# Transverse polarization in $\gamma \boldsymbol{Z}, \boldsymbol{H} Z$ production 

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#### Abstract

With the use of transverse polarization (TP), a CP-odd and T-odd observable can be constructed when the final-state particles are self-conjugate. In the case of $H Z$ production, this observable can be used to probe a certain effective four-point $e^{+} e^{-} Z H$ CP-violating coupling, not accessible without TP. Effective CP-violating ZZH coupling does not contribute to this observable. A similar observable in $\gamma Z$ production can be used to probe $e^{+} e^{-} \gamma Z$ four-point couplings.


Keywords. Transverse polarization at electron-positron collider; Higgs production; beyond standard model; $Z$ boson.

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## 1. The process $e^{+} e^{-} \rightarrow H Z$

In a theory with an extended Higgs sector and new mechanisms of CP violation, the physical Higgs bosons are not necessarily eigenstates of CP. In such a case, the production of a physical Higgs can proceed through more than one channel, and the interference between two channels can give rise to a CP-violating signal in the production. Here we consider in a general model-independent way the production of a Higgs mass eigenstate $H$ through the process $e^{+} e^{-} \rightarrow H Z$, an important mechanism for the production of the Higgs. $e^{+} e^{-} \rightarrow H Z$ gets contribution from a diagram with an $s$-channel exchange of $Z$. At the lowest order, the $Z Z H$ vertex in this diagram is simply a point-like coupling. It may be modified by interactions beyond SM by means of a form factor, and/or more complicated momentum-dependent anomalous interactions [1-3].

We consider a beyond-SM contribution represented by a four-point $e^{+} e^{-} Z H$ coupling. This is general enough to include the effects of the diagram with anomalous $Z Z H$ vertex with $s$-channel $Z$ exchange. In addition it can include effects of box diagrams, or diagrams with $t$-channel exchange of new particles. We obtain the angular distributions in the presence of polarized beams and examine how angular asymmetries can be used to constrain the form factors of the four-point coupling. Details may be found in [4]. In earlier works, it has been observed that polarization does not give any new information about the anomalous $Z Z H$ couplings when they are assumed real [3]. However, with four-point couplings, we find that there are
new CP-odd terms in the distribution when both $e^{-}$and $e^{+}$beams have transverse polarization (TP). Thus, TP would be most useful in isolating such terms.

The most general chirality conserving (CC) four-point vertex for the process $e^{-}\left(p_{1}\right)+e^{+}\left(p_{2}\right) \rightarrow Z^{\alpha}(q)+H(k)$ consistent with Lorentz invariance can be written as (we neglect the electron mass everywhere)

$$
\begin{align*}
M^{3} \Gamma_{\mathrm{CC}}^{\alpha}= & -M^{2} \gamma^{\alpha}\left(V_{1}+\gamma_{5} A_{1}\right)+\not \phi\left(V_{2}+\gamma_{5} A_{2}\right) k^{\alpha} \\
& -i \not q\left(V_{3}+\gamma_{5} A_{3}\right)\left(p_{2}-p_{1}\right)^{\alpha} . \tag{1}
\end{align*}
$$

In the above expression, $V_{i}$ and $A_{i}$ are form factors, which, for simplicity, we treat as constants. The terms containing $V_{3}$ and $A_{3}$ are CP violating, whereas the remaining are CP conserving. $M$ is a parameter with dimensions of mass, put in to render the form factors dimensionless.

We obtain, for arbitrary polarizations, analytic expressions for the angular distribution [4] of $Z$ arising from SM and from the interference between the SM amplitude and the amplitude arising from the four-point couplings of (1). We neglect terms bilinear in the four-point couplings, which are assumed small compared to the SM ones.

The $z$-axis is chosen along the direction of the $e^{-}$momentum, and the $x z$ plane to coincide with the production plane. The positive $x$ axis is chosen to be along the direction of the $e^{-}$TP. $\theta$ and $\phi$ are then the polar and azimuthal angles of the momentum of $Z$. The $e^{+}$polarization direction is chosen anti-parallel to the $e^{-}$ polarization direction.

The $V_{3}$ and $A_{3}$ terms in the distribution can be determined using a simple forward-backward asymmetry:

$$
\begin{equation*}
A_{\mathrm{FB}}\left(\theta_{0}\right)=\left(1 / \sigma_{0}\right)\left[\Delta \sigma\left(\theta_{0}, \pi / 2\right)-\Delta \sigma\left(\pi / 2, \pi-\theta_{0}\right)\right], \tag{2}
\end{equation*}
$$

with $\Delta \sigma\left(\theta, \theta^{\prime}\right) \equiv \int_{\theta}^{\theta^{\prime}}(\mathrm{d} \sigma / \mathrm{d} \theta) \mathrm{d} \theta$, where $\sigma_{0} \equiv \Delta \sigma\left(\theta_{0}, \pi-\theta_{0}\right)$ is the integrated cross section, and $\theta_{0}$ is a cut-off in the forward and backward directions needed to keep away from the beam pipe, and chosen to optimize the sensitivity. This asymmetry is odd under CP and is proportional to the combination $g_{V} \operatorname{Im} V_{3}-g_{A} \operatorname{Im} A_{3}$.

