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An SO(10) model with adjoint fermions for double seesaw neutrino masses

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Abstract

An $SO(10)$ model where the 10_H and 120_H representations are used for generating fermion masses is quite predictive, though due to the absence of $SU(2)_{L,R}$ triplet/singlet fields it cannot give rise to neutrino masses through the usual type-I or type-II seesaw mechanisms. In this paper for neutrino masses we propose an extension of such an $SO(10)$ model by adding fermions in the adjoint representation (45_F) and a symmetry breaking scalar $\overline{16}_H$. The $\overline{16}_H$ couples the adjoint fermions to the standard fermions in 16_F and induces neutrino masses through the 'double seesaw' mechanism. In order to enhance the predictivity of the model we impose $\mu - \tau$ flavour symmetry on the Yukawa matrices for 10_H and $\overline{16}_H$ whereas for the 120_H it is assumed to be antisymmetric. We discuss the conditions that the mass matrices must obey so that the model can reproduce the tri-bimaximal mixing pattern.

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I Introduction

A number of experiments with solar, atmospheric, reactor and accelerator neutrinos have now unambiguously established that these elusive particles are massive. In addition, the data imply one small and two large mixing angles in complete contrast with the quark sector where all three mixing angles are small. In Table 1 we present the best-fit values and 3σ ranges of neutrino oscillation parameters as obtained from the global oscillation analysis [1]. These values are close to the so called tri-bimaximal mixing pattern [2] which implies $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{13} = 0$.

Since in the Standard Model (SM) neutrinos are massless this compels one to transcend beyond the realms of the SM. There are also several theoretical motivations for going beyond the SM, one of which is that the SM is a product of three gauge groups and so involves three independent couplings. A Grand Unified Theory (GUT), which is a theory of strong and electroweak interactions based on a single gauge group [3], aims to unify the three forces with a single coupling constant [4]. It also unifies the matter fields by placing the quarks and leptons in the same irreducible representation of the underlying gauge group [5]. Since GUTs aim to unify quarks and leptons it is a challenge to reconcile the large mixings in the lepton sector with the small mixings in the quark sector. The issue of fermion masses and mixing in the context of GUTs has received much attention from this perspective.

Several GUT models based on gauge symmetries such as $SU(5)$, $SO(10)$, and $E(6)$ have been proposed and studied extensively. The minimal GUT group which has the same rank as $\mathcal{G}_{SM} \equiv SU(3) \otimes SU(2)_L \otimes U(1)_Y$ is $SU(5)$ [3]. $SU(5)$ requires two different representations $(\bar{5} + 10)$ to accommodate all the fermions of one generation. Moreover the minimal model does not achieve gauge coupling unification neither does it allow a neutrino mass. On the other hand, $SO(10)$ GUT has the feature of unifying all quarks and leptons within its 16-dimensional spinor representation [6]. This accounts for the 15 SM fermions and a right-handed neutrino and allows a natural implementation of the seesaw mechanism [7]. It has been shown in a number of papers that renormalizable $SO(10)$ – with and without supersymmetry (SUSY) – is quite predictive and powerful in constraining fermion mass patterns because of the underlying $SU(4)_c$ symmetry which relates the quark and lepton Yukawa couplings. In $SO(10)$, $16 \otimes 16 = 10 \oplus 120 \oplus 126$ and so Higgs fields giving mass to the 16_F can reside in the 10_H, 120_H and $\overline{126}_H$ representations. Obtaining correct masses for the quarks and the charged leptons requires at least two Higgs multiplets. It has been noted, for example in [8], that any one of the combinations $(10_H, 120_H)$, $(10_H, \overline{126}_H)$, or $(120_H, \overline{126}_H)$ can, in principle, be utilized. Among these the model with 10_H and $\overline{126}_H$ has been extensively considered as the most successful candidate for the minimal $SO(10)$ GUT [9]. $\overline{126}_H$ contains colour singlet submultiplets which transform as a triplet under $SU(2)_L$ and a singlet under $SU(2)_R$ or *vice versa*; these are the cornerstones of the seesaw mechanism [7]. Both type-I (mediated through singlets [7]) and type-II (mediated through scalar triplets [10]) seesaw have been examined for both supersymmetric [11] and non-supersymmetric [12] cases. The $\overline{126}_H$ relates the Majorana mass of the neutrinos to the Dirac mass as well as other charged fermion masses making the model predictive. It is also possible and in some cases advantageous to include all the three Higgs representations [13, 14]. The model with $10_H + 120_H$ [15, 16], on the other hand, does not have the requisite scalars to lead to neutrino masses through the seesaw mechanism. Here, neutrino mass can be obtained at two loop through the radiative seesaw mechanism due to Witten [17] by adding $16_H + \overline{16}_H$ multiplets. This model has been studied in [18] and it was shown that under plausible assumptions it predicts $b - \tau$ unification, natural occurrence of large leptonic and small quark mixing and large value for the atmospheric mixing angle. However, the radiative seesaw runs into difficulty with low-energy SUSY although it works well in the context of split SUSY

