Gauge unification in 5-D $SU(5)$ model with orbifold breaking of GUT symmetry

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ABSTRACT

We consider a 5-dimensional $SU(5)$ model wherein the symmetry is broken to the 4-dimensional Standard Model by compactification of the 5th dimension on an $S^1/(Z_2 \times Z_2')$ orbifold. We identify the members of all $SU(5)$ representations upto **75** which have zero modes. We examine how these light scalars affect gauge coupling unification assuming a single intermediate scale and present several acceptable solutions. The 5-D compactification scale coincides with the unification scale of gauge couplings and is determined via this renormalization group analysis. When $SO(10)$ is considered as the GUT group there are only two solutions, so long as a few low dimensional scalar multiplets upto **126** are included.

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1 Introduction

The $SU(5)$ model[1] unifies the strong, weak, and electromagnetic interactions in the smallest simple group. It has many other attractive features which are well recognized. But it suffers from the following two major difficulties which are actually generic to the idea of grand unification[2] itself. (i) Because quarks and leptons reside in unified multiplets and there are B- and L-violating interactions, gauge boson exchanges can result in proton decay[3]. If these gauge bosons are appropriately heavy, the decay rate will be very small. Their masses, in the usual formulation, are, however, not arbitrary but rather determined by the scale where the different gauge couplings unify. The proton decay lifetime is therefore a robust prediction of the model. No experimental signature of proton decay[4] has been found yet and the model is disfavoured. More complicated unification models involving several intermediate mass-scales can evade this problem[5]. (ii) The low energy Higgs doublet, responsible for electroweak breaking, is embedded in a **5** representation of $SU(5)$. The other members of this multiplet are color triplet scalars which must have a mass near the unification scale – since no such scalars have been observed at the electroweak scale. This leads to an unnatural mass splitting among the members of the same $SU(5)$ multiplet. This is termed the double-triplet splitting problem[6].

These two unwelcome features of the $SU(5)$ model can be tackled in an elegant way if unified $SU(5)$ symmetry exists in a 5-D world. Low energy 4-D $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry is recovered when the extra dimension is compactified on a $S^1/(Z_2 \times Z_2')$ orbifold[7]. This situation is realized when space-time is considered to be factorized into a product of 4D Minkowski space-time M^4 and the orbifold $S^1/(Z_2 \times Z_2')$. The coordinate system consists of $x^\mu = (x^0, x^1, x^2, x^3)$ and $y = x^5$. There are two distinct 4-D branes; one at $y = 0$ and another at $y = \pi R/2$. On the S^1, y=0 is identified with $y = \pi R$ (Z_2' symmetry) while $y = \pm \pi R/2$ are identified with each other (Z_2 symmetry).

As is common in models of this type, we assume that the fermions are located in the 4-D brane at $y = 0$ while the gauge bosons and the scalars are allowed to travel in the bulk. The discrete Z_2 and Z_2' symmetries, which we refer to as P and P', permit the expansion of any 5-D field ϕ in the following mode

expansions according to whether they are even or odd under (P, P'):

$$
\begin{aligned}
\phi_{++}(y) &= \sqrt{\tfrac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)} \cos \tfrac{2ny}{R}; & M_n &= \tfrac{2n}{R} \\
\phi_{-+}(y) &= \sqrt{\tfrac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)} \sin \tfrac{(2n+1)y}{R}; & M_n &= \tfrac{2n+1}{R} \\
\phi_{+-}(y) &= \sqrt{\tfrac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)} \cos \tfrac{(2n+1)y}{R}; & M_n &= \tfrac{2n+1}{R} \\
\phi_{--}(y) &= \sqrt{\tfrac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)} \sin \tfrac{(2n+2)y}{R}; & M_n &= \tfrac{2n+2}{R}
\end{aligned}
$$

Here $n = 0, 1, 2, \ldots$ and we have suppressed the x^μ dependence. The behaviour of the fields under P and P' can be read off from the subscripts in the left hand side above. For example, ϕ_{-+} is odd under P and even under P'. We have also listed the masses of the different modes. Notice that only the ϕ_{++} field can have a massless mode. One of the prime motivations of these higher dimensional $SU(5)$ models is to ensure doublet-triplet splitting within the **5** scalar multiplet of $SU(5)$ and to ensure that from within the adjoint representation (**24**) of the gauge bosons only the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge bosons remain massless. Both can be achieved by ascribing P, P' parities of $++$ to the (1,2,1/2) submultiplet in the **5** while the remaining (3,1,–1/3) states carries $+-$ parity[1]. For the sake of completeness, the decomposition of the $SU(5)$ representations upto **75** are listed in Table (1) [7]. Since P, P' commute with the Standard Model (SM) gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$, the $Z_2 \times Z_2'$ parities for the members of the higher $SU(5)$ multiplets can be built up from this assignment for the scalars in the fundamental representation[2]. These parities have been indicated in Table (1). In this way we can also assure that the (1,1,1)+(1,3,0)+(8,1,0) multiplets of **24** remain massless, breaking $SU(5)$ symmetry below the compactification scale $1/R \equiv M_X$.

