# New constraints on $R$-parity violation from $K$ and $B$ systems 

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#### Abstract

We have derived new upper limits on several products (two at a time) of lepton number violating $\lambda^{\prime}$-type couplings from the consideration of $\Delta S=2$ and $\Delta B=2$ box graphs. Each box contains one scalar lepton and one W -boson or one charged-Higgs-boson as internal lines. Most of these bounds are more stringent than previously obtained. Some of these product couplings drive $K_{L}$ (and some other $B_{d}$ ) decays to two charged leptons at enhanced rates. Some of them can explain the rare $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ event recently observed at BNL.


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Unless one assumes that lepton-number $(L)$ and baryon-number $(B)$ are conserved quantities, that are otherwise not ensured by gauge invariance, supersymmetric theories with two Higgs doublets naturally allow $R$-parity-violating ( $\not R$ ) couplings [1]. Defined as $R=(-1)^{(3 B+L+2 S)}$ (where $S$ is the spin of the particle), $R$-parity is a discrete symmetry under which all Standard Model (SM) particles are even while their superpartners are odd. Even though any concrete evidence of $L$ - or/and $B$-violation is yet to be reported, supersymmetry without $R$-parity has emerged as a fashionable area of research of late [2]. When $R$-parity is violated, the lightest supersymmetric particle does not remain stable and therefore the canonical missing energy signature of supersymmetry search is no longer valid. However, depending on the nature of $\not R$ couplings, novel supersymmetry signatures (e.g. multilepton or like-sign dilepton final states [3]) emerge that could lead to the discovery of supersymmetry in the present or future colliders. In parallel with this, it is important to take a stock of the extent to which those couplings are already constrained from existing phenomenology. In this paper, we derive multitude of new upper bounds on several combinations of the products of $\lambda^{\prime}$-type couplings (defined below), taken two at a time with different flavour indices, that contribute to the $K_{L}-K_{S}$ mass difference $\left(\Delta m_{K}\right)$ or to the $B_{q}-\bar{B}_{q}(q=d, s)$ mass differences $\left(\Delta m_{B_{q}}\right)$ at one-loop level. We compare our limits with the previous ones. We also find enhanced rates for $K_{L} \rightarrow \mu^{+} \mu^{-}$, $B_{d} \rightarrow \tau^{+} \tau^{-}$and $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$, that proceed at tree level, in the presence of some of those product couplings.

The most general Yukawa superpotential of an explicitly broken $\not R$ supersymmetric theory is given by

$$
\begin{equation*}
\mathcal{W}_{\not R}=\frac{1}{2} \lambda_{i j k} L_{i} L_{j} E_{k}^{c}+\lambda_{i j k}^{\prime} L_{i} Q_{j} D_{k}^{c}+\frac{1}{2} \lambda_{i j k}^{\prime \prime} U_{i}^{c} D_{j}^{c} D_{k}^{c} \tag{1}
\end{equation*}
$$

where $L_{i}$ and $Q_{i}$ are $\mathrm{SU}(2)$-doublet lepton and quark superfields respectively; $E_{i}^{c}, U_{i}^{c}, D_{i}^{c}$ are $\mathrm{SU}(2)$-singlet charged lepton, up- and down-quark superfields respectively; $\lambda_{i j k^{-}}$and $\lambda_{i j k}^{\prime}$-types are $L$-violating while $\lambda_{i j k}^{\prime \prime}$-types are $B$-violating Yukawa couplings. $\lambda_{i j k}$ is antisymmetric under the interchange of the first two family indices, while $\lambda_{i j k}^{\prime \prime}$ is antisymmetric under the interchange of the last two. Thus there could be 27 $\lambda^{\prime}$-type and 9 each of $\lambda$ - and $\lambda^{\prime \prime}$-type couplings. Stringent constraints on the individual couplings have been placed from the consideration of $n-\bar{n}$ oscillation [4], $\nu_{e}$-Majorana mass [5], neutrinoless double beta decay [6], charged-current universality [7], $e-\mu-\tau$ universality [7], $\nu_{\mu}$ deep-inelastic scattering [7], atomic parity violation [7, 8], $\tau$-decays [9, 10], $D$-decays [9], $Z$-decays 11, 10], $K^{+}$-decay 12] and $M^{0}-\bar{M}^{0}(M \equiv K, B, D)$ mixing 12, 8]. Products couplings (two at a time) have been constrained by considering proton stability

[^0][13], neutrinoless double beta decay [14], meson mass differences and decays 15, 16, 17], $\mu-e$ conversion [15], $\mu \rightarrow e \gamma$ [18], $b \rightarrow s \gamma$ [17, $B$ decays into two charged leptons 19, 20] and $C P$-violation [21.