	best fit	3σ range
Δm^2_{21} [10^{-5} eV^2]	7.59	7.03 - 8.27
$\lvert\Delta m^2_{31}\rvert$ [10^{-3} eV^2]	2.40	2.07 - 2.75
$\sin^2\theta_{12}$	0.318	0.27 - 0.38
$\sin^2\theta_{23}$	0.50	0.36 - 0.67
$\sin^2\theta_{13}$	0.013	≤ 0.053

Table 1: The best-fit values and the 3σ ranges of neutrino mass and mixing parameters as obtained from a global analysis of oscillation data [1]. $\Delta m^2_{ij} = m^2_i - m^2_j$.

[19]. Moreover, as has been shown in [16] the SUSY $SO(10)$ model containing 10_H and 120_H cannot reproduce the charged fermion masses correctly. On the other hand in non-SUSY $SO(10)$ the two-loop neutrino mass is very small.

In this paper we consider the generation of neutrino masses in the 10_H+120_H model embellished with a $\overline{16}_H$ by adding fermions belonging to the adjoint representation (45_F) of $SO(10)$. Such fermions couple to the usual sixteen-plet of quarks and leptons *via* the $\overline{16}_H$ and can give rise to neutrino masses through the 'double seesaw' mechanism. In models with $10_H + 120_H$ this can serve as an alternative option for generating small neutrino masses[1]. Fermions in the triplet adjoint representation of $SU(2)_L$ are also considered in the so called type-III [21] seesaw mechanism. Such models have become quite popular in the context of $SU(5)$ GUTs [22]. $SU(2)_L$ triplet fermions fit naturally into the 24-dimensional representation of $SU(5)$ and can cure two main problems of these theories, *viz.* generation of neutrino masses and unification of gauge couplings. The latter requires the mass of the fermionic triplets to be $\sim \mathcal{O}(1$ TeV) making the model testable at the LHC [23]. Presence of adjoint fermions in the context of left-right symmetric models has been considered in [24] and generation of neutrino masses and possible collider signatures were discussed. From this point of view our model can also be considered as a generalization of type-III seesaw for $SO(10)$. However as in LR symmetric models the mechanism of mass generation here is actually the 'double seesaw' mechanism.

We discuss the conditions which the Yukawa coupling matrices should satisfy for the model to have predictive power. This requires ascribing some additional flavour symmetry to the model which we choose to be the generalized $\mu - \tau$ symmetry that has been considered widely for explaining the neutrino mixing angles [25]. It predicts θ_{23} to be $\pi/4$ which is the best-fit value of this angle from global fits. In addition it implies $\theta_{13} = 0$ which is also consistent with the data. Small deviation from these exact values may be generated by breaking the $\mu - \tau$ symmetry by a small amount. Combining $\mu - \tau$ flavour symmetry with GUTs has been considered in the case of $SU(5)$ in [26] and also for $SO(10)$ [14]. Here we impose $\mu - \tau$ symmetry on the Yukawa matrix for the 10_H and $\overline{16}_H$ whereas the one for 120_H is taken to be antisymmetric. We also impose a parity symmetry leading to Hermitian Yukawa matrices. Thus we consider the model $SO(10) \otimes Z_2^{(\mu-\tau)} \otimes Z_2^{\mathcal{P}}$ [14]. Imposition of these two symmetries help in reducing the number of unknown parameters in the Yukawa sector. In addition, we make an ansatz relating the effective ν_R mass matrix arising due to the inclusion of adjoint fermions with the Yukawa matrix for 10_H. As a result the light neutrino mass matrix after seesaw mechanism obtains a simple form and can be written as a sum of two contributions. It turns out that with the above choice the neutrino mass matrix is $\mu - \tau$ symmetric so that one immediately gets $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. It is straight-forward to get the prediction for the neutrino masses and θ_{12} and obtain the

[1]It is also possible to get a double seesaw type mass matrix using singlet fields [20].

conditions on the parameters such that tri-bimaximal mixing is obtained. We also present the limiting values when one of the two contributions dominates. With the above set of assumptions one can get masses and mixing angles consistent with those presented in Table 1.