Above M_X, the mass scale of the non-SM gauge bosons, X, Y, and that of the colour triplet scalars in the **5** representation, $SU(5)$ symmetry is unbroken. This scale is determined in our analysis by the unification of the three SM gauge couplings. We assume one intermediate scale, M_I, such that all scalars which are permitted to have a zero mode, excepting the SM scalar doublet,

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[1]Here we are using the decomposition of the $SU(5)$ multiplets under $SU(3)_c \times SU(2)_L \times U(1)_Y$.

[2]In principle, one can assign arbitrary $Z_2 \times Z_2'$ parities to the submultiplets of the higher SU(5) representations. However, to reduce adhocness, here we work with the *ansatz* that once we assign the parities for the fundamental representation, those for the submultiplets of higher $SU(5)$ representations are determined by group theoretic relationships.

$SU(5) \supset$	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$\mathbf{5} \supset$	$(1,2,1/2)_{+\,+} + (3,1,-1/3)_{+\,-}$
$\mathbf{\bar{5}} \supset$	$(1,2,-1/2)_{+\,+} + (\bar{3},1,1/3)_{+\,-}$
$\mathbf{10} \supset$	$(1,1,1)_{+\,+} + (\bar{3},1,-2/3)_{+\,+} + (3,2,1/6)_{+\,-}$
$\mathbf{15} \supset$	$(1,3,1)_{+\,+} + (3,2,1/6)_{+\,-} + (6,1,-2/3)_{+\,+}$
$\mathbf{24} \supset$	$(1,1,0)_{+\,+} + (1,3,0)_{+\,+} + (3,2,-5/6)_{+\,-} + (\bar{3},2,5/6)_{+\,-} + (8,1,0)_{+\,+}$
$\mathbf{35} \supset$	$(1,4,-3/2)_{+\,+} + (\bar{3},3,-2/3)_{+\,-} + (\bar{6},2,1/6)_{+\,+} + (\overline{10},1,1)_{+\,-}$
$\mathbf{40} \supset$	$(1,2,-3/2)_{+\,+} + (3,2,1/6)_{+\,+} + (\bar{3},1,-2/3)_{+\,-} + (\bar{3},3,-2/3)_{+\,-}$
	$+(8,1,1)_{+\,-} + (\bar{6},2,1/6)_{+\,+}$
$\mathbf{45} \supset$	$(1,2,1/2)_{+\,+} + (3,1,-1/3)_{+\,-} + (3,3,-1/3)_{+\,-} + (\bar{3},1,4/3)_{+\,-}$
	$+(\bar{3},2,-7/6)_{+\,+} + (\bar{6},1,-1/3)_{+\,-} + (8,2,1/2)_{+\,+}$
$\mathbf{50} \supset$	$(1,1,-2)_{+\,+} + (3,1,-1/3)_{+\,+} + (\bar{3},2,-7/6)_{+\,-} + (\bar{6},3,-1/3)_{+\,-}$
	$+(6,1,4/3)_{+\,+} + (8,2,1/2)_{+\,-}$
$\mathbf{70} \supset$	$(1,2,1/2)_{+\,+} + (1,4,1/2)_{+\,+} + (3,1,-1/3)_{+\,-} + (3,3,-1/3)_{+\,-}$
	$+(\bar{3},3,4/3)_{+\,-} + (6,2,-7/6)_{+\,+} + (8,2,1/2)_{+\,+} + (15,1,-1/3)_{+\,-}$
$\mathbf{70'} \supset$	$(1,5,-2)_{+\,+} + (\bar{3},4,-7/6)_{+\,-} + (\bar{6},3,-1/3)_{+\,+} + (\overline{10},2,1/2)_{+\,-}$
	$+(\overline{15},1,4/3)_{+\,+}$
$\mathbf{75} \supset$	$(1,1,0)_{+\,+} + (3,1,5/3)_{+\,+} + (3,2,-5/6)_{+\,-} + (\bar{3},1,5/3)_{+\,+}$
	$+(\bar{3},2,5/6)_{+\,-} + (\bar{6},2,-5/6)_{+\,-} + (6,2,5/6)_{+\,-} + (8,1,0)_{+\,+}$
	$+(8,3,0)_{+\,+}$

