hep-ph/0511276
SINP/TNP/05-27

Remarks on flavour mixings from orbifold compactification

Gautam Bhattacharyya[1] and Amitava Raychaudhuri[2]

[1]) *Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India*
[2]) *Department of Physics, University of Calcutta, 92 A.P.C. Road, Kolkata 700009, India*

Abstract

We consider 5d SU(5) GUT models based on the orbifold $S^1/(Z_2 \times Z_2')$, and study the different possibilities of placing the SU(5) matter multiplets in three possible locations, namely, the two branes at the two orbifold fixed points and SU(5) bulk. We demonstrate that if flavour hierarchies originate solely from geometrical suppressions due to wavefunction normalisation of fields propagating in the bulk, then it is not possible to satisfy even the gross qualitative behaviour of the CKM and MNS matrices regardless of where we place the matter multiplets.

PACS Nos: 11.10.Kk, 11.30.Hv
Key Words: Flavour, Orbifold compactification

I Introduction

In order to bypass some of the problems with conventional Grand Unified Theories (GUTs), namely, the doublet-triplet splitting, too fast proton decay, etc, new proposals have been advanced which allow realization of GUT gauge symmetry in a higher dimensional orbifold [1]-[8]. The quantitative success of supersymmetric (SUSY) gauge coupling unification provides further attraction to embed SUSY in a higher dimensional SU(5) gauge symmetry. Several attempts have been made in this regard. In this short note, we consider those which have only one extra dimension compactified on the orbifold $S^1/(Z_2 \times Z_2')$. We go by the hypothesis that flavour hierarchies originate *purely* from geometrical suppression of Yukawa couplings depending on the location of the fields attached to the given vertex in the extra dimension. There are three possible locations where matter fields can be placed: the SU(5) preserving five-dimensional (5d) bulk, and the two fixed points, namely, the SU(5) preserving O brane ($y = 0$), and the standard model (SM) brane O' ($y = \pi R/2$), where y is the 5th coordinate and R is the radius of compactification. If a particular field extends in the bulk, it is associated with a wavefunction normalisation factor (< 1), whereas for a field restricted to one of the two branes there is no such suppression. So the relative size of two Yukawa couplings comes from the relative number of brane vis-a-vis bulk fields attached to them. In this brief communication, we investigate whether by placing the SU(5) multiplets of different generations ($\bar{\mathbf{5}}_\mathbf{i}$, $\mathbf{10}_\mathbf{i}$), including the right-handed singlet neutrinos ($\mathbf{N}_\mathbf{i}$), in different locations, one can generate even the overall qualitative characteristics of the Cabibbo-Kobayashi-Maskawa (CKM) and Maki-Nakagawa-Sakata (MNS) matrices: the question of quantitative success comes thereafter. We examine all possible choices for the relative placements of the multiplets of different generations at alternate locations in a systematic manner and demand that *only* locations matter in determining the relative Yukawa couplings. We observe that each such case fails to meet this test. One can, however, argue that O(1) corrections to the Yukawa couplings are in general unavoidable. Such corrections, as mentioned in [1]-[6], play a key rôle in generating flavor mixings. Clearly, this is achieved at the cost of introducing many new and unknown parameters. In this short note, we reach at a similar conclusion by looking at the problem from a different angle. We show on a case by case basis that if hierarchies emerge purely from geometrical suppressions, barring any further O(1) corrections for economy of parameters, it is not possible to simultaneously satisfy even the gross features of the three generation CKM and MNS mixings, no matter where we place the different multiplets.

II Formalism

We briefly summarise the primary consequences of orbifold compactification. Let us conceive a 5d GUT with minimal SU(5) gauge group and $N = 1$ SUSY. The 5d spacetime is factorised into a product of the 4d spacetime M^4 (labelled by the coordinates x^μ) with the extra spatial dimension compactified on the orbifold $S^1/(Z_2 \times Z_2')$ (labelled by the coordinate $y = x_5$). The inverse radius R^{-1} is chosen to be of the order of $M_{\rm GUT} = 10^{16}$ GeV. The orbifold construction proceeds by dividing S^1 first by a Z_2 transformation $y \to -y$ and then by a further division by Z_2' which acts as $y' \to -y'$ with $y' = y + \pi R/2$. After these identifications, the physical spacetime becomes the interval $[0, \pi R/2]$ with a brane located at each fixed point $y = 0$ and $y = \pi R/2$. As a result of the two reflections, the branes at $y = \pi R$ and $-\pi R/2$ are identified with those at $y = 0$ and $y = \pi R/2$, respectively. Now let us consider a generic field $\phi(x^\mu, y)$ existing in the 5d bulk. The Z_2 and Z_2' parities (called P and P', respectively) are defined for this field as

$$\begin{aligned} \phi(x^\mu, y) &\to \phi(x^\mu, -y) = P\phi(x^\mu, y), \\ \phi(x^\mu, y') &\to \phi(x^\mu, -y') = P'\phi(x^\mu, y'). \end{aligned} \qquad (1)$$