The plan of the paper is as follows. In the next section we discuss the model. In section III we compute the evolution of the gauge couplings in the context of this model and obtain the range of the intermediate as well as unification scales. In section IV we discuss the neutrino mass matrix. Finally in section V we impose $\mu - \tau$ symmetry and obtain predictions for neutrino masses and mixing angles. We end with the conclusions.

II The Model

We explore an $SO(10)$ model where the three fermion families acquire mass through the 10_H and/or 120_H. The model also includes additional fermion multiplets in the $SO(10)$ adjoint representation, 45_F, and a $\overline{16}_H$.

In this model the Yukawa terms for the fermions can be expressed as:

$$\mathcal{L} = Y_{10} 16_F 16_F 10_H + Y_{120} 16_F 16_F 120_H. \tag{1}$$

In general, Y_{10} is a complex symmetric matrix while Y_{120} is complex antisymmetric. When the 10_H and 120_H scalars obtain their vacuum expectation values (*vevs*) quarks and leptons obtain masses which can be represented as:

$$\begin{aligned} m_d &= M_0 + iM_2, \;\; m_u = c_0 M_0 + ic_2 M_2, \\ m_l &= M_0 + ic_3 M_2, \;\; m_D = c_0 M_0 + ic_4 M_2. \end{aligned} \tag{2}$$

Above, m_d (m_u) denotes the mass matrix for the d-type (u-type) quarks, m_l is the charged lepton mass matrix, whereas m_D is the Dirac mass matrix of the neutrinos. The matrices M_0 and M_2 are proportional to Y_{10} and Y_{120} respectively.

$$M_0 = M_0^T, \;\; M_2 = -M_2^T. \tag{3}$$

c_0, c_2, c_3, and c_4 are constants fixed by Clebsch-Gordan (CG) coefficients and *vev* ratios which are taken to be real. We impose a generalized parity symmetry and make appropriate choices of the *vev*s [27] which make M_0 and M_2 real thereby reducing the number of free parameters and ensuring the hermiticity of the mass matrices in eq. (2).

For neutrinos the above implies the presence of only the Dirac mass term which cannot reproduce the correct neutrino mass pattern [18]. Since the $\overline{126}_H$ field is not present the type-I and type-II seesaw mass terms are absent in this model. One can of course generate the neutrino mass through the Witten mechanism of radiative seesaw [17] but then for non-SUSY $SO(10)$ such contributions are too small [18].

In this work we propose a new mechanism to generate a neutrino mass in a non-SUSY $SO(10)$ with 10_H and 120_H. We introduce additional matter multiplets (45_F) which belong to the adjoint representation of $SO(10)$. Note that this is similar to the so called type-III seesaw mechanism where one adds additional matter fields in the adjoint representation. However, as we will see, the neutrino mass

is generated here through the 'double seesaw' mechanism. $SO(10)$ breaks to the SM through two intermediate steps:

$$SO(10) \xrightarrow{M_X} SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \xrightarrow{M_C} SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{(B-L)} \xrightarrow{M_R} \mathcal{G}_{SM}. \quad (4)$$

The Pati-Salam ($\mathcal{G}_{422} \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$) decomposition gives:

$$45 = (\Sigma_{3L}, \Sigma_{3R}, \Sigma_{4C}, \Sigma_{LRC}) = (1,3,1) \oplus (1,1,3) \oplus (15,1,1) \oplus (6,2,2). \quad (5)$$

It is useful to note the $SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ decompositions

$$\begin{aligned} (15,1,1) &\equiv (1,1,0,0) + (3,1,0,-4/3) + (\bar{3},1,0,4/3) + (8,1,0,0)\ , \quad (6) \\ (4,1,2) &\equiv (1,1,\pm\frac{1}{2},1) + (3,1,\pm\frac{1}{2},-1/3)\ . \end{aligned}$$

