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AN UPPER BOUND ON POMERANCHUK THEOREM VIOLATION IN KN SCATTERING

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A B S T R A C T

An upper bound on the asymptotic total cross-section difference $[\sigma^{K^-n}_{tot} - \sigma^{K^+n}_{tot}]$ in terms of the integrated cross-section for $K^o_L p \rightarrow K^o_S p$ scattering is derived and shown to be almost saturated by presently available data.

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As soon as the IHEP-CERN measurement [1] of total cross-sections indicated that there might be a violation of the Pomeranchuk theorem in the K nucleon system, it was realized [2] that investigation of the forward K_S^0 regeneration amplitude could provide further information on this possible violation. In this paper, we point out the existence of an upper bound on the magnitude of this violation, given in terms of the integrated K_S^0 regeneration cross-section from hydrogen, and compare this bound with existing experimental data.

The violation of the Pomeranchuk theorem is reflected in this case by the magnitude of the difference $[\sigma_{tot}^{\bar{K}^0 p} - \sigma_{tot}^{K^0 p}]$ at high energies. Let us assume that this difference tends to a finite limit at high-energy, and define

$$\Delta\sigma \equiv \lim_{s\to\infty} \left[\sigma_{tot}^{\bar{K}^0 p}(s) - \sigma_{tot}^{K^0 p}(s) \right] , \qquad (1)$$

where $s^{\frac{1}{2}}$ is the energy in the centre-of-mass system. We do not need any assumption about the behaviour of the sum of the total cross-sections. Recently an upper bound on $\Delta\sigma$ in terms of the integrated elastic cross-sections for K^-p and K^+p scattering has been derived [3], namely

$$(\Delta\sigma)^2 \leq \frac{\pi^3}{4m_\pi^2} \lim_{s\to\infty} \left[\left(\sigma_{el}^{K^-p}(s)\right)^{1/2} + \left(\sigma_{el}^{K^+p}(s)\right)^{1/2} \right]^2 , \qquad (2)$$

where m_π is the pion mass, assuming the elastic cross-sections above to tend to finite limits. In this paper we derive the following upper bound on $(\Delta\sigma)^2$ in terms of the integrated $K_L^0 p \to K_S^0 p$ cross-section σ_{reg} :

$$(\Delta\sigma)^2 \leq \frac{\pi^3}{m_\pi^2} \operatorname*{Lim\,Sup}_{s\to\infty} \left[\sigma_{reg}(s) \right] . \qquad (3)$$

We note that in (3), $\sigma_{reg}(s)$ is not assumed to tend to a limit and may have oscillations as $s \to \infty$ so long as it satisfies the bound (3). The derivation is rigorous, in the sense that it relies only on unitarity, on analyticity which can be established from the axioms of quantum field theory, and on crossing symmetry. In the form stated above, the proof of (3) does not rely on the assumption of isotopic spin invariance of the strong interactions ; if we do assume charge symmetry, we may also write, in place of (1),

$$\Delta\sigma = \lim_{s\to\infty} \left[\sigma_{tot}^{K^-n}(s) - \sigma_{tot}^{K^+n}(s) \right] . \qquad (4)$$

The new bound given by (3) and (4) can be seen to improve the previous result given by (2) by about two orders of magnitude when presently available data are used. The chief purpose of the present work is, by comparing the new bound with available experimental data, to point out that the measurement of $\sigma_{reg}(s)$ at higher energies could play a crucial role in limiting the possibility of Pomeranchuk theorem violation.

Proof of the bound

The method follows closely Ref. 3). Let $T(s,\theta)$ denote the helicity non-flip $K_L^0 p \to K_S^0 p$ amplitude, where θ is the scattering angle in the centre-of-mass, with the normalization specified by

$$\left(\frac{d\sigma}{d\Omega}\right)_{reg}^{non-flip} = \left| \frac{T(s,\theta)}{\sqrt{s}} \right|^2 . \qquad (5)$$

Neglecting CP violation, we can write

$$T(s,\theta) = \frac{1}{2} \left[T^{K^0p\to K^0p}(s,\theta) - T^{\bar{K}^0p\to\bar{K}^0p}(s,\theta) \right] . \qquad (6)$$

The forward dispersion relations then imply [3),4)]

$$\lim_{s \to i\infty} \left[\frac{T(s,0)}{s \ln s} \right] = \frac{\Delta\sigma}{8\pi^2} \qquad (7)$$