Using the notation $\phi_{\pm\pm}$ for the fields with $(P, P') = (\pm, \pm)$, we are led to the following observations regarding the 4d KK fields. $\phi_{++}^{(2n)}$ acquire a mass $2n/R$, while $\phi_{+-}^{(2n+1)}$ and $\phi_{-+}^{(2n+1)}$ acquire a mass $(2n+1)/R$ and $\phi_{--}^{(2n+2)}$ acquire a mass $(2n+2)/R$. This implies that the only fields which can have massless components are $\phi_{++}^{(2n)}$. The other interesting consequence is that only ϕ_{++} and ϕ_{+-} can have non-vanishing components on the $y = 0$ brane. In fact, compactification leads to symmetry reduction. The starting theory is 5d $N = 1$ SUSY invariant under the gauge group SU(5). From a 4d perspective, this is equivalent to $N = 2$ SUSY. We assign suitable (P, P') quantum numbers for the fields. Upon the first compactification by Z_2 the conjugated fields are projected out and the $N = 2$ SUSY reduces to $N = 1$ SUSY but still respecting the gauge SU(5); on the second compactification by Z_2' the SU(5) gauge symmetry is broken to the SM gauge group $\mathrm{SU(3)_C} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$ with an unbroken $N = 1$ SUSY[1].

The P and P' quantum numbers are to be so arranged that the 5d SU(5) gauge symmetry remains intact at O but is broken to $\mathrm{SU(3)_C} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$ at O'. This can be done by choosing $P = (+ + + + +)$ and $P' = (- - - + +)$ or $(+ + + - -)$ acting on $\mathbf{5}$. As has been noted [1, 2], with the above P' assignments it is not possible to fill up a complete SU(5) multiplet by zero mode matter. One has to introduce $\bar{\mathbf{5}}'$ and $\mathbf{10}'$ with P' assignments opposite to those in $\bar{\mathbf{5}}$ and $\mathbf{10}$ to obtain correct low energy matter content[2].

Now we come to the discussion of Yukawa couplings. All such couplings consistent with gauge symmetry and R-parity are admitted. We assume that the hierarchical structure of the effective 4d Yukawa couplings is generated solely from the different normalisation of brane and bulk fields. Let us denote[3] a Yukawa coupling involving three brane superfields by λ. The Yukawa coupling for an interaction where one of the three fields is replaced by the zero mode of a bulk field is $\lambda/\sqrt{M_* R}$, where M_* is the UV cutoff scale of the 5d theory and the appearance of M_* is related to the canonical normalisation of the zero mode kinetic terms[4]. The number of $\epsilon = 1/\sqrt{M_* R}$ factors in front of λ is given by the number of bulk zero modes in a given interaction, each bulk field contributing one such factor.

III Fermion mass matrices

We write the fermion mass matrices in the convention that the fields on the left are left-handed and those on the right are right-handed. The up quark mass matrix is given by $\overline{\mathbf{10}}_i (M_u)_{ij} \mathbf{10^c}_j$ and is hermitian. The down quark mass matrix is given by $\overline{\mathbf{10}}_i (M_d)_{ij} \mathbf{5^c}_j$. The charged lepton mass matrix M_l is simply $(M_d)^\dagger$. For illustration,

[1]The doublet-triplet splitting problem is elegantly solved as the coloured triplet Higgs do not have $(++)$ assignments unlike the Higgs doublets, as a result the former has a mass of order $1/R \sim M_{\rm GUT}$.

[2]If $\bar{\mathbf{5}}_\mathbf{1}$and $\mathbf{10_1}$are kept in SU(5) bulk, then the first generation zero mode quarks and leptons come from different SU(5) multiplets, and proton decay from broken gauge boson exchange does not exist at leading order.

[3]See, e.g. the Lagrangian in Eq. (6) of [1].

[4]$M_* R \sim 10^{2-3}$ is a good choice for gauge coupling unification [3].

M_l can be schematically represented as:

$$M_l = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} \bar{\mathbf{5}}_\mathbf{1} & \bar{\mathbf{5}}_\mathbf{2} & \bar{\mathbf{5}}_\mathbf{3} \end{array} \\ \left\{ \begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array} \right\} \end{array} \begin{array}{c} \\ \mathbf{10^c_1} \\ \mathbf{10^c_2} \\ \mathbf{10^c_3} \end{array} . \tag{2}$$

where the entries $a, b, \ldots, i$ are determined solely by the locations of the $\mathbf{10_i}$ and $\bar{\mathbf{5}}_\mathbf{i}$ fields.

