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Minimal see-saw model for atmospheric and solar neutrino oscillations

Ernest Ma[a] and D.P. Roy[b,c]

[a] Department of Physics, University of California, Riverside, California 92521, USA.
[b] Department of Physics, University of Durham, Durham, DH1 3LE, UK.
[c] Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India.

Abstract

We present a minimal see-saw model based on an extension of the standard model (SM) which includes an additional U(1), with gauge charge $B - \frac{3}{2}(L_\mu + L_\tau)$. Requirement of anomaly cancellation implies the existence of two right-handed singlet neutrinos, carrying this gauge charge, which have normal Dirac couplings to ν_μ and ν_τ but suppressed ones to ν_e. Assuming the U(1) symmetry breaking scale to be 10^{12-16} GeV, this model can naturally account for the large (small) mixing solutions to the atmospheric (solar) neutrino oscillations.

Super-Kamiokande data have recently provided convincing evidence for atmospheric neutrino oscillations [1] as well as confirmed earlier results on solar neutrino oscillations [2]. The atmospheric neutrino oscillation data seem to require a large mixing angle between ν_μ and ν_τ,

$$\sin^2 2\theta_{\mu\tau} \ > \ 0.82 \tag{1}$$

and

$$\Delta M^2 \ = \ (0.5 - 6) \ \times \ 10^{-3}\text{eV}^2. \tag{2}$$

On the other hand, the solar neutrino oscillation data can be explained by the small mixing angle matter enhanced (MSW) [3] solution between ν_e and a combination of ν_μ/ν_τ with [4]

$$\sin^2 2\theta_{e-\mu/\tau} \ = \ 10^{-2} \ - \ 10^{-3} \tag{3}$$

and

$$\Delta m^2 \ = \ (0.5 - 1) \ \times \ 10^{-5}\text{eV}^2. \tag{4}$$

This represents the most conservative solution to the solar neutrino anomaly although one can get reasonably good solutions with large mixing angle MSW and vacuum oscillations as well. One would naturally expect a near-maximal mixing between ν_μ and ν_τ (1), as required by the atmospheric neutrino data, if they were almost degenerate Dirac partners with a small mass difference given by (2). In the context of a three neutrino model however, the solar neutrino solution (4) would then require the ν_e to show a much higher level of degeneracy with one of these states, which is totally unexpected. Therefore, it is more natural to consider the three neutrino mass eigenstates as non-degenerate with

$$m_1 \ = \ (\Delta M^2)^{1/2} \ \simeq \ 0.05 \text{ eV}, \qquad \mathrm{m}_2 \ = \ (\Delta \mathrm{m}^2)^{1/2} \ \simeq \ 0.003 \text{ eV}, \qquad \mathrm{m}_3 \ll \mathrm{m}_2. \tag{5}$$

There is broad agreement on this point in the current literature on neutrino physics [5]–[10], much of which is focused on the question of reconciling this hierarchical structure of neutrino masses with at least one large mixing angle (1). It may be noted here that in a minimal scenario one needs only two neutrino masses with $m_3 \to 0$, since it has no relevance for atmospheric or solar neutrino oscillations.

The cannonical mechanism for generating neutrino masses is the so-called see-saw model, containing heavy right-handed singlet neutrinos [11], which induce small hierarchical masses for the doublet neutrinos. The standard see-saw model is based on a U(1) extension of the standard-model (SM) gauge group SU(3)$_C\times$ SU(2)$_L\times$ U(1)$_Y$, corresponding to the gauge charge B–L [12], where the anomaly cancellation requirement implies the existence of three right-handed singlet neutrinos. However it cannot explain the large mixing between the ν_μ and ν_τ states and their small mixing with ν_e, since it treats the three flavours on equal footing. Furthermore, since $m_3 = 0$ is allowed, we need only <u>two</u> heavy right-handed singlet neutrinos. Such a see-saw model was recently considered by us [13], which was based on the gauge group U(1)$_{B-3Le}$, thereby distinguishing the e flavour from μ and τ in the choice of the gauge group. We present here a <u>more economical</u> and <u>better motivated</u> model based on a slightly different U(1)$_{Y'}$ extension of the SM with

$$Y' \;=\; B - \tfrac{3}{2}\,(L_\mu + L_\tau). \tag{6}$$

This U(1) $_{Y'}$ can only be gauged together with the SM if there are <u>two</u> right-handed singlet neutrinos carrying this charge, as we see below. We now have a reason why ν_μ and ν_τ are different from ν_e, and also why the ν_e mass is zero. Contrast this with the usual B–L model [12] where there must be three singlets and the B–3L$_e$ model [13] where there is only one. In the latter, an extra singlet neutrino has to be added by hand, and it must not have any gauge interactions, hence its existence is not very well motivated.

