

# Lower Bound on the Pseudoscalar Mass in the Minimal Supersymmetric Standard Model

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## Abstract

In the Higgs sector of the Minimal Supersymmetric Standard Model, the mass of the pseudoscalar  $A$  is an independent parameter together with  $\tan\beta \equiv v_2/v_1$ . If  $m_A$  is small, then the process  $e^+e^- \rightarrow h + A$  is kinematically allowed and is suppressed only if  $\tan\beta$  is small. On the other hand, the mass of the charged Higgs boson is now near  $M_W$ , and the decay  $t \rightarrow b + h^+$  is enhanced if  $\tan\beta$  is small. Since the former has not been observed, and the branching fraction of  $t \rightarrow b + W$  cannot be too small (by comparing the experimentally derived  $t\bar{t}$  cross section from the leptonic channels with the theoretical prediction), we can infer a phenomenological lower bound on  $m_A$  of at least 60 GeV for all values of  $\tan\beta$ .

The most studied extension of the standard  $SU(2) \times U(1)$  electroweak gauge model is that of supersymmetry with the smallest necessary particle content. In this Minimal Supersymmetric Standard Model (MSSM), there are two scalar doublets  $\Phi_1 = (\phi_1^+, \phi_1^0)$  and  $\Phi_2 = (\phi_2^+, \phi_2^0)$ , with Yukawa interactions  $\overline{(u, d)}_L d_R \Phi_1$  and  $\overline{(u, d)}_L u_R \tilde{\Phi}_2$ , respectively, where  $\tilde{\Phi}_2 = i\sigma_2 \Phi_2^* = (\overline{\phi_2^0}, -\phi_2^-)$ . The Higgs sector of the MSSM has been studied in great detail[1] and it is a current topic of intensive experimental and theoretical scrutiny.[2] There are five physical Higgs bosons in the MSSM: two neutral scalars ( $h$  and  $H$ ), one neutral pseudoscalar ( $A$ ), and two charged ones ( $h^\pm$ ). Their masses and couplings to other particles are completely determined up to two unknown parameters which are often taken to be  $m_A$  and  $\tan \beta \equiv v_2/v_1$ , where  $v_i$  is the vacuum expectation value of  $\phi_i^0$ .

In the following, we will show that  $m_A > 60$  GeV for all values of  $\tan \beta$ . Our conclusion is based on a combination of theoretical and experimental inputs from a number of different observations which have become available recently.

In the MSSM, the pseudoscalar Higgs boson  $A$  and the charged Higgs bosons  $h^\pm$  are given by analogous expressions, namely

$$A = \sqrt{2}(\sin \beta \text{Im} \phi_1^0 - \cos \beta \text{Im} \phi_2^0), \quad (1)$$

$$h^\pm = \sin \beta \phi_1^\pm - \cos \beta \phi_2^\pm. \quad (2)$$

At tree level, their masses are related by  $m_{h^\pm}^2 = m_A^2 + M_W^2$ . The mass-squared matrix spanning the two neutral scalar Higgs bosons  $\sqrt{2}\text{Re}\phi_{1,2}^0$  is given by

$$\mathcal{M}^2 = \begin{bmatrix} m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & -(m_A^2 + M_Z^2) \sin \beta \cos \beta \\ -(m_A^2 + M_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta + \epsilon / \sin^2 \beta \end{bmatrix}. \quad (3)$$

In the above,  $\epsilon$  is the leading radiative correction[6] due to the  $t$  quark:

$$\epsilon = \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2} \ln \left( 1 + \frac{\tilde{m}^2}{m_t^2} \right), \quad (4)$$

where  $\tilde{m}$  is the mass parameter for the supersymmetric scalar quarks.

Let us take  $m_A = 0$  and rotate  $\mathcal{M}^2$  to the basis spanned by

$$h_1 = \sqrt{2}(\sin \beta \text{Re}\phi_1^0 - \cos \beta \text{Re}\phi_2^0), \quad h_2 = \sqrt{2}(\cos \beta \text{Re}\phi_1^0 + \sin \beta \text{Re}\phi_2^0). \quad (5)$$

We get[3]

$$\mathcal{M}^2 = \begin{bmatrix} M_Z^2 \sin^2 2\beta + \epsilon \cot^2 \beta & -M_Z^2 \sin 2\beta \cos 2\beta + \epsilon \cot \beta \\ -M_Z^2 \sin 2\beta \cos 2\beta + \epsilon \cot \beta & M_Z^2 \cos^2 2\beta + \epsilon \end{bmatrix}. \quad (6)$$

