Neutron-antineutron oscillations and SO (10) grand unification

AMITAVA RAYCHAUDHURI and PROBIR ROY
Physics Department, University of Calcutta, 92 A P C Road, Calcutta 700 009, India
*Tata Institute of Fundamental Research, Bombay 400 005, India

MS received 30 April 1982; revised 15 July 1982.

Abstract. Within the framework of the survival hypothesis for Higgs scalars we comprehensively examine the following question: could there be neutron-antineutron oscillations in SO(10) grand unified theories which would be detectable in the forthcoming experiments? In the process of answering this, we critically discuss and supplement the existing knowledge of the relevant patterns of SO(10) symmetry breakdown in relation to the said oscillations. However, our conclusions are negative with the oscillation period being $10^{16}$ years or higher.

Keywords. $\Delta B = 2$ transitions; $B-L$ violation; survival hypothesis; intermediate mass-scales; partial unification; Pati-Salam symmetry.

1. Introduction

The possibility of detecting a doubly baryon-violating $\Delta B = 2$ transition via neutron-antineutron ($n\bar{n}$) oscillations has attracted much attention over the past few years. The present experimental limit on the oscillation period is $\tau_{n\bar{n}} > 10^{9}$ sec (Wilson 1980). It appears that a period up to at least $3 \times 10^{9}$ sec $\sim 10^{12}$ years will become measurable in the foreseeable future (Baldo-Ceolin 1982; Ratti 1982; Wilson 1982). On the theoretical front, baryon nonconserving processes are now expected to occur naturally in grand unified theories (GUTs), as reviewed by Langacker (1981). It would thus be pertinent to ask whether detectable $n\bar{n}$ oscillations could occur in the currently popular grand unified models.

The minimal SU(5) theory of Georgi and Glashow (1974) obeys an exact global conservation law (Langacker 1981) on the difference between the baryon and the lepton numbers $B-L$. Consequently, $n\bar{n}$ oscillations are forbidden. $B-L$ violation can be allowed into a nonminimal SU(5) theory via an extended Higgs sector (Georgi and Jarlskog 1979), but its mass-scale is superheavy, $i.e. \gtrsim 10^{14}$ GeV. The fact (Kuo and Love 1980) that the $n\rightarrow n\bar{n}$ transition amplitude in such theories is controlled by the inverse fifth power of this scale then makes the corresponding $n\bar{n}$ oscillation period comparable to or greater than the proton lifetime (Chang and Chang 1980). This conclusion applies equally to those GUTs—based on bigger groups such as SO (10), $E_6$ etc.—where the symmetry descends via SU (5).

GUTs with intermediate (rather than superheavy) $B-L$ violating scales can, in principle, admit detectable $n\bar{n}$ oscillations. Indeed, this possibility has provided an important theoretical motivation behind the ongoing experiments searching for such an effect. In this respect, the question of detectable $n\bar{n}$ oscillations in those SO(10)
GUTS (Fritzsch and Minkowski 1975) which break along routes bypassing SU(5)
is a priori interesting on three counts. First, since B-L is a generator of SO(10), any
violation of it (as in n̅n oscillation) must be by the spontaneous symmetry breakdown
mechanism; such spontaneous violation of B-L with intermediate mass-scales is
possible in this type of SO(10) theories. Second, Higgs scalars must play an essential
part in such amplitudes and this class of SO(10) models minimally possesses a richer
spectrum of scalars than those breaking via SU(5). Third, these non-SU(5) descend-
ing patterns can also be incorporated in bigger GUTS (based on E8, say) so that the
n̅n oscillation question will have direct relevance to those scenarios as well. In this
paper we, therefore, address ourselves to the above question.

A central role, in connection with the above issue, is played by the pattern of descent
of the intermediate Pati-Salam symmetry† GPS = SU(4)C × SU(2)L × SU(2)R, a sub-
group of the grand unifying group G = SO(10), to the standard low-energy symmetry
Gstd = SU(3)C × SU(2)L × U(1)Y. It is already known†† (Mohapatra and Marshak
1980) that the transition n→n̅ involves three-coloured Higgs propagators and one
colour singlet Higgs tadpole involving a vacuum expectation value (vev). These Higgs
will have to be from a right-handed weak isospin triplet. In particular, the colour-
singlet vev has to violate both B-L and SU(2)R in order to induce a nonzero
n→n̅ amplitude which is controlled by the lower of the two corresponding scales.
Consequently, only two of various possible symmetry-breaking schemes in the descent
G→GPS→Gstd matter. Constraints, from the matching of the evolutionary SU(3)C
and SU(2)L × U(1)Y gauge coupling strengths with corresponding low-energy experi-
mental parameters, specify the magnitudes of the intermediate mass-scales in the
two schemes. We then consider the survival hypothesis (sh) for Higgs scalars which
compels the coloured Higgs masses to roughly equal the scale at which GPS is broken.
Consequently, in either scenario the n̅n oscillation period gets too long—far beyond
detectability.