The colour, $U(1)_R$, and $U(1)_{(B-L)}$ singlet members of Σ_{3R} and Σ_{4c} couple to ν_R when $\overline{16}_H$ gets a *vev* along $(1,1,-\frac{1}{2},1) \subset$(4,1,2) that breaks $U(1)_R \otimes U(1)_{B-L}$. The relevant Yukawa coupling is:

$$Y_{16} 16_F 45_F \overline{16}_H \supset Y_{16} \left[a_1 (1,1,\frac{1}{2},-1)_F (1,1,0,0)_F^{\Sigma_{3R}} + a_2 (1,1,\frac{1}{2},-1)_F (1,1,0,0)_F^{\Sigma_{4c}} \right] (1,1,-\frac{1}{2},1)_H\ . \quad (7)$$

$a_{1,2}$ are CG coefficients. The *vev* $v_R \equiv < (1,1,-\frac{1}{2},1)_H >$ sets the scale M_R.

The masses of the adjoint matter fields are generated from

$$M\ Tr(45_F^2)\ +\ \lambda\ Tr(45_F^2 210_H)\ . \quad (8)$$

Once 210_H acquires a *vev* along the (1,1,1) direction, $SO(10)$ is broken to $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. In the mass term M_N of $(1,1,0,0)_F \subset (15,1,1)_F$ and $M_{\Sigma_{3R}}$ of $(1,1,0,0)_F \subset (1,1,3)_F$, an extra contribution (from the second term of eq. (8)) is added, *i.e.,* $M_{\Sigma_{3R}} = M_N = M + \lambda < (1,1,0,0)_H >$. There is no symmetry that protects the masses of these adjoint fermions. So naturally these are very heavy ($\sim M_X$).

III Constraints from gauge coupling unification

In this section, we discuss the Renormalization Group (RG) evolution of the gauge couplings at the one-loop level, check for the scale of unification and determine the possible intermediate scales. The symmetry breaks in two stages following the steps given in (4). The contributions in the RG running from scalars at the different scales are included according to the 'extended survival hypothesis'[2] (ESH) [28] which amounts to minimal fine tuning of the parameters of the potential. Our model contains extra adjoint fermions. But these fermions are very heavy $\sim \mathcal{O}\ (M_X)$, so they do not contribute in the renormalization group evolution of the gauge couplings.

When the $SO(10)$ symmetry is broken to the Pati-Salam group [5] $\mathcal{G}_{422}$ by a 210_H multiplet through the *vev* in the $< (1,1,1) >$ direction, D-parity[3] [29] is spontaneously broken at this scale (M_C).

[2]Only those scalars are light which take part in the symmetry breaking.

[3]D-parity is a symmetry that connects the $SU(2)_L$ and $SU(2)_R$ sectors of a multiplet.

$SO(10)$ representation	Symmetry breaking	Scalars contributing to RG evolution		
		$M_Z \to M_R$ Under $\mathcal{G}_{SM}$	$M_R \to M_C$ Under $\mathcal{G}_{3211}$	$M_C \to M_X$ Under $\mathcal{G}_{422}$
10	$\mathcal{G}_{SM} \to EM$	(1,2,$\pm$1)	(1,2,$\pm\frac{1}{2}$,0)	(1,2,2)
120		...	...	(1,2,2), (15,2,2)
$\overline{\mathbf{16}}$	$\mathcal{G}_{3211} \to \mathcal{G}_{SM}$	...	(1,1,$-\frac{1}{2}$,1)	(4,1,2)
210	$\mathcal{G}_{422} \to \mathcal{G}_{3211}$	...	...	(15,1,3)

Table 2: Higgs submultiplets contributing to the RG evolution as per the extended survival hypothesis when symmetry breaking of $SO(10)$ takes place with two intermediate stages – see (4).

The gauge coupling evolution is usually stated as [4]:

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_i, g_j), \qquad (i, j = 1, \ldots, n), \tag{9}$$

where n is the number of couplings in the theory and at one-loop order

$$\beta_i(g_i, g_j) = (16\pi^2)^{-1} b_i g_i^3. \tag{10}$$

There is, however, a subtlety which must be taken into account since the gauge symmetry in the energy range M_R to M_C includes two $U(1)$ factors. According to the ESH the $SO(10)$ multiplets are split in mass with some submultiplets having mass above and some below this range. The incomplete scalar and fermion multiplets that contribute to the RG evolution at this stage lead to a mixing between these two $U(1)$ gauge groups. Thus even at the one-loop level one cannot treat the evolution of these $U(1)$ couplings in separation and in a generic scenario one must include a (2×2) matrix of $U(1)$ couplings. The details of this $U(1)$ mixing are skipped here[4]. We have computed the RG-coefficients following the proposals given in [30] at the one-loop level including the $U(1)$ mixings. The b_i are the ordinary beta-coefficients and the $\tilde{b}_j$ are the additional ones which arise due to the mixings stated above. Taking all this into account, the gauge couplings evolve as follows:

