IC/92/92
CUPP-92/3

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

MASSIVE MAJORANA NEUTRINOS IN PRE-BOUNCE SUPERNOVAE

Srubabati Goswami

Kamales Kar

and

Amitava Raychaudhuri

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

MIRAMARE-TRIESTE

IC/92/92
CUPP-92/3

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MASSIVE MAJORANA NEUTRINOS IN PRE-BOUNCE SUPERNOVAE

Srubabati Goswami
Department of Pure Physics, University of Calcutta,
92 Acharya Prafulla Chandra Road, Calcutta 700 009, India,

Kamales Kar
Saha Institute of Nuclear Physics
1/AF, Bidhannagar, Calcutta 700 064, India

and

Amitava Raychaudhuri *
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

The currently accepted models of supernova collapse rely on the standard electroweak theory and massless left-handed neutrinos. We consider the effect of massive right-handed Majorana neutrinos on this scenario. In order that they do not upset the agreement of the usual treatment with observation, we require that in the pre-bounce stage either (a) these neutrinos are trapped or (b) if they free stream they do not change the electron fraction to the extent that the explosion is prevented. From these constraints, we obtain upper and lower bounds on the right-handed interaction strengths as a function of the neutrino mass which can be translated to bounds on the right-handed gauge boson mass.

MIRAMARE – TRIESTE
June 1992

* Permanent address: Department of Pure Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India.

I INTRODUCTION

Energy transport by neutrinos plays a central role in the standard theories of stellar collapse leading to neutron star formation via supernova (SN) explosion. This has been confirmed by the neutrinos from the explosion of SN 1987A detected by the IMB and K-II facilities. From these events it is inferred that the total energy carried by the six species of neutrino-antineutrino ($\nu\bar{\nu}$) pairs amounts[1] to (2-3)$\times 10^{53}$ ergs. The total gravitational energy released in SN collapse is estimated[2] to be $\simeq$ (4-5)$\times 10^{53}$ ergs. The missing energy has been surmised to be carried away by exotic particles like the right-handed neutrino, the axion, supersymmetric particles etc. There are two ways in which this exotic energy drain can operate. For large interaction strengths coming down from infinity, the mean free path (mfp) remains less than the core radius so that the particle in question gets 'trapped' and is then subsequently emitted from a 'particle sphere' thereby cooling the core. In such cases, the luminosity increases with decreasing interaction strength. The constraint on the total energy emitted in these exotic particles ($\leq$ (2-3)$\times 10^{53}$ ergs), therefore sets a lower bound to the strength of interaction. On the other hand in the no interaction limit, the exotic particles will not be produced at all. If now the strength increases gradually, the flux of such particles grows, increasing the energy carried away by them. Thus, in this case the luminosity increases with increasing interaction strength. The constraint on the total emitted energy puts an upper bound to the strength of interaction in this regime[3].

The exotic particle on which we focus attention is the right-handed massive Majorana neutrino (ν_R) such as is present in the left-right symmetric $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ model of electroweak interactions. If there are indeed right-handed currents, such neutrinos will be produced in the core of the supernova *both prior to bounce and after bounce*. So far the discussion in the literature has remained confined to the post-bounce trapping or free streaming of exotic particles. If missing energy is carried away by such particles, one must also look into the implications of their presence during the infall stage. In this paper we examine this aspect of the problem.

In section II we summarise the role of left-handed neutrinos in the standard supernova collapse scenario. In the following section we present the essential facts of the left-right symmetric model. In section IV we develop the formalism in detail to make the paper self-contained and present the results in section V. We end in section VI with some discussions. We find that the presence of right-handed neutrinos will be consistent for two ranges of the interaction strength. For large interaction strengths the ν_R will be 'trapped' during collapse in consistency with the adiabatic description. Two constraints have been used in section IV to describe effective trapping of ν_Rs. (i) The mfp, which decreases as the interaction strength increases, must be shorter than the core radius. (ii) The ν_R diffusion time, which increases as the interaction strength increases, must be larger than the hydrodynamic timescale of collapse. These lead to a lower bound on the strength of interaction. At the opposite extreme, the strength of interaction can be zero implying a total absence of ν_Rs. As the interaction grows ν_Rs will get produced and will free stream, decreasing the electron fraction, Y_e. A stronger interaction leads to a smaller Y_e. But if Y_e is too low, the shock, which depends sensitively on it, will not have enough energy to cause the explosion[4,5]. This puts an upper bound on the interaction strength.

A fairly standard practice in the literature is to consider in this connection exotic particles of zero mass. The effect of massive Majorana ν_R's has not been considered at all. On the other hand, most left-right symmetric models can lead to right-handed Majorana neutrinos which are massive. Furthermore, the claims of the observation of a 17 keV neutrino add extra focus to this issue. Of course, it is of no interest if the ν_R is so heavy that its production during collapse is kinematically forbidden. We have restricted ourselves in this work to Majorana ν_R of mass upto 10 MeV. We calculate in detail all the relevant cross-sections pertaining to the production, interaction and diffusion of such massive neutrinos during the infall stage. We use these to map out the allowed range of values in the two parameter space of relative strength and neutrino mass. Thus we are able to set lower and upper bounds on the strength of interaction for different neutrino masses. These limits are then translated to bounds on the mass of the right-handed charged gauge boson.

II LEFT-HANDED NEUTRINOS IN SUPERNOVAE

A Stellar Collapse

Massive stars with $8M_\odot \le M \le 60M_\odot$ in the final phase of their evolution go through gravitational collapse and eventually end with type-II supernova explosions. Once silicon burning stops, the stars develop central cores of mass $1.2 - 2M_\odot$ consisting of iron-type nuclei and a sea of relativistic degenerate electrons. Since Fe has the highest binding energy per nucleon, this signals the termination of thermonuclear burning in the core. At this stage, the outward pressure ceases to have a part coming from radiation and the pressure of the relativistic electron gas alone can no longer counterbalance the inward gravitational pressure. The collapse is triggered off by the photodissociation of Fe-nuclei and/or electron capture which reduces the electron gas pressure. As the collapse proceeds the core density rises, causing an increase in the electron chemical potential. Subsequently the electron Fermi energy becomes higher than the capture threshold and facilitates electron capture by nuclei and free protons leading to the neutronization of the core. This further reduces the e^- pressure thereby accelerating the collapse. The infalling material is halted and bounces back when the core density becomes of the order of supranuclear densities, at which point the subsonic inner core consists of nuclear matter. The collision of this stiff rebounding inner core with the infalling supersonic outer core results in the propagation of a shock wave into the mantle. In the shock heated regions, thermal processes produce $\nu\bar{\nu}$ pairs. The inner core develops into the 'proto-neutron star' while the shock is believed to cause ejection of

the matter leading to the supernova fireworks, which herald the birth of the neutron star[5].

B Production, Opacity and Transport of ν_L

During the infall stage, mainly electon neutrinos are produced. Their dominant production modes are the 'neutronization' reactions (which, in the standard model, yield left-handed neutrinos):

$$\text{(i) } e^- + (Z,A) \rightarrow \nu_L + (Z\text{-}1,A)$$

$$\text{(ii) } e^- + p \rightarrow \nu_L + n$$

As positrons are much fewer, the corresponding antineutrinos cannot be produced via similar reactions. Also, since the μ and τ leptons present are negligible in number, charged current interactions leading to the production of ν_μ and ν_τ can be neglected. The thermal processes that yield $\nu\bar{\nu}$ pairs of all flavours are largely suppressed while the infall proceeds, since the temperature is not high enough.