It is well-known that in this basis, the  $h_1 ZZ$  and  $h_2 AZ$  couplings are absent, hence the nonobservation of  $e^+e^- \rightarrow h + A$  does not rule out any value of  $m_A$  if  $\tan \beta$  is small enough[4]. In this limit, the eigenstates of  $\mathcal{M}^2$  are essentially  $h_1$  and  $h_2$ . If  $h \simeq h_1$ , then it is too heavy to be produced. If  $h \simeq h_2$ , then its coupling to  $A$  is too small to have a measurable branching fraction. Note that  $\epsilon \simeq M_Z^2$ , *i.e.* (91 GeV)<sup>2</sup>, for  $m_t = 175$  GeV and  $\tilde{m} = 1$  TeV.

From the nonobservation of  $e^+e^- \rightarrow h + Z$  where the  $Z$  boson may be either real or virtual and the nonobservation of  $e^+e^- \rightarrow h + A$ , where  $h$  is an arbitrary linear combination of  $h_1$  and  $h_2$ , it is possible to obtain the MSSM exclusion region in the  $m_A - \tan \beta$  plane. One such detailed analysis[5] using only LEP1 data collected at the  $Z$  resonance shows that  $m_A$  has to be greater than about  $M_Z/2$  for  $\tan \beta > 1$ . With the higher energies available at LEP2 since then, this bound is expected to be at least 60 GeV.

To obtain a lower bound on  $m_A$  for  $\tan \beta < 1$ , we propose to use the MSSM relationship[6]

$$m_{h^\pm}^2 = m_A^2 + M_W^2 - \frac{\epsilon}{4 \sin^2 \beta} \frac{M_W^2}{m_t^2}, \quad (7)$$

where the last term is the leading radiative correction for  $\tan \beta < 1$ . We then derive bounds on  $m_A$  from the bounds on  $m_{h^\pm}$  by considering  $t$  decay. Taking  $m_t = 175$  GeV, we see that  $t \rightarrow b + h^+$  is allowed for values of  $m_{h^\pm}$  up to 170 GeV, corresponding to  $m_A$  up to about 150 GeV. The nonobservation of the above process would then translate into lower bounds on  $m_A$  as a function of  $\tan \beta$ .

In the MSSM, the charged-Higgs-boson couplings to the quarks and leptons are given by

$$\mathcal{H}_{int} = \frac{-g_2}{\sqrt{2}M_W} h^+ [\cot \beta m_{u_i} \bar{u}_i d_{iL} + \tan \beta m_{d_i} \bar{u}_i d_{iR} + \tan \beta m_{l_i} \bar{\nu}_i l_{iR}] + h.c., \quad (8)$$

where the subscript  $i$  represents the generation index, and we have used the diagonal KM matrix approximation[7]. The leading-logarithm QCD (quantum chromodynamics) correction is taken into account by substituting the quark mass parameters by their running masses evaluated at the  $h^\pm$  mass scale. The resulting decay widths are

$$\Gamma(t \rightarrow bh^+) = \frac{g_2^2 \lambda^{1/2}(1, m_b^2/m_t^2, m_{h^+}^2/m_t^2)}{64\pi M_W^2 m_t} [(m_t^2 \cot^2 \beta + m_b^2 \tan^2 \beta)(m_t^2 + m_b^2 - m_{h^+}^2) - 4m_t^2 m_b^2], \quad (9)$$

where  $\lambda$  denotes the usual Kallen function and  $\lambda^{1/2}$  is equal to the magnitude of the momentum of either decay product divided by  $m_t/2$ , and

$$\Gamma(h^+ \rightarrow \tau^+ \nu) = \frac{g_2^2 m_{h^+}}{32\pi M_W^2} m_\tau^2 \tan^2 \beta, \quad (10)$$

$$\Gamma(h^+ \rightarrow c\bar{s}) = \frac{3g_2^2 m_{h^+}}{32\pi M_W^2} (m_c^2 \cot^2 \beta + m_s^2 \tan^2 \beta). \quad (11)$$

