ZENITH ANGLE RESPONSE OF A VERTICAL MESON TELESCOPE

BY R. P. KANE AND U. R. RAO
(*Physical Research Laboratory, Ahmedabad-9*)

Received October 25, 1957
(Communicated by Prof. V. A. Sarabhai, F.A.SC.)

ABSTRACT

Expressions have been obtained for the Geometrical Sensitivity as a function of the zenith angle of a cosmic ray telescope comprising of counter trays of rectangular dimensions. The Radiation Sensitivity and Cumulative Sensitivity have also been calculated, assuming a zenith angle attenuation of the form $I_\theta = I_\theta . \cos^2\theta$ for cosmic ray intensity.

I. INTRODUCTION

MEASUREMENTS of cosmic ray intensity in the vertical direction are mostly carried out by Geiger counter telescopes which comprise of a series of counter trays of rectangular dimensions situated one above the other. All counters in the same tray are connected in parallel and the pulses from the various trays are fed to a coincidence unit which gives a sizable pulse in the output only if pulses are fed to all its inputs within a small time interval, termed as the "resolving time" of the coincidence unit. The coincidence output rate corresponds to single charged particles passing through all the counter trays except for (1) a contribution due to showers where different trays could be triggered by different particles all belonging to the same shower and hence arriving almost simultaneously and (2) a contribution due to chance coincidences where different trays could be triggered by different particles apparently unrelated to each other. For coincidence units with resolving times of the order of a few microseconds as is usually the case and for more than two counter trays in coincidence, the contribution of chance coincidences is negligible. The contribution of side showers depends on the material in the near environment of the telescope and on the separation of the trays. Under optimum conditions this does not exceed about 7% for triple coincidence telescopes, as investigated by Greisen and Nereson.[1] Thus the vast majority of counts registered by such a counter telescope are due to cosmic radiation incident in the cone within which a single particle can cross all the trays.

A vertical meson telescope is characterised by (*a*) its dimensions which may be given as length, breadth and separation of the end trays or, alternatively, the semi-angles subtended by the telescope in two mutually perpendicular planes and the separation of the end trays, (*b*) the amount of absorber used in between the top and bottom trays. Once these characteristics are known it is possible to evaluate the expected counting rate of the telescope from known values of the vertical flux of cosmic ray intensity at the place of observation and the zenith angle dependence of the same.

A vertical meson telescope offers a maximum sensitive area for particles coming in the vertical direction. A simple geometrical consideration shows, however, that the solid angle available for particles coming in inclined directions is greater than the one available for those coming in a vertical direction. Taking into account both these factors one can calculate the Geometrical Sensitivity G.S. (θ) of any telescope arrangement for different values of the inclination θ which the incoming cosmic ray trajectories make with the vertical. Parsons[2] has obtained an expression for the same in case of a meson telescope of cubical symmetry which is recommended as standard equipment for the current International Geophysical Year. His method can, however, be extended to counter telescopes of any dimensions. We have attempted here to get a general expression for telescopes of any given dimensions.

II. Geometrical Sensitivity of a Vertical Meson Telescope

Consider a geometrical arrangement (Fig. 1) in which the top and bottom trays are represented by ABCD and A′B′C′D′ respectively. Let the length AB = A′B′ = CD = C′D′ = l and breadth BC = AD = B′C′ = A′D′ = d. The separation between the two trays is AA′ = BB′ = CC′ = DD′ = a.

If the ratios l/a and d/a are denoted by Δ and δ respectively, the semi-angles of the telescope in the two vertical planes one along the length and the other perpendicular to it are $\tan^{-1}(\Delta)$ and $\tan^{-1}(\delta)$ respectively. The most inclined direction the telescope can record has a zenith angle $\theta = \tan^{-1} \sqrt{l^2 + d^2}/a = \tan^{-1} \sqrt{\Delta^2 + \delta^2}$ and corresponds to rays parallel to the diagonals AC′, BD′, CA′ or DB′.

