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R-parity-violating trilinear couplings and recent neutrino data

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Abstract

The nontrivial structure of the neutrino mass matrix, suggested by the recent Super-Kamiokande results and data from other neutrino experiments, can be reproduced in R-parity-violating supersymmetric theories. This requires sets of products of R-parity-violating trilinear couplings to take appropriately chosen values. It is shown that the existing constraints on these couplings are satisfied by these choices.
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The recent data from the Super-Kamiokande (SK) collaboration provides strong evidence in support of neutrino oscillations as an explanation of the atmospheric anomaly [1]. The observed solar neutrino deficit [2] and the LSND accelerator experiment [3] are also indicative of neutrino oscillations. Put together, all these evidences indicate a nontrivial structure of the neutrino mass matrix. In this paper we show that this mass matrix can be reproduced in the R-parity violating ($\not{R}$) Minimal Supersymmetric Standard Model [4] and requires sets of products of the couplings to take up appropriate values which are consistent with existing constraints.

'R-parity' in supersymmetry refers to a discrete symmetry which follows from the conservation of lepton-number (L) and baryon-number (B). It is defined as $R = (-1)^{(3B+L+2S)}$, where S is the intrinsic spin of the field. R is $+1$ for all standard model particles and -1 for all super-particles. However, B- and L- conservation are not ensured by gauge invariance and hence there is *a priori* no reason to set these couplings to zero.

The most general $\not{R}$ superpotential is given by,

$$W_{\not{R}} = \frac{1}{2}\lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2}\lambda''_{ijk} U^c_i D^c_j D^c_k + \mu_i L_i H_u, \tag{1}$$

where $i, j, k = 1, 2, 3$ are quark and lepton generation indices; L_i and Q_i are $SU(2)$-doublet lepton and quark superfields respectively; E^c_i, U^c_i, D^c_i are $SU(2)$-singlet charged lepton, up- and down-type quark superfields respectively; H_u is the Higgs superfield responsible for the generation of up-type quark masses; λ_{ijk} and λ'_{ijk} are L-violating while λ''_{ijk} are B-violating Yukawa couplings. λ_{ijk} is antisymmetric under the interchange of the first two generation indices, while λ''_{ijk} is antisymmetric under the interchange of the last two. Thus there could be 27 λ', 9 each of λ and λ'' couplings and 3 μ_i parameters. We assume that the generation indices correspond to the flavour basis of fermions. We note at this point that proton stability severely restricts the upper limits on the products of B- and L-violating couplings [5]. Therefore our requirement of 'not-too-small' L-violating couplings for the present analysis implies that the B-violating couplings are either zero or vanishingly small. In any case, the λ''_{ijk} couplings are not of any relevance to the present work.

Stringent constraints on individual L-violating couplings have been placed from the consideration of neutrinoless double beta decay, ν_e-Majorana mass, charged-current universality, $e - \mu - \tau$ universality, ν_μ deep-inelastic scattering, atomic parity violation, τ decays, D and K decays, Z decays, etc. Product

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couplings (two at a time), on the other hand, have been constrained by considering $\mu - e$ conversion, $\mu \to e\gamma$, $b \to s\gamma$, B decays into two charged leptons, $K_L - K_S$ and $B_q - \bar{B}_q$ ($q = d, s$) mass differences, etc. (For a collection of all these limits, see [6]).

In this work, we are concerned with the generation of neutrino masses and mixings. One of the neutrino states can develop a non-zero tree-level mass from the last term of eq. (1). This obtains if μ_α and the vacuum expectation values $\langle L_\alpha \rangle$, where $\alpha = 0, .., 3$, considered as two four component vectors, are not aligned [7]. Here, $L_0 \equiv H_d$ (the Higgs responsible for the mass generation of down-type quarks and charged leptons) and $\mu_0 \equiv \mu$ (note, $\mu H_d H_u$ appears in the R-parity-conserving superpotential). This procedure cannot generate the other terms of the mass matrix. Here we are interested in reproducing the complete mass matrix as masses and mixings of neutrinos are both essential ingredients of our analysis. So we concentrate on the alternative way in which Majorana masses are generated at 1-loop order *via* self-energy diagrams involving λ or λ' couplings (we will discuss this in detail in the following paragraphs). The complete mass matrix can be generated by considering different leptonic flavour indices attached to λ or λ'. For the sake of simplicity, we choose the μ_i terms in (1) to be zero. Although this amounts to a slight loss of generality, dropping these terms helps to examine in isolation the rôle of the trilinear couplings, the focus of our analysis, in reproducing the recent neutrino data. Moreover, the numerical impact of the tree-level contribution induced by the μ_i terms could be tuned to be very small by arranging a perfect or a close alignment between μ_α and $\langle L_\alpha \rangle$ at some high scale. The misalignment that might creep in through renormalisation group running is suppressed by loop factors [8, 9]. As a result, it is possible to arrange that the other contribution, namely, the one induced by trilinear L-violating terms, dominates. Hence for the order of magnitude estimate of the values that we assign on the product of trilinear couplings, dropping the μ_i parameters is not unjustified. We are also not interested in see-saw type contributions to neutrino masses, which involve heavy right-handed neutrinos.

