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Energy Independent Solution to the Solar Neutrino Anomaly including the SNO Data

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Abstract

The global data on solar neutrino rates and spectrum, including the SNO charged current rate, can be explained by LMA, LOW or the energy independent solution – corresponding to near-maximal mixing. All the three favour a mild upward renormalisation of the Cl rate. A mild downward shift of the B neutrino flux is favoured by the energy independent and to a lesser extent the LOW solution, but not by LMA. Comparison with the ratio of SK elastic and SNO charged current scattering rates favours the LMA over the other two solutions, but by no more than 1.5σ.

The suppression of the solar neutrino flux has been confirmed now by a number of experiments [1-4], covering a neutrino energy range of $0.2 - 20$ MeV. However there is still considerable uncertainty regarding its energy dependence. An energy independent suppression factor has been advocated by several groups over the years [5]. This was shown to be disfavoured however by the combined data on the total suppression rates [6]. More recently the weight of experimental evidence changed in favour of an energy independent solution following the SK data on the day/night spectrum, showing practically no energy dependence nor any perceptible day/night effect [3].

It was recently shown by us in [7] that an energy independent solution can describe the global data on rates and spectrum satisfactorily, with reasonable readjustments of the Cl experiment rate and the ^8B neutrino flux. This has been corroborated now by other groups [8,9]. It was also suggested in [7] that the measurement of the charged to neutral current ratio by the SNO experiment will be able to distinguish the energy independent solution from the conventional MSW solutions to the rates and spectrum data, i.e. the so called LMA and LOW solutions. Recently the SNO experiment has produced its first data [4], providing the charged current scattering rate with reasonable precission. In the absence of the neutral current data one can combine the CC rate of SNO with the elastic scattering rate of SK to distinguish the above solutions from one another. The purpose of the present exercise is two fold. Firstly we reevaluate the experimental status of the energy independent solution vis a vis the LMA and LOW solutions after the inclusion of the SNO data. Secondly we plug an important gap in the analysis of [7], which arose out of neglecting the coherent term in the solar neutrino oscillation. Inclusion of this term allows us to probe the parameter space down to $\Delta m^2 \simeq 10^{-11}$ eV^2, i.e. the vacuum oscillation region, which was not possible in [7]. We also take this opportunity to update the result using the full SK data sample of 1258 days, which became available more recently [3]. We shall see that the conclusion of [7] are not changed by this and the inclusion of the SNO CC rate in the global rates and spectrum data. On the other hand a direct comparison with the ratio of SNO (CC) and SK (ES) scattering rates does favour the LMA solution over the energy independent (and to a lesser extent the LOW) solution, but at only $\sim 1.5\sigma$ level.

Table 1 shows the suppression rate or survival probability of the solar neutrino ($P_{\nu_e \nu_e}$) from the combined Ga [1], Cl [2], SK [3] and SNO [4] experiments along with their threshold energies. The corresponding compositions of the solar neutrino flux are also indicated. The SK survival rate shown in parantheses is appropriate for the oscillation of ν_e into another active flavour ($\nu_{\mu,\tau}$), on which we shall concentrate here. It is obtained by subtracting the neutral current contribution of $\nu_{\mu,\tau}$ from the SK rate. All the survival rates are shown relative to the standard solar model (SSM) prediction of BPB00 [10].

The apparent energy dependence in the survival rates of Table 1 is conventionally explained in terms of the vacuum oscillation (VO), small and large angle MSW (SMA and LMA) as well as the LOW solutions [11]. The VO and SMA solutions, showing strong and nonmonotonic energy dependence, are essentially ruled out now by the SK day/night spectrum [3]. On the other hand the spectrum data is compatible with the LMA and LOW solutions, which predict modest and monotonic decrease of the survival rate with energy.

We shall fit the combined data on the survival rates and the SK spectrum with the energy independent solution as well as the four traditional solutions mentioned above. In order to reconcile the energy independence of the spectrum with the apparent energy dependence in the rates of Table 1, we shall consider the following variations in the Cl rate and B neutrino flux, since the Cl experiment [2] has not been calibrated while the B neutrino flux is very sensitive to the underlying solar model.

i) We shall consider an upward renormalisation of the Cl rate by 20% (i.e. 2σ), which will push it marginally above the SK and SNO rates. This is favoured not only by the energy independent solution but also the LMA and LOW solutions, showing monotonic energy dependence.

ii) We shall consider a downward variation of the B neutrino flux of BPB00 [10],

$$f_B = 5.15 \times 10^6/\mathrm{cm}^2/\mathrm{sec} \begin{pmatrix} & +.20 \\ 1.0 & \\ & -.16 \end{pmatrix} \tag{1}$$

by upto 2σ. A downward renormalisation of this flux is favoured by the energy independent solution and to a lesser extent by LOW, though not by the LMA solution. It is also favoured by some helioseismic models, e.g. the model of [12] giving $f_B = (4.16 \pm 0.76) \times 10^6/\mathrm{cm}^2/\mathrm{sec}$.