Consider cosmic rays incident along a zenith angle θ and an azimuth α, where $\alpha = 0$ for a direction parallel to the breadth 'd' of the counter trays. The beam will then be incident on the rectangular area OXD′Y of the lower tray where

$$\mathrm{OX} = (l - a \cdot \tan\theta \cdot \sin\alpha) \text{ and}$$

$$\mathrm{OY} = (d - a \cdot \tan\theta \cdot \cos\alpha) \quad (1)$$

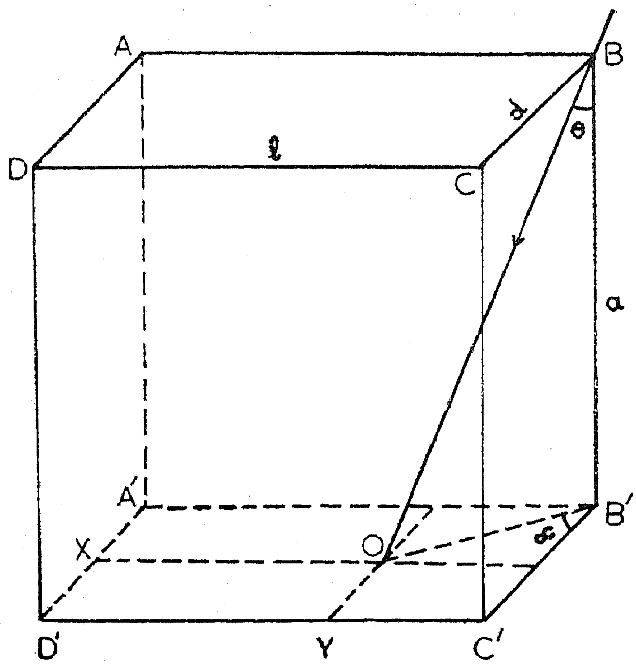

FIG. 1

The effective cross-sectional area of the telescope normal to the beam would be

$$A = \cos\theta \cdot (l - a \tan\theta \cdot \sin\alpha) \cdot (d - a \cdot \tan\theta \cdot \cos\alpha). \qquad (2)$$

Consider the shaded area (Fig. 2) normal to the direction of incidence of a beam which has a zenith angle ranging from θ to $(\theta + d\theta)$ and azimuth

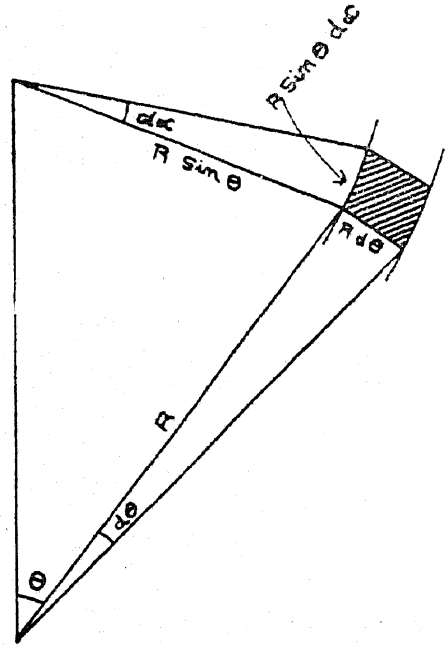

FIG. 2

from α to $(\alpha + d\alpha)$. The solid angle subtended by this beam at the centre of the telescope is given by

$$dw = \frac{(\mathrm{R}.\sin\,\theta.d\alpha).\mathrm{R}d\theta}{\mathrm{R}^2}$$

$$= \sin\,\theta.d\theta.d\alpha \tag{3}$$

The relative Geometrical Sensitivity of the telescope for directions confined to zenith angles between θ and $(\theta + d\theta)$ and azimuth between α and $(\alpha + d\alpha)$ is given by

$$\mathrm{G.S.}\,(\theta).d\theta.d\alpha = \mathrm{A}.\sin\,\theta.d\theta.d\alpha \tag{4}$$