Majorana mass terms for the left-handed neutrinos can be generated through quark-squark loop diagrams (Fig. 1(a)) which involve the λ' couplings in the following way:

$$m_{\nu_{ii'}} \approx \frac{3}{8\pi^2} \lambda'_{ijk} \lambda'_{i'kj} \frac{m_{d_j} \Delta m_k^2(d)}{m_{\tilde{q}}{}^2}. \tag{2}$$

These mass terms can also be generated through lepton-slepton loop diagrams (Fig. 1(b)) which are related to the λ couplings as:

$$m_{\nu_{ii'}} \approx \frac{1}{8\pi^2} \lambda_{ijk} \lambda_{i'kj} \frac{m_{e_j} \Delta m_k^2(l)}{m_{\tilde{l}}{}^2}. \tag{3}$$

In eqs. (2) and (3), $\Delta m^2(f)$ represents the left-right sfermion mixing term which we assume can be parametrized as $\Delta m^2(f) \approx m_f \tilde{m}$, where $\tilde{m}$ is the average squark mass $m_{\tilde{q}}$ and the average slepton mass $m_{\tilde{l}}$ in eqs. (2) and (3) respectively.

Strictly speaking, in eq. (2) some quark mixing angles ought to appear. This is because the $\not{R}$-interactions are written in the flavour basis while the states propagating in the loop diagrams are the mass eigenstates. If the entire Cabibbo-Kobayashi-Maskawa mixing is attributed to the down-type quark sector then the replacement $m_{d_j} \to \Sigma_l |V_{lj}|^2 m_{d_l}$ is needed. On the contrary, if the entire quark mixing is in the up-type sector then no changes are necessary. It also needs to be remarked that at the two vertices d-type quarks of opposite chiralities appear. The mixing of quarks in the right-handed sector cannot be probed *via* the Standard Model interactions and the choice which yields the factor mentioned earlier corresponds to taking identical mixing for both chiralities. In order not to complicate matters unnecessarilly, we have ignored this small difference in the flavour and mass bases in the following.

The mass matrices generated in this way correspond to the flavour-basis of fermions. The expressions above resemble the see-saw m_D^2/M formula to some extent but there are several differences. m_D is the neutrino Dirac mass – in GUT motivated models related to the up-type quark mass – whereas here we have charged lepton or down-type quark masses in the numerator. More importantly, the smallness of the neutrino masses in this picture is due not just to mass ratios but also to the sizes of the $\not{R}$ interactions.

We consider mixing of the three neutrinos as:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{4}$$

where ν_e, ν_μ and ν_τ are the flavour states and ν_1, ν_2 and ν_3 are mass eigenstates with masses m_{ν_1}, m_{ν_2} and m_{ν_3} respectively, which we choose to satisfy the hierarchical mass structure $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. The mixing matrix $\mathcal{U}$ is taken to be real and can be parametrised in terms of three mixing angles θ_{12}, θ_{23} and θ_{13} as

$$\mathcal{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{5}$$

where c and s stand for cosine and sine respectively of the mixing angles.

