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Neutron-antineutron oscillations and SO (10) grand unification

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Abstract. Within the framework of the survival hypothesis for Higgs scalars we comprehensively examine the following question: could there be neutron-antineutron oscillations in SO(10) grand unified theories which would be detectable in the forthcoming experiments? In the process of answering this, we critically discuss and supplement the existing knowledge of the relevant patterns of SO(10) symmetry breakdown in relation to the said oscillations. However, our conclusions are negative with the oscillation period being 10^{25} years or higher.

Keywords. $\Delta B = 2$ transitions; B−L violation; survival hypothesis; intermediate mass-scales; partial unification; Pati-Salam symmetry.

1. Introduction

The possibility of detecting a doubly baryon-violating $\Delta B=2$ transition *via* neutron-antineutron ($n\bar{n}$) oscillations has attracted much attention over the past few years. The present experimental limit on the oscillation period is $\tau_{n\bar{n}}>10^5$ sec (Wilson 1980). It appears that a period upto at least 3×10^9 sec $\sim 10^2$ years will become measurable in the foreseeable future (Baldo-Ceolin 1982; Ratti 1982; Wilson 1982). On the theoretical front, baryon nonconserving processes are now expected to occur naturally in grand unified theories (GUTs), as reviewed by Langacker (1981). It would thus be pertinent to ask whether detectable $n\bar{n}$ oscillations could occur in the currently popular grand unified models.

The minimal SU(5) theory of Georgi and Glashow (1974) obeys an exact global conservation law (Langacker 1981) on the difference between the baryon and the lepton numbers *B-L*. Consequently, nn oscillations are forbidden. *B-L* violation can be allowed into a nonminimal SU(5) theory *via* an extended Higgs sector (Georgi and Jarlskog 1979), but its mass-scale is superheavy, *i.e.* $\gtrsim 10^{14}$ GeV. The fact (Kuo and Love 1980) that the $n \to \bar{n}$ transition amplitude in such theories is controlled by the inverse fifth power of this scale then makes the corresponding $n\bar{n}$ oscillation period comparable to or greater than the proton lifetime (Chang and Chang 1980). This conclusion applies equally to those GUTs—based on bigger groups such as SO (10), E_6 etc.—where the symmetry descends *via* SU (5).

GUTs with intermediate (rather than superheavy) *B-L* violating scales can, in principle, admit detectable $n\bar{n}$ oscillations. Indeed, this possibility has provided an important theoretical motivation behind the ongoing experiments searching for such an effect. In this respect, the question of detectable $n\bar{n}$ oscillations in those SO(10)

GUTS (Fritzsch and Minkowski 1975) which break along routes bypassing SU(5) is *a priori* interesting on three counts. First, since *B-L* is a generator of SO(10), any violation of it (as in $n\bar{n}$ oscillation) must be by the spontaneous symmetry breakdown mechanism; such spontaneous violation of *B-L* with intermediate mass-scales is possible in this type of SO(10) theories. Second, Higgs scalars must play an essential part in such amplitudes and this class of SO(10) models minimally possesses a richer spectrum of scalars than those breaking *via* SU(5). Third, these non-SU(5) descending patterns can also be incorporated in bigger GUTS (based on E_6, say) so that the $n\bar{n}$-oscillation question will have direct relevance to those scenarios as well. In this paper we, therefore, address ourselves to the above question.

A central role, in connection with the above issue, is played by the pattern of descent of the intermediate Pati-Salam symmetry† $G_{\mathrm{PS}}\equiv SU(4)_{\tilde{C}}\times SU(2)_L\times SU(2)_R$, a subgroup of the grand unifying group $G\equiv SO(10)$, to the standard low-energy symmetry $G_{\mathrm{std}}\equiv SU(3)_C\times SU(2)_L\times U(1)_Y$. It is already known†† (Mohapatra and Marshak 1980) that the transition $n\to\bar{n}$ involves three-coloured Higgs propagators and one colour singlet Higgs tadpole involving a vacuum expectation value (VEV). These Higgs will have to be from a right-handed weak isospin triplet. In particular, the colour-singlet VEV has to violate both *B-L* and $SU(2)_R$ in order to induce a nonzero $n\to\bar{n}$ amplitude which is controlled by the *lower* of the two corresponding scales. Consequently, only two of various possible symmetry-breaking schemes in the descent $G\to G_{\mathrm{PS}}\to G_{\mathrm{std}}$ matter. Constraints, from the matching of the evolutionary $SU(3)_C$ and $SU(2)_L\times U(1)_Y$ gauge coupling strengths with corresponding low-energy experimental parameters, specify the magnitudes of the intermediate mass-scales in the two schemes. We then consider the survival hypothesis (SH) for Higgs scalars which compels the coloured Higgs masses to roughly equal the scale at which G_{PS} is broken. Consequently, in either scenario the $n\bar{n}$ oscillation period gets too long—far beyond detectability.