The cross-section for e^- capture on free protons is given by

$$\sigma = 4.5 \times 10^{-44} (E_\nu / 1\text{MeV})^2 \text{ cm}^2 \tag{1}$$

The average energy of the produced neutrinos is given by, $\overline{E}_\nu = \frac{5}{6}\,\mu_e$. For e^- capture by nuclei, one uses capture rates including allowed Fermi and Gamow-Teller transitions and considers the forbidden transitions when these get blocked[4,6,7].

As discussed in section II.A the collapse depends sensitively on electron capture rates. The relevant parameter is the electron to baryon ratio Y_e, which decreases as neutronisation proceeds. The rate of change of Y_e is given by Bethe *et al.*[8] (referred to as BBAL hereafter),

$$\frac{dY_e}{dt} = -X \rho Y_e{}^2 N_o c \widetilde{\sigma}_o \tag{2}$$

where, X is the energy averaged proportion of the total number of protons and electrons participating in the neutronisation reaction. ρ is the density of the stellar material in gm/cc. N_o is the Avogadro number. $\widetilde{\sigma}_o = \sigma_o\,(\,c_V^2 + 3\,c_A^2\,)/8 = 1.2 \times 10^{-44}$ cm^2. We have taken c_V= 1 and c_A= 1.25, where c_V and c_A are the vector and axial vector couplings respectively at the $n - p - W_L$ vertex. Here and in the rest of the paper $\sigma_o = (4G_F^2 m_e^2 \hbar^2)/(\pi c^2) = 1.7 \times 10^{-44}$ in cm^2. The decrease of Y_e stops at the onset of ν_L trapping which brings about β equilibrium.

The main neutrino opacity sources leading to ν_L trapping are coherent scattering by nuclei[9], and scattering by free neutrons and to a lesser extent by free protons via neutral current interactions. A detailed calculation of the above cross-sections as well as neutrino-nucleon absorption and neutrino-electron scattering at the high temperature and densities relevant to stellar collapse was done by Tubbs and Schramm[10].

Subsequently Lamb and Pethick[11] considering neutrino neutron and neutrino nucleus scattering, obtained a neutrino mean free path λ,

$$\lambda \ = \ 1.0 \times 10^{6} \rho_{12}{}^{-1} \left(\frac{1}{12} X_h \bar{A} \ + \ X_n \right)^{-1} \left(\frac{E_\nu}{10 \text{ MeV}} \right)^{-2} \text{cm.} \tag{3}$$

where, X_h and X_n are the fractions by mass of heavy nuclei and neutrons. ρ_{12} is the density of stellar material in 10^{12} gm/cc. E_ν is in MeV.

The transport of neutrinos outwards has been considered using different detailed schemes[12,7]. Widely used is the 'multigroup flux limited diffusion scheme', which solves the neutrino transport problem by numerical methods. For the purpose of setting a bound on right-handed interactions, we follow instead the semianalytic approach of BBAL. They treated the transport of neutrinos in a diffusion approximation, using the detailed multizone hydrodynamical computations of Arnett[13]. The neutrino diffusion equation used by BBAL is,

$$\frac{\partial n_\nu}{\partial t} \ = \ \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{1}{3} c \lambda_\nu \frac{\partial n_\nu}{\partial r} \right) \tag{4}$$

Using this equation BBAL showed that the neutrinos diffuse out of the material in about $\frac{1}{9}$ sec. This is much larger than the hydrodynamic time scale of collapse which is of the order of milliseconds. This indicates that the ν_L's are effectively trapped within the core prior to bounce.

III THE LEFT-RIGHT SYMMETRIC MODEL

A natural extension of the standard model which incorporates massive right-handed neutrinos is the left-right symmetric model[14] based on the gauge group $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$. The right-handed neutrinos interact by charged and neutral currents coupled to the extra gauge bosons of this model, W_R and Z' respectively. The charged current Lagrangian involving W_R is given by

$$\mathcal{L}_{c.c.} \ = \ \frac{g}{\sqrt{2}} \left(\bar{u}_R \gamma_\mu d_R \ + \ \bar{\nu}_R \gamma_\mu e_R \right) W_R^\mu + \text{h.c.} \tag{5}$$

where $g_L = g_R = g$ is the common coupling constant for $SU(2)_R$ and $SU(2)_L$. We define G_R by

$$\frac{G_R}{\sqrt{2}} \ = \ \frac{g^2}{8 M_{W_R}^2} \tag{6}$$

where, M_{W_R} is the mass of the W_R boson. It is natural to relate G_R with the Fermi coupling G_F by

$$G_R^2 \ = \ B \times G_F^2 \tag{7}$$

Thus B is a measure of the right-handed interaction relative to the left-handed one. In terms of the gauge boson masses,

$$B \ = \ \left(\frac{M_{W_L}}{M_{W_R}} \right)^4 \tag{8}$$

The extra neutral current Lagrangian of the left-right symmetric model needed for our purpose is,

$$\mathcal{L}_{n.c.} \ = \ \frac{g}{\sqrt{2}} \bar{f} \gamma_\mu \frac{1}{2} \left(\tilde{g}_V'^f \ + \ \tilde{g}_A'^f \gamma_5 \right) f \, Z'^\mu \tag{9}$$

where,

$$\tilde{g}_V'^f \ = \ \sqrt{\frac{2}{\cos 2\theta_W}} \frac{1}{\cos \theta_W} \left(T_{3L}^f \sin^2 \theta_W \ + \ T_{3R}^f \cos^2 \theta_W \ - \ 2 Q_f \sin^2 \theta_W \right) \tag{10}$$

$$\tilde{g}_A'^f \ = \ \sqrt{\frac{2}{\cos 2\theta_W}} \frac{1}{\cos \theta_W} \left(-T_{3L}^f \sin^2 \theta_W \ + \ T_{3R}^f \cos^2 \theta_W \right) \tag{11}$$

The W_R and Z satisfy the approximate mass relation[15],

$$M_{W_R}^2 \ = (M_{Z'}{}^2 \cos 2\theta_W)/2\cos^2\theta_W$$

To facilitate our presentation we later write the neutral and charged current matrix elements with the same coefficient $G_R/\sqrt{2}$. Due to the above mass relationship, this entails an extra factor of $\cos 2\theta_W/(2 \ \cos^2 \theta_W)$ for the neutral current matrix elements. We subsume this factor in the coupling constants by defining:

$$g_V'^f \ = \ \frac{1}{\cos^2\theta_W} \left(T_{3L}^f \sin^2 \theta_W \ + \ T_{3R}^f \cos^2 \theta_W \ - \ 2 Q_f \sin^2 \theta_W \right) \tag{12}$$

$$g_A'^f \ = \ \frac{1}{\cos^2\theta_W} \left(-T_{3L}^f \sin^2 \theta_W \ + \ T_{3R}^f \cos^2 \theta_W \right) \tag{13}$$

$g_V'^f$ and $g_A'^f$ for different fermions are summarised in Table 1. For the neutron and the proton they have been calculated assuming a spectator quark model.

IV RIGHT HANDED NEUTRINOS IN THE PRE-BOUNCE STAGE

A Production

In the left-right symmetric model, right-handed neutrinos can be produced by electron capture on nuclei and free protons. In a simplified analysis the effect of nuclear protons can be taken into account by considering a Fermi-gas model of the nucleus. In this treatment the cross-section for e^- capture on a free proton is appropriately modified by taking a Fermi distribution for the protons to incorporate the effect of the nucleus.