As is well known, neutrino oscillations depend on two factors: (a) the flavour eigenstates and mass eigenstates do not coincide, and (b) the mass eigenstates are not degenerate. The experimental indications of neutrino oscillations can be used to restrict the possible ranges of the mixing angles and neutrino mass splittings, Δm^2. For example, it has been shown [10] that the recent SK data, along with the CHOOZ [11] and the pre-SK solar neutrino results, can be accommodated in a three neutrino oscillation model at 99% CL, with $\sin^2\theta_{12} = 0.4$, $\sin^2\theta_{23} = 0.5$, $\sin^2\theta_{13} = 0.2$, $\Delta m^2_{32} = m^2_{\nu_3} - m^2_{\nu_2} = 8 \times 10^{-4}$ eV^2, $\Delta m^2_{21} = m^2_{\nu_2} - m^2_{\nu_1} = 1 \times 10^{-4}$ eV^2. Another group [12] claimed a good overall fit to the atmospheric SK data for $\theta_{12} = 37.6^o$, $\theta_{23} = 26.5^o$, $\theta_{13} = 10.3^o$, $\Delta m^2_{32} = 0.4$ eV^2, $\Delta m^2_{21} = 0.0003$ eV^2, including the LSND and solar neutrino results in their analysis. A similar solution is also offered in ref. [13]. It needs to be mentioned that questions about the result in ref. [12] have been raised in ref. [10] partly because matter effects were ignored in the analysis of ref. [12]. Further, the data from the Homestake experiment were also ignored in ref. [12]. We have examined the requirements for the values of $\not{R}$ couplings from *both* these fits [10, 12] and found that those implied by the results in ref. [10] are more stringent than the other one. A similar effort, but restricted to only some individual $\not{R}$ couplings and not products, has been presented in [14] using the results of [12]. We agree with the results of ref. [14] in the appropriate limits. An analysis containing only λ-type couplings along with a neutrino mass hierarchy inverted with respect to ours has been presented in ref. [15].

Once the mixing matrix for the neutrinos and the mass splittings are fixed, the structure of the neutrino mass matrix is determined. Choosing any one of the neutrino masses completely specifies this matrix. Of course, one must bear in mind that there are some experimental limits which must be respected. The (11) component of the Majorana mass matrix in the flavour basis is constrained to be less than 0.46 eV from neutrinoless double beta decay experiments [16]. The masses of ν_μ and ν_τ are constrained to be $m_{\nu_\mu} \leq 0.17$ MeV [17] and $m_{\nu_\tau} \leq 18.2$ MeV [17].

Here we have performed our analysis for $m_{\nu_1} = 0$, 0.01, and 0.1 eV taking $m_{\tilde{l}} = m_{\tilde{q}} = 100$ GeV. In Table 1 we present the values which the products of λ'-type couplings must assume in order to reproduce the neutrino mass matrix. Note that each element of the mass matrix can be generated by several different product couplings. It should be borne in mind that the presented values are *not* upper bounds. The complete mass matrix must be reproduced in order to obtain the correct mass eigenvalues and eigenstates and for each element of the mass matrix any one of the corresponding product couplings listed in Table 1 must achieve the corresponding listed value. It is noteworthy that in several of the cases presented in Table 1 there are strong existing constraints from other processes on the relevant product couplings forbidding the required value. It may be mentioned that in case the sparticle masses and mixings are determined from some model (*e.g.* supergravity) then the specific values of the product couplings that we have presented will be replaced by ranges dependent on the model parameters (*e.g.* $\mu, \tan\beta$ etc.). Further, in obtaining the results presented in Table 1 we have used as input a set of mass-splittings and mixing angles which accommodates the data at 99% CL [10]. This is not a best-fit set and, indeed, at 99% CL for each of the parameters there will be an allowed range[1]. These will, in turn, lead to allowed ranges for the product couplings rather than the specific values listed in Table 1.

Table 2 is a similar list but for products of the λ-type couplings. For ease of presentation, in this Table we have not followed an oft-used convention in which for λ_{ijk} the antisymmetry in i and j is utilised to always choose $i < j$. Note that due to the antisymmetry, this Table has fewer entries than the previous one.

[1]Such a range is unavailable in the published literature. Nevertheless, the given mass differences and mixing angles enable us to examine our primary concern in this paper, *i.e.* whether the $\not{R}$ Yukawa couplings, with assigned values allowed otherwise, are indeed capable of reproducing the observed indication of neutrino masses and mixings.