In § 2 we briefly discuss the role of scalars in the $n\to\bar{n}$ transition. Section 3 contains our study of those channels in the spontaneous breakdown of SO(10) symmetry which are relevant to the $n\to\bar{n}$ transition. In § 4 we formulate the SH for Higgs scalars and consider its implications for the above channels; in particular, we obtain the suppression of the $n\to\bar{n}$ transition amplitude. Section 5 contains concluding remarks. Some technical details on the derivation of bounds in intermediate mass-scales in the relevant channels are given in the Appendix.

2. Mass-scales controlling $n\bar{n}$ oscillation

To amplify the remarks in § 1 on the Higgs scalars controlling the $n\to\bar{n}$ transition amplitude we briefly discuss the Mohapatra-Marshak mechanism. Consider the stage (which any SO(10) GUT with intermediate *B-L* violation must go through) where the effective symmetry is $G_{\mathrm{MM}}=SU(3)_C\times U(1)_{\mathrm{B\text{-}L}}\times SU(2)_L\times SU(2)_R$ in the

†Here $\tilde{C}$ stands for the four-fold Pati-Salam colour including lepton number as the fourth colour, C for usual colour and Y for the weak hypercharge.

††There is an alternative, more complicated mechanism due to Deo (1981) which will be touched upon later.

chain $G \to G_{PS} \to \ldots \to G_{std}$, so that the weak Gell-Mann—Nishijima relation in a transparent notation is

$$Q_{EM} = I_{3L} + I_{3R} + \tfrac{1}{2}(B - L) = I_{3L} \tfrac{1}{2}\, Y.$$

Define flavour-doublet colour-triplet chiral quark fields and flavour-doublet colour-singlet chiral lepton fields

$$q_{L,R,i} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R,i}, \quad l_{L,R} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{L,R}$$

respectively. Similarly, introduce colour-sextet flavour-triplet Higgs fields† $(\Delta_L^C)^a_{ij} \equiv (\mathbf{6}, \tfrac{2}{3}, \mathbf{3}, \mathbf{1})$, $(\Delta_R^C)^a_{ij} \equiv (\mathbf{6}, -\tfrac{2}{3}, \mathbf{1}, \mathbf{3})$, with i, j as colour indices, a being a flavour index and Δ^C_{ij} symmetric in i, j; there are also colour-singlet flavour-triplet Higgs fields $\Delta_L \equiv (\mathbf{1}, 2, \mathbf{3}, \mathbf{1})$, $\Delta_R = (\mathbf{1}, 2, \mathbf{1}, \mathbf{3})$. The Yukawa interaction is

$$\mathscr{L}_Y = h\, q^T_{L,i}\, (i\tau_2)\, \mathbf{C}_D^{-1}\, \tau^a\, q_{L,j}\, (\Delta_L^C)^a_{ij} + h'\, l_L^T\, (i\tau_2)\, \mathbf{C}_D^{-1}\, \tau^a\, l_L\, \Delta^a_L$$
$$+ L \longleftrightarrow R + \text{h.c.} \tag{1}$$

and the quartic scalar interaction is

$$\mathscr{L}_S = \lambda\, \epsilon_{idp}\, \epsilon_{jeq}\, (\Delta_L^C)^a_{ij}\, (\Delta_L^C)^a_{de}\, (\Delta_L^C)^b_{pq}\, \Delta^b_L + L \longleftrightarrow R + \text{h.c.} \tag{2}$$

ϵ's being the standard antisymmetric Levi-Civita tensors and $\mathbf{C}_D$ the Dirac C-matrix.

In this theory, therefore, the $n \to \bar{n}$ $\Delta B = 2$, $\Delta L = 0$ transition, responsible for $n\bar{n}$ oscillation, is Higgs-mediated and can be diagrammatically represented as in figure 1. The amplitude is of the order of $\lambda h^3 \langle \Delta \rangle M^{-6}_{\Delta C}$, where $M_{\Delta C}$ is the mass of Δ^C and $\langle \Delta \rangle$ is the VEV of Δ. $\langle \Delta^C \rangle$ must vanish since $SU(3)_C$ is an exact symmetry, but $\langle \Delta \rangle$ does have a nonzero VEV since $U(1)_{B-L}$ is spontaneously broken. The Δ's could be either Δ_L or Δ_R. However, we know from many sources (*e.g.* Majorana masses of neutrinos) that $\langle \Delta_R \rangle \gg \langle \Delta_L \rangle$. Moreover, we shall later see while imposing the survival hypothesis for scalars that $M_{\Delta C}$ turns out