Table 1: The $SU(3)_c \times SU(2)_L \times U(1)_Y$ contents of the different $SU(5)$ representations. Also shown are the P and P' assignments.

pick up a mass at this scale. We include their contributions to the beta functions of the one loop renormalization group equations (RGE) and solve for both the intermediate scale M_I and the unification scale M_X. The beta function coefficients are given by:

$$b_i = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + n_f \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + T_s^i/3 \qquad (1)$$

We take $n_f = 3$. The above expression assumes that the scalar fields are complex. For real scalar fields one has to use $T_s^i/6$ in Eqn. (1). The T_s^i

R	light scalar multiplets	T_s^3	T_s^2	T_s^1
5	(1,2,1/2)	0	1/2	3/10
10	(1,1,1)+($\overline{3}$, 1,- 2/3)	1/2	0	7/5
15	(1,3,1)+(6,1,-2/3)	5/2	2	17/5
24	(1,3,0)+(8,1,0)	3	2	0
35	(1,4,-3/2)+($\overline{6}$,2,1/6)	5	8	28/5
40	(1,2,-3/2)+(3,2,1/6)+($\overline{6}$,2,1/6)	6	5	3
45	(1,2,1/2)+($\overline{3}$,2,-7/6)+(8,2,1/2)	7	6	38/5
50	(1,1,-2)+(3,1,-1/3)+ ($\overline{6}$,3,-1/3)+(6,1,4/3)	21/2	3	51/5
70	(1,2,1/2)+(1,4,1/2)+(6,2,-7/6)+(8,2,1/2)	11	25/2	131/10
70′	(1,5,-2)+($\overline{6}$,3,-1/3)+($\overline{15}$,1,4/3)	25	22	146/5
75	(3,1,5/3)+($\overline{3}$,1,-5/3)+(8,1,0)+(8,3,0)	13	16	10

Table 2: The contributions to the β-functions from the light members of the different $SU(5)$ representations upto **75**.

for the light scalar submultiplets of the different $SU(5)$ representations upto **75** are listed in Table (2). Defining $m_{k,l} = \ln(m_k/m_l)$ and $b_{k,l}^i$ to be the β coefficients governing evolution in the range $m_k \leftrightarrow m_l$, we get the following three solutions of the RGE.

$$2\pi\alpha_i^{-1}(M_Z) = 2\pi\alpha_X^{-1} + b_{X,I}^i m_{X,I} + b_{I,Z}^i m_{I,Z} \qquad (2)$$

Using the values of couplings at the low energy scale M_Z

$$\alpha_1(M_Z) = 0.01688, \quad \alpha_2(M_Z) = 0.03322, \quad \alpha_3(M_Z) = 0.117 \qquad (3)$$

we solve the three equations in Eqn. (2). First, we present a simple illustrative example below.

2   Simple example

Because the GUT symmetry is broken via orbifolding, let us consider the case where there are only **5**-plets of $SU(5)$ Higgs scalars at the unification scale and assume that there are n_5 of them. Then compactification allows only doublets to be light and not their triplet partners. In this case the β

coefficients are given by,

$$b^i_{X,I} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} + \frac{n_5}{3} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix} \tag{4}$$

Solving Eqn. (2) we obtain

$$\alpha_X^{-1} = 38.53, m_{I,Z} = 26.98 - 194.75/n_5, m_{X,I} = 194.75/n_5 \tag{5}$$

Because $m_{I,Z} \geq 0$ we obtain $n_5 \geq 8$. For the case of $n_5 = 8$ we get,

$$M_I = 1.39 \text{ TeV}, M_X = 5.0 \times 10^{10} \text{ TeV}. \tag{6}$$

The GUT scale M_X is rather low but it is consistent with proton decay because of the existence of $Z_2 \times Z'_2$ parity. Note that the intermediate scale is in a very attractive region phenomenologically. Eight Higgs doublets can be degenerate at this scale of 1.3-1.4 TeV. They may play an important role in the fermion mass puzzle. Further, the scale $M_X \simeq 10^{10}$ TeV is interesting from the point of view of the see-saw mechanism. The unification pattern is shown in Fig. (1).

3 More general cases

We now turn to the more general possibility where scalars in higher representations of $SU(5)$ are present. The light scalars of all $SU(5)$ multiplets upto **75** and their contributions to the beta coefficients are listed in Table (2).