We first note that the SM contribution [22] to $\Delta m_{K}$ is $\sim 2 \times 10^{-15} \mathrm{GeV}$ and, on account of the large implicit theoretical errors, it is believed to agree with the experimental value $\left(\Delta m_{K}^{\exp } \simeq 3.5 \times 10^{-15}\right.$ GeV (23]) fairly well. While placing bounds on any new physics that contributes to $\Delta m_{K}$, we require that none of the new contributions individually exceeds the experimental value. With this view, we ignore the contribution coming purely from the Minimal Supersymmetric Standard Model [24] and restrict ourselves to discussions of only $\not R$ contributions. It has been argued in that one non-zero $\not \subset$ coupling in the weak basis of fermions could lead to more such non-zero couplings in their mass basis contributing to $\Delta m_{K}$. The bounds on the weak basis couplings were estimated to be order 0.1 for 100 GeV scalar exchanges. However, such bounds crucially depend on what assumption is made during the basis rotation. In the present paper, we avoid undergoing such a basis rotation and instead right-away assume two non-zero $\not R$ couplings in their mass basis (in the same notation) in each case. Now we observe that the combinations $\lambda_{i 21}^{\prime} \cdot \lambda_{i 12}^{\prime}$ and $\lambda_{i 31}^{\prime} \cdot \lambda_{i 13}^{\prime}$ contribute to $\Delta m_{K}$ and $\Delta m_{B_{d}}\left(\Delta m_{B_{d}}^{\exp } \simeq 3.3 \times 10^{-13} \mathrm{GeV}\right.$ [23]), respectively, at tree level. These yield $\lambda_{i 21}^{\prime} \cdot \lambda_{i 12}^{\prime} \lesssim 1 \times 10^{-9}$ and $\lambda_{i 31}^{\prime} \cdot \lambda_{i 13}^{\prime} \lesssim 8 \times 10^{-8}$, when the exchanged scalar has a mass of 100 GeV . However, there are many other such product couplings that contribute to the $\Delta S=2$ or $\Delta B=2$ process via one-loop box graphs. In such cases, all four vertices of a given box diagram (corresponding to either $\Delta S=2$ or $\Delta B=2$ ) could be of the $\lambda^{\prime}$-type 17. As an illustration, if we take two $\lambda_{i 31^{-}}^{\prime}$ and two $\lambda_{i 32}^{\prime}$-vertices in a $\Delta S=2$ box graph, the effective Hamiltonian could be expressed as

$$
\begin{equation*}
\mathcal{H}_{4 \lambda^{\prime}}^{\Delta S=2}=\frac{\left(\lambda_{i 31}^{\prime} \lambda_{i 32}^{\prime}\right)^{2}}{128 \pi^{2} \tilde{m}^{2}} \mathcal{J}\left(\frac{m_{t}^{2}}{\tilde{m}^{2}}\right)\left[\bar{d} \gamma_{\mu}\left(1+\gamma_{5}\right) s\right]^{2} \tag{2}
\end{equation*}
$$