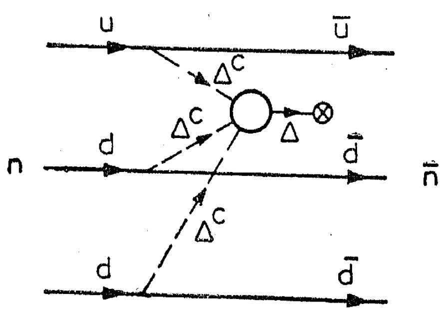

Figure 1. Higgs-mediated $n\bar{n}$ transition

†In this notation $(\mathbf{R}_1, U, \mathbf{R}_2, \mathbf{R}_3)$ denotes an irreducible representation of G_{MM} which carries a $B-L$ quantum number U and transforms as $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$ under $SU(3)_C$, $SU(2)_L$ and $SU(2)_R$ respectively.

to be roughly the same for Δ_L^C and Δ_R^C. Hence the dominant contribution to figure 1 comes from the case where $\Delta^C = \Delta_R^C \equiv (6, \frac{2}{3}\, 1, 3)$ and $\Delta = \Delta_R \equiv (1, 2, 1, 3)$ so that the controlling scales are those violating B-L and $SU(2)_R$ (by a Higgs triplet) and the mass of Δ^C. Since λ and h can at most be of order unity for perturbation theory to make sense, the $n \to \bar{n}$ transition amplitude is bounded above in order of magnitude by $\langle\Delta\rangle\, M_{\Delta^C}^{-6}$.

The above amplitude can be related to the $n\bar{n}$ oscillation period† (Riazuddin 1982). Dimensional considerations imply (ignoring wavefunction overlap effects) that

$$\tau_{n\bar{n}}^{-1} \sim \lambda h^3 \langle\Delta\rangle (M_N/M_{\Delta^C})^{-6} \lesssim \langle\Delta\rangle (M_N/M_{\Delta^C})^6, \tag{3}$$

M_N being the nucleon mass. Thus we have

$$|\langle\Delta\rangle|^{-1} (M_{\Delta^C} M_N^{-1})^6 < \tau_{n\bar{n}}\,. \tag{4}$$

Hence, for $\tau_{n\bar{n}}$ not to exceed 3×10^9 sec, one would need

$$\frac{M_N}{M_{\Delta^C}} |\langle\Delta\rangle \text{ in GeV}|^{1/6} \gtrsim 2 \times 10^{-6}. \tag{5}$$

3. Descents of SO(10) with intermediate B–L violation

Though proposed earlier (Fritzsch and Minkowski 1975), SO(10) grand unified theories became popular only three years ago (Georgi and Nanopoulos 1979). The relevant possible chains of SO(10) symmetry breakdown can be classified (Rajpoot 1980) into three primary categories (A), (B) and (C), vide flow-chart of figure 2.

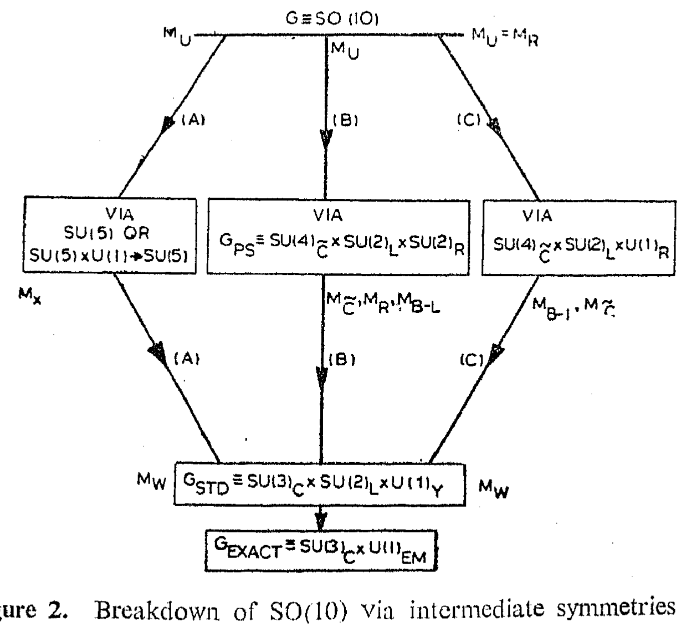

Figure 2. Breakdown of SO(10) via intermediate symmetries

†The rate of nuclear instability in matter induced by the $n \to \bar{n}$ transition is given by $\Gamma \sim \tau_{n\bar{n}}^{-2} M_N^{-1}$ so that the known lower bound of 10^{30} years on Γ^{-1} merely implies $\tau_{n\bar{n}} > 10^5$ sec.

Here M_U defines the unification mass where SO(10) first suffers spontaneous breakdown. The other masses M_X, M_C, M_R and $M_{B\text{-}L}$—shown in figure 2—define various scales corresponding to the breakdown of SU(5), SU(4)$_{\tilde{C}}$, SU(2)$_R$ and U(1)$_{B\text{-}L}$ respectively. Clearly, there can be more intermediate steps in each category. Patterns, in which one or more of the intermediate steps (considered in our general discussion) are skipped, can be recovered as special cases by raising the mass-scale(s) at the end of the step(s) to equal the one(s) at the beginning.