The definition of χ^2 used in our fits is

$$\chi^2 = \sum_{i,j} (F_i^{\mathrm{th}} - F_i^{\mathrm{exp}})(\sigma_{ij}^{-2})(F_j^{\mathrm{th}} - F_j^{\mathrm{exp}}), \tag{2}$$

where i, j run over the number of data points, and both the experimental and theoretical values of the fitted quantities are normalised relative to the BPB00 [10] prediction. The experimental values for the total rates are the ones shown in Table 1, while the SK day/night spectra are taken from [3]. The error matrix σ_{ij} contains the experimental errors as well as the theoretical errors of the BPB00 predictions along with their correlations. The latter includes the uncertainty in the B neutrino flux of eq. (1). The error matrix is evaluated using the procedure of [13]. The details of the solar code used is described in [14]. As in [7] we vary the normalisation of the SK spectrum as a free parameter to avoid double counting with the SK data on total rate. Thus there are $(2 \times 19 - 1)$ independent data points from the SK day/night spectrum along with the 4 total rates giving a total of 41 points. In addition to the best-fit parameter values and $\chi^2_{\rm min}$ we shall present the goodness of fit (g.o.f.) of a solution, which represents the probability of a correct model having a $\chi^2 \geq$ this $\chi^2_{\rm min}$. Finally for the oscillation solutions with matter effects we shall also delineate the 90%, 95%, 99% and 99.73% (3σ) allowed regions in the mass and mixing parameter ($\Delta m^2 - \tan^2\theta$) plane. These regions correspond to $\chi^2 \leq \chi^2_{\rm min} + \Delta\chi^2$, where $\Delta\chi^2 = 4.61, 5.99, 9.21$ and 11.83 respectively for two parameters and $\chi^2_{\rm min}$ is the global χ^2 minimum.

Energy Independent Solution:

Table 2 summarises the results of fitting the global rate + spectrum data with an energy independent survival probability

$$P_{\nu_e\nu_e} = 1 - \frac{1}{2}\sin^2 2\theta. \tag{3}$$

It shows that even without any readjustment to the Cl rate or the B neutrino flux the energy independent solution gives an acceptable g.o.f. of 24%. An upward renormalisation of the Cl rate by 20% improves the g.o.f. to 42%. And varying the B neutrino flux downwards improves it further to 49%, which corresponds to a renormalisation factor $X_B = 0.7$ for the B neutrino flux. Note however that the g.o.f. and the best-fit value of the mixing angle for $X_B = 1$ are very close to those for $X_B = 0.7$. This is because the large error in the B neutrino flux of eq. (1) is already incorporated into the error matrix of eq. (2). It automatically chooses a lower value of flux, close to $X_B = 0.7$, even without floating this parameter.

Traditionally the energy independent solution (3) is associated with the averaged vacuum oscillation probability at distances much larger than the oscillation wave-length, as originally suggested by Gribov and Pontecorvo [15]. As we shall see below however an effectively energy independent solution holds around the maximal mixing region over a wide range of Δm^2 even after including all the matter effects.

Region of Energy Independent Solution in the $\Delta m^2 - \tan^2\theta$ Plane:

The energy dependence of the survival probability arises from different sources in different regions of the parameter space.

i) For $\Delta m^2/E < 10^{-14}$ eV the earth regeneration effect can be safely neglected. Then the survival probability

$$P_{\nu_e\nu_e} = P_1 \cos^2\theta + P_2 \sin^2\theta + 2\sqrt{P_1 P_2}\sin\theta\cos\theta\left(\frac{\Delta m^2 L}{E}\right), \qquad (4)$$

where L is the distance between the neutrino production point at the solar core and its detection point on earth; and P_2 $(= 1 - P_1)$ is the probability of the produced ν_e emerging from the sun as the heavier mass eigen-state

$$\nu_2 = \nu_e \sin\theta + \nu_\mu \cos\theta. \qquad (5)$$

The coherent interference term, represented by the last term of eq. (4), is responsible for the nonmonotonic energy dependence of the socalled Just-So (VO) solution.