i) From M_C to M_X :

$$b_{2L} = 7/3; \;\; b_{2R} = 13; \;\; b_{4c} = -1. \tag{11}$$

ii) From M_R to M_C:

$$b_{2L} = -3; \; b_{RR} = 53/12; \; b_{3c} = -7; \; b_{(B-L)(B-L)} = 33/8; \; \tilde{b}_{R(B-L)} = \tilde{b}_{(B-L)R} = -1/4\sqrt{6}. \tag{12}$$

iii) From M_Z to M_R:

$$b_{1Y} = 21/5; \;\; b_{2L} = -3; \;\; b_{3c} = -7. \tag{13}$$

[4]See, for example, reference [30].

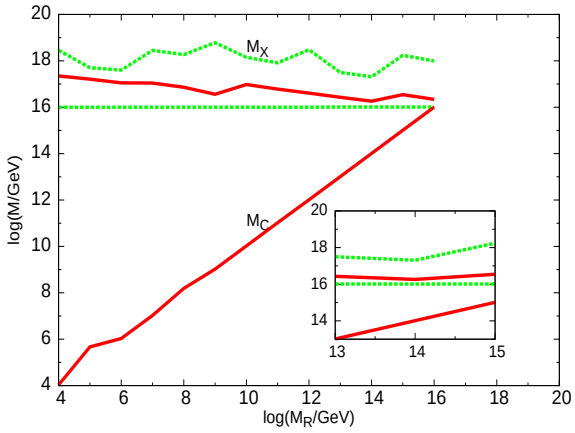

Figure 1: The allowed ranges of the unification (M_X, pale, green) and intermediate Pati-Salam (M_C, dark, red) scales as a function of the $U(1)_{(B-L)}$ breaking scale (M_R) for $SO(10)$ with two intermediate scales. The inset is a zoom of the region of interest for generating neutrino masses of the right magnitude.

The mixing of the two $U(1)$ groups adds flexibility to the model. With this, we find for every M_R a range of consistent solutions for M_C and M_X (see Fig. 1). In the plot we have exhibited the maximum and minimum values of both M_C and M_X consistent with unification. In a Grand Unified Theory low intermediate scales are always perceived with extra interest. These low intermediate scale scenarios keep alive the hope that signals of the GUT may be identified at accessible energies. In Fig. 1, we have shown that M_R and M_C can be quite low – $\sim$ 10 TeV – which is within the reach of recent colliders, such as the LHC; this is an artifact of the inclusion of the $U(1)$ mixings. The *vev* v_R of the scalar $(1, 1, -\frac{1}{2}, 1) \subset \overline{16}$, sets the scale M_R. In the next section we have shown that v_R needs to be very high ($\sim 10^{14}$ GeV) to yield the correct neutrino mass with the Yukawa couplings $\sim \mathcal{O}(1)$. In the inset of Fig. 1 we magnify this range of M_R. It is to be noted that this establishes that the proposed model of 'double-seesaw' mechanism is compatible with gauge coupling unification at a scale which is not in conflict with the present bound on the proton lifetime.

IV Neutrino Mass

The neutrino mass matrix in the basis $((\nu_L)^c, \nu_R, \Sigma_R^0, N)$ is:

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D^T & 0 & a_1 Y_{16} v_R & a_2 Y_{16} v_R \\ 0 & a_1 Y_{16}^T v_R & M_N & 0 \\ 0 & a_2 Y_{16}^T v_R & 0 & M_N \end{pmatrix} . \tag{14}$$