where $\mathcal{J}(x)=(1+x) /(1-x)^{2}+2 x \ln x /(1-x)^{3}$. In the vacuum saturation approximation, using $\left\langle K_{0}\right|\left[\bar{d} \gamma_{\mu}\left(1+\gamma_{5}\right) s\right]^{2}\left|\bar{K}_{0}\right\rangle \simeq 4 f_{K}^{2} m_{K} / 3$ and $\Delta m_{K}=2 \operatorname{Re}\left(\left\langle K_{0}\right| \mathcal{H}\left|\bar{K}_{0}\right\rangle\right)$, we obtain $\lambda_{i 31}^{\prime} \cdot \lambda_{i 32}^{\prime} \lesssim 2.8 \times 10^{-3}$ for $\tilde{m} \equiv m_{\tilde{L}_{i}}=100 \mathrm{GeV}$ (also in all our subsequent calculations, we take the mass of the exchanged scalar as 100 GeV ). Throughout we use $m_{t}=175 \mathrm{GeV}$ and $f_{K}=150 \mathrm{MeV}$. The corresponding bounds when the internal fermions are instead the $c$-quarks (or $u$-quarks) are $\lambda_{i 21}^{\prime} \cdot \lambda_{i 22}^{\prime}\left(\right.$ or $\left.\lambda_{i 11}^{\prime} \cdot \lambda_{i 12}^{\prime}\right) \lesssim 1.2 \times 10^{-3}$. On the other hand, a similar computation of the $B_{d}-\bar{B}_{d}$ mixing yields (with $f_{B_{d}}=200 \mathrm{MeV}$ ) $\lambda_{i 31}^{\prime} \cdot \lambda_{i 33}^{\prime} \lesssim 6.4 \times 10^{-3}$ and $\lambda_{i 21}^{\prime} \cdot \lambda_{i 23}^{\prime}\left(\right.$ or $\left.\lambda_{i 11}^{\prime} \cdot \lambda_{i 13}^{\prime}\right) \nwarrow 2.7 \times 10^{-3}$.

However, the main thrust of our analysis lies in computing another set of $\Delta S=2$ and $\Delta B=2$ box graphs, hitherto overlooked, in each of which there is one $\tilde{L}_{i}$ propagator between two $\lambda^{\prime}$-type vertices and one of $W^{ \pm}$(transverse $W$-boson), $G^{ \pm}$(longitudinal $W$-boson in the 't Hooft - Feynman gauge) and $H^{ \pm}$(charged Higgs) as the other non-fermionic propagator 1 The nature of the internal fermion lines are decided by the flavour indices associated with the two $\lambda^{\prime}$-vertices. These diagrams are present in any $\not R$ supersymmetric theory, and as we will see, they serve to constrain other flavour combinations of product couplings in addition to those considered above. With $H^{ \pm}$(or $G^{ \pm}$) as one scalar propagator and, as always, $\tilde{L}_{i}$ as another between the $\lambda^{\prime}$-vertices, box graphs with $t$-quarks in internal lines contribute more than those with $c$ - or $u$-quarks, despite the relatively larger Cabibbo-Kobayashi-Maskawa (CKM) suppressions associated with the former. For light quarks ( $c$ or $u$ ) as internal fermions, box graphs with internal $W^{ \pm}$ dominate over those with internal $H^{ \pm}$or $G^{ \pm}$, for the same choice of $\lambda^{\prime}$-couplings. Below we consider these possibilities case by case:
(i) Bounds from $\Delta m_{K}$ : First we choose the same two $\lambda^{\prime}$ as displayed in eq. (2), namely, $\lambda_{i 31}^{\prime}$ and $\lambda_{i 32}^{\prime}$, as the two $\not R$ vertices of a $\Delta S=2$ box. Between the other two vertices in the box could flow either of $W^{ \pm}, G^{ \pm}$and $H^{ \pm}$. Evidently $t$-quark propagates in internal fermion lines. Assuming $m_{H^{ \pm}}=\tilde{m}$ (which is a reasonable assumption for an order of magnitude estimate of the upper bounds of the $\not R$ couplings), the dominant part of the effective Hamiltonian for this process could be written as

$$
\begin{equation*}
\mathcal{H}_{2 \lambda^{\prime} ; t-t}^{\Delta S=2} \simeq \frac{G_{F} \lambda_{i 31}^{\prime} \lambda_{i 32}^{\prime}}{16 \pi^{2} \sqrt{2}} V_{t s} V_{t d}^{*}\left[\left(\cot ^{2} \beta+1\right) \frac{m_{t}^{4}}{\tilde{m}^{4}} \mathcal{I}\left(\frac{m_{t}^{2}}{\tilde{m}^{2}}\right)+\mathcal{J}\left(\frac{m_{t}^{2}}{\tilde{m}^{2}}\right)\right]\left[\bar{d}\left(1-\gamma_{5}\right) s\right]\left[\bar{d}\left(1+\gamma_{5}\right) s\right], \tag{3}
\end{equation*}
$$

where $\mathcal{I}(x)=-2 /(1-x)^{2}+(x+1) \ln x /(x-1)^{3}$ and $\cot \beta=v_{d} / v_{u}$, the ratio of the vacuum expectation values of the two Higgs bosons that are responsible for the generation of down- and up-quark masses respectively.