ii) For $\Delta m^2/E \sim 10^{-14} - 10^{-11}$ eV, the coherent term is negligible while the earth regeneration contribution can be significant. Besides over a large part of this region, represented by the MSW triangle, the ν_e is adiabatically converted into ν_2 in the sun, i.e. $P_2 = 1$. Thus the day/night averaged probability

$$\bar{P}_{\nu_e\nu_e} = \sin^2\theta + \frac{\eta_E \sin^2 2\theta}{4(1 - 2\eta_E \cos 2\theta + \eta_E^2)}, \qquad (6)$$

where

$$\eta_E = 0.66\left(\frac{\Delta m^2/E}{10^{-13}\ \text{eV}}\right)\left(\frac{g/\text{cm}^2}{\rho Y_e}\right). \qquad (7)$$

Here ρ is the matter density in the earth and Y_e the average number of electrons per nucleon [16]. The regeneration contribution is always positive and peaks around $\eta_E \sim 1$, i.e. $\Delta m^2/E \sim 3 \times 10^{-13}$ eV. This is responsible for the LOW solution.

iii) Finally for $\Delta m^2/E > 10^{-11}$ eV the survival probability can be approximated by the average vacuum oscillation probability of eq. (3). The MSW solutions (LMA and SMA) lie on the boundary of the regions ii) and iii), i.e. $\Delta m^2 \sim 10^{-5}$ eV^2 for $E \sim 1$ MeV. The survival probability $P_{\nu_e\nu_e}$ goes down from $1 - \frac{1}{2}\sin^2 2\theta$ (> 0.5) to $\sin^2\theta$ (< 0.5) in going up from Ga to SK (SNO) energy.

All the solar neutrino rates except that of SK have $\gtrsim 10\%$ error, which is also true for the SK energy spectrum. The SK normalisation has at least similar uncertainty from the B neutrino flux. Therefore we shall treat solutions, which predict survival probability $P_{\nu_e\nu_e}$ within 10% of eq. (3) over the Ga to SK energy range, as effectively energy independent solutions [7]. Moreover the predicted Ga, Cl and SNO rates will be averaged over the respective energy spectra, while the predicted SK rates will be averaged over 0.5 MeV bins, corresponding to the SK spectral data, since experimental information is available for only these averaged quantities.

Fig. 1 shows the region of effectively energy independent solution as per the above definition. The parameter space has been restricted to $\Delta m^2 < 10^{-3}$ eV^2 in view of the constraint from the CHOOZ experiment [17]. One sees that the energy independent solution (3) is effectively valid over the two quasi vacuum oscillation regions lying above and below the MSW range. Moreover it is valid over a much larger range of Δm^2 around the maximal mixing region, since the solar matter effect does not affect $P_{\nu_e\nu_e}$ at $\tan^2\theta = 1$. It is this near-maximal mixing strip that is relevant for the observed survival probability, $P_{\nu_e\nu_e} \sim 1/2$. The upper strip ($\Delta m^2 = 10^{-3} - 10^{-5}$ eV^2) spans the regions iii) and part of ii) till it is cut off by the earth regeneration effect. The lower strip ($\Delta m^2 = 10^{-7} - 5 \times 10^{-10}$ eV^2) spans parts of region ii) and i) till it is cut off by the coherent term contribution. Note that this near-maximal mixing strip represents a very important region of the parameter space, which is favoured by the socalled bimaximal mixing models of solar and atmospheric neutrino oscillations [18,19].

One can easily check that averaging over the SK energy bins of 0.5 MeV has the effect of washing out the coherent term contribution for $\Delta m^2 > 2 \times 10^{-9}$ eV^2. Hence the contour of effectively energy independent solution, shown in ref. [7] neglecting the coherent term, is correct. But including this term enables us now to trace the contour down to its lower limit. It was claimed in ref. [9] that the lower strip disappears when one includes the coherent term contribution. This may be due to the fact that their predicted rate in the SK energy range was not integrated over the corresponding bin widths of 0.5 MeV.