3.1 Small number of representations and low intermediate scales

We consider upto the **75** dimensional representation of $SU(5)$ and demand that the threshold scale, M_I, be less than 10 TeV. For the sake of economy, we also consider only those solutions where for every representation R, the number n_R is either 0 or 1. If we do not put any restriction on the number of representations, but maintain that n_R be zero or unity only, then we get 43 different solutions. In Table (3) we list those solutions for which not more than two n_R are non-zero.

Representations	M_I	M_X
with $n_R = 1$	(TeV)	(TeV)
35	0.223	1.61×10^{11}
5,35	5.70	1.38×10^{11}
24,35	0.905	5.50×10^{11}
35,40	3.61	1.84×10^{12}
45,75	3.61	1.84×10^{11}

Table 3: $SU(5)$ repesentations of scalars whose light members ensure coupling constant unification. The unification scale, M_X, and the intermediate scale, M_I, are also given.

Though the intermediate scales, M_I, and the unification scales, M_X, in the last two cases are the same, the value of α_X turns out to be 0.037 and 0.710, respectively.

Let us explain one case in more detail. Let there be only **35**-plets of $SU(5)$ at the unification scale and assume that there are n_{35} of them. Then compactification allows only $(1,4,-3/2) + (\overline{6},2,1/6)$ fields at low energy. In this case the β_i coefficients are given by,

$$b^i_{X,I} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} + \frac{n_{35}}{3} \begin{pmatrix} 28/5 \\ 8 \\ 5 \end{pmatrix} \tag{7}$$

Solving the RGE we obtain

$$\alpha_X^{-1} = 32.72, m_{I,Z} = 28.20 - 27.3073/n_{35}, m_{X,I} = 27.3073/n_{35} \tag{8}$$

Because $m_{I,Z} \geq 0$ we obtain $n_{35} \geq 1$. For the case of $n_{35} = 1$ we get,

$$M_I = 0.223 \text{ TeV}, M_X = 1.6 \times 10^{11} \text{ TeV}. \tag{9}$$

3.2 Low dimensional representations only

Another alternative which we examine is by restricting to $SU(5)$ representations upto **24** subject further to the requirements $n_5 < 8, n_{10} < 5, n_{15} <$

$5, n_{24} < 5$. Then we get the results given in Table (4) when we impose $M_I < 70$ TeV. The unification patterns of the gauge couplings for a few sample cases are shown in Fig. (1).

n_5	n_{10}	n_{15}	n_{24}	M_I (TeV)	M_X (TeV)
8	0	0	0	1.678	4.66×10^{10}
7	0	0	1	0.223	1.61×10^{11}
8	0	0	1	5.696	1.38×10^{11}
7	0	0	2	0.905	5.49×10^{11}
8	0	0	2	19.08	4.05×10^{11}
7	0	0	3	3.607	1.84×10^{12}
8	0	0	3	63.08	1.17×10^{12}
6	0	0	4	0.407	1.33×10^{13}
7	0	0	4	14.12	6.08×10^{12}
6	0	0	5	0.199	5.22×10^{13}
7	0	0	5	54.36	1.97×10^{13}
8	1	0	5	0.407	1.33×10^{13}
8	0	1	5	0.407	1.33×10^{13}

Table 4: Various $SU(5)$ scrnarios which gives low intermediate scales upto 70 TeV

4   Remarks about $SO(10)$

It might be of interest to extend this analysis to grand unification groups of higher rank. It is readily seen that the solutions will become more difficult to come by. For example, we give in Table (5) the $SU(5)$ contents of the $SO(10)$ representations upto **126**. Notice, that the inclusion of a single **126** of $SO(10)$ is equivalent to the simultaneous presence of $\overline{\mathbf{5}}$, **10**, $\overline{\mathbf{15}}$, **45**, $\overline{\mathbf{50}}$ repesentations of $SU(5)$ and there is no flexibility of including the $SU(5)$ representations individually.

R	SU(5) components	T_s^3	T_s^2	T_s^1
10	$\mathbf{5} + \overline{\mathbf{5}}$	0	1	3/5
16	$\mathbf{1} + \overline{\mathbf{5}} + \mathbf{10}$	1/2	1/2	17/10
45	$\mathbf{1} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{24}$	4	2	14/5
120	$\mathbf{5} + \overline{\mathbf{5}} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{45} + \overline{\mathbf{45}}$	15	13	93/5
126	$\mathbf{1} + \overline{\mathbf{5}} + \mathbf{10} + \overline{\mathbf{15}} + \mathbf{45} + \overline{\mathbf{50}}$	41/2	23/2	229/10

Table 5: The contributions to the β-functions from the light members of the different SO(10) representations upto **126**.

