c_v^\gamma+r_Rc_v^Z)-\beta^2\gamma^2(c_M^\gamma+r_Rc_M^Z)\right)r_Rc_a^{Z*}\right]\\
&\mp\gamma^2\left(\mathrm{Re}c_v^\gamma+r_R\mathrm{Re}c_v^Z\right)\left(\mathrm{Re}c_d^\gamma+r_R\mathrm{Re}c_d^Z\right)\Big](1+P_e)(1-P_{\overline{e}})\Big\}\,,\\
D_1^\pm &= 4\beta^2\gamma^2\Big\{\Big[(\mathrm{Re}c_v^\gamma+r_L\mathrm{Re}c_v^Z)(\mathrm{Im}c_M^\gamma\\
&+r_L\mathrm{Im}c_M^Z)\pm r_L\mathrm{Re}c_a^Z\left(\mathrm{Re}c_d^\gamma+r_L\mathrm{Re}c_d^Z\right)\Big](1-P_e)(1+P_{\overline{e}})\\
&+\Big[(\mathrm{Re}c_v^\gamma+r_R\mathrm{Re}c_v^Z)(\mathrm{Im}c_M^\gamma+r_R\mathrm{Im}c_M^Z)\\
&\pm r_R\mathrm{Re}c_a^Z\left(\mathrm{Re}c_d^\gamma+r_R\mathrm{Re}c_d^Z\right)\Big](1+P_e)(1-P_{\overline{e}})\Big\}\,.
\end{aligned}
$$

The relations

$$r_L=\frac{(\frac{1}{2}-x_W)}{(1-m_Z^2/s)\sqrt{x_W(1-x_W)}}$$

and

$$r_R=\frac{-x_W}{(1-m_Z^2/s)\sqrt{x_W(1-x_W)}}$$

are used in writing the above equation.

Note added in proof

The result shown in refs [12] and [13], that the lepton angular distributions do not depend on anomalous couplings occurring in top decay, has now been shown to be valid even when the b-quark mass is not neglected, see B Grzadkowski and Z Hioki, *Phys. Lett.* **B557**, 55 (2000).

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