[^1]In eq.(3), $H^{ \pm}$- and $G^{ \pm}$-induced contributions carry an enhancement factor ( $m_{t}^{4} / \tilde{m}^{4}$ ), the former involving $\cot ^{2} \beta$; the last term arises from $W^{ \pm}$propagation. Within the vacuum saturation approximation, employing $\left\langle K_{0}\right|\left[\bar{d}\left(1+\gamma_{5}\right) s\right]\left[\bar{d}\left(1-\gamma_{5}\right) s\right]\left|\bar{K}_{0}\right\rangle \simeq 10 f_{K}^{2} m_{K}$, we obtain, for $\cot \beta=1, \lambda_{i 31}^{\prime} \cdot \lambda_{i 32}^{\prime} \lesssim 7.7 \times 10^{-4}$. Noteworthy is the point that even when $\cot \beta \ll 1$, the Goldstone contribution holds the bounds at the same order of magnitude

An analogous process induced by $\lambda_{i 31}^{\prime}$ and $\lambda_{i 22}^{\prime}$ (and hence with $t$ - and $c$-quarks as the two internal fermions) leads to the effective Hamiltonian

$$
\begin{equation*}
\mathcal{H}_{2 \lambda^{\prime} ; t-c}^{\Delta S=2} \simeq \frac{G_{F} \lambda_{i 31}^{\prime} \lambda_{i 22}^{\prime}}{16 \pi^{2} \sqrt{2}} V_{t s} V_{c d}^{*}\left[\left(\cot ^{2} \beta+1\right) \frac{m_{t}^{2} m_{c}^{2}}{\tilde{m}^{2}} \mathcal{K}\left(\frac{m_{t}^{2}}{\tilde{m}^{2}}\right)+\mathcal{L}\left(\frac{m_{t}^{2}}{\tilde{m}^{2}}\right)\right]\left[\bar{d}\left(1-\gamma_{5}\right) s\right]\left[\bar{d}\left(1+\gamma_{5}\right) s\right] \tag{4}
\end{equation*}
$$

where $\mathcal{K}(x)=1 /(x-1)-\ln x /(x-1)^{2}$ and $\mathcal{L}(x)=1-x \mathcal{K}(x)$. The following bounds are $\lambda_{i 31}^{\prime} \cdot \lambda_{i 22}^{\prime} \leqslant 1.0 \times 10^{-4}$. Similarly, with $c$-quarks (or $u$-quarks) as internal fermions, we obtain the bounds $\lambda_{i 21}^{\prime} \cdot \lambda_{i 22}^{\prime} \curvearrowright 1.4 \times 10^{-6}$ and $\lambda_{i 11}^{\prime} \cdot \lambda_{i 12}^{\prime} \leqslant 1.4 \times 10^{-6}$. In the last two cases, there are almost no CKM-suppressions and hence such tight constraints. Bounds on other combinations are also obtained.
(ii) Bounds from $\Delta m_{B}$ : The $\Delta B=2$ processes are very similar to the $\Delta S=2$ ones considered above. Let us first consider $B_{d}-\bar{B}_{d}$ mixing with $\lambda_{i 31}^{\prime}$ and $\lambda_{i 33}^{\prime}$ at the two $\not R$ vertices (i.e. with $t$-quark in both internal fermion lines), and $W^{ \pm}$or $G^{ \pm}$or $H^{ \pm}$propagating between the other two, as before, in a given box graph. The effective Hamiltonian is exactly analogous to the one in eq. (3), and we do not display it here. However, in the present case, the CKM entries are $V_{t b}$ and $V_{t d}^{*}$ and the hadronic matrix element turns out to be $\left\langle B_{d}\right|\left[\bar{d}\left(1+\gamma_{5}\right) b\right]\left[\bar{d}\left(1-\gamma_{5}\right) b\right]\left|\bar{B}_{d}\right\rangle \simeq 7 f_{B_{d}}^{2} m_{B_{d}} / 6$. Again for $\cot \beta=1$, we obtain $\lambda_{i 31}^{\prime} \cdot \lambda_{i 33}^{\prime} \lesssim 1.3 \times 10^{-3}$. We have also derived upper bounds on other product couplings, corresponding to heavy-light $(t-c, t-u)$ or light-light $(c-c, c-u, u-u)$ combinations in the two internal fermion lines, that contribute to $B_{d}-\bar{B}_{d}$ mixing ${ }^{f}$. Analogous bounds from $B_{s}-\bar{B}_{s}$ mixing have also been derived