A somewhat similar analysis, in a two-generation $\nu_\mu - \nu_\tau$ oscillation scenario, has been performed in ref. [18]. Assuming $\Delta m^2_{\nu_\mu \nu_\tau} \simeq m^2_{\nu_\tau}$, it has been shown that λ'_{233} (also, λ'_{333}) $\sim 10^{-5}$ and λ_{233} (also, λ_{232}) $\sim 10^{-4}$ are relevant for explaining the atmospheric neutrino oscillation anomaly. Our analysis is based on three-generation neutrino oscillation and we do not consider the effects of μ_i terms. On account of mainly these two differences, the values of the $\not{R}$ product couplings that we have found are a little larger in comparison with those of ref. [18].

At this point, a discussion of the results of Tables 1 and 2 are in order. We observe that as m_{ν_1} increases, the magnitudes of the $\lambda'_{ijk}\lambda'_{ikj}$ type products increase, whereas $\lambda'_{ijk}\lambda'_{i'kj} (i \neq i')$ type products behave oppositely. The same is true for the products of the λ couplings as well. This fact is easily comprehensible in a two generation scenario. Notice that the values of the product couplings which we find get diluted with increased scalar masses as $\tilde{m}/100$ GeV, where $\tilde{m}$ is the mass of the relevant scalar. Here we list the dependences of the existing bounds for some of the individual couplings (see, ref. [6]) on the squark or slepton masses: $\lambda'_{111} \sim (m_{\tilde{u}_L, \tilde{d}_R}/100 \text{ GeV})^2$, $\lambda'_{11k(k\neq 1)} \sim (m_{\tilde{d}_{kR}}/100 \text{ GeV})$, $\lambda'_{1j1(j\neq 1)} \sim (m_{\tilde{q}_{jL}}/100 \text{ GeV})$, $\lambda'_{1jj} \sim (m_{\tilde{d}_j}/100 \text{ GeV})^{1/2}$, $\lambda'_{21k} \sim (m_{\tilde{d}_{kR}}/100 \text{ GeV})$, $\lambda'_{231} \sim (m_{\tilde{\nu}_{\tau L}}/100 \text{ GeV})$, $\lambda_{133} \sim (m_{\tilde{\tau}}/100 \text{ GeV})^{1/2}$ and all other $\lambda_{ijk} \sim (m_{\tilde{e}_{kR}}/100 \text{ GeV})$. For λ'_{132}, λ'_{22k}, $\lambda'_{23k(k\neq 1)}$, λ'_{31k} and λ'_{33k} the dependences are more complicated. For $\lambda'_{i12}\lambda'_{i21}$-, and $\lambda'_{i13}\lambda'_{i31}$-combinations, on the other hand, it is not necessary to take products of individual couplings. These are constrained from tree level $\Delta S = 2$ and $\Delta B = 2$ processes respectively. The bounds are $\lambda'_{i12}\lambda'_{i21} \lesssim 1 \times 10^{-9}$ $(m_{\tilde{\nu}_L}/100 \text{ GeV})^2$ and $\lambda'_{i13}\lambda'_{i31} \lesssim 8 \times 10^{-8}$ $(m_{\tilde{\nu}_L}/100 \text{ GeV})^2$ [6]. As listed in the Tables, the product couplings relevant for our studies of the neutrino mass matrix are bounded from their contributions to other physical processes. It would not be out of place to stress here that the processes from which these bounds are obtained involve exchanged scalars which are not the same and their masses are generally uncorrelated. Notice that we have followed the usual practice of comparing the magnitudes of product couplings from different processes assuming a benchmark value of 100 GeV for whichever scalar is involved. Needless to say, such a comparison should be made in a guarded way since these scalars could be highly non-degenerate. As an example, if one considers R-parity breaking in gauge-mediated supersymmetry breaking models [19], where squarks are much heavier than sleptons on account of the former's strong coupling dependence in comparison with the latter's weak, all bounds which depend on squark masses become significantly weaker as one cannot admit a squark mass as low as 100 GeV in the phenomenological description of such models.

We make a remark in passing that a heavier neutrino state can in principle decay radiatively into a lighter state *via* graphs involving trilinear L-violating couplings. Such decays are cosmologically troublesome [20]. However, on account of the low mass (order eV) of even the heaviest state and the smallness of the couplings, the decay proceeds at an extremely slow rate so that it does not take place within the present lifetime of the universe (order 10^{18} s). Finally, we point out that although quite a few non-zero L-violating couplings are required to be present simultaneously to reproduce the complete mass matrix in our analysis, they do not, with the kind of numbers they need to assume, trigger any forbidden or highly suppressed process at an unwanted rate. It has to be mentioned that if one attempts to realise R-parity as an extension of the conventional flavour problem in supersymmetry in the context of an abelian [21] or a non-abelian [22] flavour group, many non-zero $\not{R}$-couplings naturally appear together with appropriate suppressions dictated by the flavour symmetry.