In evaluating the cross-section of the processes which we will encounter, the fact that the initial and final particles have a distribution in energy and the role of Fermi blocking need to be kept in mind. This has the effect of modifying the initial and final momentum space densities as,

$$dn(p_i) = [d^3p_i/(2\pi)^3]\, f(E_i)$$

$$dn(p_f) = [d^3p_f/(2\pi)^3]\, (\, 1 - f(E_f)\,)$$

where $f(E)$ denotes the Fermi distribution function. For initial state electrons, an extra factor of 2 has to be included to take spin degeneracy into account. For initial state neutrinos of a fixed helicity this factor is absent. For low particle densities the fermion distribution function $f(E) \ll 1$. On the other hand if the particle density is high, the fermions are degenerate and for such extreme degeneracy,

$$f(E) = \begin{cases} 1 & \text{if } E \leq E_F \\ 0 & \text{if } E > E_F \end{cases}$$

where E_F is the Fermi energy.

The process $e^- p \to \nu_R n$ has the following matrix element :

$$M = \frac{G_R}{\sqrt{2}}\, \bar{u}(p_{\nu_e})\, \gamma_\mu (1+\gamma_5)\, u(p_e)\, \bar{u}(p_n)\, \gamma^\mu (c'_V + c'_A \gamma_5)\, u(p_p) \tag{14}$$

where c'_V and c'_A are the generalisations of the usual charge current couplings to the right-handed interaction. From (14)

$$\begin{aligned} \overline{|M|^2} = {} & 8\, G_R^2\, \{ (c'_V + c'_A)^2\, (p_{\nu_e}.p_n)\, (p_e.p_p) + (c'_V - c'_A)^2\, (p_{\nu_e}.p_p)\, (p_e.p_n) \\ & - m_n m_p\, (c'^2_V - c'^2_A)\, (p_{\nu_e}.p_e)\, \} \end{aligned} \tag{15}$$

Here and in the rest of the paper $\overline{|M|^2}$ denotes the squared matrix element averaged over initial spins and summed over final spins.

Using eq.(15) and assuming low densities of neutrinos and neutrons at the point of their production, we get the cross-section for the above process in the rest frame of the initial proton, as,

$$\sigma = \frac{G_R^2}{2\pi}\, E_e^2\, (\, 1 - \frac{W}{E_e}\,)\, \sqrt{(\, 1 - \frac{2W}{E_e} + \frac{W^2 - m_\nu^2}{E_e^2}\,)}\, (\, c'^2_V + 3c'^2_A\,) \tag{16}$$

where $W = m_n - m_p$, throughout the paper. E_e denotes the energy of the electron in the rest frame of the initial proton. The assumptions involved in obtaining eq.(16) are:

(i) $m_p,\, m_n, \gg W,\, E_e,\, E_\nu,\, m_\nu$

(ii) The mass of the electron is neglected.

The average energy of such massive neutrinos is given by,

$$\overline{E_\nu} = \frac{\int_{W+m_\nu}^{\infty} E_\nu\, \sigma(\, e_R^- p \to \nu n\,)\, f(\, E_e\,)\, g(\, E_e\,)\, dE_e}{\int_{W+m_\nu}^{\infty} \sigma(\, e_R^- p \to \nu n\,)\, f(\, E_e\,)\, g(\, E_e\,)\, dE_e} \tag{17}$$

where E_ν denotes the energy of the neutrino in the rest frame of the initial proton. Under the approximations discussed above it can be expressed in terms of the electron energy as:

$$E_\nu \approx E_e \left(1 - \frac{W}{E_e} \right) \tag{18}$$

$g(E_e)$ is the density of states of the initial electrons which goes as E_e^2. $f(E_e)$ is the distribution function of the initial electrons, which are completely degenerate. Substituting for the various factors eq.(17) then becomes,

$$\overline{E_\nu} = \frac{\int_{W+m_\nu}^{\mu_e} E_e^5\, (\, 1 - \frac{W}{E_e})^2 \sqrt{(\, 1 - \frac{2W}{E_e} + \frac{W^2 - m_\nu^2}{E_e^2}\,)}\, dE_e}{\int_{W+m_\nu}^{\mu_e} E_e^4\, (\, 1 - \frac{W}{E_e}) \sqrt{(\, 1 - \frac{2W}{E_e} + \frac{W^2 - m_\nu^2}{E_e^2}\,)}\, dE_e} \tag{19}$$

Keeping terms upto order (W^2/E_e^2) and (m_ν^2/E_e^2) the above simplifies to,

$$\overline{E_\nu} = \frac{5}{6}\, \mu_e \left[1 - \frac{11}{10}\frac{W}{\mu_e} + \frac{1}{12}\frac{W^2}{\mu_e^2} + \frac{1}{12}\frac{m_\nu^2}{\mu_e^2} \right] \tag{20}$$

which reduces to $(5/6)\, \mu_e$, the standard result in the limit $m_\nu \to 0$, $W \to 0$.

B Depletion of electron fraction

To find the rate of decrease of Y_e in the presence of right-handed neutrinos one must extend eq.(2) to,

$$\frac{dY_e}{dt} = -\, \rho\, Y_e^{\,2}\, N_o\, c\, \tilde{\sigma}_o\, (\, X^{\nu_L} + BX^{\nu_R}\,) \tag{21}$$

In the above, we have taken $c'_V = 1$ and $c'_A = 1.25$. X^{ν_L} and X^{ν_R} are the energy averaged proportions of the total number of protons and electrons which take part in the production of ν_L and ν_R respectively and, in general,

$$\tilde{\sigma}_o X = \int\int\int \frac{dQ}{F}\, \overline{|M|^2}\, g(\epsilon_p)\, d\epsilon_p\, g(\epsilon_e)\, d\epsilon_e \qquad (22)$$

where F is the initial flux and dQ is the phase space factor for the final state. $g(\epsilon_p)\, d\epsilon_p$ is the fraction of the total number of protons which participate in the reaction. $g(\epsilon_e)\, d\epsilon_e$ is the corresponding quantity for the electrons. Substituting for $\overline{|M|^2}$ from eq.(15), putting in the other factors, and performing the integration over the neutron 3-momentum one gets,

$$X^{\nu_R} = \frac{1}{m_e^2}\int p_\nu E_\nu dE_\nu \int \frac{d\epsilon_p}{k_F{}^2/3m_p}\int \frac{3E_e^2}{\mu_e^3} dE_e\, \delta(\, E_\nu + E_n - E_e - E_p\,) \qquad (23)$$

Here ϵ_p denotes the kinetic energy of the proton, considered as nonrelativistic. E denotes the total energy in each case. It should be noted that the electrons are ultra-relativistic so that $\epsilon_e \approx E_e$. Further, we used the approximation $m_p, m_n \gg E_e, E_\nu, m_\nu, m_p - m_n$. The δ function in eq.(23) is equivalent to the energy conservation relation,

$$E_e + \epsilon_p + m_p = \epsilon_n + m_n + E_\nu \qquad (24)$$

Following BBAL we now define,

$$E_e = \mu_e - \Delta_e; \quad \epsilon_p = \mu_p - \Delta_p; \quad \epsilon_n = \mu_n + \Delta_n; \qquad (25)$$

where μ's are the chemical potentials. Δ_n is the excitation energy of the daughter nucleus. In general, the Δ's are positive quantities. Then, the conservation of energy relation implies,

$$E_e = E_\nu + \Delta_p + Q \qquad (26)$$

where,

$$Q = \hat{\mu} + \Delta_n + W \qquad (27)$$

$\hat{\mu}$ denoting $\mu_n - \mu_p$. Now performing the integration over electron energy in eq. (23), one gets finally

$$X^{\nu_R} = \frac{3}{m_e^2\, (k_F^2/3m_p)\, \mu_e^3}\int_{m_\nu}^{\Delta} (\, E_\nu^2 - m_\nu^2\,)^{\frac{1}{2}} E_\nu dE_\nu \int_0^{\Delta - E_\nu} d\Delta_p\, (\, E_\nu + \Delta_p + W\,)^2 \qquad (28)$$

where,

$$\Delta = \Delta_e + \Delta_p + E_\nu = \mu_e - Q = \mu_e - \hat{\mu} - \Delta_n - W \qquad (29)$$