The total relative Geometrical Sensitivity of the telescope corresponding to a zenith angle θ is

$$\mathrm{G.S.}\,(\theta) = \int_{\alpha_1}^{\alpha_2} \mathrm{A}.\sin\,\theta.d\alpha$$

$$= \sin\,\theta.\cos\,\theta\,[l\,d.(\alpha_2 - \alpha_1) - a.\tan\,\theta.l.(\sin\,\alpha_2 - \sin\,\alpha_1) + a.\tan\,\theta.d.(\cos\,\alpha_2 - \cos\,\alpha_1) - (a^2/4).\tan^2\theta.(\cos\,2\alpha_2 - \cos\,2\alpha_1) \tag{5}$$

Putting $(d/a) = \delta$ and $(l/a) = \Delta$,

$$\frac{\mathrm{G.S.}\,(\theta)}{a^2} = \sin\,\theta.\cos\,\theta\,[\Delta.\delta.(\alpha_2 - \alpha_1) - \Delta.\tan\,\theta.(\sin\,\alpha_2 - \sin\,\alpha_1) + \delta.\tan\,\theta.(\cos\,\alpha_2 - \cos\,\alpha_1) - (\tan^2\,\theta/4).(\cos\,2\alpha_2 - \cos\,2\alpha_1)] \tag{6}$$

where α_1 to α_2 is the range of azimuth for which the effective cross-sectional area A has positive values.

Assuming that the breadth 'd' is less than the length 'l', the zenith angle θ can be grouped into three ranges for each of which a set of values of α_1 and α_2 would be effective.

Case 1.—$0 < \theta \leqslant \tan^{-1}(\delta)$. For this range, the azimuth values range from 0 to 2π. Since, however, all the quadrants are symmetrical, it is sufficient to integrate from 0 to $\pi/2$. Thus $\alpha_1 = 0$ and $\alpha_2 = \pi/2$.

Case 2.—$\tan^{-1}(\delta) < \theta \leqslant \tan^{-1}(\Delta)$. For values of θ in this range (*see* Fig. 3) the particle would miss the telescope for azimuth $\alpha = 0$. The cross-sectional area A will first become positive for values of azimuth given by

$$\cos \alpha_1 = \frac{d}{D} = \frac{d/a}{D/a} = (d/a).\cot \theta = \delta.\cot \theta$$

Thus the lower limit is $\alpha_1 = \cos^{-1}(\delta.\cot \theta)$. The upper limit for integration is still $\pi/2$ as in case 1.

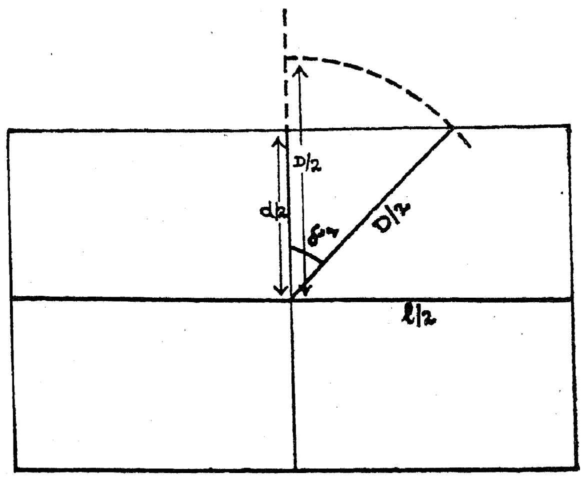

FIG. 3

Case 3.—$\tan^{-1}(\Delta) < \theta \leqslant \tan^{-1}\sqrt{\Delta^2 + \delta^2}$. For this region (*see* Fig. 4) the particles will have a positive cross-sectional area for values of α between α_1 and α_2, where

$$\alpha_1 = \cos^{-1}(\delta.\cot \theta) \text{ and } \alpha_2 = \pi/2 - \cos^{-1}(\Delta.\cot \theta).$$