In conclusion, we have shown that in supersymmetry it is possible to reproduce the neutrino mass matrix, as determined by the latest experimental data, through loop diagrams which involve products of $\not{R}$ trilinear couplings provided sets of these products take on specific values. All the existing constraints on these parameters are satisfied. Indeed, this might not be the only source of neutrino mass generation of which there is a wide latitude of possibilities existing in the literature. Still our mechanism is an 'existence proof' in support of the observed data. Our effort to relate the smallness of neutrino masses to the smallness of the $\not{R}$ Yukawa couplings might provide an indication of their common ancestral link rooted to some underlying flavour theory.

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Table 1: Values (*not* bounds) of the λ'-type product couplings for different possible ν_1 masses. In deriving our numbers, we have used the results of ref. [10]. The products marked with '*' are obtained by multiplying the upper bounds on the individual couplings.

Mass matrix elements	Combinations	$m_{\nu_1} = 0$ eV	$m_{\nu_1} = 0.01$ eV	$m_{\nu_1} = 0.1$ eV	Existing bounds
M_{11}	$\lambda'_{111}\lambda'_{111}$	$9.7\ \ 10^{-4}$	$1.6\ \ 10^{-3}$	$1.1\ \ 10^{-2}$	$1.2\ \ 10^{-7}$[6]*
	$\lambda'_{112}\lambda'_{121}$	$4.8\ \ 10^{-5}$	$8.2\ \ 10^{-5}$	$5.3\ \ 10^{-4}$	$1\ \ 10^{-9}$[6]
	$\lambda'_{113}\lambda'_{131}$	$1.1\ \ 10^{-6}$	$1.8\ \ 10^{-6}$	$1.2\ \ 10^{-5}$	$8\ \ 10^{-8}$[6]
	$\lambda'_{122}\lambda'_{122}$	$2.4\ \ 10^{-6}$	$4.1\ \ 10^{-6}$	$2.7\ \ 10^{-5}$	$4\ \ 10^{-4}$[6]*
	$\lambda'_{123}\lambda'_{132}$	$5.3\ \ 10^{-8}$	$9.0\ \ 10^{-8}$	$5.8\ \ 10^{-7}$	$1.4\ \ 10^{-2}$[6, 23]*
	$\lambda'_{133}\lambda'_{133}$	$1.1\ \ 10^{-9}$	$2.0\ \ 10^{-9}$	$1.3\ \ 10^{-8}$	$4.9\ \ 10^{-7}$[6]*
$M_{12} = M_{21}$	$\lambda'_{111}\lambda'_{211}$	$1.1\ \ 10^{-3}$	$7.3\ \ 10^{-4}$	$1.4\ \ 10^{-4}$	$5\ \ 10^{-8}$[6]