Retaining only terms upto $(m_\nu{}^2/E_\nu^2)$ the above simplifies to,

$$X^{\nu_R} = \frac{1}{m_e^2(k_F^2/3m_p)\mu_e}\frac{1}{6}(\, \mu_e - Q\,)^4 \left[\left(1 + \frac{2}{5}\frac{Q}{\mu_e} + \frac{1}{10}\frac{Q^2}{\mu_e^2}\right) - \frac{9}{4}\frac{m_\nu{}^2\mu_e^2}{(\mu_e - Q)^4}\left(1 - \frac{28}{3}\frac{Q}{\mu_e} + 20\frac{Q^2}{\mu_e^2} - 16\frac{Q^3}{\mu_e^3} + \frac{13}{8}\frac{Q^4}{\mu_e^4}\right)\right] \qquad (30)$$

In the limit $m_\nu \rightarrow 0$ this reduces to the known result [BBAL]:

$$X^{\nu_L} = \frac{1}{m_e^2(k_F^2/3m_p)\mu_e}\frac{1}{6}(\, \mu_e - Q\,)^4 \left[1 + \frac{2}{5}\frac{Q}{\mu_e} + \frac{1}{10}\frac{Q^2}{\mu_e^2}\right] \qquad (31)$$

Now, for homologous collapse, during which the density structure of the collapsing core remains self-similar, the change in density with time is governed by the following equation:

$$\frac{d(ln\rho)}{dt} = 224\, (\rho_{10}F)^{\frac{1}{2}} \qquad (32)$$

ρ_{10} is the density of stellar material in units of 10^{10} gm/cc. F is the fraction of the electron degeneracy pressure disappearing with neutronisation. Following BBAL we take an average $F = 0.2$. Then the change of electron fraction with density is given by,

$$\frac{dY_e}{d(\ln\rho)} = -\frac{5.33\times 10^{-4}}{c_f}(\rho_{10})^{\frac{1}{2}}Y_e{}^2\frac{\Delta^4}{\mu_e}\left[\left(1 + \frac{2}{5}\frac{Q}{\mu_e} + \frac{1}{10}\frac{Q^2}{\mu_e^2}\right)(1 + B) - \frac{9}{4}\frac{m_\nu{}^2\mu_e^2}{\Delta^4}B\left(1 - \frac{28}{3}\frac{Q}{\mu_e} + 20\frac{Q^2}{\mu_e^2} - 16\frac{Q^3}{\mu_e^3} + \frac{13}{8}\frac{Q^4}{\mu_e^4}\right)\right] \qquad (33)$$

Putting $B = 0$, the above yields the equation governing the change of Y_e with ρ for ν_L's alone. Solving this, BBAL obtained values of Y_e at different densities. In particular, at the density where ν_L trapping sets in, they obtained Y_e=0.32. It was later shown[4,6,7] that for nuclei with neutron number greater than 40, the final state neutron shell is full, preventing the allowed Gamow-Teller transition to that state, leading to the phenomenon of neutron shell blocking. This causes e^- capture to proceed via forbidden transitions and on free protons, with consequent decrease in the rate of neutronisation. As a result the final electron fraction in the region of trapping rises. The scaling factor c_f in eq.(33) is introduced to take into account this decrease in the neutronisation rate. The calculation of BBAL with no shell blocking corresponds to $c_f = 1$. Now, Y_e is related to $\hat{\mu}$ and μ_e by[8]

$$\hat{\mu} = 250(0.5 - Y_e) - 50Y_e(1 - Y_e)^{\frac{1}{3}}\, (3 - 5Y_e)\text{MeV} \qquad (34)$$

and,

$$\mu_e = 11.1\, (\rho_{10}Y_e)^{\frac{1}{3}}\text{MeV} \qquad (35)$$

so that eq.(33) can be converted to

$$\frac{d\hat{\mu}}{d\mu_e} = \frac{0.4}{c_f}\, (\rho_{10})^{\frac{1}{2}}Y_e{}^2\frac{\Delta^4}{\mu_e^2}\left[\left(1 + \frac{2}{5}\frac{Q}{\mu_e} + \frac{1}{10}\frac{Q^2}{\mu_e^2}\right)(1 + B) - \frac{9}{4}\frac{m_\nu{}^2\mu_e^2}{\Delta^4}\, B\left(1 - \frac{28}{3}\frac{Q}{\mu_e} + 20\frac{Q^2}{\mu_e^2} - 16\frac{Q^3}{\mu_e^3} + \frac{13}{8}\frac{Q^4}{\mu_e^4}\right)\right] \qquad (36)$$

With rising density, unless Δ/μ_e decreases $(d\hat{\mu}/d\mu_e)$ will increase rapidly with density making Δ, the maximum neutrino energy, negative - see eq.(29). Thus for Δ to remain positive, Δ/μ_e must decrease, leading to $(d\hat{\mu}/d\mu_e) \rightarrow 1$ for large densities.

C Opacity sources and mean free path

The main contribution to ν_R opacities comes from the processes

(i) Free nucleon scattering

$$\nu_R + n \xrightarrow{Z'} \nu_R + n$$

$$\nu_R + p \xrightarrow{Z'} \nu_R + p$$

(ii) Coherent scattering by nuclei,

$$\nu_R + (Z,A) \xrightarrow{Z'} \nu_R + (Z,A)$$

These two opacity sources exist only by virtue of the weak neutral currents mediated by the extra Z′ boson of the left-right symmetric models, since the standard model Z does not couple to ν_R. These processes are essentially elastic. Further, there are the inelastic processes,

(iii) Electron-neutrino scattering,

$$e^- + \nu_R \xrightarrow{W_R/Z'} e^- + \nu_R$$

which can be mediated both by the extra W_R and Z' bosons.

(iv) Charged current nucleon absorption,

$$\nu_R + n \xrightarrow{W_R} p + e^-$$

The process (iv) is, however, blocked for low energy neutrinos ($E_\nu \ll \mu_e$) by an inhibition factor (due to electron degeneracy in the final state), which is $\exp(-\mu_e/kT)$ for $E_\nu \ll \mu_e$. Since for $E_\nu \gg \mu_e$ this inhibition factor is unity, high energy neutrinos can undergo absorption, thereby rendering the process important in neutrino thermalisation. As for ν-e scattering, since the electrons are extremely degenerate, they cannot loose energy and it is the neutrinos which must downscatter. However, with the build up of neutrino degeneracy, such scattering is reduced drastically except for the highest energy neutrinos. So ν-e scattering is not included in the consideration of ν-opacities. If such a collision occurs, the neutrinos and not the electrons will downscatter, thereby loosing energy, because it is the electrons which have initially the higher degree of degeneracy. Thus, this process is also important in thermalising the energy of the neutrinos. In contrast, as pointed out by Lamb and Pethick[11], since neutrino nucleon and neutrino nucleus coherent scattering are elastic, ν degeneracy has little effect on these processes.

To obtain an expression for the mfp of ν_R's one has to calculate the probabilities of various processes contributing to opacity. We use the following expression for the collision probability[11]

$$\frac{1}{\tau} = \frac{n}{(2\pi)^2}\int \frac{d^3p_3}{2E_3}\frac{d^3p_4}{2E_4}\delta^4(p_3+p_4-p_1-p_2)\overline{|M|^2}\frac{(1-\widehat{p}_1.\widehat{p}_3)}{4E_1E_2} \tag{37}$$

where n is the number density of scattering particles. In this section and for subsequent scattering processes p_1, p_3 denote the four momenta of the initial and final neutrino, while, p_2, p_4 denote those of the scatterer. All heavy particles are assumed to be nondegenerate. The total collision probability is given by,

$$\tau_{tot}^{-1} = \tau_{coh}^{-1} + \tau_n^{-1} + \tau_p^{-1} \tag{38}$$

where τ_{coh}^{-1} is the collision probability for neutrino nucleus coherent scattering and τ_n^{-1} and τ_p^{-1} are the collision probabilities with free neutron and proton respectively.