Let us consider and quickly dispense with categories (A) and (C) first. In (A), standard SU(5) proton decay arguments (Langacker 1981) imply that M_X is at least 10^{14} GeV and M_U could be anywhere between M_X and the Planck mass 10^{19} GeV. Since SU(5) containing SU(3)$_C$ × SU(2)$_L$ × U(1)$_Y$ does not contain either U(1)$_{B\text{-}L}$ or SU(2)$_R$ fully but G=SO(10) does, scales violating *B-L* and right weak isospin and hence the order of magnitude of $\langle\Delta\rangle$ must be between M_X and M_U. By the survival hypothesis (see §4) $M_{\Delta C}$ will also be of the same order. The left side of equation (5) being less than 10^{-11}, the $n\bar{n}$ oscillation period will be too long ($\tau_{n\bar{n}} \gtrsim 10^{38}$ years)—far beyond detectability. Making $\langle\Delta\rangle \gg M_{\Delta C}$, $\tau_{n\bar{n}}$ could be lowered—but only down to about 10^{33} years. This pushes $\langle\Delta\rangle$ upto the Planck mass but keeps $M_{\Delta C}$ at 10^{14} GeV. In (C), the right weak isospin group SU(2)$_R$ is broken at M_R=M_U. Since $M_{\Delta C}$ will have to be of this order by the survival hypothesis argument of the next section and $\langle\Delta\rangle \sim M_{B\text{-}L}$ which is less, again the $n \to \bar{n}$ transition amplitude is suppressed to a level much below possible detection†. We are thus left only with schemes which come within the aegis of category (B).

In (B), the simplest and most popular way to induce the breakdown $G \to G_{\mathrm{PS}}$ is through a Higgs scalar in the†† $\{\mathbf{54}\}$ representation. Since $\{\mathbf{54}\} \supset [\mathbf{1}, \mathbf{1}, \mathbf{1}]$, a VEV accruing to the latter will leave G_{PS} unbroken while breaking G. At this stage

$$Q_{\mathrm{EM}} = \sqrt{\tfrac{2}{3}}\, T_{15} + T_{3L} + T_{3R},$$

where $T_{15} = \sqrt{\tfrac{3}{8}}\ (B\text{-}L)$ is the 15th generator of SU(4)$_{\tilde{C}}$. The fermion quartet of SU(4)$_{\tilde{C}}$ is

$$\psi_{L,\,R,\,\alpha} = \begin{array}{ll} q_{L,\,R,\,i} & (\alpha = i = 1, 2, 3) \\ l_{L,\,R} & (\alpha = 4) \end{array}$$

and there are Higgs fields $\left[\Delta_L^{\tilde{C}}\right]^a_{\alpha\beta} \equiv [\mathbf{10}, \mathbf{3}, \mathbf{1}]$ and $\left[\Delta_R^{\tilde{C}}\right]^a_{\alpha\beta} \equiv [\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}]$. Here α, β go from 1 to 4 ($\Delta^{\tilde{C}}_{\alpha\beta}$ being symmetric in α, β). The SU(4)$_{\tilde{C}}$—invariant quartic scalar coupling is

$$\mathscr{L}_S = \lambda\, \epsilon_{\alpha\beta\gamma\delta}\, \epsilon_{\epsilon\rho\sigma\tau} \left[\Delta_L^{\tilde{C}}\right]^a_{\alpha\epsilon} \left[\Delta_L^{\tilde{C}}\right]^a_{\beta\rho} \left[\Delta_L^{\tilde{C}}\right]^b_{\gamma\sigma} \left[\Delta_L^{\tilde{C}}\right]^b_{\delta\tau} + L \longleftrightarrow R + \text{h.c.} \qquad (6)$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the four-dimensional completely antisymmetric tensor density.

†See also the discussion at the end of § 4.

††$\{\mathbf{R}\}$ denotes an irreducible representation of G and $[\mathbf{R}, \mathbf{R}', \mathbf{R}'']$ means the same for G_{PS}, transforming as $\mathbf{R}$, $\mathbf{R}'$ and $\mathbf{R}''$ under SU(4)$_{\tilde{C}}$, SU(2)$_L$ and SU(2)$_R$ respectively.

In the symmetry breakdown $G \to G_{PS} \to G_{std.}$ the $\Delta_R \equiv (\mathbf{1, 2, 1, 3})$, which must come into play to induce the $n \to \bar{n}$ transition, is contained only in the $\{\mathbf{126}\}$ of SO(10).

$$G \supset G_{PS} \supset SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R,$$

$$\{\mathbf{126}\} \supset [\overline{\mathbf{10}}, 1, 3] \supset (\mathbf{1, 2, 1, 3}). \tag{7}$$