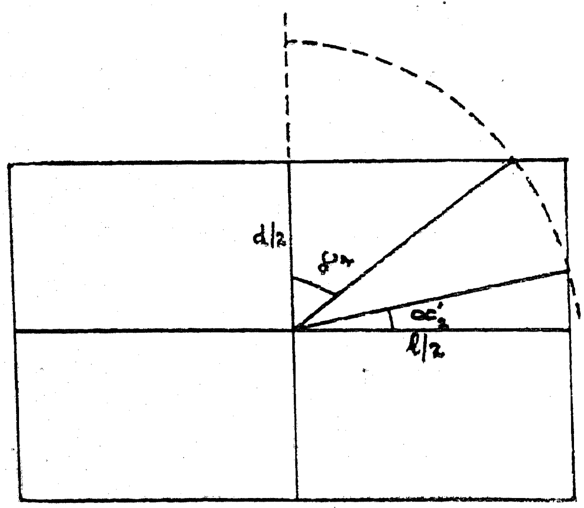

FIG. 4

For isotropic radiation incident upon the telescope, G.S. (θ) is directly proportional to the number of particles coming at a zenith θ. If, however, there is a zenith angle dependence of the cosmic radiation of the form

$$I_\theta = I_0 \cdot \cos^\lambda \theta \tag{7}$$

the Radiation Sensitivity R.S. (θ) is given by

$$\text{R.S.}(\theta) = \cos^\lambda \theta . \text{G.S.}(\theta). \tag{8}$$

III. Results

In Figs. 5, 6 and 7, the relative Geometrical Sensitivity and Radiation Sensitivity for various values of the semi-angles $\tan^{-1}(\delta)$ and $\tan^{-1}(\Delta)$ of a telescope are plotted. The value chosen for λ is 2.

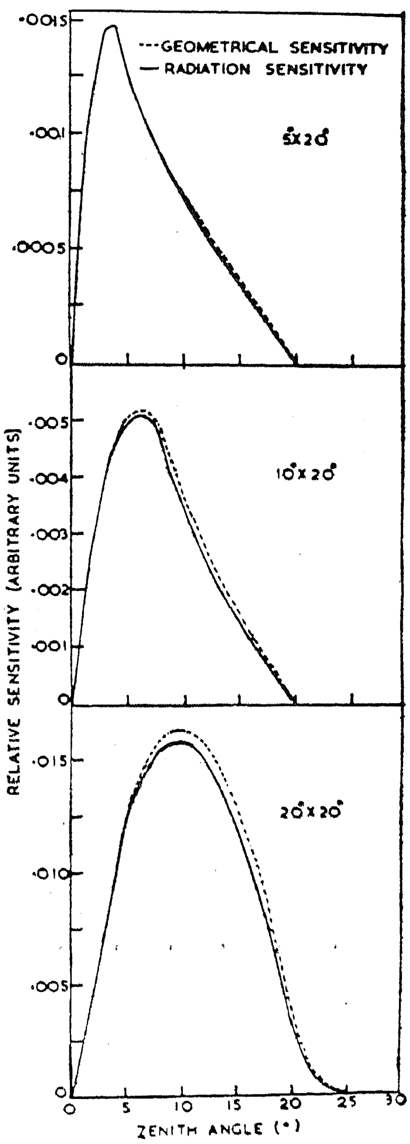

FIG. 5. Zenith angle dependence of Geometrical Sensitivity and Radiation Sensitivity for a telescope having a semi-angle of 20° in one plane and 5°, 10° and 20° in a perpendicular plane.

