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# BOUNDS ON POLARIZATION IN $\pi N$ SCATTERING FROM ISOTOPIC SPIN INVARIANCE

G.V. Dass, J. Froyland, F. Halzen, André Martin,
C. Michael and S.M. Roy *)

CERN - Geneva

## A B S T R A C T

Isotopic spin bounds on the polarization in $\pi^- p \to \pi^0 n$ scattering in terms of the polarizations in $\pi^+ p \to \pi^+ p$ and $\pi^- p \to \pi^- p$ scattering and the unpolarized differential cross-sections for $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$, and $\pi^- p \to \pi^0 n$ scattering are derived, and compared with recent experimental data. Similar bounds on the P, R and A parameters for elastic scattering are also given.

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*) On leave from the Tata Institute of Fundamental Research, Bombay.

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Recent experiments on polarization in $\pi^- p \to \pi^0 n$ scattering [1] at pion lab. momenta of 5 and 8 GeV/c show large positive polarization at momentum transfers t between -0.3 and -0.5 $(\mathrm{GeV/c})^2$ in contradiction with the negative polarization expected there on the basis of some phenomenological models [2]. It would obviously be interesting if one could make some "model-independent" statements about the expected polarization.

We point out here that isotopic spin invariance alone implies the following bounds on polarization in $\pi N$ scattering,

$$\left[\sqrt{\sigma_+(1+P_+)} - \sqrt{\sigma_-(1+P_-)}\right]^2 \leq 2\sigma_0(1+P_0) \leq \left[\sqrt{\sigma_+(1+P_+)} + \sqrt{\sigma_-(1+P_-)}\right]^2, \quad (1)$$

and

$$\left[\sqrt{\sigma_+(1-P_+)} - \sqrt{\sigma_-(1-P_-)}\right]^2 \leq 2\sigma_0(1-P_0) \leq \left[\sqrt{\sigma_+(1-P_+)} + \sqrt{\sigma_-(1-P_-)}\right]^2, \quad (2)$$

valid at any energy and momentum transfer, where $\sigma_+$, $\sigma_-$ and $\sigma_0$ denote respectively the unpolarized differential cross-sections for $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$ and $\pi^- p \to \pi^0 n$ scattering, and $P_+$, $P_-$ and $P_0$ denote the polarizations in $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$ and $\pi^- p \to \pi^0 n$ scattering respectively. The relations (1) and (2) follow from the triangular inequalities applied to the usual pion-nucleon amplitudes [3] (f+ig) and (f-ig) respectively. We now summarize our conclusions obtained from these bounds.

1. Bounds on $P_0$

a) Near-forward region.

We have plotted the bounds on $P_0$ predicted by the inequalities (1) and (2) using experimental values [4] for $\sigma_+$, $\sigma_-$, $\sigma_0$, $P_+$ and $P_-$ in Figs. 1 and 2. The recent data [1] on $P_0$ at 5 GeV/c are compared in Fig. 1 with the bounds at 6 GeV/c because the quality of the interpolations used to obtain $\sigma_+$, $\sigma_-$ and $\sigma_0$ at the same energy is optimized [5] at 6 GeV/c. Clearly, the data are consistent with these bounds. Barring very strong energy dependence between 5 and 6 GeV/c, large negative polarizations for $0.2(\mathrm{GeV/c})^2 \lesssim (-t) \lesssim 0.4(\mathrm{GeV/c})^2$ are ruled out by the bounds. Absorption model calculations for $P_0$ have been reviewed by Carreras and White [6]. Finite energy sum rules [7] and amplitude analysis [5] indicate $R_p \equiv (\nu B/A')_P \approx (\nu B/A')_{P'} \approx 1$ [in the notation of Ref. 6)]. The prediction of the model for $R_p \approx 1$ is given in Fig. 2 of Ref. 6), and violates not only experimental data [1] but also our isospin bounds. It should be pointed out that the phenomenological models violating the bounds of Fig. 1 do of course have isotopic spin invariance built in; but they simply do not fit $\sigma_+$, $\sigma_-$, $\sigma_o$, $P_+$ and $P_-$ well enough. This has also been noted recently by Halzen and Michael, Ref. 5).