	$\lambda'_{112}\lambda'_{221}$	$5.5\ \ 10^{-5}$	$3.7\ \ 10^{-5}$	$7.1\ \ 10^{-6}$	$3.6\ \ 10^{-3}$[6]*
	$\lambda'_{113}\lambda'_{231}$	$1.2\ \ 10^{-6}$	$7.9\ \ 10^{-7}$	$1.5\ \ 10^{-7}$	$4.4\ \ 10^{-3}$[6]*
	$\lambda'_{121}\lambda'_{212}$	$5.5\ \ 10^{-5}$	$3.7\ \ 10^{-5}$	$7.1\ \ 10^{-6}$	$3.2\ \ 10^{-3}$[6]*
	$\lambda'_{122}\lambda'_{222}$	$2.8\ \ 10^{-6}$	$1.8\ \ 10^{-6}$	$3.5\ \ 10^{-7}$	$3.6\ \ 10^{-3}$[6]*
	$\lambda'_{123}\lambda'_{232}$	$6.0\ \ 10^{-8}$	$4.0\ \ 10^{-8}$	$7.7\ \ 10^{-9}$	$1.4\ \ 10^{-2}$[6, 23]*
	$\lambda'_{131}\lambda'_{213}$	$1.2\ \ 10^{-6}$	$7.9\ \ 10^{-7}$	$1.5\ \ 10^{-7}$	$3.2\ \ 10^{-3}$[6]*
	$\lambda'_{132}\lambda'_{223}$	$6.0\ \ 10^{-8}$	$4.0\ \ 10^{-8}$	$7.7\ \ 10^{-9}$	$6.1\ \ 10^{-2}$[6]*
	$\lambda'_{133}\lambda'_{233}$	$1.3\ \ 10^{-9}$	$8.6\ \ 10^{-10}$	$1.7\ \ 10^{-10}$	$2.5\ \ 10^{-4}$[6]*
$M_{13} = M_{31}$	$\lambda'_{111}\lambda'_{311}$	$4.5\ \ 10^{-4}$	$4.6\ \ 10^{-4}$	$1.1\ \ 10^{-4}$	$3.5\ \ 10^{-5}$[6]*
	$\lambda'_{112}\lambda'_{321}$	$2.2\ \ 10^{-5}$	$2.3\ \ 10^{-5}$	$5.4\ \ 10^{-6}$	$7.2\ \ 10^{-3}$[6, 23]*
	$\lambda'_{113}\lambda'_{331}$	$4.9\ \ 10^{-7}$	$5.0\ \ 10^{-7}$	$1.2\ \ 10^{-7}$	$9.6\ \ 10^{-3}$[6]*
	$\lambda'_{121}\lambda'_{312}$	$2.2\ \ 10^{-5}$	$2.3\ \ 10^{-5}$	$5.4\ \ 10^{-6}$	$3.5\ \ 10^{-3}$[6]*
	$\lambda'_{122}\lambda'_{322}$	$1.1\ \ 10^{-6}$	$1.2\ \ 10^{-6}$	$2.7\ \ 10^{-7}$	$7.2\ \ 10^{-3}$[6, 23]*
	$\lambda'_{123}\lambda'_{332}$	$2.4\ \ 10^{-8}$	$2.5\ \ 10^{-8}$	$5.9\ \ 10^{-9}$	$1.9\ \ 10^{-2}$[6, 23]*
	$\lambda'_{131}\lambda'_{313}$	$4.9\ \ 10^{-7}$	$5.0\ \ 10^{-7}$	$1.2\ \ 10^{-7}$	$3.5\ \ 10^{-3}$[6]*
	$\lambda'_{132}\lambda'_{323}$	$2.4\ \ 10^{-8}$	$2.5\ \ 10^{-8}$	$5.9\ \ 10^{-9}$	$1.4\ \ 10^{-1}$[6, 23]*
	$\lambda'_{133}\lambda'_{333}$	$5.3\ \ 10^{-10}$	$5.4\ \ 10^{-10}$	$1.3\ \ 10^{-10}$	$3.4\ \ 10^{-4}$[6]*
M_{22}	$\lambda'_{211}\lambda'_{211}$	$1.4\ \ 10^{-3}$	$2.0\ \ 10^{-3}$	$1.1\ \ 10^{-2}$	$8.1\ \ 10^{-3}$[6]*
	$\lambda'_{212}\lambda'_{221}$	$7.0\ \ 10^{-5}$	$1.0\ \ 10^{-4}$	$5.4\ \ 10^{-4}$	$1\ \ 10^{-9}$[6]