To get further insight into the oscillation phenomenon in the maximal mixing region we have plotted in Fig. 2 the predicted survival rates at maximal mixing against Δm^2 for the Ga, Cl, SK and SNO experiments. In each case the rate has been averaged over the corresponding energy spectrum. This is similar to the Fig. 7 of Gonzalez-Garcia, Pena-Garay, Nir and Smirnov [16]. As in [16] the predictions have been shown relative to the central value of the B neutrino flux of BPB00 along with those differing by $\pm 20\%$ from the central value, i.e. $X_B = 1 \pm 0.2$. We have found that these curves are in good agreement with the corresponding ones of [16]. The two regions of $< 10\%$ energy dependence are indicated by vertical lines. One can easily check that in these regions the central curves lie within 10% of the energy independent prediction $R = 0.5$ (only the SK rate is higher due to the neutral current contribution). One can clearly see the violent oscillation in the Just-So (VO) region at the left and the LOW solution corresponding to the earth regeneration peak of the Ga experiment in the middle. The LMA and SMA are not identifiable since they do not occur at maximal mixing angle. It should be noted that the gap between the two energy independent regions due to the earth regeneration effect in Fig. 1 is a little narrower than here. This is due a cancellation between the positive contribution from the earth regeneration effect and the negative contribution from the solar matter effect at $\tan^2\theta < 1$. It ensures that the resulting survival rate agrees with the energy independent solution (3) over a somewhat larger range of Δm^2 slightly below the maximal mixing angle.

The observed rates from the Ga, Cl, SK and SNO experiments are shown in Fig. 2 as horizontal lines along with their 1σ errors. With 20% downward renormalisation of the B neutrino flux ($X_B = 0.8$) the energy independent prediction is seen to agree with the SK rate and also approximately with SNO. It is higher than the Cl rate by about 2σ. The agreement with the Ga rate can be improved by going to a little smaller θ and compensating the resulting deviation from the other rates by a somewhat smaller X_B as in Table 2. Nonetheless the maximal mixing solution for $X_B = 0.8$, shown here, is in reasonable agreement with the observed rates over the energy independent regions. The earth regeneration effect can be seen to improve the agreement with the Ga experiment for the LOW solution.

The SMA, LMA, LOW and VO Solutions:

Tables 3 and 4 summarises the results of fits to the global rates + spectrum data in terms of the conventional oscillation solutions. Table 3 shows solutions to the data with observed and renormalised Cl rate with the neutrino flux of BPB00 ($X_B = 1$), while Table 4 shows the effects of renormalising this B neutrino flux downwards by 25% ($X_B = 0.75$). The corresponding 90%, 95%, 99% and 99.73% (3σ) contours are shown in Fig. 3.

As we see from these tables and Fig. 3 the SMA solution gives rather poor fit in each case, with no allowed contour at 3σ level. This result agrees with the recent fits of [20] to the global data including SNO. The fit of [21] to these data shows a small allowed region for SMA solution at 3σ level due to a slightly different method of treating the normalisation in the SK spectrum data, as explained there.

The LMA and LOW solutions give good fits to the original data set, which improve further with the upward renormalisation of the Cl rate by 2σ. This is because the monotonic decrease of rate with energy, implied by these solutions, favours the Cl rate to be marginally higher than the SNO and SK rates as mentioned earlier. For the renormalised Cl case, downward renormalisation of the B neutrino flux by 25% is seen to give a modest increase (decrease) of g.o.f. for the best LOW (LMA) solution. On the other hand the allowed ranges increase in both cases as we see from Fig. 3b and c. Combined together they imply that the downward renormalisation of the B neutrino flux modestly favours the LOW solution but makes little difference to the LMA. Its main effect on these two solutions is increasing their allowed ranges in the parameter space. Comparing Fig. 3c with Fig. 1 shows that much of these enlarged ranges of validity correspond to effectively energy independent solution. It is intersting to note that the best-fit values of parameters in the LMA region are same for Cl observed and Cl renormalised cases while the χ^2_{min} improves for the latter. This shows that the best-fit already chose a probability at Cl energy, which is a little higher than that at SK/SNO energy. Renormalising the Cl rate brought that point up to the fitted curve without changing the best-fit parameters.

While the best vacuum solution seems to show remarkably high g.o.f. particularly for renormalised Cl rate and B neutrino flux, its regions of validity are two miniscule islands just below the lower energy independent strip of Fig. 3b,c. This solution has also been obtained in the global fits of ref. [20,21] as well as the SK fit to their rate and spectrum data [3]. The position and size of its range of validity suggest this to be a downward fluctuation of the effectively energy independent quasi vacuum oscillation rather than a genuine VO solution of Just-So type. To get further insight into this solution we have analysed the resulting energy dependence. It shows practically no energy dependence below 5 MeV, but a 15% fall over the $5 - 12$ MeV range. The latter seems to follow the SK spectral points rather closely amidst large fluctuation. To check the stability of this trend we have repeated the fit to the SK spectral data points, plotted over 8 broad energy bins shown in [3], which show much less fluctuation than the 2×19 points sample. The solution completely disappears from this fit. This confirms that the above VO solution is simply an artifact of the sampling of the SK spectral data.

