EXCITED STATES OF ⁷¹Ge ABOVE THE NEUTRON EMISSION THRESHOLD AND SOLAR NEUTRINO CAPTURE RATES FOR Ga DETECTORS

Kamales Kar

Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Calcutta 700 064, India.

Sukhendusekhar Sarkar* Inter-University Consortium for DAE Facilities, Calcutta Centre, Sector - III, Block LB, Plot 8, Bidhannagar, Calcutta 700 091, India.

Srubabati Goswami and Amitava Raychaudhuri Department of Pure Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India.

ABSTRACT

In Gallium detectors for solar neutrinos, the capture rate due to Gamow-Teller transitions to excited states of ^{71}Ge beyond the neutron emission threshold is usually neglected. We make a model calculation to estimate its effect and find that this yields an additional contribution which may be as much as 0.4 SNU, even larger than that from the Isobaric Analog State in ^{71}Ge reached by Fermi transitions, which is normally included in the standard predictions.

*Present address: Department of Physics, North Bengal University, Raja Rammohanpur, West Bengal 734 430, India.

Research in the area of the solar neutrino deficit problem has reached a very exciting stage. On one side, several groups are working on making both the theoretical as well as the observed neutrino capture rates for the different detectors more precise. On the other side, explanations of this shortfall invoking non-zero neutrino mass and mixing in the two- and three-flavour scenarios are being pursued vigourously. Out of the four detectors which have already published data, measurements using Gallium (by the GALLEX and SAGE groups) have the lowest threshold (0.233 MeV) and are able to capture neutrinos produced through all the reactions in the sun, including the most abundant pp neutrinos. The theoretical prediction for the total neutrino capture rate in the Ga detectors is 132^{+7}_{-6} SNU (Solar Neutrino Unit) using the Standard Solar Model (SSM) of Bahcall and Ulrich [1] whereas it is 137^{+8}_{-7} in the SSM of Bahcall and Pinsonneault [2] which includes He and heavy element diffusion. To calculate the total capture rates one integrates the solar neutrino flux times the capture cross-section over the range of neutrino energy and performs summation over all the neutrino sources. Evaluation of the neutrino capture cross-section as a function of the energy needs the knowledge of the Gamow-Teller (GT) strengths to excited states of the daughter nucleus ^{71}Ge . The theoretical calculations of these GT strengths depend sensitively on the choice of the interaction Hamiltonian [3]. The SSM predictions of ref. [1] and [2] use the GT transition strengths deduced from the forward angle (p, n) reaction data [4] from the analogous hadronic process ${}^{71}Ga(p,n){}^{71}Ge$. The neutron emission threshold in ⁷¹Ge is 7.4 MeV, and, in the prevalent practice, one does not consider any ν capture which excites states above this energy through the GT operator. This is based on the assumption that for such states the partial width for neutron emission is much larger than the γ -decay width. In this letter we go beyond this assumption and probe the effect that an inclusion of the above states might have on the final solar neutrino capture rate. To set this in perspective, the Isobaric Analogue State (IAS) in ^{71}Ge is observed at an energy of 8.932 MeV and Champagne *et al.* [5] measured the upper limit of the ratio of the gamma decay width to the total width for the IAS to be about 10%. Based on this, Bahcall and Ulrich [1] add 10% of the capture rate coming from the Fermi strength and this turns out to yield 0.2 SNU for the solar neutrinos. But the *n*-emission from the IAS to low-lying states of ^{70}Ge with T = 3 is suppressed by isospin conservation. On the other hand, for the states below the IAS (i.e., below 8.932 MeV of excitation in ^{71}Ge) with $T = \frac{7}{2}$, connected by the GT transition, there is no such inhibition due to isospin conservation and hence the neutron emission probability is expected to be much larger. In this work we consider the contribution, albeit small, coming from GT-transitions to states above 7.4 MeV using information from betadelayed particle emission experiments. We make a model calculation for this and to the best of our knowledge no earlier estimate of this possible additional contribution has been made. Efforts are on for the experimental measurement of the ratio of the gamma decay width to the total width for these states.

The total capture rate R for ⁷¹Ga is given by $R = \Sigma R_i$, where R_i is the capture

rate for neutrinos from the i^{th} source. R_i is obtained by convoluting the calculated cross-section with $\phi_{\nu_i}(q)$, the flux of neutrinos of energy q from the source i [6]. The normalisation of the latter is fixed by the SSM under consideration and one has:

$$R_i = C_i \frac{\int \phi_{\nu_i}(q) \ \sigma(q) \ dq}{\int \phi_{\nu_i}(q) \ \sigma_B(q) \ dq}$$
(1)

where σ and σ_B denote the absorption cross-sections as calculated from this work and as given in ref. [6] respectively. C_i is the prediction in SNU for neutrinos from the i^{th} source according to the SSMs of ref. [1] or [2] depending on which model is used as the reference point.