We have listed all of our bounds in Table 1. While comparing our bounds with the previous ones (the latter obtained by multiplying the bounds on the individual couplings), it has to be borne in mind that, keeping $\tilde{m}$ fixed, pushing $m_{H^{ \pm}}$to higher values indeed weaken our limits but not the previous ones. However, the fact remains that all existing bounds, that we compare with, have been derived assuming a mass of 100 GeV for whichever scalar is exchanged, and that way our assumption, that one more scalar (the charged Higgs boson) has the same mass, is not unreasonable. It should be noted that we have not indulged ourselves into computing the effects of QCD corrections for our order of magnitude estimates.

Next we proceed to study the decays of neutral $K$ - and $B$-mesons (in particular, those of $K_{L}$ and $B_{d}$ ) into two charged leptons. In $\not R$ scenario, and particularly for some choices of product couplings considered above, these decays can take place at tree level and hence their branching ratios could be substantially amplified over their loop-driven SM predictions. As has been clarified in 19, the most general operators that contribute to the decay of a neutral meson $M(=p \bar{q})$ into two charged leptons $l \bar{l}$, could only be of the forms $c_{P}\left(\bar{q} \gamma_{5} p\right)\left(\bar{l} \gamma_{5} l\right), c_{P}^{\prime}\left(\bar{q} \gamma_{5} p\right)(\bar{l} l)$ and $c_{A}\left(\bar{q} \gamma_{\mu} \gamma_{5} p\right)\left(\bar{l} \gamma_{\mu} \gamma_{5} l\right)$. In our case, the branching ratio of $K_{L}$ decaying to $\mu^{+} \mu^{-}$(tree level $\tilde{u}_{L j}$-exchanged decay) is given by

$$
\begin{equation*}
B\left(K_{L} \rightarrow \mu^{+} \mu^{-}\right)=\frac{\left(\lambda_{2 j 1}^{\prime} \lambda_{2 j 2}^{\prime}\right)^{2}}{128 \pi}\left(\frac{f_{K} m_{\mu}}{m_{\tilde{u}_{L j}}^{2}}\right)^{2} \sqrt{1-4 \frac{m_{\mu}^{2}}{m_{K}^{2}}}\left(m_{K} \tau_{K_{L}}\right) \tag{5}
\end{equation*}
$$

where $\tau_{K_{L}}\left(\simeq 5.17 \times 10^{-8} s\right)$ is the $K_{L}$-lifetime. When the $\not R$ couplings are switched on, the above branching ratio could be $\sim 217\left(\lambda_{2 j 2}^{\prime} \cdot \lambda_{2 j 1}^{\prime}\right)^{2}$ for $m_{\tilde{u}_{L j}}=100 \mathrm{GeV}$. Inserting the bound $\lambda_{231}^{\prime} \cdot \lambda_{232}^{\prime} \lesssim 7.7 \times 10^{-4}$, obtained for $m_{\tilde{L}_{e}}=100 \mathrm{GeV}, B\left(K_{L} \rightarrow \mu^{+} \mu^{-}\right)$could touch $1.3 \times 10^{-4}$ overshooting the measured value $\left((7.2 \pm 0.5) \times 10^{-9}\right.$ [23]) by four or five orders of magnitude. So either we have to take $m_{\tilde{t}_{L}}=1.2 \mathrm{TeV}$ to compute the above branching ratio, or, for $m_{\tilde{t}_{L}}=100 \mathrm{GeV}$, we obtain a more stringent constraint $\lambda_{231}^{\prime} \cdot \lambda_{232}^{\prime} \lesssim 5.8 \times 10^{-6}$. The process $B\left(K_{L} \rightarrow e^{+} e^{-}\right)$is helicity suppressed and from the experimental bound ( $\lesssim 4.1 \times 10^{-11}$ at $90 \%$ CL