The experimental fact that $P_o$ is generally positive where $\sigma_o$ is small, is qualitatively understood from the left-hand side of the inequality (1), and the experimental facts that $0 < P_+ \approx (P_+ - P_-)/2$, $|\sigma_+ - \sigma_-| \ll (\sigma_+ + \sigma_-)$ and $\sigma_o \ll \sigma_+$ for $(-t) \sim 0.5\ (GeV/c)^2$, in the energy region we have considered.

b) Near-backward region

The recent determinations [8] of $P_\pm$ at 6 GeV/c and the fact that good data [9] on $\sigma_+$, $\sigma_-$ and $\sigma_o$ exist at the same energy makes a similar analysis possible in the backward direction. The bounds on $P_o$ at 6 GeV/c and $|u| \lesssim 0.3 (GeV/c)^2$ are not very restrictive; but for $0.45\ (GeV/c)^2 \lesssim -u \lesssim 0.65\ (GeV/c)^2$ the bounds rule out large positive values of $P_o$. For example, at $u = -0.63\ (GeV/c)^2$, we find $-0.77 \pm 0.16 \leq P_o \leq 0.04 \pm 0.25$.

c) $0.625\ (GeV/c)^2 \lesssim -t \lesssim 1.5\ (GeV/c)^2$

In this region the data available on $\sigma_+$ and $\sigma_-$ are not accurate enough for our purpose. Now, Halzen and Michael [5] have found that the Barger-Phillips amplitudes [7] are quite close to those deduced from experimental data for $(-t) \lesssim 0.625\ (GeV/c)^2$ at 6 GeV/c. This encourages us to use them to interpolate the experimental data on $\sigma_+$ and $\sigma_-$ at larger momentum transfers. The bounds then obtained for $P_o$ are shown by the dotted curves in Fig. 1b. We have no quantitative estimate of the reliability of this interpolation, and hence cannot indicate the errors on the dotted curves.

## 2. Bounds on R and A parameters

Similar restrictions on the R and A parameters [10] in $\pi N$ scattering follow by applying the triangular inequalities to the pairs of amplitudes (f,g) and (f+g, f-g) respectively. We obtain then the inequalities (1) and (2) with the following replacements

$$\begin{array}{ll} (R) & P_+, P_-, P_o \rightarrow R_+, R_-, R_o \\ (A) & P_+, P_-, P_o \rightarrow A_+, A_-, A_o \end{array} , \qquad (3)$$

where the subscripts +, −, o denote the reactions $\pi^+ p \rightarrow \pi^+ p$, $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^o n$ respectively. These inequalities can be used to set bounds on $R_o$ and $A_o$ when sufficiently accurate data on $R_\pm$ and $A_\pm$ become available.

## 3. Bounds on the P, R and A parameters in terms of unpolarized cross-sections alone

We have also derived isotopic spin bounds on the P, R and A parameters in terms of unpolarized cross-sections alone. For example, we obtain the result valid at all energies and scattering angles

$$\left| \frac{X_+ - X_-}{2} \right| \leq \sqrt{\frac{\sigma_0}{\sigma_+ \sigma_-} \left[ \sigma_+ + \sigma_- - \sigma_0 - \frac{(\sigma_+ - \sigma_-)^2}{4\sigma_0} \right]} , \qquad (4)$$

where

$$X_{\frac{+}{0}} \equiv P_{\frac{+}{0}} , \text{or } R_{\frac{+}{0}} , \text{or } A_{\frac{+}{0}} . \qquad (5)$$

We may remark that the bounds given by (4) and (5) are the best possible ones, given only the unpolarized cross-sections, because one can construct amplitudes which saturate these bounds. The square bracket in (4), and hence the upper bound, vanishes whenever the unpolarized cross-sections saturate the bounds on them due to the triangular inequalities. Since such a saturation has recently [11] been noticed in the near-backward region for lab. momenta up to about 1.5 GeV/c, we expect that the above bounds will be very restrictive in this region.

Further, in the near-forward region we find that these bounds on $(P_+ - P_-)$ are very close to the experimental values (Table I). The bounds on $(R_+ - R_-)$ and $(A_+ - A_-)$ in this table are to be tested against future experiments.

Białas and Svensson [12] and Le Bellac [13] obtained from some high-energy models the bound

$$\left| \frac{P_+ - P_-}{2} \right| \leq 2 \sqrt{\frac{\sigma_0}{(\sigma_+ + \sigma_-)}} , \qquad (6)$$

and had hoped to test their models in this way. However, the inequality (4) with $X_\pm = P_\pm$, follows from isospin alone and implies their bound (6). Hence, it cannot discriminate between models.