(i) Neutrino nucleon scattering

The matrix element for neutrino nucleon scattering is given by,

$$M = \frac{G_R}{2\sqrt{2}}\,\bar{u}(p_3)\,\gamma_\mu\,(1+\gamma_5)\,u(p_1)\,\bar{u}(p_4)\,\gamma^\mu\,(g_V'^{Nn} + g_A'^{Nn}\gamma_5)\,u(p_2) \tag{39}$$

Here, Nn denotes a nucleon (either a proton or neutron). $g_V'^{Nn}$ and $g_A'^{Nn}$ are the vector and axial vector couplings respectively of the nucleons to Z′. For neutrinos $g_V' = g_A' = \frac{1}{2}$. Then,

$$\begin{aligned}\overline{|M|^2} = {} & 4\,G_R^2\,[\,(g_V'^{Nn}+g_A'^{Nn})^2\,(p_1.p_2)\,(p_3.p_4) + (g_V'^{Nn}-g_A'^{Nn})^2\,(p_3.p_2)\,(p_1.p_4) \\ & - m_{Nn}^2\,\{(g_V'^{Nn})^2-(g_A'^{Nn})^2\}\,(p_1.p_3)\,]\end{aligned} \tag{40}$$

In the rest frame of the initial nucleon eq.(40) becomes,

$$\begin{aligned}\overline{|M|^2} = {} & 4\,G_R^2\,[\,(g_V'^{Nn}+g_A'^{Nn})^2\,m_{Nn}^2\,E_\nu^2 + (g_V'^{Nn}-g_A'^{Nn})^2\,m_{Nn}^2\,E_\nu^2 - \\ & m_{Nn}^2\,\{(g_V'^{Nn})^2-(g_A'^{Nn})^2\}\,\{\,E_\nu^2\,(1-\widehat{p}_1.\widehat{p}_3) + m_\nu^2\,(\widehat{p}_1.\widehat{p}_3)\,\}\,]\end{aligned} \tag{41}$$

Here m_{Nn} is the nucleon mass. The assuumption in obtaining eq.(41) from eq.(40) is to treat the nucleons as nonrelativistic, which leads to $E_1 \simeq E_3 = E_\nu$ and $E_2 \simeq E_4 = m_{Nn}$ i.e. the nucleons are assumed to take up negligible energy although they participate in momentum transfer.

Using eq.(37) the collision probability is then,

$$\begin{aligned}\frac{1}{\tau_{Nn}} = {} & \frac{1}{24}\,B\,n_{Nn}\,\sigma_o\,c\left(\frac{E_\nu}{m_e}\right)^2 [\,5\,(g_A'^{Nn})^2 + (g_V'^{Nn})^2 + \\ & \frac{m_\nu^2}{2E_\nu^2}\,\{(g_V'^{Nn})^2 - (g_A'^{Nn})^2\}]\,\sqrt{1-\frac{m_\nu^2}{E_\nu^2}} \quad \text{in sec}^{-1}\end{aligned} \tag{42}$$

where, σ_o has been defined after eq.(2) and m_e, m_ν and E_ν in eq.(42) are in MeV.

(ii) Coherent scattering off nucleus

For coherent scattering with spinless nuclei the axial vector coupling is zero. For such processes,

$$\overline{|M|^2} = 4 G_R^2 (g_V'^{Nu})^2 m_A^2 E_\nu^2 \left[1 + \left(1 + \frac{m_\nu^2}{E_\nu^2}\right)(\widehat{p}_1 . \widehat{p}_3)\right] \quad (43)$$

where m_A is the mass of the nucleus and $g_V'^{Nu}$ is given by,

$$g_V'^{Nu} = Z g_V'^{p} + N g_V'^{n}$$

where $g_V'^{p}$ and $g_V'^{n}$ are the vector couplings of the proton and the neutron respectively. Z and N denote the number of protons and neutrons respectively. Then, using eq. (12) and the overall charge neutrality of stellar matter one can express $g_V'^{Nu}$ as,

$$g_V'^{Nu} = \frac{A}{\cos^2\theta_W} [-\frac{1}{2} (1 - Y_e) + (\frac{1}{2} - 2 \sin^2\theta_W) Y_e] \quad (44)$$

Recall that Y_e denotes the electron to baryon ratio. Then,

$$\frac{1}{\tau_{coh}} = \frac{1}{24} B (\sum_{A>1} n_A A^2) \sigma_o c \left(\frac{E_\nu}{m_e}\right)^2 (\widehat{g}_V'^{Nu})^2 \left(1 + \frac{m_\nu^2}{2E_\nu^2}\right) \sqrt{1 - \frac{m_\nu^2}{E_\nu^2}} \quad \text{in sec}^{-1} \quad (45)$$

where, $\widehat{g}_V'^{Nu}$ is $g_V'^{Nu}/A$. n_A is the number density of nuclei with atomic mass number A.

(iii) Total collision probability

From eq.(38) we obtain the total collision probability as,

$$\begin{aligned} \tau_{tot}^{-1} = & \frac{1}{24} B \sigma_o c \left(\frac{E_\nu}{m_e}\right)^2 \sqrt{1 - \frac{m_\nu^2}{E_\nu^2}} \\ & \Big[(\sum_{A>1} n_A A^2)(\widehat{g}_V'^{Nu})^2 \left(1 + \frac{m_\nu^2}{2E_\nu^2}\right) \\ & + n_n \{ 5 (g_A'^{n})^2 + (g_V'^{n})^2 + \frac{m_\nu^2}{2E_\nu^2} [(g_V'^{n})^2 - (g_A'^{n})^2] \} \\ & + n_p \{ 5 (g_A'^{p})^2 + (g_V'^{p})^2 + \frac{m_\nu^2}{2E_\nu^2} [(g_V'^{p})^2 - (g_A'^{p})^2] \} \Big] \quad \text{in sec}^{-1} \end{aligned} \quad (46)$$

where, n denotes a neutron and p a proton. However the contribution from neutrino proton scattering is negligible compared to the other two contributions and in the subsequent calculation we will neglect it. Then finally one gets,

$$\begin{aligned} \tau_{tot}^{-1} = & \frac{1}{24} B \sigma_o c \frac{\rho}{m_n} \left(\frac{E_\nu}{m_e}\right)^2 \Big[X_h \bar{A} \widehat{g}_V'^{Nu^2} \left(1 + \frac{m_\nu^2}{2E_\nu^2}\right) + \\ & X_n \left\{ 5 (g_A'^{n})^2 + (g_V'^{n})^2 + \frac{m_\nu^2}{2E_\nu^2} [(g_V'^{n})^2 - (g_A'^{n})^2] \right\} \Big] \end{aligned} \quad (47)$$

where ρ is the density of the stellar material, X_h and X_n are the mass fractions of nuclei and free neutrons respectively.

$$\bar{A} \equiv \frac{\sum_{A>1} X_A A}{\sum_{A>1} X_A} \quad ; \quad \sum_{A>1} X_A = X_h \ .$$