In § 2 we briefly discuss the role of scalars in the n→n̅ transition. Section 3 contains
our study of those channels in the spontaneous breakdown of SO(10) symmetry which
are relevant to the n→n̅ transition. In § 4 we formulate the sh for Higgs scalars and
consider its implications for the above channels; in particular, we obtain the
suppression of the n→n̅ transition amplitude. Section 5 contains concluding remarks.
Some technical details on the derivation of bounds in intermediate mass-scales in the
relevant channels are given in the Appendix.

2. Mass-scales controlling n̅n oscillation

To amplify the remarks in § 1 on the Higgs scalars controlling the n→n̅ transition
amplitude we briefly discuss the Mohapatra-Marshak mechanism. Consider the
stage (which any SO(10) GUT with intermediate B-L violation must go through)
where the effective symmetry is GMM = SU(3)C × U(1)B-L × SU(2)L × SU(2)R in the

†Here C stands for the four-fold Pati-Salam colour including lepton number as the fourth colour,
C for usual colour and Y for the weak hypercharge.

††There is an alternative, more complicated mechanism due to Deo (1981) which will be touched
upon later.
chain $G \rightarrow G_{PS} \rightarrow \ldots \rightarrow G_{std}$, so that the weak Gell-Mann—Nishijima relation in a transparent notation is

$$Q_{EM} = I_{3L} + I_{3R} + \frac{1}{3} (B - L) = I_{3L} \frac{1}{3} Y.$$ Define flavour-doublet colour-triplet chiral quark fields and flavour-doublet colour-singlet chiral lepton fields

$$q_{L,R,i} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R,i}, \quad l_{L,R} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{L,R}$$ respectively. Similarly, introduce colour-sextet flavour-triplet Higgs fields† ($\Delta C_{ij}^a = (6, \bar{3}, 1, 1, 3)$, $\Delta R_{ij}^a = (6, -\bar{3}, 1, 1, 3)$, with $i, j$ as colour indices, $a$ being a flavour index and $\Delta C_{ij}^a$ symmetric in $i, j$; there are also colour-singlet flavour-triplet Higgs fields $\Delta_L = (1, 2, 3, 1), \Delta_R = (1, 2, 1, 3)$. The Yukawa interaction is

$$\mathcal{L}_Y = \bar{H} \cdot \left( i \tau_3 C_D^{-1} q_{L,i} + k' \tau_3 C_D^{-1} l_{L} \Delta_L^a \right) + h.c.$$ and the quartic scalar interaction is

$$\mathcal{L}_S = \lambda \epsilon_{idp} \epsilon_{jef} \Delta C_{ij}^{a} \Delta C_{ij}^{a} \Delta C_{ij}^{a} \Delta C_{ij}^{a} + h.c.$$ £s being the standard antisymmetric Levi-Civita tensors and $C_D$ the Dirac $C$-matrix.

In this theory, therefore, the $n \rightarrow \tilde{n}$, $\Delta B = 2, \Delta L = 0$ transition, responsible for $n\bar{n}$ oscillation, is Higgs-mediated and can be diagrammatically represented as in figure 1. The amplitude is of the order of $\lambda \alpha \langle \Delta \rangle M_{\Delta C}^8$, where $M_{\Delta C}$ is the mass of $\Delta^C$ and $\langle \Delta \rangle$ is the vev of $\Delta$. $\langle \Delta C \rangle$ must vanish since $SU(3)_C$ is an exact symmetry, but $\langle \Delta \rangle$ does have a nonzero vev since $U(1)_{B-L}$ is spontaneously broken. The $\Delta$'s could be either $\Delta_L$ or $\Delta_R$. However, we know from many sources (e.g. Majorana masses of neutrinos) that $\langle \Delta_R \rangle \gg \langle \Delta_L \rangle$. Moreover, we shall later see while imposing the survival hypothesis for scalars that $M_{\Delta C}$ turns out