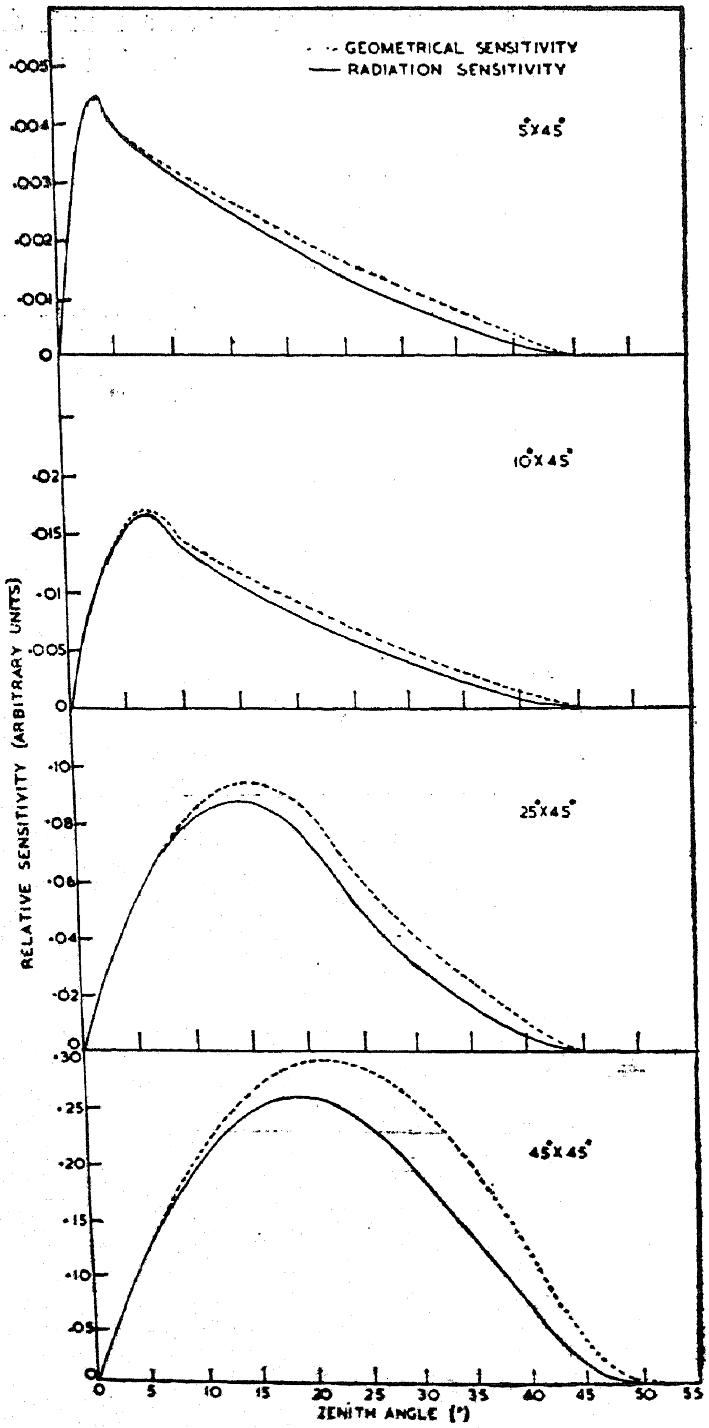

FIG. 6. Zenith angle dependence of Geometrical Sensitivity and Radiation Sensitivity for a telescope having a semi-angle of 45° in one plane and 5°, 10°, 25° and 45° in a perpendicular plane.