Assuming that the only other competing channel is the standard-model decay  $t \rightarrow bW^+$ , the  $t \rightarrow bh^+$  branching fraction is then

$$B = \frac{\Gamma(t \rightarrow bh^+)}{\Gamma(t \rightarrow bh^+) + \Gamma(t \rightarrow bW^+)}, \quad (12)$$

where

$$\Gamma(t \rightarrow bW^+) = \frac{g_2^2 \lambda^{1/2}(1, m_b^2/m_t^2, M_W^2/m_t^2)}{64\pi M_W^2 m_t} [M_W^2(m_t^2 + m_b^2) + (m_t^2 - m_b^2)^2 - 2M_W^4]. \quad (13)$$

It is clear from Eq. (9) that  $B$  has a minimum at  $\tan \beta = (m_t/m_b)^{1/2} \simeq 6$ , but it becomes large for  $\tan \beta < 1$  and  $\tan \beta > m_t/m_b$ . Thus we expect to see a sizeable  $t \rightarrow bh^+$  signal in these two regions if  $m_{h^+} < m_t$ .

We see from Eqs. (10) and (11) that  $\tau^+ \nu$  is the dominant decay mode of  $h^+$  if  $\tan \beta \gg 1$ . Thus an excess of  $t\bar{t}$  events in the  $\tau$  channel compared to the standard-model prediction

constitutes a viable  $h^\pm$  signal in the large  $\tan\beta$  region. A recent analysis[8] of the CDF  $t\bar{t}$  data in the  $\tau l$  channel ( $l = e, \mu$ ) has led to a mass bound of  $m_{h^\pm} > 100$  GeV for  $\tan\beta > 40$ . A similar bound has also been obtained from the same  $t\bar{t}$  data in the inclusive  $\tau$  channel[9].

The above method is not applicable in the small  $\tan\beta$  region, where  $h^+$  is expected to decay mainly into  $c\bar{s}$ , *i.e.* two jets. On the other hand, we can use the so-called disappearance method to look for the presence of  $t \rightarrow bh^+$  decay in both the small and large  $\tan\beta$  regions[7] as described below. The key observation is that  $h^\pm$  couples negligibly to the light fermions, particularly  $e$  and  $\mu$ , whereas the  $W$  boson couples to them with full strength universally. Since the  $e$  and  $\mu$  decay modes play an important role in the detection of  $t\bar{t}$  events at the Tevatron, the experimentally derived  $t\bar{t}$  cross section is sensitive to the branching fraction  $B$  of Eq. (12). After all, if  $t$  decays into  $bh^+$ , there would not be any energetic  $e$  or  $\mu$  in the final state, as would be possible with the  $W$  boson.

The experimental  $t\bar{t}$  cross sections obtained by the CDF and D0 collaborations[10, 11] are weighted averages of their measured cross sections in the (I) dilepton ( $ll$ ) and (II) lepton plus multijet ( $lj$ ) channels, using the standard formula

$$\sigma = \frac{\Sigma(\sigma_i/\delta_i^2)}{\Sigma(1/\delta_i^2)}. \quad (14)$$

They are summarized below.

$$\text{CDF : } \sigma_{ll} = 8.5^{+4.4}_{-3.4} \text{ pb, } \sigma_{lj} = 7.2^{+2.1}_{-1.7} \text{ pb} \Rightarrow \sigma_{\text{CDF}} = 7.5^{+1.9}_{-1.6} \text{ pb.} \quad (15)$$

$$\text{D0 : } \sigma_{ll} = 6.3 \pm 3.3 \text{ pb, } \sigma_{lj} = 5.1 \pm 1.9 \text{ pb} \Rightarrow \sigma_{\text{D0}} = 5.5 \pm 1.8 \text{ pb.} \quad (16)$$

The  $\sigma_{lj}$  of CDF is a weighted average of the measured cross sections using the SVX and SLT  $b$ -tagging methods; that of D0 is a weighted average of those using kinematic cuts and SLT  $b$ -tagging. In both cases, the weight of the SLT method is rather low. From Eqs. (15) and (16), we see that for both CDF and D0,  $\delta_{lj} \simeq \delta_{ll}/2$ , hence

$$\sigma \simeq \frac{\sigma_{ll} + 4\sigma_{lj}}{5}. \quad (17)$$

Furthermore, since the CDF and D0 cross sections have essentially identical errors, we can take a simple average of the two:

$$\sigma_{\text{CDF+D0}} = 6.5 \begin{array}{c} +1.3 \\ -1.2 \end{array} \text{ pb.} \quad (18)$$

Here we have combined the two errors using  $\delta^{-2} = \delta_1^{-2} + \delta_2^{-2}$ , since they are largely statistical.