The distribution with TP of beams has new $\phi$-dependent terms occurring with a factor of $P_{\mathrm{T}} \bar{P}_{\mathrm{T}}$. Hence both beams need to have TP for nontrivial azimuthal dependence. We define two asymmetries, which serve to measure two different combinations of CP-violating couplings:

$$
\begin{align*}
& A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)=\frac{1}{\sigma_{0}} \sum_{n=0}^{3}(-1)^{n} \int_{\pi n / 2}^{\pi(n+1) / 2} \mathrm{~d} \phi A_{\mathrm{FB}}\left(\theta_{0} ; \phi\right),  \tag{3}\\
& A_{\mathrm{FB}}^{\prime \mathrm{T}}\left(\theta_{0}\right)=\frac{1}{\sigma_{0}} \sum_{n=0}^{3}(-1)^{n} \int_{\pi(2 n-1) / 4}^{\pi(2 n+1) / 4} \mathrm{~d} \phi A_{\mathrm{FB}}\left(\theta_{0} ; \phi\right), \tag{4}
\end{align*}
$$

where $A_{\mathrm{FB}}\left(\theta_{0} ; \phi\right)$ is a differential form of the asymmetry in eq. (2), before integration over $\phi$, and for nonzero TP. The integrals in the above can be easily evaluated. The result shows that $A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)$ and $A_{\mathrm{FB}}^{\prime \mathrm{T}}\left(\theta_{0}\right)$ are respectively proportional to $g_{V} \operatorname{Re} A_{3}+g_{A} \operatorname{Re} A_{3}$ and $g_{V} \operatorname{Im} V_{3}+g_{A} \operatorname{Im} A_{3}$. Both are proportional to $P_{\mathrm{T}} \bar{P}_{\mathrm{T}}$. We find that if one considers only a modification of the $Z Z H$ vertex as in


Figure 1. The asymmetry $A_{\mathrm{FB}}^{\mathrm{T}}$ for transverse polarizations $P_{\mathrm{T}}=0.8$ and $\bar{P}_{\mathrm{T}}=$ 0.6 plotted against $\theta_{0}$ for $\operatorname{Re} V_{3}=0.1$, $\operatorname{Re} A_{3}=0$.

Figure 2. The asymmetry $A_{\mathrm{FB}}^{\prime} \mathrm{T}$ for transverse polarizations $P_{\mathrm{T}}=0.8$ and $\bar{P}_{\mathrm{T}}=$ 0.6 plotted against $\theta_{0}$ for $\operatorname{Im} V_{3}=0.1$, $\operatorname{Im} A_{3}=0$.
refs $[1-3], A_{\mathrm{FB}}^{\mathrm{T}}$ and $A_{\mathrm{FB}}^{\prime} \mathrm{T}$ vanish. This result for $A_{\mathrm{FB}}^{\mathrm{T}}$ was obtained in [3]. Thus, observation of a nonzero asymmetry would signal the presence of a CP-violating four-point interaction.

We now describe our numerical results. We use the value $M=1 \mathrm{TeV}$. This choice is simply for convenience, and is not simply related to any assumption about the scale of new physics - a change in $M$ can always be compensated by changes $i_{\bar{P}}$ the form factors. For the linear collider, we assume $\sqrt{s}=500 \mathrm{GeV}, P_{\mathrm{T}}=0.8$, $\bar{P}_{\mathrm{T}}=0.6$, and an integrated luminosity of $500 \mathrm{fb}^{-1}$. We show results for three values of the Higgs mass, $m_{\mathrm{H}}=150 \mathrm{GeV}, 200 \mathrm{GeV}$ and 300 GeV .

The asymmetries $A_{\mathrm{FB}}^{\mathrm{T}}$ and $A_{\mathrm{FB}}^{\prime \mathrm{T}}$ are shown as functions of $\theta_{0}$ in figures 1 (for $\left.\operatorname{Re} V_{3}=0.1, \operatorname{Re} A_{3}=0\right)$ and $2\left(\right.$ for $\left.\operatorname{Im} V_{3}=0.1, \operatorname{Im} A_{3}=0\right)$. We now examine the accuracy in which these couplings can be constrained. We find that the $90 \%$ confidence level (C.L.) limit that can be placed on $\operatorname{Re} V_{3}$ from the asymmetry $A_{\mathrm{FB}}^{\mathrm{T}}$ for a typical value of $\theta_{0}=45^{\circ}$ ranges from $3.9 \times 10^{-2}$ for $m_{\mathrm{H}}=150 \mathrm{GeV}$, to $1.3 \times 10^{-1}$ for $m_{\mathrm{H}}=300 \mathrm{GeV}$. The corresponding limits on $\operatorname{Re} A_{3}$ are a factor $\left|g_{A} / g_{V}\right| \approx 8.3$ higher. In case of $A_{\mathrm{FB}}^{\prime} \mathrm{T}$ it is $\operatorname{Im} A_{3}$ which has the limits mentioned above for $\operatorname{Re} V_{3}$, and limits on $\operatorname{Im} V_{3}$ are a factor of about 8.3 higher.