The capture rates $\sigma(E_i)$ of neutrinos from the ground state of ^{71}Ga to a state of ^{71}Ge at an excitation energy E_i has two parts – one coming from the Fermi transition and the other from GT transitions – and can be written as,

$$\sigma(E_i) = \sigma_F(E_i) + \sigma_{GT}(E_i) \tag{2}$$

where $\sigma_F(E_i)$ and $\sigma_{GT}(E_i)$ are given by,

$$\sigma_F(E_i) = 0.1 (\pi c^3 \hbar^4)^{-1} g_V^2 B_F(E_i) p_e \epsilon F(Z, \epsilon)$$
(3)

$$\sigma_{GT}(E_i) = (\pi c^3 \hbar^4)^{-1} g_A^2 B_{GT}(E_i) p_e \epsilon F(Z, \epsilon)$$
(4)

In eqs. (3) and (4) g_V , g_A are the vector and axial vector coupling constants, p_e and ϵ are the momentum and energy of the emitted electron and $F(Z,\epsilon)$ is the β -decay Fermi function for which we use the analytic form given by Schenter and Vogel [7]. $B_F(E_i)$ and $B_{GT}(E_i)$ are the squared Fermi and GT matrix elements for ⁷¹Ge states at energy E_i respectively. The total Fermi strength of (N-Z) is concentrated at E_{IAS} (~ $\delta(E_i - E_{IAS})$) and the net contribution is proportional to $B_F(IAS)$. For the GT strength distribution the usual practice is to use $B_{GT}(E_i)$ from (p, n) reactions. Here we are interested in the GT strength beyond the neutron emission threshold for which no (p, n) study results are available and we use a simple theoretical model for the strength distribution based on the configuration structures of the initial and final states which get connected by the one body GT transition operator. The model for the GT strength distribution is based on a formalism constructed for calculating the beta decay/electron capture rates in stellar conditions in [8] and used recently in [9] for calculating the neutrino capture rates for the ^{71}Ga detectors. In this method, one uses pure shell model configurations to describe the ground state of Ga as well as the ground and excited states of ^{71}Ge . For example the ground state of ^{71}Ga is taken as $(1f_{\frac{7}{4}})^8$ $(2p_{\frac{3}{4}})^3$ for the protons and a completely filled fp shell for the neutrons. Out of the eight possible GT transitions: (1) $2p_{\frac{1}{2}}(n) \rightarrow 2p_{\frac{3}{2}}(p)$, (2) $2p_{\frac{1}{2}}(n) \rightarrow 2p_{\frac{1}{2}}(p)$, (3) $1f_{\frac{5}{2}}(n) \to 1f_{\frac{5}{2}}(p), \ (4) \ 1f_{\frac{5}{2}}(n) \to 1f_{\frac{7}{2}}(p), \ (5)^{2}2p_{\frac{3}{2}}(n) \to 2p_{\frac{3}{2}}(p), \ (6)^{2}2p_{\frac{3}{2}}(n) \to 2p_{\frac{1}{2}}(p), \ (7) \ 1f_{\frac{7}{2}}(n) \to 1f_{\frac{7}{2}}(p), \ (8) \ 1f_{\frac{7}{2}}(n) \to 1f_{\frac{5}{2}}(p) \ \text{connecting the ground state of} \ ^{71}Ga \ \text{ to}$ eight pure configurations in ^{71}Ge , (4) and (7) are completely blocked. Each remaining configuration has many individual states in it and the strength distribution to each configuration gives rise to a resonance taken to be of the Gaussian form in this model. The centroid energy of the strength Gaussians is given by,

$$E_{cent} = \Delta E_{s.p.} + \Delta E_{2-body} + \Delta E_{pairing} \tag{5}$$

where $\Delta E_{s.p.}$ and $\Delta E_{pairing}$ are the differences in the total single particle energies and pairing energies, respectively, of the excited and the ground state configurations of ^{71}Ge . ΔE_{2-body} is the contribution coming from the 2-body potential and is treated in a simplified TDA model as in [8]. The total GT strength to each configuration is given by the sum rule expression of the product of single particle transition strength, the number of neutron particles and the fraction of proton holes [9]. The widths (σ) of the strength Gaussians to different configurations are all taken equal and is a free parameter of the model. One finds that $\sigma = 1.55$ MeV gives the total capture rate as 132.83 SNU (137.17 SNU) close to the standard values according to ref. [1] ([2]). So, for the purpose of comparison, we use the strength width of 1.55 MeV.