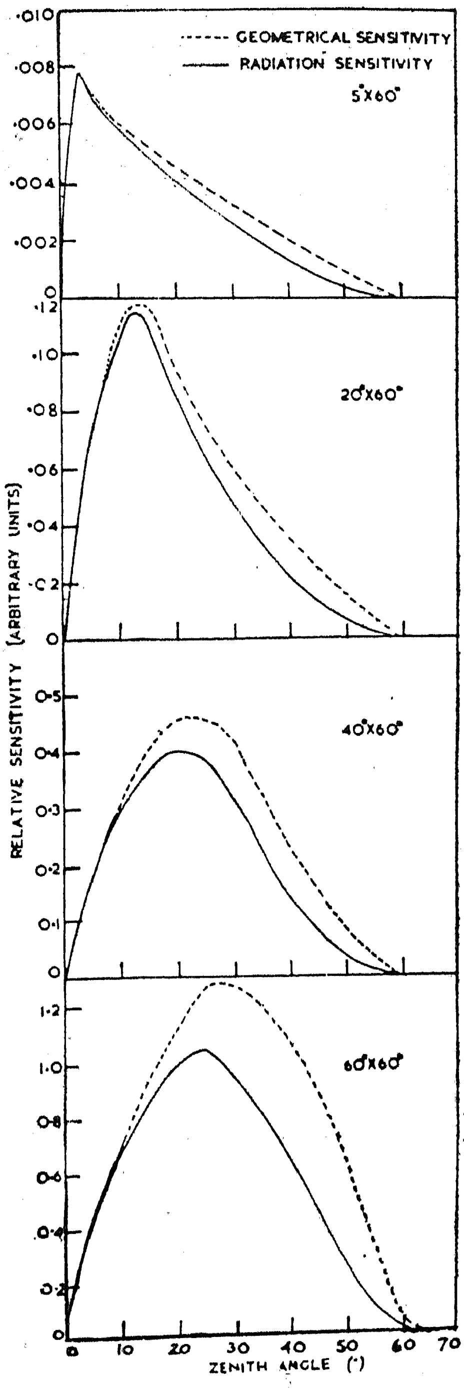

FIG. 7. Zenith angle dependence of Geometrical Sensitivity and Radiation Sensitivity for a telescope having a semi-angle of 60° in one plane and 5°, 20°, 40° and 60° in a perpendicular plane.

The total counting rate of a telescope is given by

$$N = 4I_0 \cdot \int_0^{\tan^{-1}\sqrt{\Delta^2+\delta^2}} R.S.(\theta) \cdot d\theta \qquad (9)$$

The percentage contribution to the total rate of particles confined to the zenith angles between 0 and any value θ_0 is given by the Cumulative Sensitivity C.S. as

$$\text{C.S.} = [100 \cdot \int_0^{\theta_0} \text{R.S.}\,(\theta).d\theta] \div [\int_0^{\tan^{-1}\sqrt{\Delta^2+\delta^2}} \text{R.S.}\,(\theta).d\theta] \tag{10}$$

The Cumulative Sensitivity for various zenith angles and for various values of $\tan^{-1}(\delta)$ and $\tan^{-1}(\Delta)$ is shown in Figs. 8, 9 and 10.

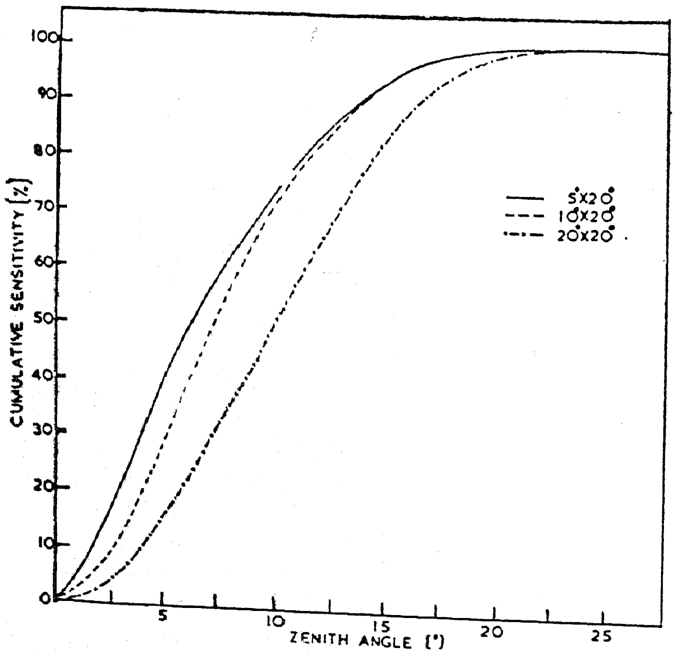

FIG. 8. Percentage Cumulative Sensitivity of a telescope having a semi-angle of 20° in one plane and 5°, 10° and 20° in a perpendicular plane.

A striking feature revealed by the plots of Cumulative Sensitivity is that the bulk of the radiation is confined to comparatively small zenith angles in spite of large opening of the telescope. Thus, for a telescope having semi-angles as large as 60° in both the East-West and North-South planes, about 70% of the radiation is confined to zenith angles less than about 35°. The bias towards smaller zenith angles is increased still further if one of the semi-angles of the telescope is small. For example, a telescope having semi-angles 20° and 60°, counts 70% of its counting rate within zenith angles of 0 to 25°.