We note that the dilepton channel (I) corresponds to the leptonic ( $e, \mu$ ) decay of both the  $t$  and  $\bar{t}$  quarks, whereas the lepton plus multijet channel (II) corresponds to the leptonic decay of one, say  $t \rightarrow bl^+\nu$ , and the hadronic decay of the other. For the standard-model decay  $t \rightarrow bW^+$ , the respective branching fractions are  $2/9$  and  $2/3$ , whereas for the postulated decay  $t \rightarrow bh^+$ , they are 0 and a function which rises rapidly to 1 for  $\tan\beta < 1$ . Thus the relative contributions of different final states to the two channels are  $WW : Wh^\pm : h^\pm h^\mp = 1 : 0 : 0$  for  $(ll)$  and  $1 : 3/4 : 0$  for  $(lj)$ . [We have used the maximum value of  $3/4$  corresponding to very small  $\tan\beta$ . This is a conservative approach, because any smaller value will give us a better bound on  $m_{h^\pm}$  as explained below.] We have then a suppression factor relative to the standard model of

$$f_{ll} = (1 - B)^2 \simeq 0.5 \quad (\text{for } B = 0.3), \quad (19)$$

$$f_{lj} = (1 - B)^2 + 2B(1 - B)(3/4) \simeq 0.8 \quad (\text{for } B = 0.3). \quad (20)$$

Since the relative weights of the  $(ll)$  and  $(lj)$  channels are 1:4, Eqs. (19) and (20) correspond to an effective suppression factor of

$$f = 0.74 \quad (\text{for } B = 0.3). \quad (21)$$

We note that for large  $\tan\beta$ ,  $h^\pm$  decays mainly into  $\tau$ , hence it would be hard for the  $Wh^\pm$  final state to pass the  $n_{\text{jet}} \geq 3$  cut required for the  $(lj)$  channel. This implies an extra suppression factor of about  $1/3$  for the  $Wh^\pm$  contribution, hence  $f$  is about 0.7 already for  $B = 0.2$ , *i.e.* our bound is conservative because it assumes  $B = 0.3$ .

Finally the theoretical estimates of the  $t\bar{t}$  cross section including higher-order QCD corrections are 4.13 to 5.48 pb[12], and 5.10 to 5.59 pb[13]. These ranges are not identical, but the two estimates are in reasonable agreement as to their upper bounds. We shall thus assume for our purpose that

$$\sigma(t\bar{t}) \leq 5.6 \text{ pb.} \quad (22)$$

Combining this with the suppression factor of Eq. (21), we obtain an upper bound of

$$\sigma \leq 4.1 \text{ pb} \quad (23)$$

for the weighted cross section of Eq. (17). This is  $2\sigma$  lower than the combined CDF and D0 estimate of Eq. (18), as well as the CDF estimate of Eq. (15). Hence we can take  $B = 0.3$  as a  $2\sigma$  upper bound for the branching fraction of  $t \rightarrow bh^+$  decay. In Figure 1 we plot the exclusion regions of  $m_{h^\pm}$  as a function of  $\tan\beta$  using  $B = 0.3$ . We also show the exclusion region obtained in Ref. [8], which used the ‘‘appearance’’ method of looking for  $\tau$ , instead of the ‘‘disappearance’’ method of not finding  $e$  or  $\mu$  discussed here.

To convert a bound on  $m_{h^\pm}$  to one on  $m_A$ , we use the full expression including all one-loop radiative corrections[6] in place of Eq. (7) which is approximate and valid only for  $\tan\beta < 1$ . In Figure 2 we plot the exclusion regions of  $m_A$  as a function of  $\tan\beta$  deduced from  $t$  decay and  $t\bar{t}$  production corresponding to Fig. 1. We note that the radiative correction is negative for small  $\tan\beta$  which increases the  $m_A$  bound, and is positive for large  $\tan\beta$  which decreases it. We note also that at extreme values of  $\tan\beta$ , near 0.2 and 100, the Yukawa couplings involved are becoming too large for a perturbative calculation to be reliable. We then add a line at  $m_A = 60$  GeV for  $\tan\beta > 1$  as a conservative upper limit from the combined LEP data[5, 14]. Our conclusion is simple: in the Minimal Supersymmetric Standard Model, combining what we know from LEP and the Tevatron and using a conservative estimate of the theoretical  $t\bar{t}$  cross section, the pseudoscalar mass  $m_A$  is now known to be greater than 60 GeV for all values of  $\tan\beta$ .