The two extra right-handed singlet neutrinos have normal Dirac couplings to ν_μ and ν_τ but suppressed ones to ν_e because they do so through a different Higgs doublet which has a naturally small vacuum expectation value (vev) as we see below. This ensures the desired mixing pattern of (1) and (3). Moreover, one can get the induced neutrino masses in the desired range of (5), assuming a U(1) symmetry breaking scale of $\sim 10^{12-16}$ GeV. Thus the model can naturally account for the large (small) mixing solutions to the atmospheric (solar) neutrino oscillations.

The $SU(3)_C \times SU(2) \times U(1)_Y \times U(1)_{Y'}$ gauge charges of the quarks and leptons, including the two singlet neutrinos, are listed below

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \sim \left(3, 2, \tfrac{1}{6}, \tfrac{1}{3}\right); \quad u_{iR} \sim \left(3, 1, \tfrac{2}{3}, \tfrac{1}{3}\right); \quad d_{iR} \sim \left(3, 1, \tfrac{-1}{3}, \tfrac{1}{3}\right);$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim \left(1, 2, \tfrac{-1}{2}, 0\right); \quad e_R \sim (1, 1, -1, 0);$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \sim \left(1, 2, \tfrac{-1}{2}, \tfrac{-3}{2}\right); \quad \mu_R, \tau_R \sim \left(1, 1, -1, \tfrac{-3}{2}\right);$$

$$\nu_{1R}, \nu_{2R} \sim \left(1, 1, 0, \tfrac{-3}{2}\right). \tag{7}$$

The cancellation of anomalies has been discussed in [14] in the context of an analogous U(1) extension of the SM. Since the number of $SU(2)_L$ doublets remain unchanged (even), the global SU(2) chiral gauge anomaly [15] is absent. The presence of the two right-handed singlet neutrinos ensures that the quarks and leptons transform vectorially under the $U(1)_{Y'}$. Consequently the mixed gravitational-gauge anomaly [16] is absent. It also ensures the absence of the $[SU(3)_C]^2\ U(1)_{Y'}$ and $[U(1)_{Y'}]^3$ axial-vector anomalies [17]. The other axial vector triangle anomalies are cancelled as follows

$$[SU(2)]^2 U(1)_{Y'} \ :\ (3)(3)(\tfrac{1}{3}) + (2)(\tfrac{-3}{2}) = 0, \tag{8}$$

$$\begin{aligned}[U(1)_{Y'}]^2 U(1)_Y \ :\ & (3)(3)(\tfrac{1}{3})^2[2(\tfrac{1}{6}) - (\tfrac{2}{3}) - (\tfrac{-1}{3})] \\ & + (2)(\tfrac{-3}{2})^2[2(\tfrac{-1}{2}) - (-1)] = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} \mathrm{U}(1)_{Y'}[\mathrm{U}(1)_Y]^2 \quad : \quad & (3)(3)(\tfrac{1}{3})[2(\tfrac{1}{6})^2 - (\tfrac{2}{3})^2 - (\tfrac{-1}{3})^2] \\ & + (2)(\tfrac{-3}{2})[2(\tfrac{-1}{2})^2 - (-1)^2] \;=\; 0, \end{aligned} \tag{10}$$

where the first two entries in each equation refer to numbers of quark colours and generations. Thus the Y' symmetry can be gauged along with the others.

The minimal scalar sector of the model consists of the SM Higgs doublet and a neutral singlet,

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim \left(1, 2, \tfrac{1}{2}, 0\right), \quad \chi^0 \sim (1, 1, 0, 3). \tag{11}$$

The latter couples to the singlet pairs $\nu_i \nu_j$, while the former is responsible for their Dirac couplings to ν_μ and ν_τ. This will be adequate for atmospheric neutrino oscillations but not for solar neutrino, as ν_e will completely decouple from the other neutrinos. Therefore we shall assume another Higgs doublet and a singlet,

$$\begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} \sim \left(1, 2, \tfrac{1}{2}, \tfrac{-3}{2}\right), \quad \zeta^0 \sim \left(1, 1, 0, \tfrac{-3}{2}\right). \tag{12}$$

The doublet shall account for the suppressed Dirac couplings of the singlet neutrinos to ν_e. The singlet does not couple to the fermions; but is required to avoid an unwanted pseudo-Goldstone boson [14]. This comes about because there are 3 global U(1) symmetries, corresponding to rotating the phases of ϕ, η and χ^0 independently in the Higgs potential, while only 2 local U(1) symmetries get broken. The addition of the singlet ζ^0 introduces two more terms in the Higgs potential, $\eta^\dagger \phi \zeta^0$ and $\chi^0 \zeta^0 \zeta^0$, so that the extra global symmetry is eliminated.