When one includes the contribution from all states beyond the neutron threshold of 7.4 MeV in ^{71}Ge the capture rate becomes 133.56 SNU (138.005 SNU), an increase of 0.73 SNU (0.84 SNU) over the usual SSM prediction of ref. [1] ([2]). But this is clearly an overestimation as these states mostly decay by particle emission and not by gamma emission. To get a realistic estimate we use information collected from beta delayed particle emission experiments. With Γ_{γ} , Γ_{μ} being the partial widths of gamma and particle decay respectively, the total width $\Gamma = \Gamma_{\gamma} + \Gamma_{\mu}$. Denoting the particle separation energy by B_{μ} , " Γ_{μ}/Γ is zero below B_{μ} and remains small immediately above that, then increases in the space of a few hundred keV to nearly its maximum value, rises more slowly and finally decreases again at much higher excitations" [10]. This indicates that Γ_{γ}/Γ first falls slowly, then very rapidly and then slowly goes towards zero. We model this by a Gaussian fall-off and include that as a multiplicative factor in the GT strength distribution beyond 7.4 MeV. The width of this Gaussian, σ_{γ} , is varied in the range 0.3 - 0.7 MeV, consistent with beta-delayed neutron emission experiments. We exhibit the Gamow-Teller strength distribution in Fig. 1 both without and with the modulation beyond the particle emission threshold. The results of [5] indicate that Γ_{γ}/Γ for the IAS falls off to 10% at an excitation energy of 1.532 MeV with respect to the neutron emission threshold. In the case under consideration the suppression will be much larger due to lack of isospin constraints, as noted earlier. For comparison, we choose $\sigma_{\gamma} = 0.5$ MeV which makes $\exp(-x^2/\sigma_{\gamma}^2) = 8.4 \times 10^{-5}$ at x = 1.532 MeV. Modulating the contribution beyond n-emission in this manner, the total capture rate is 133.09 (137.46) SNU, an increase by 0.26 (0.30) SNU over the standard value using the SSM of ref. [1] ([2]). The contribution from the different sources are given in Table 1, where we see that the increase comes almost entirely from ${}^{8}B$ neutrinos. We have checked that the increase in the capture rate does not depend very sensitively on the parameter σ_{γ} : varying σ_{γ} from 0.3 to 0.7 MeV changes the total neutrino capture rate from 132.99 (137.36) SNU to 133.16 (137.55) SNU using the solar model of ref. [1] ([2]).

The work discussed so far assumes that the model space for the distribution of the GT strength includes only the four fp shell orbitals, namely $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$. However, the experimental pick-up reaction data suggest that the $1g_{9/2}$ neutron orbital is about 10% occupied [11] in the ground state of ^{71}Ga and one also sees that ^{71}Ge has a low-lying excited state of $9/2^+$ at an excitation of only 0.1984 MeV [12]. Also, theoretical estimates using spectral distribution methods and with different interaction Hamiltonians in the $(2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2})$ shell model space show that the $1g_{9/2}$ orbit is about 16 - 20 % occupied [13] in the ground state of ^{71}Ga . Inclusion of the $1g_{9/2}$ orbital in shell model calculations makes the model space too large to handle. On the other hand, using our schematic model of the GT strength distribution one can extend the calculation to include it rather easily [14]. Here the ground stae of ^{71}Ga is a pure configuration with the fp neutron orbitals full except $1f_{5/2}$ which has 4 particles instead of 6 and the two extra particles are put in $1g_{9/2}$. This configuration gives rise to two additional transitions other than the ones listed earlier, namely $1g_{9/2}(n) \rightarrow 1g_{9/2}(p)$ and $1g_{9/2}(n) \rightarrow 1g_{7/2}(p)$. For the GT strength distribution this will give rise to seven Gaussians and one half-Gaussian instead of five Gaussians and a half-Gaussian considered earlier [14]. The centroids of these extra strength Gaussians are evaluated in the same manner and they are given the same width parameters as others. The β^- sum rule strength, S_{β^-} , increases in this space to 221/7 from the earlier value of 3(N-Z) - i.e., 27 – giving rise to S_{β^+} of 32/7. With this form of GT strength distribution one can again investigate how much the neutrino capture rate increases for the Ga detectors if one includes the contribution beyond the neutron emission threshold in ^{71}Ge . Using the same model for the ratio of gamma decay width to neutron emission width, one gets the capture rates in this fpq model spaces as given in Table 2 for the solar models of ref. [1] and [2]. These numbers are for the same value of the strength width parameter of 1.55 MeV and σ_{γ} = 0.5 MeV as in the fp case. We see that the increase in the total capture rate from contributions beyond the *n*-emission threshold is 0.32 SNU for fluxes of ref. [1] and 0.36 SNU for the solar model of ref. [2]. Varying σ_{γ} from 0.3 MeV to 0.7 MeV changes the total rate from 133.182 (137.572) SNU to 133.386 (137.806) SNU using the solar model of ref. [1] ([2]).