The mfp, λ, is given by

$$\lambda = \text{velocity} \times \tau$$

For massive neutrinos the velocity is $c \ [1 - (m_\nu^2/E_\nu^2)]^{\frac{1}{2}}$. Therefore from eq.(47) the mfp of massive neutrinos is given by,

$$\begin{aligned} \lambda_\nu = & \frac{5.88 \times 10^8}{B} \rho_{12}^{-1} \Big[X_h \bar{A} (\widehat{g}_V'^{Nu})^2 \left(1 + \frac{m_\nu^2}{2E_\nu^2}\right) + \\ & X_n \{ 5 (g_A'^{n})^2 + (g_V'^{n})^2 + \frac{m_\nu^2}{2E_\nu^2} [(g_V'^{n})^2 - (g_A'^{n})^2] \} \Big]^{-1} E_\nu^{-2} \quad \text{in cm.} \end{aligned} \quad (48)$$

m_ν and E_ν are in MeV. The above can be written as,

$$\lambda_\nu = \lambda / B \quad (49)$$

The condition of trapping of right-handed neutrinos will then imply, $\lambda_\nu \leq$ the core radius, i.e. in terms of B,

$$B \geq \lambda/(\text{core radius}) \quad (50)$$

D Diffusion

In the case of massive neutrinos the diffusion equation (see eq.(4)) becomes,

$$\frac{\partial n_\nu}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{1}{3} c \left(1 - \frac{m_\nu^2}{E_\nu^2}\right)^{\frac{1}{2}} \lambda_\nu \frac{\partial n_\nu}{\partial r} \right] \quad (51)$$

where λ_ν is given by eq.(48). Now, for $m_\nu \neq 0$,

$$n_\nu \sim \widetilde{\mu}_\nu^3 \quad (52)$$

where,

$$\widetilde{\mu}_\nu \;=\; (\;\mu_\nu^2 \;-\; m_\nu^2)^{\frac{1}{2}} \tag{53}$$

Since effectively only the neutrinos near the top of the Fermi sea diffuse, E_ν can be replaced by μ_ν in eqs. (48) and (51). Neglecting, as in BBAL, the variations of ρ_{12} with r within a fixed zone, one obtains then the diffusion equation for massive neutrinos, in terms of $\widetilde{\mu}_\nu$ as,

$$\begin{aligned}\frac{\partial \widetilde{\mu}_\nu^3}{\partial t} \;=\; & \frac{5.88\times 10^8}{B}\,\rho_{12}^{-1}\,c\,\frac{1}{r^2}\frac{\partial}{\partial r}\left[\,r^2\,\left(1-\frac{m_\nu^2}{\widetilde{\mu}_\nu{}^2+m_\nu{}^2}\right)^{\frac{1}{2}}\right.\\ & \times\Big(X_h\overline{A}\,(\widetilde{g}_V'{}^{Nu})^2 \;+\; X_n\;\left[\,5\,(g_A'{}^n)^2 \;+\; (g_V'{}^n)^2\,\right] \;+\\ & \frac{m_\nu{}^2}{2(\widetilde{\mu}_\nu{}^2+m_\nu{}^2)}\;\left[\,X_h\overline{A}\,(\widetilde{g}_V'{}^{Nu})^2 \;+\; X_n\;\left(\,(g_V'{}^n)^2 \;-\; (g_A'{}^n)^2\,\right)\right]\Big)^{-1}\\ & \left.\times\left(\,1+\frac{m_\nu{}^2}{\widetilde{\mu}_\nu{}^2}\,\right)^{-1}\frac{\partial\widetilde{\mu}_\nu}{\partial r}\,\right]\end{aligned} \tag{54}$$

As we are interested only in the order of magnitude of the bounds on the right-handed interaction strengths we replace $\widetilde{\mu}_\nu^2$ in the correction terms by, $\langle\widetilde{\mu}_\nu^2\rangle$ where, $\langle\widetilde{\mu}_\nu^2\rangle$ denotes the value of $\widetilde{\mu}_\nu^2$ at a suitable average density. Setting $\langle\widetilde{\mu}_\nu^2\rangle \;=\; \overline{E}_\nu^2$ we then get

$$\widetilde{\mu}_\nu{}^2\frac{\partial\widetilde{\mu}_\nu}{\partial t} \;=\; \frac{1}{3}\lambda_o\overline{v}\;\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\widetilde{\mu}_\nu}{\partial r}\right) \tag{55}$$

where,

$$\begin{aligned}\lambda_o \;=\; & \frac{5.88\times 10^8}{B}\rho_{12}^{-1}\;\left[\;X_h\bar{A}\,(\widetilde{g}_V'{}^{Nu})^2\left(\;1+\frac{m_\nu^2}{2\overline{E}_\nu^2}\;\right)\;+\right.\\ & \left.X_n\;\left\{5\;(g_A'{}^n)^2+(g_V'{}^n)^2+\frac{m_\nu^2}{2\overline{E}_\nu^2}\;\left((g_V'{}^n)^2-(g_A'{}^n)^2\right)\right\}\right]^{-1}\;\left(1+\frac{m_\nu{}^2}{\overline{E}_\nu{}^2}\right)^{-1}\end{aligned} \tag{56}$$

with $\overline{E_\nu}$ given by eq.(17). $\overline{v}$ is given by,

$$\overline{v} \;=\; c\,\sqrt{1-(m_\nu{}^2/\overline{E}_\nu^2)} \tag{57}$$

Eq.(55) can be solved by the method of separation of variables by assuming,

$$\widetilde{\mu}_\nu \;=\; \widetilde{\mu}_o\tilde{\psi}(r)\tilde{\phi}(t) \tag{58}$$

This gives a solution for $\tilde{\phi}$ as,

$$\tilde{\phi} \;=\; (1-at)^{\frac{1}{2}} \tag{59}$$

and a Lane-Emden type equation for $\tilde{\psi}(r)$. $(-a/2)$ is the separation constant. The properties of the Lane-Emden function give,

$$a \;=\; \frac{2\lambda_o\overline{v}}{3\widetilde{\mu}_o{}^2}\;\left(\frac{6.9}{R}\right)^2 \tag{60}$$

R denotes the trapping radius at which $\tilde{\psi}(r)$ and hence $\widetilde{\mu}_\nu$ and therefore the neutrino density n_ν is zero at all times. $\widetilde{\mu}_o$ is related to the chemical potential at the centre i.e. at r=0, t=0 by $\widetilde{\mu}_o \;=\; (\mu_o^2 - m_\nu{}^2)^{\frac{1}{2}}$ and is determined from the relation

$$\widetilde{\mu}_o \;=\; \left(\frac{\sigma_o^{\nu_R}}{\sigma_o^{\nu_L}}\right)^{\frac{1}{3}}\mu_o^{\nu_L} \tag{61}$$

where $\sigma_o^{\nu_R}$ is the production cross-section of right-handed neutrinos and $\sigma_o^{\nu_L}$ denotes that for ν_L's. $\mu_o^{\nu_L}$ is 133 MeV [BBAL]. $\sigma_o^{\nu_R}$ is determined from eq.(16) with E_ν as the energy at production. For this we use eq.(20) with μ_e taken as its value at $r=0$. From eq.(59) it is seen that at $t \;=\; 1/a$, $\tilde{\phi}(t)=0$ i.e. $\widetilde{\mu}_\nu$ and hence $n_\nu = 0$. $1/a$ is the diffusion time, which is the time in which the neutrino density vanishes at all r. Thus,

$$t_{diff} \;=\; (3\widetilde{\mu}_o{}^2\;R^2)/(2\lambda_o\overline{v}\;(6.9)^2) \tag{62}$$

Now, trapping implies that the diffusion time so defined should be larger than the hydrodynamic time scale of collapse (t_d) which is 1.6 milliseconds[5]. Quantitatively, from eqs. (56) and (61)

$$t_{diff}^{\nu_R} \;=\; B^{\frac{5}{3}}\times t_{diff} \geq t_d \tag{63}$$

i.e.

$$B \;\geq\; \left(\frac{t_d}{t_{diff}}\right)^{\frac{3}{5}} \tag{64}$$

V Results

In section IV we have developed all the necessary concepts and formulae needed to look into the consequences of the presence of right-handed neutrinos in the pre-bounce stage of the supernova. In order to now set bounds on the interaction strength one requires the numerical values of the various input parameters which characterize the composition of the collapsing star. For this purpose, we adopt, the input parameters from Arnett's results[13] simulating a type-II supernova upto core bounce, starting from an initial mass of 8 $M_\odot$. In the multizone scheme, the core is divided into different mass zones for which the zone radii, ρ, X_h, X_n, μ_e, Y_e, $\bar{A}$ are given in Table-I of Ref. (13). Though the bounds that we obtain are dependent on these input values, we expect that use of the results of other similar simulations will not change the results by a very great deal.