For completeness we summarise in Table 5 the best fits of the above solutions with free B neutrino flux normalisation. The SMA solution favours a very low B neutrino flux ($X_B \simeq 0.5$), which raises the SK and SNO rates more than the Cl, thus accentuating the nonmonotonic energy dependence of Table 1. Still the g.o.f. of the SMA solution is rather marginal. On the other hand the LMA solution favours $X_B > 1$, which suppresses the SK and SNO rates more than the Cl, resulting in a monotonic decrease of rate with energy. But the corresponding g.o.f. are no better than those of Table 3. The results of the LOW and VO fits are similar to those of Table 4.

Comparison with the Ratio of SK and SNO Rates:

With the measurement of both charged and neutral current scattering rates at SNO it will be possible to discriminate between the above solutions, since the B neutrino flux factors out from their ratio [7]. In the absence of the neutral current data from SNO one can try to make a similar comparison with the ratio of SK elastic and SNO charged current scattering rates,

$$R^{el}_{SK} = X_B P_{\nu_e\nu_e} + r(1 - P_{\nu_e\nu_e})X_B, \; r = \sigma_{nc}/\sigma_{cc} \simeq 0.17, \tag{8}$$

$$R^{cc}_{SNO} = X_B P_{\nu_e\nu_e}, \tag{9}$$

where we have assumed a common survival rate neglecting the small difference between the SK and SNO energy spectra [22]. One can eliminate $P_{\nu_e\nu_e}$ from the two rates; and the resulting B neutrino flux can be seen to be in good agreement with the BPB00 estimate [4]. Alternatively one can factor out the flux from the ratio

$$R^{es}_{SK}/R^{cc}_{SNO} = 1 - r + r/P_{\nu_e\nu_e}. \tag{10}$$

Table 6 shows the best fit values of the above ratio for the LMA, LOW and the energy independent solutions along with the corresponding predictions for $R^{cc}_{SNO}/R^{nc}_{SNO} = P_{\nu_e\nu_e}$. The predictions of the maximal mixing solution is also shown for comparison. While the predictions for the CC/NC ratio differ by $\sim 50\%$ they differ by only about $\sim 15\%$ in the case of the ES/CC ratio. The observed ratio R^{es}_{SK}/R^{cc}_{SNO} is seen to favour the LMA over the LOW and energy independent solutions; but even the largest discrepancy is only $\sim 1.5\sigma$. With the expected sample of several thousand CC and NC events from SNO one expects to reduce the 1σ error for each of these ratios to about 5%. Then one will be able to discriminate between the three solutions meaningfully, particularly with the help of the CC/NC ratio from SNO.

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Table 1: The ratio of the observed solar neutrino rates to the corresponding BPB00 SSM predictions.

experiment	$\frac{obsvd}{BPB00}$	composition
Cl	0.335 ± 0.029	B (75%), Be (15%)
Ga	0.584 ± 0.039	pp (55%), Be (25%), B (10%)
SK	0.459 ± 0.017	B (100%)
	(0.351 ± 0.017)	
SNO(CC)	0.347 ± 0.027	B (100%)

Table 2: The best-fit value of the parameter, the $\chi^2_{\rm min}$ and the g.o.f from a combined analysis of rate and spectrum with the energy independent solution given by eq. (3).

	X_B	$\sin^2 2\theta$ $\begin{pmatrix}\tan^2\theta \\ \text{or} \\ \cot^2\theta\end{pmatrix}$	$\chi^2_{\rm min}$	g.o.f
Chlorine	1.0	0.93(0.57)	46.06	23.58%
Observed	0.72	0.94(0.60)	44.86	27.54%
Chlorine	1.0	0.87(0.47)	41.19	41.83%
Renormalised	0.70	0.88(0.48)	38.63	48.66%

Table 3: The best-fit values of the parameters, the $\chi^2_{\rm min}$ and the g.o.f from a combined analysis of the Cl, Ga, SK and SNO CC rates and the SK day-night spectrum in terms of ν_e oscillation into an active neutrino, including the matter effects. X_B is kept fixed at the SSM value (=1.0).