## 2. The process $e^{+} e^{-} \rightarrow \gamma Z$

$\gamma Z$ production has a significant SM cross section at the planned ILC energies. We discuss contributions to the differential cross section due to general, modelindependent, gauge and Lorentz invariant, chirality conserving (CC) four-point interactions. Details may be found in [5]. In particular, one of these interactions generates precisely the same contributions as from anomalous CP-violating triplegauge boson vertices studied in a similar context [6].

The process considered is $e^{-}\left(p_{-}, s_{-}\right)+e^{+}\left(p_{+}, s_{+}\right) \rightarrow \gamma\left(k_{1}, \alpha\right)+Z\left(k_{2}, \beta\right)$. The chirality-conserving anomalous four-point vertex for this process contains 12 form factors [5], which we label as $v_{i}, a_{i}(i=1, \ldots, 6)$. We assume for simplicity that the form-factors are all constants. The combinations $v_{1}, v_{2}-v_{5}, v_{3}-v_{4}$ and $a_{1}, a_{2}-$ $a_{5}, a_{3}-a_{4}$ are CP conserving, while $v_{2}+v_{5}, v_{3}+v_{4}, v_{6}$ and $a_{2}+a_{5}, a_{3}+a_{4}, a_{6}$ are CP violating.


Figure 3. The asymmetries $A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)$ (solid line), $A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)$ (dashed line) and $A_{\text {FB }}\left(\theta_{0}\right)$ (dotted line), plotted as functions of the cut-off $\theta_{0}$ for $\operatorname{Re} v_{6}=\operatorname{Im} v_{6}=1$.


Figure 4. $90 \%$ C.L. limit on Re $v_{6}$ from the asymmetry $A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)$ (solid line), and on $\operatorname{Im} v_{6}$ from $A_{\mathrm{FB}}^{\prime \mathrm{T}}\left(\theta_{0}\right)$ (dashed line) and $A_{\mathrm{FB}}\left(\theta_{0}\right)$ (dotted line), plotted as functions of the cut-off $\theta_{0}$.

We use the same CP-odd asymmetries as in the previous section, with $\theta$ and $\phi$ referring to the polar and azimuthal angles of $\gamma$. Two of these again require both beams to have TP. Figure 3 shows the asymmetries as functions of the cut-off when the anomalous couplings $\operatorname{Re} v_{6}$ (for the case of $A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)$ ) and $\operatorname{Im} v_{6}$ (for the case of $A_{\mathrm{FB}}^{\prime} \mathrm{T}\left(\theta_{0}\right)$ and $\left.A_{\mathrm{FB}}\left(\theta_{0}\right)\right)$ alone are set to unity. $90 \%$ C.L. limits on couplings, denoted by $\delta$, are shown in figure 4 for a choice of $26^{\circ}$ for $\theta_{0}$. The results for other CP-violating combinations of couplings can be deduced from this case. From the asymmetry $A_{\mathrm{FB}}^{\mathrm{T}}\left(\theta_{0}\right)$, we get the limits (in units of $10^{-3}$ ) $\left|\operatorname{Re} v_{3,4}\right| \leq 0.21$, $\left|\operatorname{Re} v_{6}\right| \leq$ 3.1, and $\left|\operatorname{Re} a_{3,4}\right| \leq 3.1,\left|\operatorname{Re} a_{6}\right| \leq 46$. The asymmetry $A_{\mathrm{FB}}^{\prime} \mathrm{T}\left(\theta_{0}\right)$ yields limits which are obtained from these by the replacement of the real part by imaginary part and interchange of $v_{i}$ and $a_{i}$. The asymmetry $A_{\mathrm{FB}}\left(\theta_{0}\right)$ with unpolarized beams yields the limits (in units of $10^{-3}$ ) $\operatorname{Im} v_{2,5} \leq 0.93, \operatorname{Im} v_{6} \leq 14$ and $\operatorname{Im} a_{2,5} \leq 0.064$, $\operatorname{Im} a_{6} \leq 0.96$.

## 3. Conclusions

In conclusion, we have considered in all generality the role of chirality conserving four-point couplings due to new physics in the processes $e^{+} e^{-} \rightarrow H Z$ and $e^{+} e^{-} \rightarrow$ $\gamma Z$ with polarized beams. We found that in the presence of TP, there is a CPodd and T-odd contribution to the angular distribution, dependent on coupling combinations which cannot be determined without TP. Moreover, in the case of $H Z$ production, this contribution does not exist when only $Z Z H$ anomalous couplings are considered. Hence such a term, if observed, would be a unique signal of CPviolating four-point interaction.

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\text { Transverse polarization in } e^{+} e^{-} \rightarrow \gamma Z \text { and } e^{+} e^{-} \rightarrow H Z
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