Both χ^0 and ζ^0 are expected to acquire large vev's and masses at the scale of the $\mathrm{U}(1)_{Y'}$ symmetry breaking. In contrast, the doublet η is required to have a positive mass squared term in order to avoid SU(2) breaking at this scale. Nonetheless it can acquire a small but non-zero vev as the SU(2) symmetry gets broken [8]. This can be estimated from the relevant part of the Higgs potential

$$m_\eta^2 \eta^\dagger \eta \;+\; \lambda (\eta^\dagger \eta)(\chi^\dagger \chi) \;+\; \lambda' (\eta^\dagger \eta)(\zeta^\dagger \zeta) \;-\; \mu \eta^\dagger \phi \zeta. \tag{13}$$

Although we start with a positive mass squared term for η, after minimisation of the potential we find that this field has acquired a small vev,

$$\langle \eta \rangle \;=\; \mu \langle \phi \rangle \, \langle \zeta \rangle / M_\eta^2, \tag{14}$$

where $M_\eta^2 = m_\eta^2 + \lambda \langle \chi \rangle^2 + \lambda' \langle \zeta \rangle^2$ represents the physical mass of η and $\langle \phi \rangle \simeq 10^2$ GeV. The size of the soft term is bounded by the Y' symmetry breaking scale, i.e. $\mu \leq \langle \zeta \rangle$. In order to account for the small mixing angle of ν_e (3), we shall require

$$\langle \eta \rangle / \langle \phi \rangle \;\sim\; 10^{-2}. \tag{15}$$

This would correspond to assuming $\mu \sim \langle \zeta \rangle / 100$, or alternatively $\mu \sim \langle \zeta \rangle$ and $M_\eta \simeq m_\eta \simeq 10 \langle \zeta \rangle$. In either case one can get the desired vev with a reasonable choice of the mass parameters.

As usual we shall be working in the basis where the charged lepton mass matrix, arising from their couplings to the SM Higgs boson ϕ, is diagonal. This defines the flavour basis for the doublet neutrinos. Since the two singlet neutrinos are decoupled from the charged leptons, their Majorana mass matrix can be diagonalised independently. We shall denote their mass eigenvalues as M_1 and M_2. While the overall size of these masses will be at the Y' symmetry breaking scale, we shall assume a modest hierarchy between them,

$$M_1 / M_2 \;\sim\; 1/20, \tag{16}$$

in order to account for the desired mass ratio for the doublet neutrinos (5). The above hierarchy between the singlet neutrino masses compares favourably with those observed in the quark and charged lepton sectors.

Thus we have the following 5×5 neutrino mass matrix in the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu_1^c, \nu_2^c)$:

$$M = \begin{pmatrix} 0 & 0 & 0 & f_e^1\langle\eta\rangle & f_e^2\langle\eta\rangle \\ 0 & 0 & 0 & f_\mu^1\langle\phi\rangle & f_\mu^2\langle\phi\rangle \\ 0 & 0 & 0 & f_\tau^1\langle\phi\rangle & f_\tau^2\langle\phi\rangle \\ f_e^1\langle\eta\rangle & f_\mu^1\langle\phi\rangle & f_\tau^1\langle\phi\rangle & M_1 & 0 \\ f_e^2\langle\eta\rangle & f_\mu^2\langle\phi\rangle & f_\tau^2\langle\phi\rangle & 0 & M_2 \end{pmatrix}, \tag{17}$$

where $\nu_{1,2}^c$ denote antiparticles of the right-handed singlet neutrinos and the f's are the Higgs Yukawa couplings. The induced mass matrix for the doublet neutrinos is easy to calculate in our basis of a diagonal Majorana mass matrix. It is given by the see-saw formula in this basis,