	$\lambda'_{213}\lambda'_{231}$	$1.5\ \ 10^{-6}$	$2.2\ \ 10^{-6}$	$1.2\ \ 10^{-5}$	$8\ \ 10^{-8}$[6]
	$\lambda'_{222}\lambda'_{222}$	$3.5\ \ 10^{-6}$	$5.0\ \ 10^{-6}$	$2.7\ \ 10^{-5}$	$3.2\ \ 10^{-2}$[6]*
	$\lambda'_{223}\lambda'_{232}$	$7.6\ \ 10^{-8}$	$1.1\ \ 10^{-7}$	$5.8\ \ 10^{-7}$	$6.5\ \ 10^{-2}$[6]*
	$\lambda'_{233}\lambda'_{233}$	$1.6\ \ 10^{-9}$	$2.4\ \ 10^{-9}$	$1.3\ \ 10^{-8}$	$1.3\ \ 10^{-1}$[6]*
$M_{23} = M_{32}$	$\lambda'_{211}\lambda'_{311}$	$9.9\ \ 10^{-4}$	$8.0\ \ 10^{-4}$	$1.7\ \ 10^{-4}$	$9\ \ 10^{-3}$[6]*
	$\lambda'_{212}\lambda'_{321}$	$5.0\ \ 10^{-5}$	$4.0\ \ 10^{-5}$	$8.6\ \ 10^{-6}$	$1.8\ \ 10^{-2}$[6, 23]*
	$\lambda'_{213}\lambda'_{331}$	$1.1\ \ 10^{-6}$	$8.7\ \ 10^{-7}$	$1.9\ \ 10^{-7}$	$4.3\ \ 10^{-2}$[6]*
	$\lambda'_{221}\lambda'_{312}$	$5.0\ \ 10^{-5}$	$4.0\ \ 10^{-5}$	$8.6\ \ 10^{-6}$	$1.8\ \ 10^{-2}$[6]*
	$\lambda'_{222}\lambda'_{322}$	$2.5\ \ 10^{-6}$	$2.0\ \ 10^{-6}$	$4.3\ \ 10^{-7}$	$2.2\ \ 10^{-1}$[6, 23]*
	$\lambda'_{223}\lambda'_{332}$	$5.4\ \ 10^{-8}$	$4.3\ \ 10^{-8}$	$9.3\ \ 10^{-9}$	$8.6\ \ 10^{-2}$[6]*
	$\lambda'_{231}\lambda'_{313}$	$1.1\ \ 10^{-6}$	$8.7\ \ 10^{-7}$	$1.9\ \ 10^{-7}$	$2.2\ \ 10^{-2}$[6]*
	$\lambda'_{232}\lambda'_{323}$	$5.4\ \ 10^{-8}$	$4.3\ \ 10^{-8}$	$9.3\ \ 10^{-9}$	$1.3\ \ 10^{-1}$[6, 23]*
	$\lambda'_{233}\lambda'_{333}$	$1.2\ \ 10^{-9}$	$9.4\ \ 10^{-10}$	$2.0\ \ 10^{-10}$	$1.7\ \ 10^{-1}$[6]*
M_{33}	$\lambda'_{311}\lambda'_{311}$	$1.9\ \ 10^{-3}$	$2.2\ \ 10^{-3}$	$1.1\ \ 10^{-2}$	$1\ \ 10^{-2}$[6]*
	$\lambda'_{312}\lambda'_{321}$	$9.3\ \ 10^{-5}$	$1.1\ \ 10^{-4}$	$5.4\ \ 10^{-4}$	$1\ \ 10^{-9}$[6]
	$\lambda'_{313}\lambda'_{331}$	$2.0\ \ 10^{-6}$	$2.4\ \ 10^{-6}$	$1.2\ \ 10^{-5}$	$8\ \ 10^{-8}$[6]
	$\lambda'_{322}\lambda'_{322}$	$4.6\ \ 10^{-6}$	$5.5\ \ 10^{-6}$	$2.7\ \ 10^{-5}$	$4\ \ 10^{-2}$[6]*
	$\lambda'_{323}\lambda'_{332}$	$1.0\ \ 10^{-7}$	$1.2\ \ 10^{-7}$	$5.8\ \ 10^{-7}$	$6.1\ \ 10^{-2}$[6, 23]*
	$\lambda'_{333}\lambda'_{333}$	$2.2\ \ 10^{-9}$	$2.6\ \ 10^{-9}$	$1.3\ \ 10^{-8}$	$2.3\ \ 10^{-1}$[6]*