![Figure 1. Higgs-mediated $n\bar{n}$ transition](image)

†In this notation $(R_1, U, R_2, R_3)$ denotes an irreducible representation of $G_{MM}$, which carries a $B-L$ quantum number $U$ and transforms as $R_1, R_4$ and $R_5$ under $SU(3)_C$, $SU(2)_L$ and $SU(2)_R$ respectively.
to be roughly the same for $\Delta_L^C$ and $\Delta_R^C$. Hence the dominant contribution to figure 1 comes from the case where $\Delta_L^C = \Delta_R^C = (6, \frac{3}{2}, 1, 3)$ and $\Delta = \Delta_R = (1, 2, 1, 3)$ so that the controlling scales are those violating $B-L$ and $SU(2)_R$ (by a Higgs triplet) and the mass of $\Delta^C$. Since $\lambda$ and $h$ can at most be of order unity for perturbation theory to make sense, the $n \rightarrow \bar{n}$ transition amplitude is bounded above in order of magnitude by $\langle \Delta \rangle M_{\Delta^C}^0$.

The above amplitude can be related to the $n\bar{n}$ oscillation period (Riazuddin 1982). Dimensional considerations imply (ignoring wavefunction overlap effects) that

$$\frac{\tau_{nn}^{-1}}{\langle \Delta \rangle (M_N/M_{\Delta^C})^{-6} \lesssim \langle \Delta \rangle (M_N/M_{\Delta^C})^6,}$$

$M_N$ being the nucleon mass. Thus we have

$$\left| \langle \Delta \rangle \right|^{-1} (M_{\Delta^C} M_{N})^{-6} \lesssim \tau_{nn}^{-1}.$$  

Hence, for $\tau_{nn}$ not to exceed $3 \times 10^9$ sec, one would need

$$\frac{M_N}{M_{\Delta^C}} |\langle \Delta \rangle| \text{ in } \text{GeV}^{-1/6} \gtrsim 2 \times 10^{-6}.$$  

3. Descents of SO(10) with intermediate B-L violation

Though proposed earlier (Fritzsch and Minkowski 1975), SO(10) grand unified theories became popular only three years ago (Georgi and Napolopulos 1979). The relevant possible chains of SO(10) symmetry breakdown can be classified (Rajpoot 1980) into three primary categories (A), (B) and (C), vide flow-chart of figure 2.

Figure 2. Breakdown of SO(10) via intermediate symmetries

$\dagger$The rate of nuclear instability in matter induced by the $n \rightarrow \bar{n}$ transition is given by $\Gamma \sim \tau_{nn}^{-2} M_N^{-1}$, so that the known lower bound of $10^{38}$ years on $\Gamma^{-1}$ merely implies $\tau_{nn} > 10^8$ sec.
Here $M_U$ defines the unification mass where SO(10) first suffers spontaneous breakdown. The other masses $M_X, M_C, M_R$ and $M_{B-L}$—shown in figure 2—define various scales corresponding to the breakdown of SU(5), SU(4)$_C$, SU(2)$_R$ and U(1)$_{B-L}$ respectively. Clearly, there can be more intermediate steps in each category. Patterns, in which one or more of the intermediate steps (considered in our general discussion) are skipped, can be recovered as special cases by raising the mass-scale(s) at the end of the step(s) to equal the one(s) at the beginning.

Let us consider and quickly dispense with categories (A) and (C) first. In (A), standard SU(5) proton decay arguments (Langacker 1981) imply that $M_X$ is at least $10^{14}$ GeV and $M_U$ could be anywhere between $M_X$ and the Planck mass $10^{18}$ GeV. Since SU(5) containing SU(3)$_C \times$ SU(2)$_L \times$ U(1)$_Y$ does not contain either U(1)$_{B-L}$ or SU(2)$_R$ fully but $G =$SO(10) does, scales violating $B-L$ and right weak isospin and hence the order of magnitude of $\langle \Delta \rangle$ must be between $M_X$ and $M_U$. By the survival hypothesis (see §4) $M_{\Delta C}$ will also be of the same order. The left side of equation (5) being less than $10^{-11}$, the $\tilde{n}$ oscillation period will be too long (2 years $\gtrsim 10^{38}$ years)—far beyond detectability. Making $\langle \Delta \rangle \gtrsim M_{\Delta C}$, $\tau_{\tilde{n}}$ could be lowered—but only down to about 10$^{38}$ years. This pushes $\langle \Delta \rangle$ up to the Planck mass but keeps $M_{\Delta C}$ at $10^{14}$ GeV. In (C), the right weak isospin group SU(2)$_R$ is broken at $M_R = M_U$.