	Nature of Solution	Δm^2 in eV^2	$\tan^2\theta$	χ^2_{min}	g.o.f
	SMA	5.28×10^{-6}	3.75×10^{-4}	51.14	9.22%
Cl	LMA	4.70×10^{-5}	0.38	33.42	72.18%
Obsvd.	LOW	1.76×10^{-7}	0.67	39.00	46.99%
	VO	4.64×10^{-10}	0.57	38.28	50.25%
	SMA	4.94×10^{-6}	2.33×10^{-4}	50.94	9.54%
Cl	LMA	4.70×10^{-5}	0.38	30.59	82.99%
Renorm.	LOW	1.99×10^{-7}	0.77	34.26	68.57%
	VO	4.61×10^{-10}	0.59	32.14	77.36%

Table 4: Best fits to the combined rates and SK day-night spectrum data in terms of ν_e oscillation into active neutrino with a fixed $X_B = 0.75$.

	Nature of Solution	Δm^2 in eV^2	$\tan^2\theta$	χ^2_{min}	g.o.f
	SMA	5.28×10^{-6}	3.75×10^{-4}	48.39	14.40%
Cl	LMA	4.65×10^{-5}	0.49	38.90	47.44%
Obsvd.	LOW	1.74×10^{-7}	0.71	39.91	42.95%
	VO	4.55×10^{-10}	0.44	37.17	55.36%
	SMA	8.49×10^{-6}	1.78×10^{-4}	50.77	9.82%
Cl	LMA	4.64×10^{-5}	0.51	34.48	67.61%
enorm.	LOW	2.09×10^{-7}	0.81	33.47	71.97%
	VO	4.59×10^{-10}	0.53	30.63	82.86%

Table 5: Best fits to the combined rates and SK day-night spectrum data in terms of ν_e oscillation into active neutrino with X_B free.

	Nature of Solution	X_B	Δm^2 in eV^2	$\tan^2\theta$	χ^2_{min}	g.o.f
	SMA	0.51	5.25×10^{-6}	3.44×10^{-4}	46.83	15.41%
Cl	LMA	1.18	4.73×10^{-5}	0.33	32.32	72.89%
Obsvd.	LOW	0.88	1.75×10^{-7}	0.67	38.75	43.57%
	VO	0.70	4.55×10^{-10}	0.44	37.24	50.44%
	SMA	0.48	4.66×10^{-6}	2.32×10^{-4}	46.18	17.01%
Cl	LMA	1.15	4.71×10^{-5}	0.36	30.32	80.80%
Renorm.	LOW	0.83	2.03×10^{-7}	0.79	33.18	69.17%
	VO	0.75	4.63×10^{-10}	0.55	30.56	79.92%

Table 6: The values of the ratios R^{ES}_{SK}/R^{CC}_{SNO} and $R^{CC}_{SNO}/R^{NC}_{SNO}$ at the best-fit values for the LMA,LOW and energy independent solutions for the renormalised Cl and X_B =1.0 case. Also shown are the predictions for the maximal mixing ($P_{ee} = 0.5$) solution and the experimental value of R^{ES}_{SK}/R^{CC}_{SNO}.

	Δm^2	$\tan^2\theta$	$R^{CC}_{SNO}/R^{NC}_{SNO}$	R^{ES}_{SK}/R^{CC}_{SNO}	Expt. value of R^{ES}_{SK}/R^{CC}_{SNO}
LMA	4.7×10^{-5}	0.38	0.30	1.36	
LOW	1.99×10^{-7}	0.77	0.45	1.19	1.33 ± 0.13
energy-independent	-	0.47	0.56	1.13	
maximal mixing	-	1.0	0.5	1.15	

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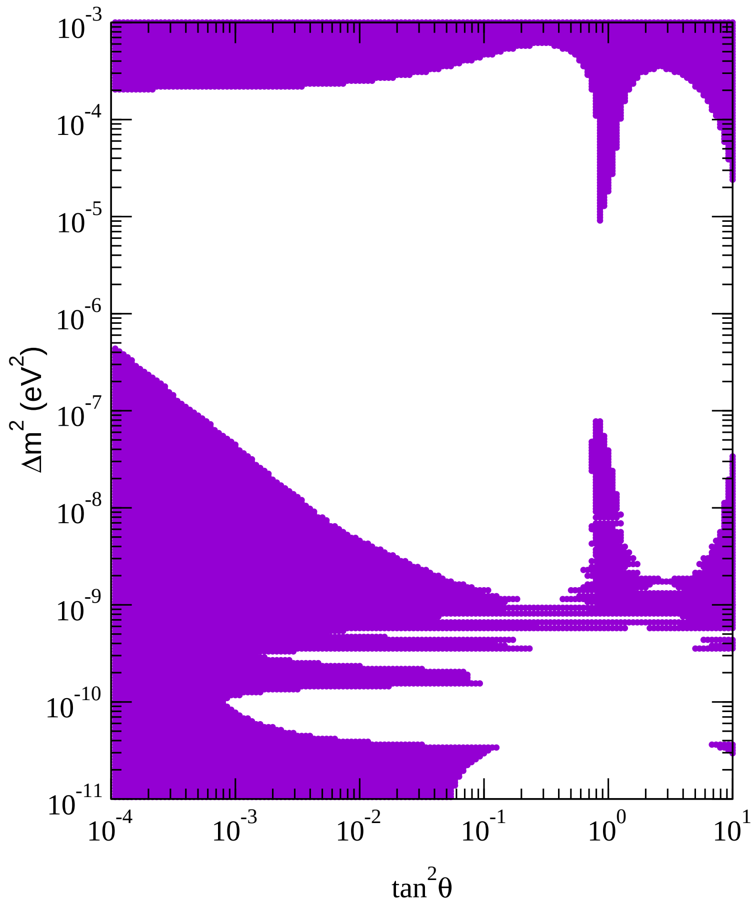