Thus $\langle \Delta \rangle$ will be related to the scale of the breakdown $G_{std.} \xrightarrow[\{126\}]{} G_{PS}$. This requirement of the spontaneous breakdown of G_{PS} being induced by the $[\overline{\mathbf{10}}, \mathbf{1, 3}]$ in the $\{\mathbf{126}\}$ selects only two out of all possible chains in the descent $G_{PS} \to G_{std.}$. One can actually say in a more general vein that any chain of SO(10) symmetry breakdown where $(\mathbf{1, 2, 1, 3}) \subset [10, 1, 3] \subset \{\mathbf{126}\}$ does not acquire a VEV can be rightaway excluded from our considerations. It is further evident that the first breakdown of $SU(4)_{\tilde{C}}$ towards $SU(3)_C$, in the case of our interest, must be induced by the $a = 4 = \beta$ member of $[\Delta_R^{\tilde{C}}]^a_{\alpha\beta}$. Indeed, $[\Delta^{\tilde{C}}]^a_{44}$ are identical to the colour-singlet Higgs fields Δ^a introduced in § 2.

The two channels, referred to above and selected out of all possible patterns† for the breakdown of G_{PS} to $G_{std.}$, are the following:

Channel (a)

$$G_{PS} \xrightarrow[M_{\tilde{C}}]{\{\mathbf{45}\} \supset [\mathbf{15, 1, 1}] \supset (\mathbf{1, 0, 1, 1})} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$$
$$\xrightarrow[M_R]{\{\mathbf{45}\} \supset [\mathbf{1, 1, 3}] \supset (\mathbf{1, 0, 1, 3})} SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow[M_{B-L}]{\{\mathbf{126}\} \supset [\overline{\mathbf{10}}, 1, 3] \supset (1, 2, 1, 3,)} G_{std.} \tag{8}$$

Channel (b)

$$G_{PS} \xrightarrow[M_R]{\{\mathbf{45}\} \supset [1, 1, 3]} SU(4)_{\tilde{C}} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow[M_{\tilde{C}}]{\{\mathbf{45}\} \supset [15, 1, 1] \supset (1, 0, 1, 1)} SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow[M_{B-L}]{\{\mathbf{126}\} \supset [\overline{10}, 1, 3] \supset (1, 2, 1, 3)} G_{std.} \tag{9}$$

In either channel $M_{\tilde{C}}$ is the scale for $SU(4)_{\tilde{C}} \to SU(3)_C \times U(1)_{B-L}$, M_R for $SU(2)_R \to U(1)_R$ and M_{B-L} for $U(1)_{B-L} \times U(1)_R \to U(1)_Y$. Evidently, the $\langle \Delta \rangle$ of our interest is of the order of M_{B-L} since $M_R \geqslant M_{B-L}$.

†One may skip intermediate steps, *e.g.* in channel (a) $G_{PS} \to SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R \to G_{std.}$ can be reached by putting $M_R = M_{B-L}$.

Channel (a) has been studied quite extensively in the limit $M_{B\text{-}L} = M_R$ (del Aguila and Ibañez 1981; Rizzo and Senjanovic 1982). In the more general case a careful analysis of the evolution of the strong coupling α_S as well as the Weinberg angle θ_W down to laboratory energies reveals (vide Appendix) that there are two cases for $M_{\tilde{C}}$ and M_R:

Case (a1) $M_{\tilde{C}} \gtrsim 10^{13}$ GeV, $M_{B\text{-}L} = g\langle\Delta\rangle \gtrsim 10^6$ Gev, $M_{B\text{-}L} \leqslant M_R \leqslant M_{\tilde{C}}$
Case (a2) $M_{\tilde{C}} \gtrsim 10^{10}$ GeV, $M_{B\text{-}L} = g\ \langle\Delta\rangle \sim 10^2$ GeV, $M_{B\text{-}L} \leqslant M_R \leqslant M_{\tilde{C}}$ (10)
Here g is the (L-R symmetric) weak gauge coupling. Channel (b) has not been considered before. Again there are two cases (see Appendix):
Case (b1) 5×10^{13} GeV $> M_{\tilde{C}} > 10^{13}$ GeV, $M_{B\text{-}L} = g\ \langle\Delta\rangle = 10^6$ GeV, $M_R \geqslant M_{\tilde{C}}$, (11)
Case (b2) 5×10^{13} GeV $> M_{\tilde{C}} > 10^{10}$ GeV, $M_{B\text{-}L} = g\ \langle\Delta\rangle = 10^2$ GeV $M_R \geqslant M_{\tilde{C}}$.