A microscopic calculation of the ratio of the gamma decay width to the total width is being attempted. In conclusion, we stress that the contribution to the neutrino capture rate of the excited states of ^{71}Ge beyond the neutron emission threshold are not negligible and are actually larger than the Fermi transition contributions usually included. Taking into account the contribution beyond the *n*-emission threshold in ^{71}Ge along with the extension of the space to fpg orbits give an increase as large as 0.37 SNU over the conventional theoretical calculation done including the fp shell orbits only.

ACKNOWLEDGEMENTS: This work has been supported by the Eastern Centre for Research in Astrophysics, India. The research of A.R. is also supported in part by the Council of Scientific and Industrial Research, India and the Department of Science and Technology, India while S.G. acknowledges a fellowship from the former agency. Table 1: Neutrino capture rate on ^{71}Ga without (columns A and C) and with (columns B and D) contributions above the neutron emission threshold using Bahcall-Ulrich and Bahcall-Pinsonneault solar neutrino fluxes. Here σ_{γ} has been chosen to be 0.5 MeV.

Solar Neutrino	Bahcall - Ulrich SSM [1]		Bahcall – Pinsonneault SSM [2]	
source	А	В	С	D
pp	67.781	67.781	66.727	66.727
pep	5.211	5.211	5.211	5.211
hep	0.043	0.045	0.043	0.045
^{7}Be	34.109	34.109	37.490	37.490
^{8}B	11.335	11.593	13.036	13.333
^{13}N	4.823	4.823	4.823	4.823
^{15}O	9.433	9.433	9.742	9.742
^{17}F	0.093	0.093	0.093	0.093
Total	132.828	133.088	137.165	137.464

Solar Neutrino	Bahcall - Ulrich SSM [1]		Bahcall - Pinsonneault SSM [2]	
source	А	В	С	D
pp	67.781	67.781	66.727	66.727
pep	5.211	5.211	5.211	5.211
hep	0.044	0.047	0.044	0.047
^{7}Be	34.109	34.109	37.490	37.490
^{8}B	11.485	11.798	13.208	13.568
^{13}N	4.823	4.823	4.823	4.823
^{15}O	9.433	9.433	9.742	9.742
^{17}F	0.093	0.093	0.093	0.093
Total	132.979	133.295	137.338	137.701

Table 2:	Same as in	Table 1	but for	the fpg case.

References

- [1] J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988).
- [2] J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 67, 1 (1995).
- [3] T. Oda and K. Muto in 'Nuclear Beta Decays and Neutrino', T. Kotani, H. Ejiri and E. Takasugi (eds.), World Scientific, (1985) p459.
- [4] D. Krofcheck *et al.*, Phys. Rev. Lett. **55**, 1051 (1985).
- [5] A.E. Champagne *et al.*, Phys. Rev. **C38**, 900 (1988).
- [6] J.N. Bahcall, 'Neutrino Astrophysics', Cambridge University Press, (1989).
- [7] G.K. Schenter and P. Vogel, Nucl. Sci. Eng. 83, 393 (1983).
- [8] G.M. Fuller, W.A. Fowler and M.J. Newman, Ap. J. **252**, 715 (1982).
- [9] K. Kar, S. Goswami and A. Raychaudhuri, Int. J. Mod. Phys. **10A**, 133 (1995).
- [10] J.C. Hardy, 'Nuclear Spectroscopy and Reactions', Part C, Academic Press, New York, (1974), p 407.
- [11] G.J. Mathews, S.D. Bloom, G.M. Fuller and J.N. Bahcall, Phys. Rev. C32, 796 (1985).
- [12] M.R. Bhat and D.E. Alburger, Nucl. Data Sheets 53, 36 (1988).
- [13] V. Potbhare and N. Tressler, Nucl. Phys. A530, 171 (1990); S. Sarkar and K. Kar, Proc. Symp. Nucl. Phys. (DAE India), B36 (1993).
- [14] S. Sarkar and K. Kar, J. Phys. **G21**, 411 (1995).

FIGURE CAPTION

Fig.1. The Gamow-Teller strength, B_{GT} , (solid line) as a function of the excitation energy of the *Ge* nucleus. Also shown is the neutron-emission threshold in *Ge* at 7.4 MeV. Modulating the distribution beyond the particle emission threshold by a Gaussian suppression factor of width 0.7 MeV (0.3 MeV) results in the long-dashed (short-dashed) curve.