The left-handed fermionic triplets, Σ_{3L}, having a mass matrix identical to M_N, do not mix with other fermions since the left-handed analogue of v_R is chosen to be zero. From the mass matrix (14) it is seen that the masses of the light neutrinos are obtained by integrating out the heavy triplet and singlet fermions. Thus we can have type-III and type-I seesaw mechanism in succession. The right-handed neutrino mass term is generated once the heavy triplet fermion Σ_{3R}^0 and N are integrated out – an

effective type I + III seesaw. Assuming $M_N \gg v_R Y_{16} \gg m_D$, the right-handed neutrino mass matrix is:

$$M_R = v_R^2 Y_{16} M_M^{-1} Y_{16}^T, \tag{15}$$

where,

$$M_M^{-1} = (a_1^2 + a_2^2) M_N^{-1} \ , \tag{16}$$

and the light neutrino mass matrix after an effective type-I seesaw becomes:

$$m_\nu = m_D M_R^{-1} m_D^T \ . \tag{17}$$

Substituting for m_D from eq. (2) one arrives at the general expression of m_ν as

$$m_\nu = c_0^2 M_0 M_R^{-1} M_0 - c_0 c_4 M_0 M_R^{-1} M_2 + c_4 c_0 M_2 M_R^{-1} M_0 + c_4^2 M_2 M_R^{-1} M_2 \ . \tag{18}$$

Typical values for the various parameters are $v_R \sim 10^{14}$ GeV, $M_N \sim 10^{15}$ GeV, and $c_i \sim \mathcal{O}(1)$, $Y_i \sim \mathcal{O}(1)$ which gives $M_R \sim 10^{12}$ GeV. Then with $m_D \sim 100$ GeV one gets $m_\nu \sim 1$ eV.

With three neutrino generations, the model has 6 real parameters in M_0 and 3 in M_2. In addition there are 5 *vev*s $(c_0, c_2, c_3, c_4, v_R)$. Besides, there are additional parameters in Y_{16} and M_N. However the low energy neutrino mass matrix is characterized by 9 parameters. Neutrino oscillation experiments have so far determined and/or bounded 5 of these. The general case is obviously not sufficiently constrained. One way to address this lacuna requires invoking some flavour symmetry. We consider this to be the $\mu - \tau$ symmetry.

V $\mu - \tau$ symmetry and allowed textures

$\mu - \tau$ symmetry has been considered widely for explaining the large atmospheric mixing angle in the neutrino sector [25]. In addition it gives $\theta_{13} = 0$ which is also consistent with the current global fits[5]. We impose the condition of a generalized $\mu - \tau$ symmetry on the Yukawa matrices stemming from 10_H and $\overline{16}_H$. This implies that these matrices are invariant under the exchange of the second and third rows and columns. This reduces the number of unknown parameters in the Yukawa sector. However, this symmetry cannot be exact in the quark and lepton sector. This is accomplished by the term M_2 in the fermion mass matrices which originates from the 120_H which is taken to be antisymmetric under the exchange of $2 \leftrightarrow 3$ and breaks $\mu - \tau$ symmetry spontaneously. In addition we had imposed a generalized parity symmetry [27] which makes the complex matrices M_0 and M_2 real thereby reducing the number of free parameters. Thus the model that we consider is $SO(10) \otimes Z_2^{\mu-\tau} \otimes Z_2^{\mathcal{P}}$ [14]. However it is to be mentioned that if we assume exact $\mu - \tau$ (anti)symmetry in (M_2) M_0 then a generalized CP-invariance holds [14] and the CKM matrix comes out as real. This can be rectified either by assuming some of the *vev*s to be complex or by allowing a small explicit breaking of $\mu - \tau$ symmetry in M_0. This induces CP-violation phases in both U_{CKM} and U_{PMNS} [14]. We work in the basis where the charged lepton mass matrix is diagonal and the PMNS matrix is solely determined by the mixing in the neutrino sector[6].

[5] Recent global fits have found indication for non-zero θ_{13} although this is only a 1σ effect. A small non-zero value of θ_{13} can be induced by breaking the $\mu - \tau$ symmetry.

[6] For the purpose of this paper we only consider the implications of this model for neutrino masses. The predictions for the charged lepton and quark masses would require a detailed fit which we do not discuss in this work.

The structures for M_0 and M_2 under the above symmetries are given by

$$M_0 = \begin{pmatrix} a' & b' & b' \\ b' & c' & d' \\ b' & d' & c' \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & x' & -x' \\ -x' & 0 & y' \\ x' & -y' & 0 \end{pmatrix} \ . \tag{19}$$