Since $M_{\Delta C}$ will have to be of this order by the survival hypothesis argument of the next section and $\langle \Delta \rangle \sim M_{B-L}$ which is less, again the $n \rightarrow \tilde{n}$ transition amplitude is suppressed to a level much below possible detection. We are thus left only with schemes which come within the aegis of category (B).

In (B), the simplest and most popular way to induce the breakdown $G \rightarrow G_{PS}$ is through a Higgs scalar in the$^\dagger$ $\{54\}$ representation. Since $\{54\} \supset [1, 1, 1]$, a vev accruing to the latter will leave $G_{PS}$ unbroken while breaking $G$. At this stage

$$Q_{EM} = \sqrt{3} T_{15} + T_{3L} + T_{3R},$$

where $T_{15} = \sqrt{3} (B-L)$ is the 15th generator of SU(4)$_C$. The fermion quartet of SU(4)$_C$ is

$$Q_{EM} = \sqrt{3} T_{15} + T_{3L} + T_{3R},$$

$q_L, R, i (a = i = 1, 2, 3)$

$$\psi_{L, R, a} = l_{L, R},$$

and there are Higgs fields $[\Delta L]_{a\beta} = [10, 3, 1]$ and $[\Delta R]_{a\beta} = [10, 1, 3]$. Here $a, \beta$ go from 1 to 4 ($\Delta C_{ab}$ being symmetric in $a, \beta$). The SU(4)$_C$—-invariant quartic scalar coupling is

$$S_S = \lambda \epsilon_{abc} \epsilon_{def} [\Delta L]_{a\beta} [\Delta C]^a_{\beta\gamma} [\Delta L]^b_{\gamma\delta} [\Delta _L]^c_{\delta\tau} + L \leftrightarrow R + h.c.$$ (6)

where $\epsilon_{abc}$ is the four-dimensional completely antisymmetric tensor density.

$^\dagger$ See also the discussion at the end of § 4.

$^\dagger$ $\{R\}$ denotes an irreducible representation of $G$ and $\{R, R', R''\}$ means the same for $G_{PS}$ transforming as $R, R'$ and $R''$ under SU(4)$_C$, SU(2)$_L$ and SU(2)$_R$ respectively.
In the symmetry breakdown $G \rightarrow G_{PS} \rightarrow G_{std.}$ the $\Delta_R \equiv \{1, 2, 1, 3\}$, which must come into play to induce the $n \rightarrow \eta$ transition, is contained only in the $\{126\}$ of SO(10).

$$G \supset G_{PS} \supset SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$$

$$\{126\} \supset [10, 1, 3] \supset (1, 2, 1, 3).$$

(7)

Thus $\langle \Delta \rangle$ will be related to the scale of the breakdown $G_{std.} \supset \{126\} \rightarrow G_{PS}$. This requirement of the spontaneous breakdown of $G_{PS}$ being induced by the $[10, 1, 3]$ in the $\{126\}$ selects only two out of all possible chains in the descent $G_{PS} \rightarrow G_{std.}$. One can actually say in a more general vein that any chain of SO(10) symmetry breakdown where $(1, 2, 1, 3) \subset [10, 1, 3] \subset \{126\}$ does not acquire a vev can be rightway excluded from our considerations. It is further evident that the first breakdown of $SU(4)_C$ towards $SU(3)_C$, in the case of our interest, must be induced by the $a = 4 = \beta$ member of $[\Delta_R^{\alpha}]_{ab}$. Indeed, $[\Delta_R^{\beta}]_{ab}$ are identical to the colour-singlet Higgs fields $\Delta^a$ introduced in § 2.

The two channels, referred to above and selected out of all possible patterns for the breakdown of $G_{PS}$ to $G_{std.}$, are the following:

**Channel (a)**

$$G_{PS} \{45\} \supset [15, 1, 1] \supset (1, 0, 1, 1)$$

$$M_C \supset [1, 1, 3] \supset (1, 0, 1, 3)$$

$$M_R \supset (10, 1, 3) \supset (1, 2, 1, 3)$$

$$G_{std.}$$

(8)

**Channel (b)**

$$G_{PS} \{45\} \supset [1, 1, 3]$$

$$M_C \supset [15, 1, 1] \supset (1, 0, 1, 1)$$

$$M_R \supset (10, 1, 3) \supset (1, 2, 1, 3)$$

$$G_{std.}$$

(9)

In either channel $M_C$ is the scale for $SU(4)_C \rightarrow SU(3)_C \times U(1)_{B-L}$, $M_R$ for $SU(2)_R \rightarrow U(1)_R$ and $M_{B-L}$ for $U(1)_{B-L} \times U(1)_R \rightarrow U(1)_Y$. Evidently, the $\langle \Delta \rangle$ of our interest is of the order of $M_{B-L}$ since $M_R \gg M_{B-L}$.