## 4. Improved bounds on the P, R and A parameters for elastic scattering

The above bounds on $X_\pm$ in terms of unpolarized cross-sections can be improved if we use also the data on $X_\mp$ and $X_0$. For example, we obtain

$$\left| \pm \left( \frac{X_+ - X_-}{2} \right) + \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \left\{ 1 \pm \left( \frac{X_+ + X_-}{2} \right) - \frac{2\sigma_0}{\sigma_+ + \sigma_-} (1 \pm X_0) \right\} \right|$$
$$\leqslant 4 \left[ \frac{\sigma_0 \sigma_+ \sigma_-}{(\sigma_+ + \sigma_-)^3} (1 \pm X_0) \left\{ 1 \pm \left( \frac{X_+ + X_-}{2} \right) - \frac{\sigma_0}{\sigma_+ + \sigma_-} (1 \pm X_0) \right\} \right]^{1/2} , \qquad (7)$$

which gives quite restrictive bounds on $(X_+ - X_-)/2$ in the regions where $\sigma_0/(\sigma_+ + \sigma_-)$ is small, as illustrated in Table I.

## 5. Comparison with triangular inequalities for unpolarized cross-sections

The triangular inequalities for the unpolarized $\pi N$ cross-sections alone have been checked against experiment recently, with interesting results [11), 14)]. We note that the inequalities (1) and (2) together are more restrictive in the sense that they imply the corresponding inequalities for the unpolarized cross-sections. One may also remark that if the data show a violation of the inequality for the unpolarized cross-sections, it necessarily implies that the upper bound for $P_0$ from the relations (1) and (2) is smaller than the lower bound for $P_0$. However, the violation of (1) and (2) does not necessarily imply the violation of the triangular inequalities for the unpolarized cross-sections [e.g., at 5.2 GeV/c at $t = -0.225$ $(\text{GeV/c})^2$ in Fig. 2].

It is also interesting to test these inequalities at lower energies, where it can provide valuable information for phase-shift analyses; we know in advance that the bounds on polarization, so obtained, would be restrictive because of the near saturation of the isotopic spin bounds on the unpolarized differential cross-sections near the backward direction [11)], and on the unpolarized integrated cross-sections [14)].

We thank R.J.N. Phillips for informing us that applications of the triangular inequalities to obtain bounds on the polarization parameters in $\pi N \to K \Sigma$ [15)] and in $NN \to NN$ [16)] were considered before.

TABLE I

| Lab. momentum (GeV/c) | (-t) $(GeV/c)^2$ | Experimental value [4] of $(P_+-P_-)/2$ | Upper bound on $\lvert(P_+-P_-)/2\rvert$, $\lvert(R_+-R_-)/2\rvert$, and $\lvert(A_+-A_-)/2\rvert$ from Eq. (4) | Upper bound on $(P_+-P_-)/2$ from Eq. (7) | Lower bound on $(P_+-P_-)/2$ from Eq. (7) |
|---|---|---|---|---|---|
| 5.2 | 0.275 | 0.17 ± 0.02 | 0.19 ± 0.02 | | |
| 6.0 | 0.125 | 0.17 ± 0.01 | 0.20 ± 0.01 | 0.18 ± 0.04 | -0.17 ± 0.04 |
| 6.0 | 0.25 | 0.15 ± 0.01 | 0.18 ± 0.01 | 0.17 ± 0.04 | -0.08 ± 0.04 |
| 6.0 | 0.375 | 0.11 ± 0.01 | 0.14 ± 0.02 | 0.13 ± 0.04 | -0.03 ± 0.04 |
| 6.0 | 0.50 | 0.04 ± 0.01 | 0.09 ± 0.04 | 0.07 ± 0.04 | +0.01 ± 0.04 |
| 10.0 | 0.122 | 0.13 ± 0.01 | 0.16 ± 0.01 | | |
| 13.3 | 0.12 | *0.14 ± 0.03 | 0.15 ± 0.01 | | |
| 13.3 | 0.45 | *0.06 ± 0.02 | 0.07 ± 0.03 | | |

Bounds on $(P_+-P_-)/2$ given by the relations (4) and (7) are compared with experimental data. The asterisks in the last two rows indicate that the polarization data are at 14.0 GeV/c. The bounds on $\lvert(R_+-R_-)/2\rvert$ and $\lvert(A_+-A_-)/2\rvert$ given in Column 4 are to be compared with future experiments.