Fig 1: The region of effectively energy independent solution in $\Delta m^2 - \tan^2\theta$ parameter space where the solar neutrino survival probability agrees with eq. (3) to within 10% over the range of Ga to SK energies.

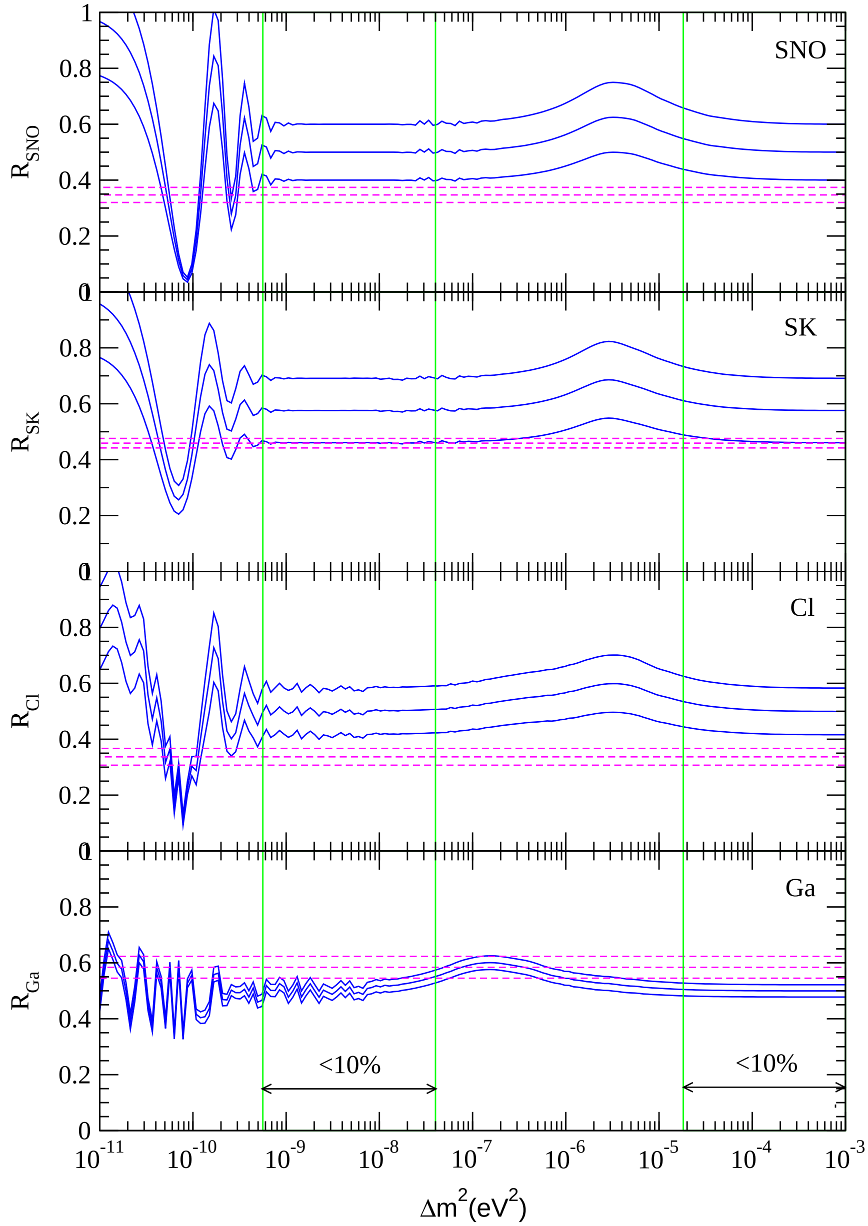

Fig. 2: The predicted survival rates at maximal mixing against Δm^2 for Cl, Ga, SK and SNO experiments (See text for details.)