The neutrino mass[5] is given by $\bar{\mathbf{5}}_i(M_\nu)_{ij}\mathbf{N}_j$. In general, M_d, M_l and M_ν are not hermitian, so they are diagonalised by biunitary transformations. The CKM matrix is given by $V_{\text{CKM}} = V_u^\dagger V_d$, where V_u diagonalises M_u as $V_u^T M_u V_u = \text{diag}(m_u, m_c, m_t)$. Similarly, V_d diagonalises $M_d(M_d)^\dagger$. In the same way, the MNS matrix is given by $V_{\text{MNS}} = V_\nu^\dagger V_l$, where V_ν and V_l diagonalise $M_\nu(M_\nu)^\dagger$ and $M_l(M_l)^\dagger$ respectively[6].

IV Results

In this section we consider one by one the possible cases distinguished by the locations of the different SU(5) matter multiplets and examine whether the gross features of the CKM and MNS mixing matrices can be reproduced based on the geometric suppression factors alone.

1. $\bar{\mathbf{5}}_\mathbf{1}$, $\bar{\mathbf{5}}_\mathbf{2}$, $\bar{\mathbf{5}}_\mathbf{3}$ all at the same location is not allowed: In this situation, as far as the contributions to M_l are concerned, there will be no difference between lepton flavours. $M_l(M_l)^\dagger$ will have a democratic structure because of the symmetry $\bar{\mathbf{5}}_\mathbf{i} \leftrightarrow \bar{\mathbf{5}}_\mathbf{j}$ for all i, j = 1,2,3. $M_\nu(M_\nu)^\dagger$ will also share the same structure. Consequently, V_l and V_ν will be identical and the MNS matrix will be the identity matrix. Note that it is only required that $\bar{\mathbf{5}}_\mathbf{1}$, $\bar{\mathbf{5}}_\mathbf{2}$, $\bar{\mathbf{5}}_\mathbf{3}$ should not be at the same location; it does not matter whether this is the SU(5) brane, the SM brane, or the bulk.

2. All $\mathbf{10_i}$ cannot be placed in the same location: This alternative can be ruled out on grounds very similar to the previous one. Because of the permutation symmetry between the $\mathbf{10_i}$, now M_u will be a democratic matrix. This property will also be shared by $M_d(M_d)^\dagger$. Thus, V_u and V_d will be identical and the quark sector will remain unmixed.

3. No two $\bar{\mathbf{5}}_\mathbf{i}$ in the same location is allowed: Let $\bar{\mathbf{5}}_\mathbf{1}$ and $\bar{\mathbf{5}}_\mathbf{2}$ share a location. In this case, the first and second rows and columns of $M_l(M_l)^\dagger$ will be identical. This will also be the case for $M_\nu(M_\nu)^\dagger$. Applying a unitary transformation – the same for both matrices – they can be brought to block diagonal forms with one state – say '1' – decoupled from the other two – '2' and '3'. This common unitary transformation will have no impact on V_{MNS}. Therefore, for this case, the lepton sector mixing will be among two generations only. This disagrees with the form determined by the data.

4. Two $\mathbf{10_i}$ in the same location is not allowed: The argument in this case is the same as that of the previous one excepting that one now has to appeal to the matrices M_u and $M_d(M_d)^\dagger$. Now V_{CKM} will mix only two generations – a situation contradicting experimental requirements on flavour mixing.

5. In view of the possibilities excluded above, the three $\mathbf{10_i}$ must be in different locations and so should be the three $\bar{\mathbf{5}}_\mathbf{i}$. We now show that placing ($\bar{\mathbf{5}}_\mathbf{i}$, $\mathbf{10_i}$) pair-wise in the same location for every i is not allowed. As noted before, M_u is a hermitian matrix. When $\bar{\mathbf{5}}_\mathbf{i}$ and $\mathbf{10_i}$ are in the same location, then M_u and M_d become proportional since according to our starting hypothesis the entries in the matrix are determined by geometrical considerations alone. Therefore, they are both diagonalised by the same unitary transformation and mixing in the quark sector will vanish.

6. The only remaining possibility is that of having ($\bar{\mathbf{5}}_\mathbf{j}$, $\mathbf{10_i}$) pair-wise placed at the same location for $i \neq j$. But even this case is ruled out. This is because this alternative can be brought to the form of the previous case by mere redefinitions of rows/columns in M_u and $M_d(M_d)^\dagger$.

[5]One can also generate Majorana mass matrix of the form $\bar{\mathbf{5}}_i(M_M)_{ij}\bar{\mathbf{5}}_j$ for light neutrinos via see-saw mechanism by integrating out the heavy $\mathbf{N}_i$ fields. Even then our conclusions below will go through.