In designing a telescope for a particular investigation, the research worker is interested in the directional sensitivity of his instrument. The present work enables the investigator to decide the shape of a counter tray which would satisfy requirements of directional sensitivity in the most appropriate

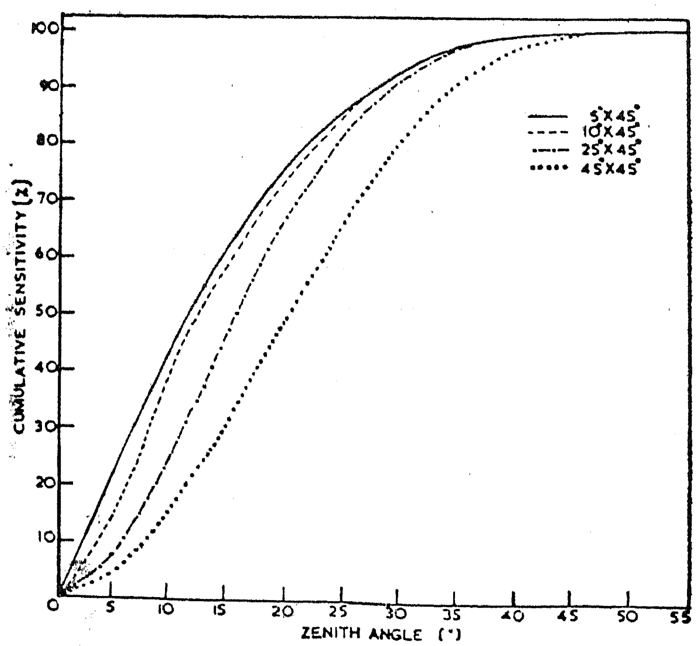

FIG. 9. Percentage Cumulative Sensitivity of a telescope having a semi-angle of 45° in one plane and 5°, 10°, 25° and 45° in a perpendicular plane.

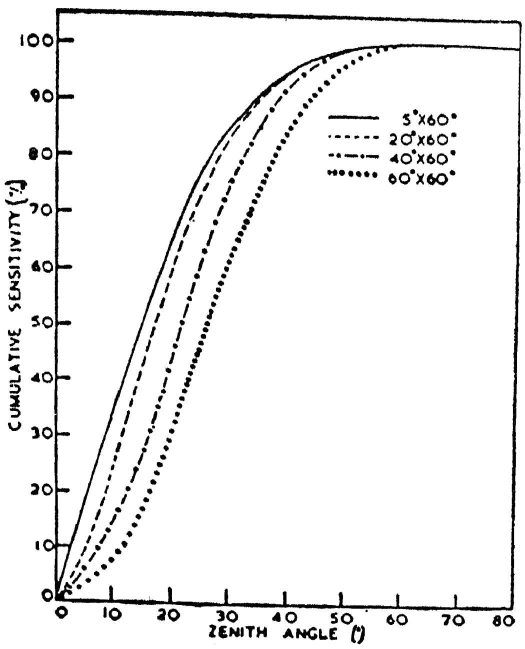

FIG. 10. Percentage Cumulative Sensitivity of a telescope having a semi-angle of 60° in one plane and 5°, 20°, 40° and 60° in a perpendicular plane.

way. If, for example, the requirements specify an 80% response due to particles within 10° inclination with zenith, irrespective of azimuth of direction of arrival, a counter tray of square geometry obviously provides the optimum solution. If, however, we are interested in a narrow angle of 5° in the E.-W. plane, a rectangular tray of dimensions 1 : 4 in the E.-W. and N.-S. planes would still provide an 80% response restricted to 10° with zenith.

The authors are grateful to Prof. V. A. Sarabhai for helpful discussions and to the Atomic Energy Commission of India for financial assistance.

REFERENCES

1. Greisen, K. and Nereson, N. *Phys. Rev.*, 1942, **62**, 316.
2. Parsons, N. R. .. *Rev. Sci. Instru.*, 1957, **28**, 265.