Table 2: Values (*not* bounds) of the λ-type product couplings for different possible ν_1 masses. In deriving our numbers, we have used the results of ref. [10]. The products marked with '*' are obtained by multiplying the upper bounds on the individual couplings.

Mass matrix elements	Combinations	$m_{\nu_1} = 0$ eV	$m_{\nu_1} = 0.01$ eV	$m_{\nu_1} = 0.1$ eV	Existing bounds
M_{11}	$\lambda_{122}\lambda_{122}$	$6.6\ 10^{-6}$	$1.1\ 10^{-5}$	$7.2\ 10^{-5}$	$2.5\ 10^{-3}$[6]*
	$\lambda_{123}\lambda_{132}$	$3.9\ 10^{-7}$	$6.6\ 10^{-7}$	$4.3\ 10^{-6}$	$3\ 10^{-3}$[6]*
	$\lambda_{133}\lambda_{133}$	$2.3\ 10^{-8}$	$3.9\ 10^{-8}$	$2.5\ 10^{-7}$	$9\ 10^{-6}$[6]*
$M_{12} = M_{21}$	$\lambda_{121}\lambda_{212}$	$1.6\ 10^{-3}$	$1.0\ 10^{-3}$	$2.0\ 10^{-4}$	$7\ 10^{-7}$[6]
	$\lambda_{123}\lambda_{232}$	$4.4\ 10^{-7}$	$2.9\ 10^{-7}$	$5.7\ 10^{-8}$	$3\ 10^{-3}$[6]*
	$\lambda_{131}\lambda_{213}$	$9.3\ 10^{-5}$	$6.2\ 10^{-5}$	$1.2\ 10^{-5}$	$3\ 10^{-3}$[6]*
	$\lambda_{133}\lambda_{233}$	$2.6\ 10^{-8}$	$1.7\ 10^{-8}$	$3.4\ 10^{-9}$	$1.8\ 10^{-4}$[6]*
$M_{13} = M_{31}$	$\lambda_{121}\lambda_{312}$	$6.4\ 10^{-4}$	$6.6\ 10^{-4}$	$1.6\ 10^{-4}$	$3\ 10^{-3}$[6]*
	$\lambda_{122}\lambda_{322}$	$3.1\ 10^{-6}$	$3.1\ 10^{-6}$	$7.4\ 10^{-7}$	$3\ 10^{-3}$[6]*
	$\lambda_{131}\lambda_{313}$	$3.8\ 10^{-5}$	$3.9\ 10^{-5}$	$9.2\ 10^{-6}$	$1.8\ 10^{-4}$[6]*
	$\lambda_{132}\lambda_{323}$	$1.8\ 10^{-7}$	$1.9\ 10^{-7}$	$4.4\ 10^{-8}$	$3.6\ 10^{-3}$[6]*
M_{22}	$\lambda_{211}\lambda_{211}$	$4.2\ 10^{-1}$	$6.1\ 10^{-1}$	3.2	$2.5\ 10^{-3}$[6]*
	$\lambda_{213}\lambda_{231}$	$1.2\ 10^{-4}$	$1.7\ 10^{-4}$	$9.1\ 10^{-4}$	$3\ 10^{-3}$[6]*
	$\lambda_{233}\lambda_{233}$	$3.3\ 10^{-8}$	$4.8\ 10^{-8}$	$2.6\ 10^{-7}$	$3.6\ 10^{-3}$[6]*
$M_{23} = M_{32}$	$\lambda_{211}\lambda_{311}$	$3.0\ 10^{-1}$	$2.4\ 10^{-1}$	$5.2\ 10^{-2}$	$3\ 10^{-3}$[6]*
	$\lambda_{212}\lambda_{321}$	$1.4\ 10^{-3}$	$1.1\ 10^{-3}$	$2.5\ 10^{-4}$	$3\ 10^{-3}$[6]*
	$\lambda_{231}\lambda_{313}$	$8.4\ 10^{-5}$	$6.7\ 10^{-5}$	$1.5\ 10^{-5}$	$1.8\ 10^{-4}$[6]*
	$\lambda_{232}\lambda_{323}$	$4.0\ 10^{-7}$	$3.2\ 10^{-7}$	$6.9\ 10^{-8}$	$3.6\ 10^{-3}$[6]*
M_{33}	$\lambda_{311}\lambda_{311}$	$5.6\ 10^{-1}$	$6.6\ 10^{-1}$	3.2	$3.6\ 10^{-3}$[6]*
	$\lambda_{312}\lambda_{321}$	$2.7\ 10^{-3}$	$3.2\ 10^{-3}$	$1.5\ 10^{-2}$	$3.6\ 10^{-3}$[6]*
	$\lambda_{322}\lambda_{322}$	$1.3\ 10^{-5}$	$1.5\ 10^{-5}$	$7.3\ 10^{-5}$	$3.6\ 10^{-3}$[6]*

Figure 1: The one loop diagrams contributing to Majorana mass terms for the left-handed neutrinos. Figures 1(a) and 1(b) involve λ'- and λ-type couplings respectively.