Furthermore, the forward amplitude is subject to the following upper bound in the physical region [3),5)]

$$\left| \frac{T(s+i\epsilon,0)}{s \ln s} \right| \underset{s\to\infty}{\leq} \frac{1}{8\sqrt{\pi}\, m_\pi} \left[\sigma_{reg}^{non\text{-}flip}(s) \right]^{1/2} , \qquad (8)$$

where we have defined

$$\sigma_{reg}^{non\text{-}flip} = \int d\Omega \left(\frac{d\sigma}{d\Omega} \right)_{reg}^{non\text{-}flip} . \qquad (9)$$

From the odd crossing symmetry and the real analyticity of $T(s,0)$, it follows that $|T(-s+i\epsilon,0)/s \ln s|$ also has the upper bound given by the right-hand side of (8). Using the Phragmen-Lindelöf theorem [6)] applied to the function $|T(s,0)/s \ln s|$ which is bounded by const $\times$ s for $s \to \infty$ in complex directions [7)], and Eqs. (7) and (8), we obtain the bound

$$(\Delta\sigma)^2 \leq \frac{\pi^3}{m_\pi^2} \operatorname{Lim\,Sup}_{s\to\infty} \left[\sigma_{reg}^{non\text{-}flip}(s) \right] . \qquad (10)$$

Noticing that $\sigma_{reg}^{non\text{-}flip}(s) \leq \sigma_{reg}(s)$, we obtain the bound (3).

Use of the bound

In order to make use of the bound (3) one would have to assume that, at high energy, σ_{reg} is always at least as large as its asymptotic value, or, if it has indefinite oscillations, at least as large as its lim sup. The basis for this assumption is the observation [8)] that σ_{reg} is in fact falling rapidly with energy, and that, unlike the forward regeneration cross-section, σ_{reg} need not rise at high-energy even in the case of Pomeranchuk theorem violation.

If one believes that $\sigma^{K^-n}_{tot}$ and $\sigma^{K^+n}_{tot}$ have attained their asymptotic values at the highest energies at which they have been measured, it would appear that $\Delta\sigma$ had a value of about 2.25 mb [9]. From the inequality (3), this would require a lower limit of 8 μb for σ_{reg}. In the Figure, we have indicated the measured values of σ_{reg} from the experiment of Brody et al. [8] which extends up to about p_{lab} = 8 GeV/c. As can be seen, these data are all above the lower bound ; however, if the trend of the data, as indicated by the dashed line, continues to higher energy, the bound will be attained at p_{lab} = 12 GeV/c, and violated by a factor of 20 at p_{lab} = 50 GeV/c.

Preliminary results from the Dubna-Serpukhov collaboration [10] indicate that the decrease of the forward regeneration cross-section does continue, at least up to p_{lab} = 36 GeV/c. If one arbitrarily assumes the same angular distribution at this energy as is seen at the highest energies in the experiment of Ref. 8), then one obtains, from the forward cross-section measurement of Ref. 10), a value of σ_{reg} which is indicated by the open data point in the Figure. This point should not be taken very seriously, both because the 36 GeV/c data is preliminary and also because there is no angular distribution available at this energy. If there is shrinkage above p_{lab} = 8 GeV/c, then the 36 GeV/c point would lie lower. In fact, there is a considerable amount of shrinkage seen in the data of Ref. 8), which is why the extrapolation of σ_{reg} from that experiment lies considerably below the open point in the Figure.

In the derivation of the inequality (3) we have neglected the corrections due to electromagnetism. An estimate using a method similar to that of Sarma and Sehgal [11] suggests that the one-photon exchange contribution to σ_{reg} is of the order of 0.05 μb ; hence we expect that the contribution to σ_{reg} due to interference between the strong and the electromagnetic amplitudes would also be small.

In conclusion, we have shown that if the observed rapid decrease of σ_{reg} continues to higher energies, then the value of $\Delta\sigma$ must be considerably less than 2.25 mb. Conversely, if $\Delta\sigma$ be as large as measurements of total cross-sections indicate that it might, then we would expect the energy dependence of σ_{reg} to flatten out at a value not much less than 10 μb by about p_{lab} = 12 GeV/c.