[6]The relation $M_l = (M_d)^\dagger$ and the hermiticity of M_u are valid in the SU(5) limit and will not be applicable when fermion multiplets are located on the SM brane O'. Our conclusions below are not affected by this.

V Discussions and Conclusions

In this brief note, we have considered a SUSY SU(5) theory defined on a 5d space where the extra dimension is an $S^1/(Z_2 \times Z_2')$ orbifold. We have used the hypothesis that the entries of the fermion mass matrices are determined entirely by geometrical factors determined by the locations of the SU(5) multiplets. We have shown that though there are many alternate possibilities of locating the various fermion multiplets, in no case can one reproduce even the qualitative nature of the CKM and MNS mixing matrices.

One way to get around this impasse is to invoke O(1) corrections to the entries of the mass matrices. By this is meant that geometric factors only determine the scale of an entry but its exact value is arbitrary. This would plague the arguments we used above. In such an event a qualitative or even a quantitative success can be achieved. But this is at the cost of a huge arbitrariness since we have to acquiesce in a host of new parameters.

Some of the existing analyses have relied on the O(1) corrections to generate nontrivial mixings. In [4], $\mathbf{10_1}$ has been kept at the O' brane, $\mathbf{10_2}$ in the SU(5) bulk, while $\mathbf{10_3}$ has been placed at the O brane. All $\mathbf{\bar{5}_i}$ have been kept in the SU(5) bulk. Going by our hypothesis, this case will render the MNS mixing to be trivial *a la* our case (1) in section IV. In [3], the placements are the following: $\mathbf{\bar{5}_1}$ and $\mathbf{10_1}$ in bulk, $\mathbf{\bar{5}_2}$ at O, $\mathbf{10_2}$ in bulk, $\mathbf{\bar{5}_3}$ and $\mathbf{10_3}$ at O. This possibility contradicts reality as per our case (3) or (4). In [6], all $\mathbf{\bar{5}_i}$ have been placed at O, while $\mathbf{10_1}$ and $\mathbf{10_2}$ reside in bulk with $\mathbf{10_3}$ at O. Again, following our case (1), this option fails to reproduce observed data. The placement of matter fields in the above noted analyses have been motivated from different considerations: suppressions of proton decay, $m_b = m_\tau$ at GUT scale, etc. We must admit one aspect at this stage. All these analyses do emphasize the need of O(1) corrections to Yukawa couplings to reproduce the data, i.e. the different entries of the mass matrices constructed in these analyses should be taken merely as mass scales with the tacit assumption that there are hidden O(1) uncertainty factors multiplying those entries. In any case, our study is neither intended to nor does it in any way undermine the different scenarios that the above mentioned analyses deal with, rather we arrive at a similar conclusion. Our modest intention is to demonstrate by exhaustion that geometrical suppressions (depending on localisation) alone, barring any O(1) corrections to Yukawa couplings, cannot reproduce even the qualitative features of quark and lepton mixings in the 5d SUSY GUT context. Any attempt to build a realistic model would be at the cost of economy of parameters. Generalisation to 6d models with orbifolds of the structure $T^2/(Z_2 \times Z_2')^2$ opens up more options for placing matter fields in different locations [1]. This allows further spatial separation of these fields which help create textures that can admit hierarchical masses with appropriate mixings. We do not deal with this in the present analysis.

Acknowledgements

We acknowledge hospitality at Abdus Salam ICTP, Trieste, where part of the work has been done. This research has been supported, in part, by the DST, India, project number SP/S2/K-10/2001.

References

[1] L. Hall, J. March-Russell, T. Okui and D. R. Smith, JHEP **0409** (2004) 026 [arXiv:hep-ph/0108161].

[2] A. Hebecker and J. March-Russell, Nucl. Phys. B **613** (2001) 3 [arXiv:hep-ph/0106166].

[3] Y. Nomura, Phys. Rev. D **65** (2002) 085036 [arXiv:hep-ph/0108170].

[4] A. Hebecker and J. March-Russell, Phys. Lett. B **541** (2002) 338 [arXiv:hep-ph/0205143].

[5] L. J. Hall and Y. Nomura, Phys. Rev. D **64** (2001) 055003 [arXiv:hep-ph/0103125].

[6] W. F. Chang and J. N. Ng, JHEP **0310** (2003) 036 [arXiv:hep-ph/0308187].

[7] G. Altarelli and F. Feruglio, Phys. Lett. B **511** (2001) 257 [arXiv:hep-ph/0102301].

[8] G. Bhattacharyya and K. Sridhar, J. Phys. G **29** (2003) 993 [arXiv:hep-ph/0111345].