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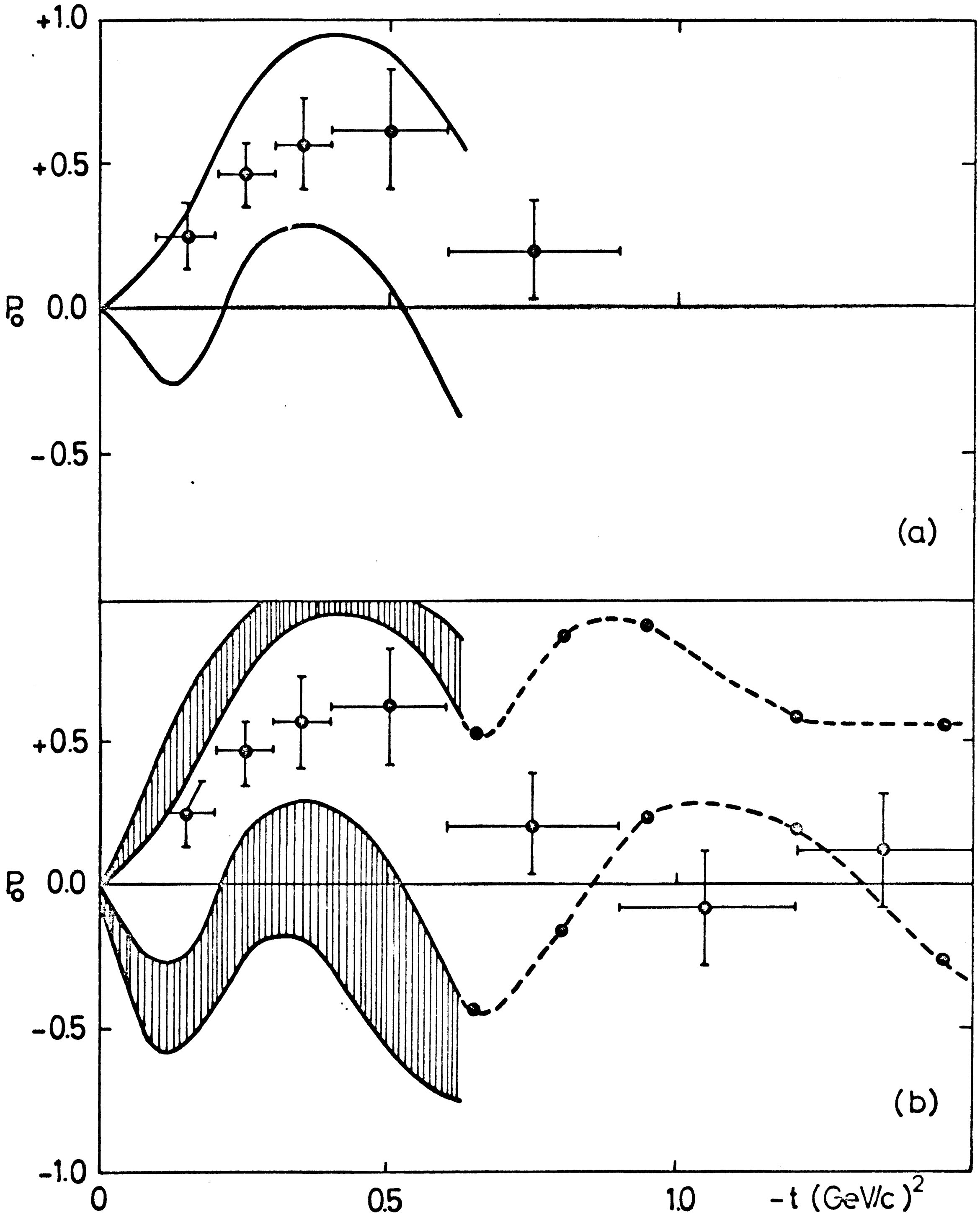
FIGURE CAPTIONS

Figure 1a : Upper and lower bounds (solid lines) on $\pi^- p \to \pi^0 n$ polarization at 6 GeV/c obtained from isospin invariance are plotted together with data at 5 GeV/c.

Figure 1b : Same as Fig. 1a. The shaded area shows an estimate of the errors on the isospin bounds. Beyond $t = -0.625$ $(\text{GeV/c})^2$, Barger and Phillips amplitudes [8] instead of data were used to estimate $\sigma_+$ and $\sigma_-$. The bounds so obtained are given by the dashed curves.