[^2][23]) one obtains $\lambda_{1 j 1}^{\prime} \cdot \lambda_{1 j 2}^{\prime} \lesssim 8.6 \times 10^{-5}$ for $m_{\tilde{u}_{L j}}=100 \mathrm{GeV}$. From a similar consideration, $B\left(B_{d} \rightarrow \tau^{+} \tau^{-}\right)$ could be estimated as $\sim 30\left(\lambda_{3 j 1}^{\prime} \cdot \lambda_{3 j 3}^{\prime}\right)^{2}$. With $\lambda_{311}^{\prime} \cdot \lambda_{313}^{\prime} \lesssim 3.6 \times 10^{-3}$ (from $\Delta m_{B_{d}}$ ), we infer that $B\left(B_{d} \rightarrow\right.$ $\tau^{+} \tau^{-}$) could be as high as $3.9 \times 10^{-4}$ for a $100 \mathrm{GeV} \tilde{u}_{L}$. This can be tested in future $B$-factories (the present experimental limit on this branching ratio has been estimated 19 to be $1.5 \times 10^{-2}$ from LEP data analysis).

Finally we turn our attention to the rare decay $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$, an evidence (only one event though) of which, citing a branching ratio $B\left(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}\right)=4.2_{-3.5}^{+9.7} \times 10^{-10}$, has recently been reported by the E787 Collaboration at BNL [25]. The product couplings $\lambda_{i j 1}^{\prime} \cdot \lambda_{i j 2}^{\prime}$, constrained above by $K_{L}-K_{S}$ mixing and $K_{L} \rightarrow \mu^{+} \mu^{-}$(or $e^{+} e^{-}$), drive this interaction at tree level. The SM contribution [26] is at least one order of magnitude smaller than the experimental $1 \sigma$ upper limit $\left(1.4 \times 10^{-9}\right)$. Assuming the dominance of tree level $\not R$ contribution, we obtain $B\left(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}\right)=\left[\lambda_{i j 1}^{\prime} \lambda_{i j 2}^{\prime} /\left(4 G_{F} m_{\tilde{d}_{L j}}^{2} V_{u s}^{*}\right)\right]^{2} B\left(K^{+} \rightarrow \pi^{0} \nu \bar{e}\right)$. Requiring then that it saturates the $1 \sigma$ upper limit yields $\lambda_{i j 1}^{\prime} \cdot \lambda_{i j 2}^{\prime} \lesssim 1.6 \times 10^{-5}$ for $m_{\tilde{d}_{L j}}=100 \mathrm{GeV}$, where we have used $B\left(K^{+} \rightarrow \pi^{0} \nu \bar{e}\right)=0.0482$ [23]. A look at Table 1 reveals that for three combinations, corresponding to $i=1,2$, or 3 and $j=3$, the bounds are improved (indeed for a mass of 100 GeV of a different scalar). Turning the argument around, those three product couplings individually are capable of reproducing the rare event seen at BNL.

To conclude, we have derived new upper limits on several combinations of $\lambda^{\prime}$-couplings, product of two at a time, by considering one-loop box graphs (with one $\tilde{L}_{i}$ and one $W^{ \pm} / G^{ \pm} / H^{ \pm}$as internal lines) contributing to $\Delta m_{K}$ or $\Delta m_{B}$. Most of our bounds are significantly tighter than the previous ones. Meson decays to two charged leptons (in particular, $K_{L} \rightarrow \mu^{+} \mu^{-}$and $B_{d} \rightarrow \tau^{+} \tau^{-}$) are enhanced in the presence of some of those product couplings. Some could explain the rare $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ event seen at BNL.