4. Survival hypothesis for Higgs scalars and its consequences

We have to estimate $M_{\Delta C}$ and $\langle\Delta_R\rangle$ for the different patterns discussed in the previous section so that a lower bound on $\tau_{n\bar{n}}$ becomes extractable. To that end, we need the survival hypothesis (SH) for Higgs scalars. This hypothesis was first introduced (Georgi 1979; Barbieri and Nanopoulos 1980) for fermions in a GUT to eliminate the disease of unnatural adjustment of parameters (in addition to that required to maintain gauge hierarchy) in understanding fermion masses. A clear review has been given by Langacker (1981), so we need merely state it in the most general form. Given the chain

$$G \xrightarrow[M_U]{} G_1 \xrightarrow[M_1]{} G_2 \xrightarrow[M_2]{} \ldots G_r \xrightarrow[M_r]{} G_{r+1} \xrightarrow[M_{r+1}]{} \ldots . \quad G_{\text{std.}} \xrightarrow[M_W]{} G_{\text{exact}}, \tag{12}$$

where $G \supset G_1 \supset G_2 \ldots G_r \supset G_{r+1} \ldots G_{\text{std.}} \supset G_{\text{exact}}$ and $M_U \geqslant M_1 \geqslant M_2 \ldots M_r \geqslant M_{r+1} \ldots M_W$, any fermion mass term that is invariant under $G_{r+1}, \ldots, G_{\text{std.}}$ (but not under G, $G_1, \ldots, G_r$) has a corresponding mass of order M_r.

Despite the usefulness of the above SH in studying fermion masses in GUTs, it is of little consequence with respect to Higgs masses, so one needs an SH for Higgs scalars or else once again unnatural fine tuning of parameters becomes obligatory—this time in the Higgs sector. There have to be some differences (del Auguila and Ibañez 1981) though, because a Higgs scalar *participating* in a symmetry breakdown step can have a mass of the order of the scale of that breakdown. The SH for Higgs scalars can be stated generally in the following form (Mohapatra and Popovič 1981; Raychaudhuri and Sarkar 1982). Return to the chain of equation (12) and concentrate on the step $G_r \to G_{r+1}$. Let the Higgs scalar H_r ($\langle H_r\rangle \neq 0$) responsible for this breaking be a member of the irreducible representation R_r of the group G_r. (Of course, H_r is uncharged, colourless and a singlet under G_{r+1}). R_r is contained in some irreducible representation R_r^0 of the grand unifying group G. Moreover,

$$R_r^0 \supset R_r^1 \supset R_r^2 \ldots\ldots R_r^{r-1} \supset R_r.$$

Here R_r^j $(j<r)$ is an irreducible representation of the intermediate symmetry G_j $(\supset G_r)$ which contains the representation R_r of G_r. Now (i) all members of R_r have to acquire masses of the order of M_r; (ii) all Higgs scalars contained in R_r^j but not in R_r^{j+1} have to acquire masses of the order of the scale M_j at which G_j breaks into G_{j+1} (these scalars form complete irreducible representations of G_{j+1}).

Let us consider the implications of this hypothesis *vis-a-vis* the $\{\mathbf{126}\}$ Higgs for the descent patterns of our interest. In category (B) the full irreducible multiplet $[\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}]$ of $G_{\rm PS}$ plays the pivotal role. Of that the submultiplet $(\mathbf{6}, \frac{2}{3}, \mathbf{1}, \mathbf{3})$ under $SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$ does not acquire any VEV while $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{3})$ does. The latter VEV is of order M_{B-L} which is what sets the scale for the masses of the three fields $(\Delta_R)^a$. In contrast, all members of the former submultiplet—*i.e.* $(\Delta_R^C)_{ij}^a$—acquire masses of the order of the scale characterizing the first breakdown of† G_{PS}. We can now examine the two specific channels:

Channel (a)

Here $M_{\Delta C} \sim M_{\tilde{C}}$—the mass-scale for the breaking $G_{\rm PS} \to SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$. Moreover, for the two solutions, with $g \sim 10^{-1}$ and using equations (10) and (3), we have

$$(a1)\;\; M_{\Delta C} \sim 10^{13}\ \text{GeV},\ \langle \Delta \rangle \sim 10^{7}\ \text{GeV},\ \tau_{n\bar{n}} \gtrsim 10^{39}\ \text{years}, \tag{13}$$

$$(a2)\;\; M_{\Delta C} \sim 10^{10}\ \text{GeV},\ \langle \Delta \rangle \sim 10^{3}\ \text{GeV},\ \tau_{n\bar{n}} \gtrsim 10^{25}\ \text{years}.$$

Channel (b)

Here G_{PS} breaks at M_R $(\geqslant M_{\tilde{C}})$ and sets the scale for $M_{\Delta C}$. Thus we have

$$(b1)\;\; M_{\Delta C} \sim 10^{13}\ \text{GeV},\ \langle \Delta \rangle \sim 10^{7}\ \text{GeV},\ \tau_{n\bar{n}} \gtrsim 10^{39}\ \text{years}; \tag{14}$$

$$(b2)\;\; M_{\Delta C} \sim 10^{10}\ \text{GeV},\ \langle \Delta \rangle \sim 10^{3}\ \text{GeV},\ \tau_{n\bar{n}} \gtrsim 10^{25}\ \text{years}.$$

Some brief remarks on category (C) (Rajpoot 1980) are called for. This chain is but a special case of channel (b) in category (B)—obtainable from the latter in the limit $M_R = M_U$. Rajpoot's study of category (*C*), in terms of the constraints from $\sin^2 \theta_W$ and α_S, led to the determination $M_U \sim 10^{18}$ GeV, $M_{\tilde{C}} \sim 10^{10}$ GeV and no constraints on M_{B-L} except $M_{B-L} \leqslant M_{\tilde{C}}$ for this case. Clearly, $g\,\langle \Delta \rangle \sim M_{B-L}$ so that $\langle \Delta \rangle \lesssim 10^9$ GeV. Since G_{PS} breaks at M_U, the survival hypothesis dictates $M_{\Delta C}$ to be $\sim 10^{10}$ GeV and $\tau_{n\bar{n}} \gtrsim 10^{67}$ years. Thus our cursory rejection of this category in § 3 was justified.