---

\*One may skip intermediate steps, e.g. in channel (a) $G_{PS} \rightarrow SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R \rightarrow G_{std.}$ can be reached by putting $M_R = M_{B-L}$.
Channel (a) has been studied quite extensively in the limit $M_{B-L} = M_R$ (del Aguila and Ibañez 1981; Rizzo and Senjanovic 1982). In the more general case a careful analysis of the evolution of the strong coupling $\alpha_S$ as well as the Weinberg angle $\theta_W$ down to laboratory energies reveals (vide Appendix) that there are two cases for $M_C$ and $M_R$:

Case (a1) $M_C > 10^{13}$ GeV, $M_{B-L} = g \langle \Delta \rangle > 10^6$ GeV, $M_{B-L} \leq M_R \leq M_C$

Case (a2) $M_C > 10^{10}$ GeV, $M_{B-L} = g \langle \Delta \rangle \sim 10^2$ GeV, $M_{B-L} \leq M_R \leq M_C$ \hspace{1cm} (10)

Here $g$ is the (L-R symmetric) weak gauge coupling. Channel (b) has not been considered before. Again there are two cases (see Appendix):

Case (b1) $5 \times 10^{13}$ GeV > $M_C > 10^{13}$ GeV, $M_{B-L} = g \langle \Delta \rangle = 10^6$ GeV, $M_R \geq M_C$,

Case (b2) $5 \times 10^{13}$ GeV > $M_C > 10^{10}$ GeV, $M_{B-L} = g \langle \Delta \rangle = 10^2$ GeV $M_R \geq M_C$.

4. Survival hypothesis for Higgs scalars and its consequences

We have to estimate $M_{\Delta C}$ and $\langle \Delta_R \rangle$ for the different patterns discussed in the previous section so that a lower bound on $\tau_{mn}$ becomes extractable. To that end, we need the survival hypothesis (sh) for Higgs scalars. This hypothesis was first introduced (Georgi 1979; Barbieri and Nanopoulos 1980) for fermions in a GUT to eliminate the disease of unnatural adjustment of parameters (in addition to that required to maintain gauge hierarchy) in understanding fermion masses. A clear review has been given by Langacker (1981), so we need merely state it in the most general form. Given the chain

$$G_{M_U} \rightarrow G_{M_1} \rightarrow G_{M_2} \rightarrow \cdots \rightarrow G_{M_r} \rightarrow G_{M_{r+1}} \rightarrow \cdots$$

(12)

where $G \supseteq G_1 \supseteq G_2 \supseteq \cdots \supseteq G_r$, $G_{\text{std.}} \supseteq G_{\text{exact}}$, and $M_U \geq M_1 \geq M_2 \cdots \geq M_r \geq M_{r+1} \cdots M_W$, any fermion mass term that is invariant under $G_{r+1}, \ldots$, $G_{\text{std.}}$ (but not under $G_r, G_{r+1}, \ldots, G_r$) has a corresponding mass of order $M_r$.

Despite the usefulness of the above SH in studying fermion masses in GUTs, it is of little consequence with respect to Higgs masses, so one needs an SH for Higgs scalars or else once again unnatural fine tuning of parameters becomes obligatory—this time in the Higgs sector. There have to be some differences (del Aguila and Ibañez 1981) though, because a Higgs scalar participating in a symmetry breakdown step can have a mass of the order of the scale of that breakdown. The SH for Higgs scalars can be stated generally in the following form (Mohapatra and Popović 1981; Raychaudhuri and Sarkar 1982). Return to the chain of equation (12) and concentrate on the step $G_r \rightarrow G_{r+1}$. Let the Higgs scalar $H_r (\langle H_r \rangle \neq 0)$ responsible for this breaking be a member of the irreducible representation $R_r$ of the group $G_r$ (Of course, $H_r$ is uncharged, colourless and a singlet under $G_{r+1}$). $R_r$ is contained in some irreducible representation $R_0$ of the grand unifying group $G$. Moreover,