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Table 1: New upper limits on product couplings for $m_{\tilde{L}_{i}}=m_{H^{ \pm}}=100 \mathrm{GeV}$. The combination marked with "*" ("**") has a stronger upper limit as $8.6 \times 10^{-5}\left(5.8 \times 10^{-6}\right)$ for $m_{\tilde{t}_{L}}=100 \mathrm{GeV}$ (irrespective of the charged Higgs boson mass), that follows from the consideration of $K_{L} \rightarrow e^{+} e^{-}\left(\mu^{+} \mu^{-}\right)$. Similarly, the products marked with ' $\dagger$ ' have a stronger constraint $1.6 \times 10^{-5}$ for $m_{\tilde{b}_{L}}=100 \mathrm{GeV}$ (again irrespective of the charged Higgs boson mass), following from $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$.

| From | $\lambda_{i j k}^{\prime} \cdot \cdot \lambda_{\text {lmn }}^{\prime}$ | Our limits | Previous limits | $\lambda_{i j k}^{\prime} \cdot \lambda_{l m n}^{\prime}$ | Our limits | Previous limits |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta m_{K}$ | (131).(132)[*, t] | $7.7 \times 10^{-4}$ | $1.2 \times 10^{-2}$ | (231).(232)[**, †] | $7.7 \times 10^{-4}$ | $7.9 \times 10^{-2}$ |
|  | (331).(332)[†] | $7.7 \times 10^{-4}$ | $2.3 \times 10^{-1}$ | (131).(122) | $1.0 \times 10^{-4}$ | $7.0 \times 10^{-4}$ |
|  | (231).(222) | $1.0 \times 10^{-4}$ | $4.0 \times 10^{-2}$ | (331).(322) | $1.0 \times 10^{-4}$ | $9.6 \times 10^{-2}$ |
|  | (121).(122) | $1.4 \times 10^{-6}$ | $7.0 \times 10^{-4}$ | (221).(222) | $1.4 \times 10^{-6}$ | $3.2 \times 10^{-2}$ |
|  | (321).(322) | $1.4 \times 10^{-6}$ | $4.0 \times 10^{-2}$ | (111).(112) | $1.4 \times 10^{-6}$ | $7.0 \times 10^{-6}$ |
|  | (211).(212) | $1.4 \times 10^{-6}$ | $8.1 \times 10^{-3}$ | (311).(312) | $1.4 \times 10^{-6}$ | $1.0 \times 10^{-2}$ |
|  | (122).(111) | $6.1 \times 10^{-6}$ | $7.0 \times 10^{-6}$ | (222).(211) | $6.1 \times 10^{-6}$ | $1.6 \times 10^{-2}$ |
|  | (322).(311) | $6.1 \times 10^{-6}$ | $2.0 \times 10^{-2}$ | (132).(121) | $1.1 \times 10^{-4}$ | $1.2 \times 10^{-2}$ |
|  | (232).(221) | $1.1 \times 10^{-4}$ | $6.5 \times 10^{-2}$ | (332).(321) | $1.1 \times 10^{-4}$ | $9.6 \times 10^{-2}$ |
|  | (132).(111) | $4.7 \times 10^{-4}$ | $1.2 \times 10^{-4}$ | (232).(211) | $4.7 \times 10^{-4}$ | $3.2 \times 10^{-2}$ |
|  | (332).(311) | $4.7 \times 10^{-4}$ | $4.8 \times 10^{-2}$ | (131).(112) | $2.4 \times 10^{-5}$ | $7.0 \times 10^{-}$ |
|  | (231).(212) | $2.4 \times 10^{-5}$ | $2.0 \times 10^{-2}$ | (331).(312) | $2.4 \times 10^{-5}$ | $4.8 \times 10^{-2}$ |
| $\Delta m_{B_{d}}$ | (131).(133) | $1.3 \times 10^{-3}$ | $2.4 \times 10^{-5}$ | (231).(233) | $1.3 \times 10^{-3}$ | $7.9 \times 10^{-2}$ |
|  | (331).(333) | $1.3 \times 10^{-3}$ | $2.3 \times 10^{-1}$ | (131).(123) | $1.8 \times 10^{-4}$ | $7.0 \times 10^{-3}$ |
|  | (231).(223) | $1.8 \times 10^{-4}$ | $4.0 \times 10^{-2}$ | (331).(323) | $1.8 \times 10^{-4}$ | $9.6 \times 10^{-2}$ |