We consider a model with three adjoint fermion multiplets, *i.e.,* the model consists of $(3\nu_L + 3\nu_R + 3N + 3\Sigma_R)$. Thus, Y_{16} and M_N are also (3×3) matrices which we take to be $\mu - \tau$ symmetric. It follows from eq. (15) that M_R also respects this symmetry. Thus we have both M_0 and M_R to be $\mu - \tau$ symmetric. In order to make the model predictive we make the further assumption that M_R and M_0 are proportional, *i.e.,*

$$K M_R = M_0. \tag{20}$$

where K is a constant. m_ν in eq. (18) then takes the form

$$m_\nu = K c_0^2 M_0 + K c_4^2 M_2 M_0^{-1} M_2 = M_1 + M_1' \ . \tag{21}$$

The number of free real parameters in the theory are now 4 from M_0, 2 in M_2, and 4 real *vevs.* Because of eq. (20) M_R adds just one further parameter. Thus in total we have 11 real parameters. The *vev* ratios c_2 and c_3 do not affect eq. (21) and thus we have 9 parameters involved in the neutrino sector. Some of these appear only as overall scale factors.

We note that although M_2 is $\mu - \tau$ antisymmetric the product $M_2 M_0^{-1} M_2$ possesses $\mu - \tau$ symmetry. Thus m_ν is $\mu - \tau$ symmetric. This immediately implies $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Therefore the mixing matrix in the basis where the charged lepton mass matrix is diagonal is given as,

$$U_{\rm PMNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}. \tag{22}$$

which can be brought to the standard $U_{\rm PMNS}$ form by a suitable redefinition of fermion phases. We have

$$m_\nu = U_{\rm PMNS} M_{dia} U_{\rm PMNS}^T, \tag{23}$$

where $M_{dia} = Diag(m_1, m_2, m_3)$. m_1, m_2, m_3, the mass eigenvalues are real [7], and are given as

$$m_1 = \frac{X - \sqrt{X^2 - 4(d-c)Y}}{2(d-c)}, \quad m_2 = \frac{X + \sqrt{X^2 - 4(d-c)Y}}{2(d-c)}, \quad m_3 = \frac{Y}{2b^2 - ac - ad}. \tag{24}$$

Here

$$\begin{aligned} X &= -ac - c^2 + ad + d^2 + 2x^2 + y^2; \\ Y &= 2b^2 c - ac^2 - 2b^2 d + ad^2 + 2cx^2 + 2dx^2 + 4bxy + ay^2, \end{aligned} \tag{25}$$

and

$$a = K c_0^2 a', \quad b = K c_0^2 b', \quad c = K c_0^2 c', \quad d = K c_0^2 d', \quad x = K c_4^2 x', \quad y = K c_4^2 y' \ . \tag{26}$$

Note that the eigenstate m_3 is determined to be the one associated with the eigenvector $(0, 1/\sqrt{2}, -1/\sqrt{2})$. Whether this is the highest mass state or the lowest mass state *i.e.* whether

[7] Since the mass matrices have real entries, complex roots can appear only in conjugate pairs leading to unacceptable degenerate neutrinos. We take the eigenvalues to be all non-negative.

the hierarchy is normal or inverted will depend on the values of the parameters. We further require $\Delta m^2_{21} > 0$ from the solar data. This implies that for our choice of m_2 and m_1

$$\frac{X}{(d-c)^2}\sqrt{X^2 - 4(d-c)Y} \quad > \quad 0 \tag{27}$$

Using eqs. (22) and (23) we obtain,

$$\tan\theta_{12} = \frac{1}{\sqrt{2}}\frac{(a-m_1)(c-d) - 2x^2}{b(c-d)+xy} \quad . \tag{28}$$

The condition for tri-bimaximal mixing implies

$$(a - m_1 - b)(c-d) = 2x^2 + xy \quad . \tag{29}$$

V.1 10_H dominance

In this case, $a, b, c, d \gg x, y$. The light neutrino mass matrix m_ν is approximated as $Kc_0^2 M_0$ with M_0 defined in eq. (19). In this limit the mass eigenvalues are given as,

$$m_1 = \frac{1}{2}(f_1 - R),\ m_2 = \frac{1}{2}(f_1 + R),\ m_3 = c - d \quad , \tag{30}$$

with

$$R = +\sqrt{8b^2 + f_2^2} \quad , \tag{31}$$

where,

$$f_1 = a + c + d, \quad f_2 = -a + c + d \quad . \tag{32}$$

Again, m_3 is identified as the eigenvalue for the state eigenvector $(0, 1/\sqrt{2}, -1/\sqrt{2})$. Since the solar data has determined the ordering of the 1 and 2 mass states to Then the mass squared differences can be expressed as,

$$\Delta m^2_{21} = f_1 R \qquad \Delta m^2_{31} = (f_1 R - a^2 - 4b^2 + c^2 + d^2 - 6cd)/2 \quad . \tag{33}$$

Again, the mass ordering will depend on the values of the parameters. In general both normal and inverted hierarchy are possible. In addition, the solar neutrino data require $\Delta m^2_{21} > 0$ which implies $f_1 R > 0$ for the above selection of states.