$$R_0 \supseteq R_{r} \supseteq R_{r}^2 \cdots \supseteq R_{r}^{r-1} \supseteq R_r,$$
Here $R_j^i (j < r)$ is an irreducible representation of the intermediate symmetry $G_j (\supseteq G_r)$ which contains the representation $R_r$ of $G_r$. Now $(i)$ all members of $R_r$ have to acquire masses of the order of $M_r$; $(ii)$ all Higgs scalars contained in $R_j^i$ but not in $R_{j+1}^i$ have to acquire masses of the order of the scale $M_j$ at which $G_j$ breaks into $G_{j+1}$ these scalars form complete irreducible representations of $G_{j+1}$.

Let us consider the implications of this hypothesis vis-a-vis the $\{126\}$ Higgs multiplet for the descent patterns of our interest. In category (B) the full irreducible multiplet $[10, 1, 3]$ of $G_{PS}$ plays the pivotal role. Of that the submultiplet $(6, \frac{3}{2}, 1, 3)$ under $SU(3)C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$ does not acquire any vev while $(1, 2, 1, 3)$ does. The latter vev is of order $M_{B-L}$ which is what sets the scale for the masses of the three fields ($\Delta_R^a$). In contrast, all members of the former submultiplet—i.e. ($\Delta_R^C_{ij}$)—acquire masses of the order of the scale characterizing the first breakdown of $G_{PS}$. We can now examine the two specific channels:

**Channel (a)**

Here $M_{\Delta C} \sim M_C$—the mass-scale for the breaking $G_{PS} \supseteq SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$. Moreover, for the two solutions, with $g \sim 10^{-1}$ and using equations (10) and (3), we have

(a1) $M_{\Delta C} \sim 10^{13}$ GeV, $\langle \Delta \rangle \sim 10^7$ GeV, $\tau_{1\tilde{n}} \gtrsim 10^{39}$ years, \hspace{1cm} (13)

(a2) $M_{\Delta C} \sim 10^{10}$ GeV, $\langle \Delta \rangle \sim 10^9$ GeV, $\tau_{1\tilde{n}} \gtrsim 10^{25}$ years.

**Channel (b)**

Here $G_{PS}$ breaks at $M_R (\supseteq M_C)$ and sets the scale for $M_{\Delta C}$. Thus we have

(b1) $M_{\Delta C} \sim 10^{13}$ GeV, $\langle \Delta \rangle \sim 10^7$ GeV, $\tau_{1\tilde{n}} \gtrsim 10^{39}$ years; \hspace{1cm} (14)

(b2) $M_{\Delta C} \sim 10^{10}$ GeV, $\langle \Delta \rangle \sim 10^9$ GeV, $\tau_{1\tilde{n}} \gtrsim 10^{25}$ years.

Some brief remarks on category (C) (Rajpoot 1980) are called for. This chain is but a special case of channel (b) in category (B)—obtainable from the latter in the limit $M_R = M_U$. Rajpoot's study of category (C), in terms of the constraints from $\sin^2 \theta_W$ and $a_s$, led to the determination $M_U \sim 10^{18}$ GeV, $M_C \sim 10^{10}$ GeV and no constraints on $M_{B-L}$ except $M_{B-L} \ll M_C$ for this case. Clearly, $g \langle \Delta \rangle \sim M_{B-L}$ so that $\langle \Delta \rangle \lesssim 10^9$ GeV. Since $G_{PS}$ breaks at $M_U$, the survival hypothesis dictates $M_{\Delta C}$ to be $\sim 10^{10}$ GeV and $\tau_{1\tilde{n}} \gtrsim 10^{67}$ years. Thus our cursory rejection of this category in § 3 was justified.

1Clearly, this argument applies equally to $\Delta_C^{126}_{ij}$ whose masses are of the same scale. This justifies our remarks on this point in § 2.
5. Concluding discussion

We really have a no-go result regarding the detectability of $n\tilde{n}$ oscillations in SO(10) GUTs. Of course, we have exclusively considered $n\tilde{n}$ oscillations in terms of the Mohapatra-Marshak mechanism of figure 1. Other possible mechanisms have also been suggested (Kuo and Love 1980; Deo 1981) for which the detailed analysis would have to be somewhat different. Nevertheless, the basic fact of the $n \to \tilde{n}$ transition amplitude being controlled roughly by something like $g$ $\langle \Delta \rangle$ $M_{\Delta C}^{\alpha}$ persists so that the imposition of the survival hypothesis in SO (10) inevitably pushes up $\tau_{n\tilde{n}}$.