|  | (111).(113) | $3.6 \times 10^{-3}$ | $7.0 \times 10^{-6}$ | (211).(213) | $3.6 \times 10^{-3}$ | $8.1 \times 10^{-3}$ |
|  | (311).(313) | $3.6 \times 10^{-3}$ | $1.0 \times 10^{-2}$ | (121).(113) | $3.1 \times 10^{-4}$ | $7.0 \times 10^{-4}$ |
|  | (221).(213) | $3.1 \times 10^{-4}$ | $1.6 \times 10^{-2}$ | (321).(313) | $3.1 \times 10^{-4}$ | $2.0 \times 10^{-2}$ |
|  | (111).(123) | $1.6 \times 10^{-2}$ | $7.0 \times 10^{-5}$ | (211).(223) | $1.6 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |
|  | (311).(323) | $1.6 \times 10^{-2}$ | $2.0 \times 10^{-2}$ | (121).(123) | $1.4 \times 10^{-3}$ | $7.0 \times 10^{-3}$ |
|  | (221).(223) | $1.4 \times 10^{-3}$ | $3.2 \times 10^{-2}$ | (321).(323) | $1.4 \times 10^{-3}$ | $4.0 \times 10^{-2}$ |
| $\Delta m_{B_{s}}$ | (132).(133) | $2.5 \times 10^{-3}$ | $2.4 \times 10^{-4}$ | (232).(233) | $2.5 \times 10^{-3}$ | $1.3 \times 10^{-1}$ |
|  | (332).(333) | $2.5 \times 10^{-3}$ | $2.3 \times 10^{-1}$ | (132).(113) | $1.5 \times 10^{-3}$ | $6.8 \times 10^{-3}$ |
|  | (232).(213) | $1.5 \times 10^{-3}$ | $3.2 \times 10^{-2}$ | (332).(313) | $1.5 \times 10^{-3}$ | $4.8 \times 10^{-2}$ |
|  | (112).(113) | $1.4 \times 10^{-1}$ | $4.0 \times 10^{-4}$ | (212).(213) | $1.4 \times 10^{-1}$ | $8.1 \times 10^{-3}$ |
|  | (312).(313) | $1.4 \times 10^{-1}$ | $1.0 \times 10^{-2}$ | (122).(113) | $1.2 \times 10^{-2}$ | $4.0 \times 10^{-4}$ |
|  | (222).(213) | $1.2 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | (322).(313) | $1.2 \times 10^{-2}$ | $2.0 \times 10^{-2}$ |
|  | (112).(123) | $3.2 \times 10^{-2}$ | $4.0 \times 10^{-3}$ | (212).(223) | $3.2 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |
|  | (312).(323) | $3.2 \times 10^{-2}$ | $2.0 \times 10^{-2}$ | (122).(123) | $2.7 \times 10^{-3}$ | $4.0 \times 10^{-3}$ |
|  | (222).(223) | $2.7 \times 10^{-3}$ | $3.2 \times 10^{-2}$ | (322).(323) | $2.7 \times 10^{-3}$ | $4.0 \times 10^{-2}$ |


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[^1]:    ${ }^{1} \lambda^{\prime \prime}$-vertices with an internal $W$-boson have been considered in 17 .

[^2]:    ${ }^{2}$ In the effective Hamiltonian, we have neglected a term proportional to $\tan ^{2} \beta$, arising from the charged Higgs coupling, that multiplies light quark masses on external legs. Even for very large $\tan \beta \sim 40$, the numerical contribution of such a term is insignificant compared to the Goldstone contribution. This is true even in the case of $B_{s}-\bar{B}_{s}$ mixing that we will discuss subsequently.
    ${ }^{3}$ For our order of magnitude estimate, we have neglected the imaginary parts that could arise when both internal fermions are sufficiently light. Also all $\lambda^{\prime}$-couplings have been assumed to be real in the present analysis.
    ${ }^{4}$ We have used $f_{B_{s}}=230 \mathrm{MeV}$ and $\Delta m_{B_{s}}^{\exp } \simeq 3.9 \times 10^{-12} \mathrm{GeV}$ (which is actually a lower limit) 23.