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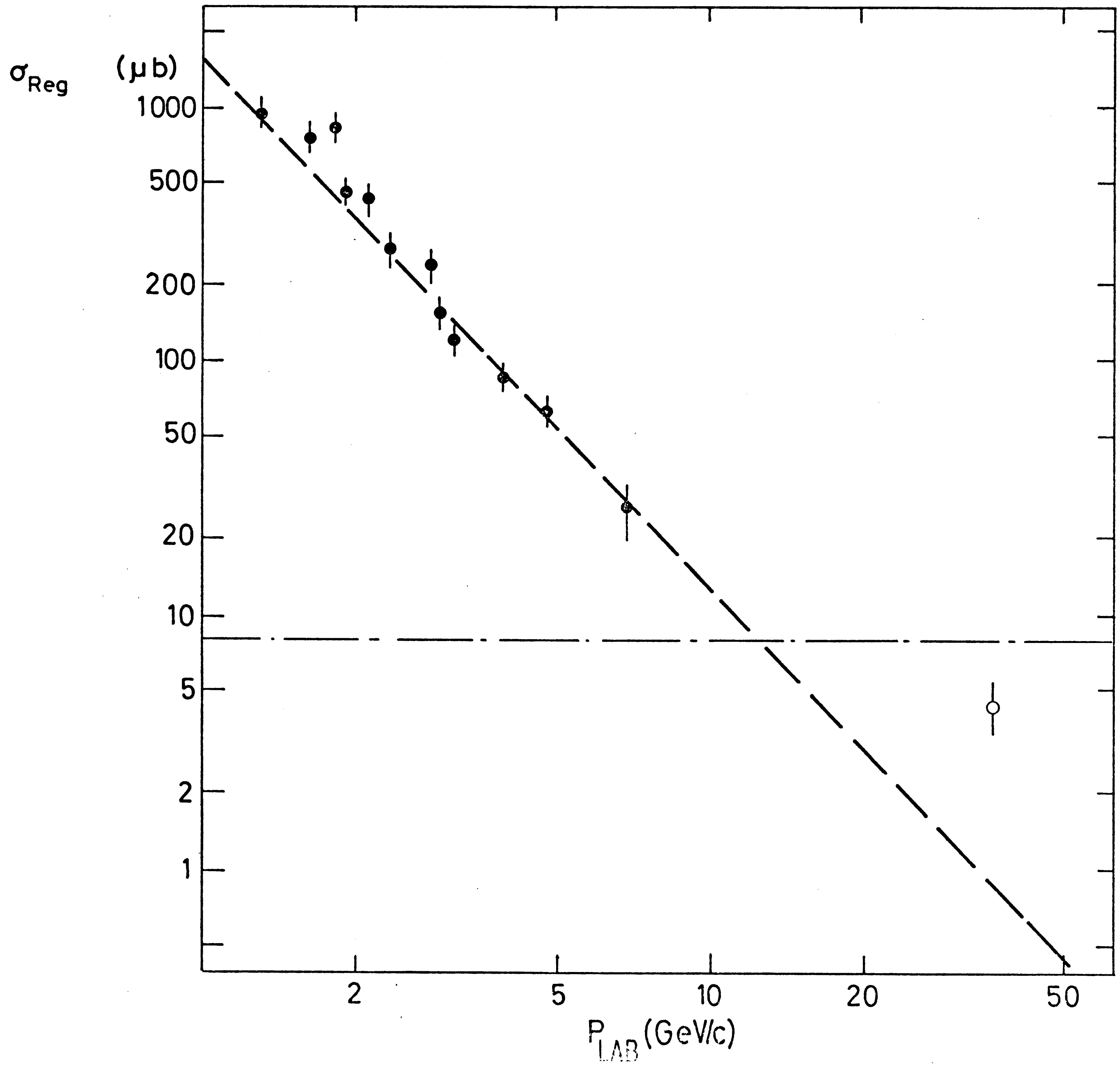
FIGURE CAPTION

The $K_L^o p \to K_S^o p$ integrated cross-section, compared with its lower bound of 8 μb from (3), assuming that $\sigma_{tot}^{K^-n} - \sigma_{tot}^{K^+n} \to$ $\to$ 2.25 mb. Solid data points are from Ref. 8) ; the open point is explained in the text.


σ_Reg (μb)
1000
500
200
100
50
20
10
5
2
1
2
5
10
20
50
P_LAB (GeV/c)