$$m_{ij} = \frac{D_{1i}D_{1j}}{M_1} + \frac{D_{2i}D_{2j}}{M_2}, \tag{18}$$

where i, j denote the 3 neutrino flavours and D represents the 2×3 Dirac mass matrix at the bottom left of (17). We get

$$m = \begin{pmatrix} c_1^2 + c_2^2 & c_1a_1 + c_2a_2 & c_1b_1 + c_2b_2 \\ c_1a_1 + c_2a_2 & a_1^2 + a_2^2 & a_1b_1 + a_2b_2 \\ c_1b_1 + c_2b_2 & a_1b_1 + a_2b_2 & b_1^2 + b_2^2 \end{pmatrix}, \tag{19}$$

where

$$a_{1,2} = \frac{f_\mu^{1,2}\langle\phi\rangle}{\sqrt{M_{1,2}}}, \quad b_{1,2} = \frac{f_\tau^{1,2}\langle\phi\rangle}{\sqrt{M_{1,2}}}, \quad c_{1,2} = \frac{f_e^{1,2}\langle\eta\rangle}{\sqrt{M_{1,2}}}. \tag{20}$$

We shall assume all the Yukawa couplings to be of the same order of magnitude, which means that the elements of a mass matrix arising from the same Higgs vev are expected to be of similar size. There is of course no conflict between the assumption of democratic mass matrix elements and hierarchical mass eigenvalues [6]. In fact the latter requires large cancellations in the determinant, which in turn implies democratic elements of the mass matrix. This appears to be a reasonable assumption, although we shall use it only for a limited purpose – i.e. to ensure that the hierarchies resulting from the ratios of the Higgs vev's (15) and the singlet mass eigenvalues (16) are not washed out by violent fluctuations in the Higgs couplings. Then these hierarchies imply

$$a_1, b_1 \gg a_2, b_2, c_1 \gg c_2. \tag{21}$$

This leads to a texture of the mass matrix (19), where the {11} element is doubly suppressed and the remaining elements of the first row and first column are singly suppressed [7]. It is a reflection of the hierarchy (15) in the Dirac mass matrix, which will show up in the hierarchy of the two mixing angles (1) and (3). On the other hand the hierarchy (16) of Majorana mass eigenvalues will be reflected in a similar hierarchy between the non-zero eigenvalues of (19), which correspond to the two neutrino masses of (5).

One can easily check that the determinant of the mass-matrix (19) vanishes, so that one of its eigenvalues is zero. The other two eigenvalues are

$$\begin{aligned} m_{1,2} = \frac{1}{2}\Big[& a_1^2 + a_2^2 + b_1^2 + b_2^2 + c_1^2 + c_2^2 \\ & \pm \sqrt{(a_1^2 + a_2^2 + b_1^2 + b_2^2 + c_1^2 + c_2^2)^2 - 4\left\{(a_1 b_2 - b_1 a_2)^2 + (a_1 c_2 - c_1 a_2)^2 + (b_1 c_2 - c_1 b_2)^2\right\}}\Big]. \end{aligned} \tag{22}$$

¿From (21) and (22) we get

$$m_1 \simeq a_1^2 + b_1^2, \tag{23}$$

$$m_2 \simeq \frac{(a_1 b_2 - a_2 b_1)^2}{a_1^2 + b_1^2}, \tag{24}$$

i.e.

$$m_2/m_1 \sim M_1/M_2. \tag{25}$$

Thus the assumed hierarchy of the Majorana masses (16) do account for the relative size of the two neutrino masses of (5). Moreover the required size of m_1 or m_2 will give the overall scale of the Y' symmetry breaking Majorana mass, i.e.

$$M_2 \sim (f^2_{\mu,\tau})^2 \langle\phi\rangle^2/m_2 \sim (f^2_{\mu,\tau})^2\, 10^{16} \text{ GeV}. \tag{26}$$

Assuming the size of the Yukawa couplings to be similar to the top Yukawa coupling (~ 1), we then have

$$M_2 \sim 10^{16} \text{ GeV}, \tag{27}$$

i.e. close to a possible grand unification scale. On the other hand, assuming the Yukawa couplings to be similar in size to that of thr τ lepton ($\sim 10^{-2}$) would imply

$$M_2 \sim 10^{12} \text{ GeV}. \tag{28}$$

Thus within the lattitude of the Yukawa coupling given above, the Y' symmetry breaking scale could be anywhere in the range 10^{12-16} GeV.