**Note Added:** After the completion of our paper, we found out that the ALEPH Collaboration has just recently obtained[15] the bound  $m_A > 62.5$  GeV for  $\tan\beta > 1$ .

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## Figure Captions

Fig. 1. Exclusion regions at 95% confidence level in the  $m_{h^\pm} - \tan\beta$  plane using  $B = 0.3$  (solid lines) for  $t \rightarrow bh^+$  as explained in the text. The dashed line corresponds to the method used in Ref. [8].

Fig. 2. Exclusion regions at 95% confidence level in the  $m_A - \tan\beta$  plane. Regions I and III correspond to those depicted in Fig. 1 with  $m_{h^\pm}$  converted to  $m_A$  taking into account the MSSM one-loop radiative corrections. Region II represents a conservative estimate of the expected limit from LEP1 and LEP2 for  $\tan\beta > 1$  (dotted line). A slightly higher value of 62.5 GeV for  $\tan\beta > 1$  has just recently been obtained by the ALEPH Collaboration[15].

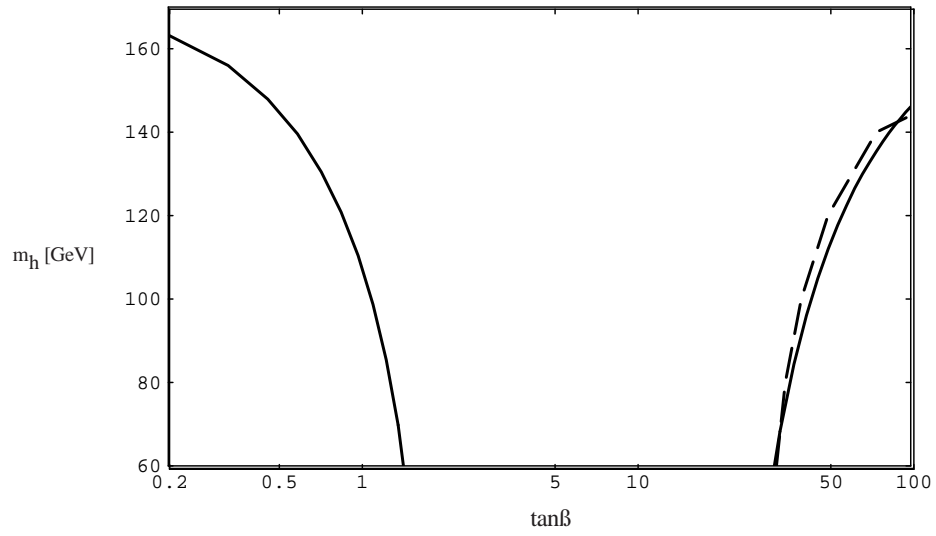


Fig. 1

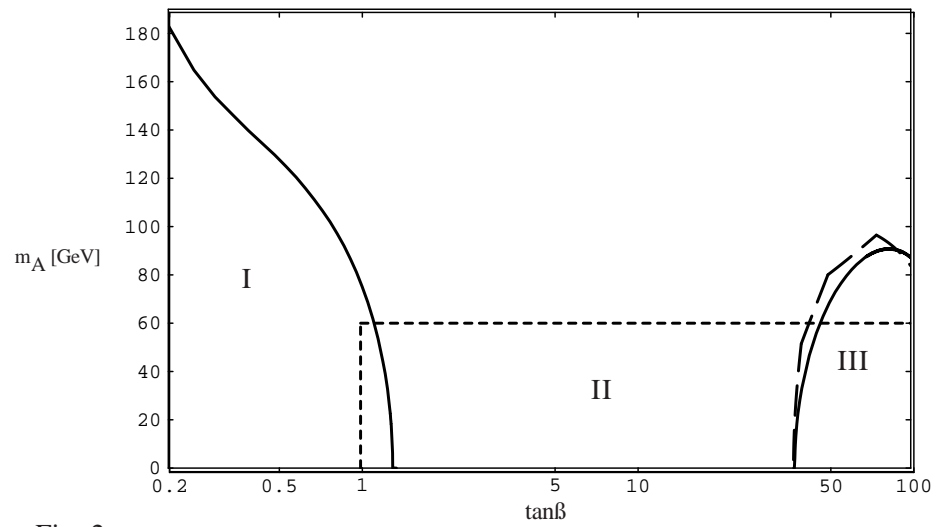


Fig. 2