†Clearly, this argument applies equally to $(\Delta_L^C)_{ij}^a$ whose masses are of the same scale. This justifies our remarks on this point in § 2.

5. Concluding discussion

We really have a no-go result regarding the detectability of $n\bar{n}$ oscillations in SO(10) GUTs. Of course, we have exclusively considered $n\bar{n}$ oscillations in terms of the Mohapatra-Marshak mechanism of figure 1. Other possible mechanisms have also been suggested (Kuo and Love 1980; Deo 1981) for which the detailed analysis would have to be somewhat different. Nevertheless, the basic fact of the $n \to \bar{n}$ transition amplitude being controlled roughly by something like† $g \langle \Delta \rangle M_{\Delta C}^{-6}$ persists so that the imposition of the survival hypothesis in SO (10) inevitably pushes up $\tau_{n\bar{n}}$.

In a sense our general result was anticipated by del Aguila and Ibañez (1981). However, their formulation of the survival hypothesis was incomplete. Further, they were not concerned *a priori* with $n\bar{n}$ oscillations and had not focused on it. Nor had they considered in detail all possible chains of symmetry-descent (relevant to $n\bar{n}$ oscillations) so as to be able to completely rule out the detection of such phenomena in SO(10) grand unification. In particular, under category (B), they had made a detailed examination only of channel (a). Our analysis of channel (b), as given here, has—to our knowledge—not appeared in the literature before.

Among the remaining popular simple group GUTs, a fair amount of work has been done on the maximal SU (16). Two types of descent have been investigated. (1) The chain SU (16) $\to$ SU (12) $\times$ SU $(4)_L$ $\times$ U $(1)_{B-L}$ ...... SU $(3)_C$ $\times$ SU $(2)_L$ $\times$ SU $(2)_R$ $\times$ U $(1)_{B-L}$ $\to G_{\text{std}}$ has been studied quite extensively by Pati *et al* (1981), but the conclusion regarding the detectability of $n\bar{n}$ oscillation is pessimistic (Mohapatra and Popović 1981). (2) The sequence SU (16) $\to$ SU (8) $\times$ SU (8) ...... G_{PS} ...... G_{std} has also been studied but once again the $n \to \bar{n}$ transition is found to be strongly suppressed (Mohapatra and Popović 1981; Raychaudhuri and Sarkar 1982).

In E_6 grand unification most of the schemes studied so far involve the SU(5) route. In these scenarios detectable $n\bar{n}$ oscillations are ruled out *a priori*. It has been suggested (Fukugita *et al* 1982) that, in the relatively unexplored chain

$$E_6 \to SU(6) \to SU(5) \times U(1) \;\ldots\ldots\; G_{\text{std}}.$$

with a low mass-scale (10^4–10^5 GeV) for the extra U (1), such phenomena may be possible *via* the low mass-scale. However, the model has not been considered in detail. In particular, the implications of the SH for Higgs scalars and of the contributions from those scalars to the evolutionary gauge coupling strengths have not been taken into account. It is not clear without a careful and complete analysis whether the claim of detectable $n\bar{n}$ oscillation in this scenario will survive such accounting. Specifically, it seems to us that, since the scalars mediating the $n \to \bar{n}$ transition carry SU $(3)_C$ colour, their masses by the SH will be characteristic of the SU (6)—breaking scale; that being superheavy, $n\bar{n}$ oscillations will be suppressed. Perhaps one should keep an open mind till a detailed treatment emerges.

Among semi-simple groups $[SU(4)]^4$ has been studied, but the conclusion in regard to $n\bar{n}$ oscillations is pessimistic (Marshak *et al* 1980). Of course, a really large GUT

†There could be two such masses M and $\bar{M}$ and one could have $M^{-4}\,\bar{M}^{-2}$ instead of $M_{\Delta C}^{-6}$, but the survival hypothesis will force both $\bar{M}$ and M to be superlarge.

—such as that based on SU(48)—may admit detectable $n\bar{n}$ oscillations, but most smaller ones do not seem to do so. In particular, our conclusion is that such phenomena are not possible in SO(10) GUTs. Betting on the success of the ongoing experiments would amount to taking a long shot.