A Upper bound on the strength of interactions

Using the results of Arnett, BBAL showed that, the massless left-handed neutrinos are trapped in zone 12 corresponding to a density of 1.35×10^{12} gm/cc. Once ν_L trapping sets in, the electron fraction Y_e reaches its saturation value. The final mass (M_f) of the unshocked inner core and also the energy of the shock at the time of bounce depend sensitively on this saturation value: $(M_f/M_\odot) \propto (Y_{ef})^2$ and $E_{shock} \sim (Y_{ef})^{\frac{10}{3}}\ (Y_{ef} - Y_{ei})$ where, Y_{ei} and Y_{ef} are the initial and final electron fractions respectively. A large saturation value of Y_e gives more energy to the shock for a fixed Y_{ei} (=0.42 in Arnett's model). Furthermore, with a larger value of Y_{ef}, the mass of the inner core increases, which means that there is less outlying material through which the shock wave passes and dissipates its energy. Thus, unless the final Y_e is greater than some lower limit the development of the SN explosion in the subsequent stages faces a severe problem[16]. In line with this, we impose the condition that in the presence of the additional depletion mode via the channel $e^-p \rightarrow \nu_R n$, a change of Y_{ef} more than 1% cannot be accepted if the Supernova mechanism is to remain viable. Then setting the value of the left hand side of eq.(36) as unity, which is a valid approximation for the twelfth zone, the allowed values of B for different m_ν ranging from 0-10 MeV are found out numerically. Since Y_e decreases with increasing B, an upper bound on B is obtained in this way. The result is shown in Fig 1. In this calculation we take the BBAL value of 3 MeV for Δ_n. The scale factor c_f is determined numerically from eq.(36) setting $B = 0$ and $m_\nu = 0$, and it is found that for c_f=129.9 the final Y_e at trapping rises to 0.373 (as required in view of nuclear blocking) from BBAL's value of 0.32. Fig 1 shows that B decreases with increasing m_ν. This is because with the parameters of zone 12 the coefficient of m_ν^2 in eq.(36) is negative. The upper bound on B we obtain is

$$B \leq (0.183 - 0.107)$$

for $m_\nu = 0$ - 10 MeV. From eq.(8) this gives the following lower bound for the W_R mass:

$$M_{W_R} \geq (122.5 - 139.9)\text{GeV}$$

B Lower bound on the strength of interactions

As already discussed, the lower bound on B can be estimated in two different ways: from the constraint on the mean free path and from the restriction on the diffusion time. The first alternative uses eqs.(48) and (49) and the condition of trapping is imposed. In a zone by zone investigation using Arnett's description, this would imply that the mfp in the zone in question be less than its width. λ is calculated substituting values of X_h, X_n and $\bar{A}$ from Arnett's Table-I. $g_V'^n$ and $g_A'^n$ are taken from Table 1 of this work. $g_V'^{Nu}$ for each zone is calculated using eq.(44) and Arnett's Y_e values. For E_ν

an average value of 10 MeV is chosen. Putting the value of the width of a zone in place of the core radius in eq.(50) a lower bound emerges for B for that zone as a function of m_ν. This procedure is carried out numerically. The results are summarised in table 2 and Fig 1. From the last column of table 2 we see that for a given m_ν the lower bound on B increases as we move on to outer zones upto zone 6. Then there is a kink in the value of B in zone 7 after which it again rises with decreasing density. Now, the lower bound on B is given by the ratio of mfp and the zone width, see eq.(50). The width decreases upto the third zone and then increases. The mfp being inversely proportional to the density, increases monotonically as we go outwards. This immediately explains the increase in B upto the fourth zone. Beyond this, the % increase in mfp always outstrips the % increase in the width except for the seventh zone. This explains the variation of B with density.

The minimum value for the lower bound at each m_ν comes from the second zone because this has the highest density and hence the lowest mfp. However it should be borne in mind that trapping in the second zone is only of academic interest. Most of the neutrino production is beyond this zone. In fig. 1 we have plotted the curve for the lower bound obtained from zone 12 - the zone where the massless ν_L are trapped. It is seen from the curve that as m_ν increases, the lower bound on B decreases. This behaviour is found to be true in all zones. The reason is that the collision probability given by eq.(47) decreases as m_ν grows thereby tending to increase the mfp. However, this decrease is rather slow $\sim [\,1 - (m_\nu{}^4/4E_\nu^4)]$. On the other hand, with increasing mass the velocity decreases $\sim [\,1 - (m_\nu{}^2/2E_\nu^2)]$ tending to diminish the mfp. The net effect is a slow decrease in mfp with m_ν. Hence, for a particular zone the lower bound on B decreases with increasing m_ν.

Let us now turn to the bound obtained from considerations of diffusion time. In this case a lower bound on B is obtained from eq.(63) numerically. As before, the condition of trapping is applied in each zone with the radius of the zone under consideration as the trapping radius (R). The results obtained are presented in table 2 and Fig.1. Table 2 shows the variation of B as one goes outwards from zone to zone for fixed values of m_ν. B is found to be minimum in the zone 8. Now, change of B with density is essentially governed by $B \sim (\lambda_o/R^2)^{\frac{2}{5}}$. It follows that, the sign of $\Delta B/B$ is determined by that of $\Delta\lambda_o/\lambda_o - 2\Delta R/R$. Now, the division of zones is such that the physical conditions remain more or less the same within a zone. To ensure this the zones have larger $\Delta R/R$ near the centre. For this reason, upto zone 8 B decreases but beyond this $\Delta\lambda_o/\lambda_o$ begins to predominate. This explains the nature of the variation of B with density for a given m_ν. It should be noted that the variation of B with density obtained from the consideration of mfp is somewhat different from that now obtained from an analysis of the diffusion time. We consider the latter to be more reliable since it is obtained from an analysis which additionally incorporates the effect of the transport of neutrinos.

For purposes of comparison of the different bounds, in Fig.1 we plot the curves showing the variation of B with m_ν for zone 12. The allowed and disallowed regions are pointed out. From this last diffusion analysis the lower bound on B obtained is,

$$B \geq (0.42 - 0.37)$$

for m_ν ranging from 0 to 10 MeV. From eq.(8) this can be translated to an upper bound on the mass of the W_R boson,

$$M_{W_R} \leq (99.4\text{-}102.5) \text{ GeV}.$$

VI Discussion

In this paper we have derived upper and lower bounds on the interaction of right-handed Majorana neutrinos in the *pre-bounce* stage of the supernova collapse. The consistency of observations with the current theories of the Supernova based on charged and neutral current interactions of the standard electroweak model may be jeopardised by new interactions of neutrinos. In particular, if the right-handed neutrinos free stream during collapse then they must not reduce the electron fraction, Y_e, to a degree that the explosion is prevented. This sets an upper bound to the interaction strength in the free streaming regime. On the other side, in the trapping regime the interaction must be above a lower limit in order to ensure that that the neutrinos are indeed trapped. In our analysis, we have determined the impact of the *neutrino mass* on these bounds. For $m_\nu \leq 10$ MeV the effect is not big.