In a sense our general result was anticipated by del Aguila and Ibañez (1981). However, their formulation of the survival hypothesis was incomplete. Further, they were not concerned a priori with $n\tilde{n}$ oscillations and had not focused on it. Nor had they considered in detail all possible chains of symmetry-descent (relevant to $n\tilde{n}$ oscillations) so as to be able to completely rule out the detection of such phenomena in SO(10) grand unification. In particular, under category (B), they had made a detailed examination only of channel (a). Our analysis of channel (b), as given here, has—to our knowledge—not appeared in the literature before.

Among the remaining popular simple group GUTs, a fair amount of work has been done on the maximal SU (16). Two types of descent have been investigated. (1) The chain SU (16) $\to$ SU (12) $\times$ SU (4)$_L \times$ U (1)$_{B-L}$ $\ldots$ SU (3)$_C \times$ SU (2)$_L \times$ SU (2)$_R \times$ U (1)$_{B-L}$ $\rightarrow$ $G_{\text{std}}$ has been studied quite extensively by Pati et al (1981), but the conclusion regarding the detectability of $n\tilde{n}$ oscillation is pessimistic (Mohapatra and Popović 1981). (2) The sequence SU (16) $\rightarrow$ SU (8) $\times$ SU (8) $\ldots$. $G_{PS}$ $\ldots$ $G_{\text{std}}$ has also been studied but once again the $n \to \tilde{n}$ transition is found to be strongly suppressed (Mohapatra and Popović 1981; Raychaudhuri and Sarkar 1982).

In $E_6$ grand unification most of the schemes studied so far involve the SU(5) route. In these scenarios detectable $n\tilde{n}$ oscillations are ruled out a priori. It has been suggested (Fukugita et al 1982) that, in the relatively unexplored chain

$$E_6 \rightarrow SU(6) \rightarrow SU(5) \times U(1) \ldots \rightarrow G_{\text{std}},$$

with a low mass-scale ($10^4$–$10^5$ GeV) for the extra U (1), such phenomena may be possible via the low mass-scale. However, the model has not been considered in detail. In particular, the implications of the st for Higgs scalars and of the contributions from those scalars to the evolutionary gauge coupling strengths have not been taken into account. It is not clear without a careful and complete analysis whether the claim of detectable $n\tilde{n}$ oscillation in this scenario will survive such accounting. Specifically, it seems to us that, since the scalars mediating the $n \to \tilde{n}$ transition carry SU (3)$_C$ colour, their masses by the st will be characteristic of the SU (6)—breaking scale; that being superheavy, $n\tilde{n}$ oscillations will be suppressed. Perhaps one should keep an open mind till a detailed treatment emerges.

Among semi-simple groups [SU(4)]$^4$ has been studied, but the conclusion in regard to $n\tilde{n}$ oscillations is pessimistic (Marshak et al 1980). Of course, a really large GUT

---

There could be two such masses $M$ and $\overline{M}$ and one could have $M^{-4} \overline{M}^{-4}$ instead of $M_{\alpha C}^{-4}$, but the survival hypothesis will force both $\overline{M}$ and $M$ to be superlarge.
—such as that based on SU(48)—may admit detectable \( n\bar{n} \) oscillations, but most smaller ones do not seem to do so. In particular, our conclusion is that such phenomena are not possible in \( \text{SO}(10) \) \textit{GUTs}. Betting on the success of the ongoing experiments would amount to taking a long shot.

Acknowledgements

We thank the \( n\bar{n} \)-oscillation speakers at the ICOBAN conference (11-14 January 1982, Tata Institute of Fundamental Research, Bombay) for encouragement. We have been informed that a similar analysis has been independently done by R N Mohapatra and G Senjanović.