If we permit all SO(10) representations upto **126** and consider no more than upto 8 of any single representation then we find just two allowed solutions:

1) $n_{10} = 8$, other $n_i = 0 \Rightarrow M_I = 1.68$ TeV, $M_X = 4.67 \times 10^{10}$ TeV

2) $n_{10} = 6$, $n_{16} = 1$, other $n_i = 0 \Rightarrow M_I = 295$ TeV, $M_X = 7.99 \times 10^9$ TeV

5 Conclusions and Discussion

In this work, we have examined the light scalar modes that survive when a 5-dimensional $SU(5)$ model reduces to the 4-dimensional SM through the orbifold compactification route. The scalars which are permitted to have zero modes are assumed to pick up a mass at some scale M_I intermediate between the electroweak and Planck scales. They contribute to the beta coefficients in the $M_I < \mu < M_X$ regime. The compactification scale, M_X, above which $SU(5)$ is unbroken, is determined by the unification of the gauge couplings. This analysis also determines M_I. We identify solutions for which M_I is in an interesting phenomenological range and can be probed at the next generation colliders. This analysis is somewhat similar in spirit to the approach chosen for supersymmetric-GUTs where the SUSY scale is fixed by gauge unification.
It is seen from Table (2) that P is (+) for all the multiplets. Thus it does not play any role in the present analysis. However, we would like to keep the option of generalizing this method to the supersymmetric case where P has a non-trivial role.

The scale of degenerate scalars, M_I, should be treated as an approximate one in the sense that in reality some spread in the masses around it can be expected. The standard model doublet has a mass at the electroweak scale. This should not be viewed as an unnatural fine tuning as some relevant Yukawa couplings can be of order $10^{-2} - 10^{-3}$.

When the unification symmetry is assumed to be of higher rank, then the number of acceptable solutions reduces dramatically. For $SO(10)$ just two solutions can be obtained, so long as we stick to the low dimensional representations of the symmetry group.

At first sight it might seem that we are introducing too many scalar degrees of freedom. However, this appears less dramatic when we compare it to the Minimal Supersymmetric Standard Model where too a large number of scalars are required.

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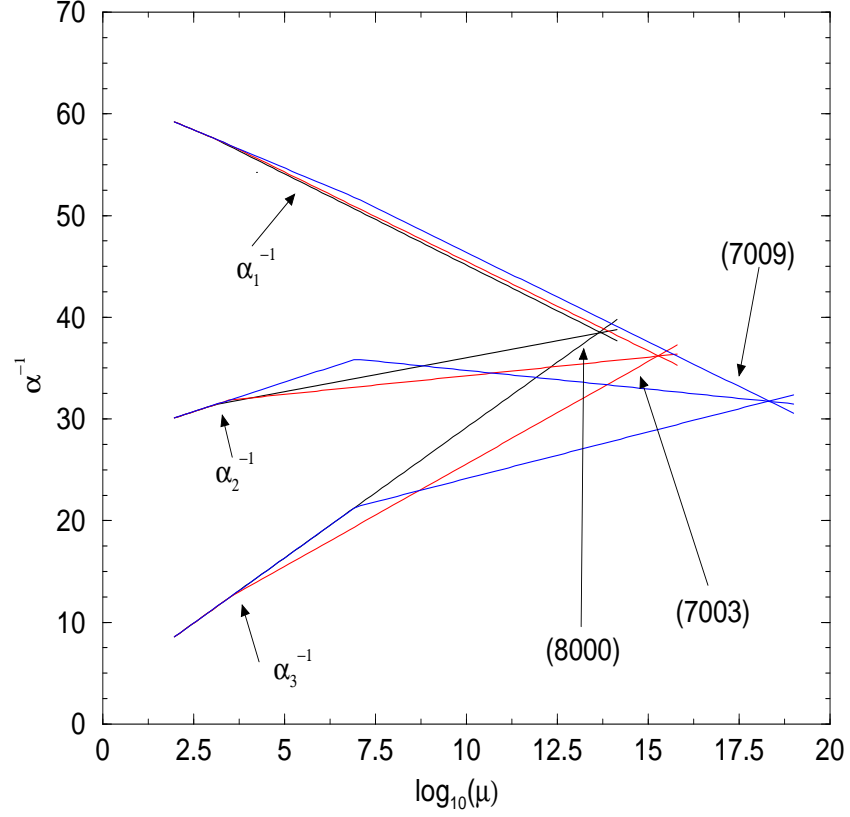
Figure 1: Gauge unification in various models. Labels of cases are $(n_5, n_{10}, n_{15}, n_{24})$. As a first approximation we have used one intermediate scale which is given by the mass scale of extra scalars allowed by $S^1/Z_2 \times Z_2'$ compactifications.