For our analysis we have used the results of Arnett[13] on the evolution of a type-II Supernova of initial mass 8 $M_\odot$ upto core bounce. Our conclusions are not expected to change vastly if the results of other simulations are used instead. It should, however, be kept in mind that in Ref. 13 the supernova evolution was assumed to be governed by the standard electroweak interactions. A more complete analysis along our lines would require the incorporation of right-handed interactions in the Supernova simulations.

Our results indicate that the upper bound in the free streaming regime is strongly dependent on the final value of the electron fraction Y_{ef}. The importance of Y_{ef} in the context of Supernova explosions is well known. In all calculations, Y_{ef} depends sensitively on the model used, i.e. on the input equation of state and on estimates of the nuclear capture rates. We therefore conclude that an accurate evaluation of Y_{ef} is important not only to answer the key question of the success of the prompt explosion mechanism but also to fix strong bounds on new interactions.

The bounds on the coupling constants that we find can be translated to an excluded zone for the mass of the right-handed charged gauge boson. The limits that we obtain are very weak, either $M_{W_R} \leq \sim 100$ GeV or $M_{W_R} \geq \sim 130$ GeV. Much stronger bounds on M_{W_R} can be derived from particle physics: $M_{W_R} > 520$ GeV from its non-observation at the Tevatron[17] and $M_{W_R} > 1.6$ TeV from the $K_L - K_S$ mass difference[18]. (These bounds, however, require additional assumptions regarding the

mass of the neutrino or the quark mixing matrix in the right-handed sector.) In fact, an analysis of the post-collapse energy emission from the Supernova has also been used to set stronger bounds[3]. Thus the pre-bounce evolution of the supernova does not provide a good testing ground for the interaction of right-handed neutrinos.

ACKNOWLEDGEMENTS: The authors are grateful to Alak Ray for discussions and help. S.G. wishes to thank Dipanjan Rai Chaudhuri for discussions and inspiration. S.G. is supported by the Council of Scientific and Industrial Research, India while A.R. has been supported in part by the Department of Atomic Energy and the Department of Science and Technology, India. A.R. wishes to thank Prof. Abdus Salam, IAEA and UNESCO for supporting his visit to the International Centre for Theoretical Physics, Trieste where part of the work was done.

REFERENCES

1. A. Burrows and J.M. Lattimer, Ap. J. **318**, L63 (1987).
2. R.N. Mohapatra and P.B. Pal, 'Massive neutrinos in physics and astrophysics', World Scientific (1991).
3. R. Barbieri and R.N. Mohapatra, Phys. Rev. **D39**, 1229 (1989); J.A. Grifols and E. Masso, Nucl. Phys. **B331** 244 (1990); J.A. Grifols, E. Masso and T.G. Rizzo, Phys. Rev **D42**, 3293 (1990); G. Raffelt and D. Seckel, Phys. Rev. Lett. **60**, 1793 (1988); M.S. Turner, Phys. Rev. Lett. **60**, 1797 (1988); J.E. Ellis and K.A. Olive, Phys. Lett. **B193**, 525 (1987).
4. G.M. Fuller, Ap. J. **252**, 741 (1982).
5. H.A. Bethe, Rev. Mod. Phys. **62**, 802 (1990); S.L. Shapiro and S.A. Teukolsky, 'Black holes, white dwarfs, and neutron stars', John Wiley (1983).
6. J. Cooperstein and J. Wambach, Nucl. Phys. **A420**, 591 (1984); S.M. Bruenn and W.C. Haxton, Ap. J. **376**, 1678 (1991).
7. A. Ray, S.M. Chitre and K. Kar, Ap. J. **285**, 766 (1984).
8. H.A. Bethe, G.E. Brown, J. Applegate and J.M. Lattimer, Nucl. Phys. **A324**, 487 (1979).
9. D.Z. Freedman, Phys. Rev. **D9**, 1389 (1974); T. Mazurek, Astrophys. Space Sci. **35**, 117 (1975); K. Sato, Prog. Theor. Phys. **54**, 1352 (1975).
10. D.L. Tubbs and D.N. Schramm, Ap. J. **201**, 467 (1975).
11. D.Q. Lamb and C.J. Pethik, Ap. J. **209**, L77 (1976)
12. S.W. Bruenn, Ap. J. Suppl., **58**, 771 (1985); R.L. Bowers and J.R. Wilson, Ap. J. Suppl. **50**, 115 (1982); E.S. Myra, Phys. Rep. **163**, 127 (1988).
13. W.D. Arnett, Ap. J. **218**, 815 (1977).
14. J.C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974); R.N. Mohapatra and J.C. Pati, Phys. Rev. **D11**, 366, 2588 (1975); G. Senjanović and R.N. Mohapatra, Phys. Rev. **D12**, 1502 (1975).
15. This relation is, strictly speaking, true only for $M_{W_R} \gg M_{W_L}$.
16. The sensitivity of the supernova mechanism to Y_e is well known. See, for example, J. Cooperstein, Phys. Reports **163**, 95 (1988) and Bethe, (Ref. 5) p807.
17. J. Freeman, in Proc. Workshop in High Energy Physics Phenomenology II, A. Datta, P. Ghose and A. Raychaudhuri (eds), World Scientific (1992).
18. G.Beall, M. Bander and A. Soni, Phys. Rev. Lett. **48**, 848 (1982); G. Ecker and W. Grimus, Nucl. Phys. **B258**, 328 (1985).

Particle	g'_V	g'_A
ν_R	1/2	1/2
e	−0.05	−0.35
u	0.25	0.35
d	−0.45	−0.35
p	0.05	0.35
n	0.65	−0.35

Table 1

The vector and axial vector couplings of the leptons and quarks with the extra neutral gauge boson, Z'.

m_ν	Zone No.	Density ρ_{12}	Width	Lower bound on B: Diffusion time	Lower bound on B: Mean free path
0.0	2	15.2	6.3	.755	.046
	3	14.2	2.2	.579	.142
	4	13.3	2.1	.466	.166
	5	12.2	2.1	.396	.178
	6	11.0	2.3	.350	.184
	7	9.57	2.7	.312	.180
	8	7.86	3.2	.299	.197
	9	5.88	4.3	.305	.221
	10	3.62	6.4	.335	.273
	11	2.25	5.0	.364	.539
	12	1.35	6.1	.416	.750
5.0	2	15.2	6.3	.749	.041
	3	14.2	2.2	.573	.127
	4	13.3	2.1	.462	.148
	5	12.2	2.1	.393	.159
	6	11.0	2.3	.346	.164
	7	9.57	2.7	.308	.160
	8	7.86	3.2	.295	.176
	9	5.88	4.3	.301	.197
	10	3.62	6.4	.329	.244
	11	2.25	5.0	.356	.480
	12	1.35	6.1	.403	.668

Table 2: Lower bounds on $B = (G_R^2/G_F^2)$ obtained from the constraints on the diffusion time and the mean free path of a right-handed neutrino for the different zones of Arnett (Ref. 13). Note that in this reference k denotes the zone between the mass points $(k-1)$ and k. m_ν is in MeV. The density, ρ_{12}, is in units of 10^{12} gm/cc and the width in 10^5 cm.

FIGURE CAPTION

Figure 1: The allowed regions for $B = (G_R^2/G_F^2)$ as a function of the neutrino mass. The parameters of Arnett's (Ref. 13) zone 12 are used. The upper bound curve (c) is obtained from the constraint on Y_e. The two lower bound curves marked (a) and (b) are obtained from the mean free path and diffusion time constraints respectively.