\textbf{Appendix: Bounds on the intermediate mass scales}

The low energy predictions for the neutral current parameter \( \sin^2 \theta_W \) and the QCD fine structure constant \( \alpha_S \) can be calculated \textit{via} the Georgi-Quinn-Weinberg equations. The final results depend on the route of descent through the intermediate mass-scales. It turns out that the fermionic contributions drop out of these expressions. We use complex (rather than real) scalar fields so that each scalar contribution has an extra factor of 2. The scalars responsible for the symmetry breakings have been indicated in equations (8) and (9). The last step in the symmetry breaking (\( G_{\text{std}} \rightarrow G_{\text{exact}} \)) is driven by the Higgs fields \((1, 0, 2, 2) \subset [15, 2, 2] \subset \{126\} \) and \((1, -2, 3, 1) \subset [10, 3, 1] \subset \{126\} \). The effect of all these scalars, as determined by the \( \alpha_s \) and the fact that those heavier than a certain mass-scale decouple from evolutions in ranges below that scale, are included in the calculations. Our basic equations are of the form (all masses scaled by GeV).

\[
\sin^2 \theta_W = \frac{3}{8} + \frac{\alpha}{16\pi} \left[ a \ln M_U + b \ln M_{\tilde{C}} + c \ln M_R + d \ln M_{B-L} + e \ln M_L \right]
\]

\[
1 - \frac{8}{3} \frac{\alpha}{a_S} = \frac{\alpha}{2\pi} \left[ a' \ln M_U + b' \ln M_{\tilde{C}} + c' \ln M_R + d' \ln M_{B-L} + e' \ln M_L \right]
\]

(A.1)

\textit{Channel (a) (}\( M_{\tilde{C}} \gg M_R \)):

Here

\[
a = \frac{32}{3}, \quad b = -\frac{94}{3}, \quad c = -17, \quad d = 2, \quad e = \frac{317}{3};
\]

\[
a' = \frac{40}{3}, \quad b' = 6, \quad c' = \frac{17}{3}, \quad d' = -\frac{2}{3}, \quad e' = -\frac{73}{3}.
\]

There are two cases (Rizzo and Senjanović 1982; Parida and Raychaudhuri 1981):
\( n \bar{n} \) oscillations in SO(10)

(a1) \( M_{B-L} \sim 10^6 \text{ GeV} \) and \( \sin^2 \theta_W = 0.23 \). Using (A.1) and the inequality \( M_U \geq M_C \geq M_R \), we have

\[
13.7 + \frac{2}{75} \log \frac{M_{B-L}}{100} > \log M_R,
\]

\[
\log M_C > 12.9 + \frac{6}{113} \log \frac{M_{B-L}}{100}.
\] (A.2)

(a2) \( M_{B-L} \approx 100 \text{ GeV}, \sin^2 \theta_W \approx 0.27 \). We obtain a new lower bound on \( M_C \):

\[
13.7 + \frac{2}{75} \log \frac{M_{B-L}}{100} > \log M_C > 9.8,
\] (A.3)

while the bound on \( M_R \) is unaltered.

The results of (A.2) and (A.3) are presented in the text in equation (10).

Channel (b) \( (M_R \geq M_C) \):

Here

\[
a = \frac{32}{3}, \quad b = -6, \quad c = \frac{127}{3}, \quad d = 2, \quad e = \frac{107}{3},
\]

\[
a' = \frac{40}{3}, \quad b' = \frac{22}{3}, \quad c' = \frac{13}{3}, \quad d' = -\frac{2}{3}, \quad e' = -\frac{73}{3}.
\]

Now it is possible to set both lower and upper bounds on \( M_C \) in the two possible cases

(b1) Here

\[
13.7 + \frac{2}{75} \log \frac{M_{B-L}}{100} > \log M_C > 12.9 + \frac{6}{113} \log \frac{M_{B-L}}{100}
\] (A.4)

(b2) Now

\[
13 + \log 5 > \log M_C > 9.8.
\] (A.5)

The results have been used in eq. (11) of the text.
After the submission of our manuscript a related paper appeared, Lust D, Maseiro A and Roncadelli M (1982) Phys. Rev. (RC) D25 3096. Most of their conclusions tally with ours. However, we emphatically disagree with their last suggestion of “arranging” detectable $\sin^2\theta$ oscillations in SO (10) through the choice $M_{\Delta_R} \sim 10^6$ GeV. As discussed in our § 4, the latter would violate the $\sin^2\theta$ for Higgs scalars and would require highly unnatural and nonminimal fine tuning of parameters in the Higgs sector.

References

del Aguila F and Ibáñez L E 1981 Nucl. Phys. B177 60
Georgi H 1979 Nucl. Phys. B156 126
Marshak R E, Mohapatra R N and Riazuddin 1980 Proceedings, Muon Physics Workshop (TRIUMF)
Raychaudhuri A and Sarkar U 1982 Calcutta University Report No. CUPP/82-2, unpublished