The mixing angles are given as,

$$\theta^\nu_{13} = 0 \ , \quad \theta^\nu_{23} = \pi/4 \ , \quad \tan\theta^\nu_{12} = \frac{(R - f_2)}{2\sqrt{2}b} \ . \tag{34}$$

Tri-bimaximal mixing implies $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\tan^2\theta_{12} = 1/2$. We see that the requirements for θ_{13} and θ_{23} are already satisfied. If in addition we impose

$$f_2 = b \quad \Longrightarrow \quad R = 3b,\ f_1 = (2a + b), \tag{35}$$

tri-bimaximal mixing is obtained. In this limit

$$\Delta m^2_{21} = 3\ b\ (2a + b) \qquad \Delta m^2_{31} = (c-d)^2 - (a-b)^2 \quad . \tag{36}$$

V.2 120_H dominance

In this limit $a, b, c, d \ll x, y$ and the low energy neutrino mass matrix is given as

$$m_\nu = M_4 = K c_4^2 M_2 \; M_0^{-1} \; M_2 \;\; . \tag{37}$$

The $U_{\rm PMNS}$ continues to be given by eq. (22). The eigenvalues, in terms of the parameters defined in eq. (26), are given as,

$$m_1 = 0, \;\; m_2 = \frac{2x^2 + y^2}{d - c}, \;\; m_3 = \frac{2cx^2 + 2dx^2 + 4bxy + ay^2}{2b^2 - ac - ad} \;\; . \tag{38}$$

Since the eigenvector $(0, 1/\sqrt{2}, -1/\sqrt{2})$ belongs to the eigenvalue m_3 so that the zero eigenvalue has to be associated with the eigenstate m_1. Therefore this case corresponds to the normal hierarchy. Since $m_1 = 0$, $\Delta m^2_{21} = m^2_2$ and $\Delta m^2_{31} = m^2_3$. Then, using eqs. (22) and (23) one obtains the 1-2 mixing angle as,

$$\tan\theta_{12} = -\frac{\sqrt{2}x}{y} \tag{39}$$

Thus, the mixing matrix in this case is completely determined by the parameters of M_2. The condition for obtaining exact tri-bimaximal mixing is $y = -2x$.

VI Conclusions

We consider a non-SUSY $SO(10)$ model in which the fermion masses originate from Yukawa couplings to 10_H and 120_H. In such a model the usual type-I and type-II seesaw mass terms which originate from $\overline{126}_H$ are not present. Here, it is possible to generate the neutrino mass at two loops by the radiative seesaw mechanism [17]. But for non-SUSY $SO(10)$ the contribution is very small.

In this paper we suggest a new possibility to generate neutrino masses in a non-SUSY $SO(10)$ model with $10_H + 120_H$ using fermions in the 45_F representation and an additional $\overline{16}_H$ scalar multiplet. Constraints from gauge coupling unification requires these $vev < \overline{16}_H >$ to be in the range $\sim 10^4 - 10^{16}$ GeV. However from the standpoint of generation of naturally small neutrinos masses the range $\sim 10^{13} - 10^{15}$ GeV is preferred. We show that in this case one can generate small neutrino masses through the 'double seesaw' mechanism. Predictions for mixing angles require further imposition of a flavour symmetry which we chose to be the $\mu - \tau$ symmetry for the Yukawa matrices due to 10_H and $\overline{16}_H$ whereas for the one originating from 120_H we assume the matrix to be $\mu - \tau$ antisymmetric. We further assume the right-handed matrix (M_R) due to the heavy fields to be proportional to the one (M_0) originating from 10_H. With this the light neutrino mass matrix is given by the sum of two terms which are both $\mu - \tau$ symmetric. This automatically satisfies $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. We present the neutrino masses and θ_{12} obtained from this model and determine the condition for satisfying tri-bimaximality. We also discuss the limiting values when one of the terms dominate. For the 10_H-dominance case both hierarchies are possible whereas if the 120_H dominates the hierarchy can only be normal.

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