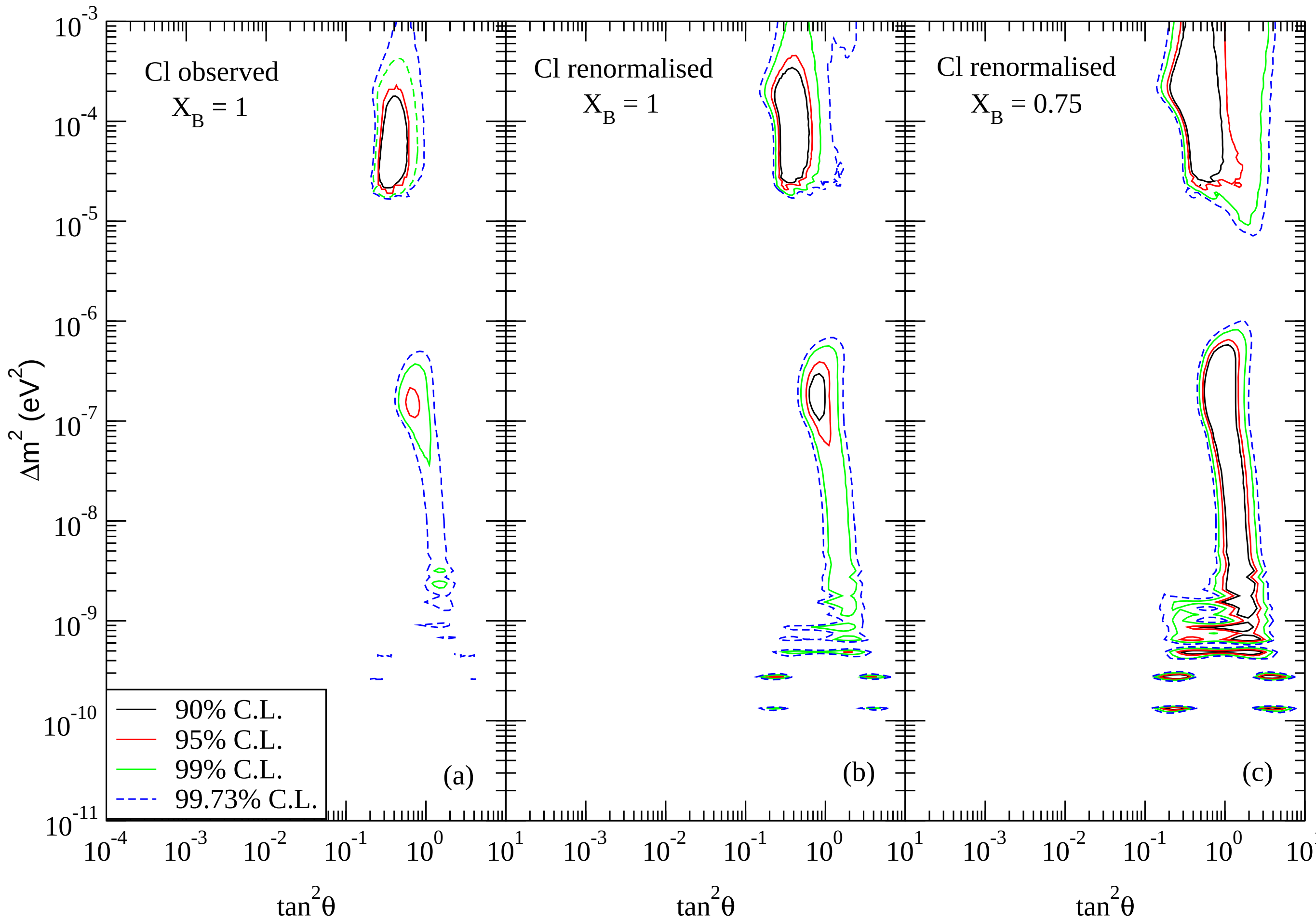

Fig. 3: The 90, 95 and 99 and 99.73% C.L. allowed area from the global analysis of the total rates from Cl (observed and 20% renormalised), Ga,SK and SNO (CC) detectors and the 1258 days SK recoil electron spectrum at day and night, assuming MSW conversions to active neutrinos.