Finally we can calculate the eigenvectors corresponding to the three eigenvalues, m_1, m_2 and $m_3(=0)$. This gives the following mixing matrix connecting the flavour eigenstates to the mass eigenstates, written in increasing order of mass :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & \dfrac{-c_2\sqrt{a_1^2+b_1^2}}{a_1b_2-b_1a_2} & \dfrac{c_1}{\sqrt{a_1^2+b_1^2}} \\ \dfrac{b_1c_2-c_1b_2}{a_1b_2-b_1a_2} & \dfrac{b_1}{\sqrt{a_1^2+b_1^2}} & \dfrac{a_1}{\sqrt{a_1^2+b_1^2}} \\ \dfrac{c_1a_2-a_1c_2}{a_1b_2-b_1a_2} & \dfrac{-a_1}{\sqrt{a_1^2+b_1^2}} & \dfrac{b_1}{\sqrt{a_1^2+b_1^2}} \end{pmatrix} \begin{pmatrix} \nu_3 \\ \nu_2 \\ \nu_1 \end{pmatrix}. \tag{29}$$

The large mixing angle, responsible for atmospheric neutrino oscillations, corresponds to

$$\tan\theta_{\mu\tau} = a_1/b_1 = f^1_\mu/f^1_\tau, \tag{30}$$

i.e. it is given by the ratio of the SM Higgs Yukawa couplings to ν_μ and ν_τ along with the lighter singlet. Assuming these Yukawa couplings to be equal implies maximal mixing,

$\theta_{\mu\tau} = 45°$. Moreover, any value of their ratio in the range

$$0.6 \; < \; f_1/f_2 \; < \; 1.6 \tag{31}$$

will ensure the large mixing angle (1) required by data, which corresponds to $32° < \theta_{\mu\tau} < 58°$. Thus one can get the required mixing angle for atmospheric neutrino oscillation without any fine tuning of the Yukawa couplings.

The small mixing angle, responsible for solar neutrino oscillations, corresponds to the mixing of the ν_e with the lighter mass eigenstate ν_2, i.e.

$$\sin\theta_{e-\mu/\tau} \; \simeq \; \frac{c_2\sqrt{a_1^2 + b_1^2}}{a_1 b_2 - b_1 a_2} \; \sim \; \frac{\langle\eta\rangle}{\langle\phi\rangle}. \tag{32}$$

Thus the ratio (15) of the two Higgs vev's can account for the required size of the mixing angle (3), i.e.

$$\sin\theta_{e-\mu/\tau} \; = \; (1.6 - 5) \; \times \; 10^{-2}. \tag{33}$$

It should be noted that in this model, one also expects a similar size of ν_e mixing with the heavier mass eigenstate ν_1. This is allowed by all current experiments, including CHOOZ [18], although it has been assumed to be zero in some mixing models. Hopefully this mixing angle can be probed by future reactor and long baseline accelerator experiments.

Notice that η also couples e_R to μ_L and τ_L, which introduces small non-diagonal elements in the charged lepton mass matrix. However, as shown in [13], its contribution to the ν_e mixing angle is very small ($\sin\theta_{e-\mu/\tau} \leq 10^{-3}$). The theoretical origin of our proposed $\mathrm{U}(1)_{Y'}$ is not obvious. It spans all three quark families but only two lepton families. A possibility is that at the putative grand unification scale, what exists is a remnant of a string theory which already breaks down to the SM together with this extra U(1). The low-energy consequence of our model is identical to that of the SM, including the effective Higgs sector, except for neutrino masses.

In summary, we have considered a see-saw model based on a new U(1) extension of the SM gauge group, corresponding to the gauge charge $B - 3/2(L_\mu + L_\tau)$. The requirement of anomaly cancellation implies the existence of two right-handed singlet neutrinos, carrying this gauge charge, which have normal Dirac couplings to ν_μ and ν_τ, but suppressed ones to ν_e. Consequently they induce see-saw masses to two doublet neutrino states, which are large admixtures of ν_μ and ν_τ with small ν_e components. Moreover, one can get the right size of these neutrino masses for explaining the large (small) mixing solutions to the atmospheric (solar) neutrino oscillations, if the scale of this U(1) symmetry breaking is in the range of 10^{12-16} GeV. The necessity of two and only two singlet neutrinos of the μ and τ variety in this model tells us why $\nu_\mu - \nu_\tau$ mixing is large and why ν_e is massless. Thus it represents what appears to be a minimal see-saw model for explaining these oscillations.

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