Figure 2 : Isospin bounds on $\pi^- p \to \pi^0 n$ polarization at $p_{lab} = 5.21$, 6.00, 10.00 and 13.3 GeV/c. The upper bounds are marked with black circles and the lower bounds with open circles. The straight-lines, used to join the points, are simply to guide the eye.




ISOSPIN BOUNDS ON $\pi^- p \to \pi^0 n$ POLARIZATION $P_0$ AT 6 GeV/c
DATA AT 5 GeV/c

FIG. 1

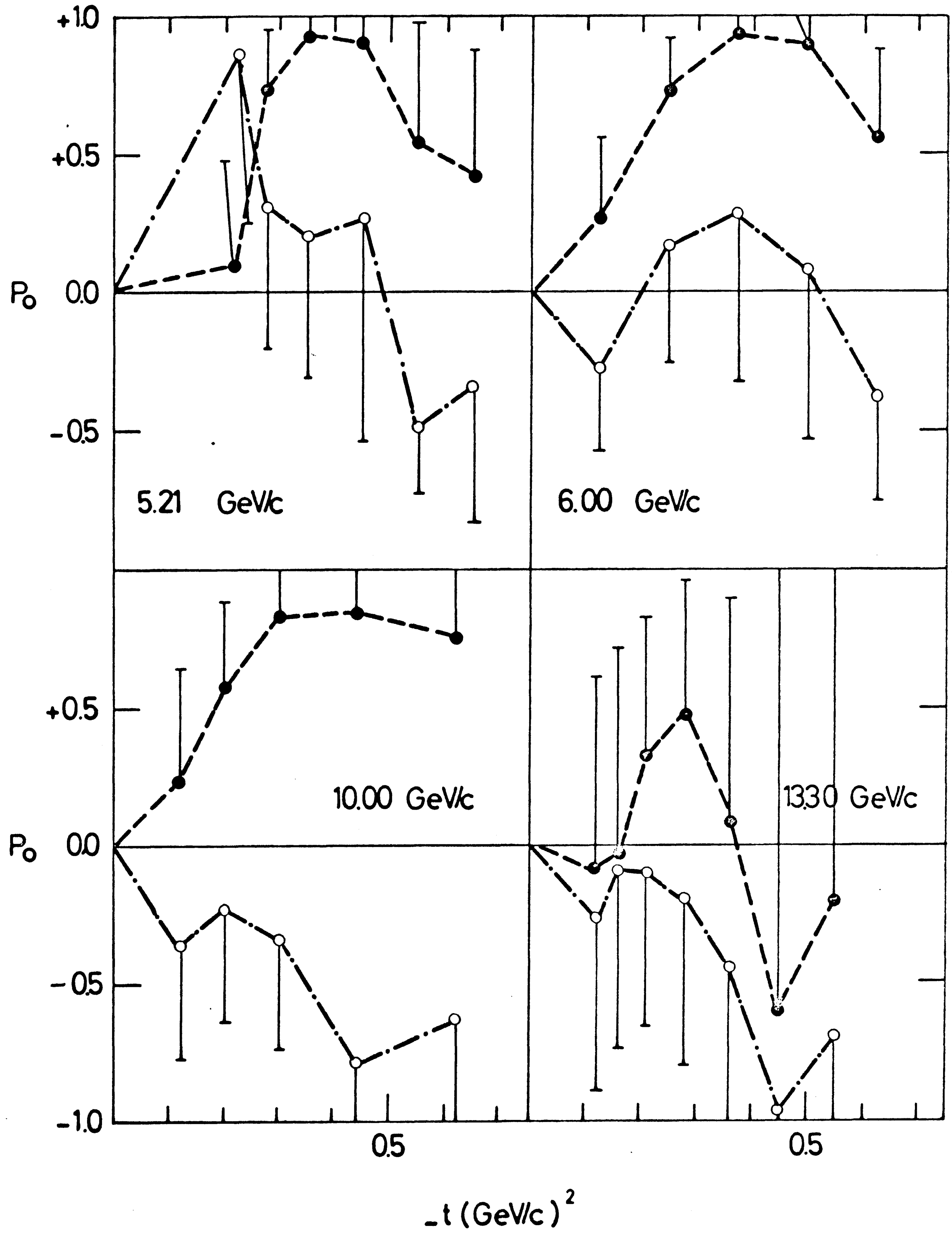

+1.0
+0.5
0.0
-0.5
-1.0
P₀
5.21 GeV/c
6.00 GeV/c
10.00 GeV/c
13.30 GeV/c
0.5
0.5
-t (GeV/c)²

FIG. 2