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We thank the $n\bar{n}$-oscillation speakers at the ICOBAN conference (11-14 January 1982, Tata Institute of Fundamental Research, Bombay) for encouragement. We have been informed that a similar analysis has been independently done by R N Mohapatra and G Senjanović.

Appendix: Bounds on the intermediate mass scales

The low energy predictions for the neutral current parameter $\sin^2\theta_W$ and the QCD fine structure constant α_S can be calculated *via* the Georgi-Quinn-Weinnberg equations. The final results depend on the route of descent through the intermediate mass-scales. It turns out that the fermionic contributions drop out of these expressions. We use complex (rather than real) scalar fields so that each scalar contribution has an extra factor of 2. The scalars responsible for the symmetry breakings have been indicated in equations (8) and (9). The last step in the symmetry breaking ($G_{\text{std}} \to G_{\text{exact}}$) is driven by the Higgs fields $(\mathbf{1}, 0, \mathbf{2}, \mathbf{2}) \subset [\mathbf{15}, \mathbf{2}, \mathbf{2}] \subset \{\mathbf{126}\}$ and $(\mathbf{1}, -2, \mathbf{3}, \mathbf{1}) \subset [\mathbf{10}, \mathbf{3}, \mathbf{1}] \subset \{\mathbf{126}\}$. The effect of all these scalars, as determined by the SH and the fact that those heavier than a certain mass-scale decouple from evolutions in ranges below that scale, are included in the calculations. Our basic equations are of the form (all masses scaled by GeV).

$$\sin^2\theta_W = \frac{3}{8} + \frac{\alpha}{16\pi}[a \ln M_U + b \ln M_{\tilde{C}} + c \ln M_R + d \ln M_{B-L} + e \ln M_L]$$

$$1 - \frac{8}{3}\frac{\alpha}{\alpha_S} = \frac{\alpha}{2\pi}[a' \ln M_U + b' \ln M_{\tilde{C}} + c' \ln M_R + d' \ln M_{B-L} + e' \ln M_L] \tag{A.1}$$

Channel (*a*) ($M_{\tilde{C}} \geqslant M_R$):

Here

$$a = \frac{32}{3},\ b = -\frac{94}{3},\ c = -17,\ d = 2,\ e = \frac{107}{3};$$

$$a' = \frac{40}{3},\ b' = 6,\ c' = \frac{17}{3},\ d' = -\frac{2}{3},\ e' = -\frac{73}{3}.$$

There are two cases (Rizzo and Senjanović 1982; Parida and Raychaudhuri 1981):

(a1) $M_{B-L} \sim 10^6$ GeV and $\sin^2\theta_W = 0{\cdot}23$. using (A.1) and the inequality $M_U \geqslant M_{\tilde{C}} \geqslant M_R$, we have

$$13{\cdot}7 + \frac{2}{75}\log\frac{M_{B-L}}{100} > \log M_R,$$

$$\log M_{\tilde{C}} > 12{\cdot}9 + \frac{6}{113}\log\frac{M_{B-L}}{100}. \tag{A.2}$$

(a2) $M_{B-L} \simeq 100$ GeV, $\sin^2\theta_W \simeq 0{\cdot}27$. We obtain a new lower bound on $M_{\tilde{C}}$:

$$13{\cdot}7 + \frac{2}{75}\log\frac{M_{B-L}}{100} > \log M_{\tilde{C}} > 9.8, \tag{A.3}$$

while the bound on M_R is unaltered.
The results of (A.2) and (A.3) are presented in the text in equation (10).

Channel (*b*) ($M_R \geqslant M_{\tilde{C}}$):

Here

$$a = \frac{32}{3},\ b = -6,\ c = \frac{127}{3},\ d = 2,\ e = \frac{107}{3};$$

$$a' = \frac{40}{3},\ b' = \frac{22}{3},\ c' = \frac{13}{3},\ d' = -\frac{2}{3},\ e' = -\frac{73}{3}.$$

Now it is possible to set both lower and upper bounds on $M_{\tilde{C}}$ in the two possible cases

(b1) Here

$$13.7 + \frac{2}{75}\log\frac{M_{B-L}}{100} > \log M_{\tilde{C}} > 12.9 + \frac{6}{113}\log\frac{M_{B-L}}{100} \tag{A.4}$$

(b2) Now

$$13 + \log 5 > \log M_{\tilde{C}} > 9{\cdot}8. \tag{A.5}$$

The results have been used in eq. (11) of the text.

After the submission of our manuscript a related paper appeared, Lüst D, Maseiro A and Roncadelli M (1982) *Phys. Rev.* (RC) **D25** 3096. Most of their conclusions tally with ours. However, we emphatically disagree with their last suggestion of "arranging" detectable $n\bar{n}$ oscillations in SO (10) through the choice $M_{\Delta_R^c} \sim 10^5$ Gev. As discussed in our § 4, the latter would violate the SH for Higgs scalars and would require highly unnatural and nonminimal fine tuning of parameters in the Higgs sector.

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