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2006 Nanotechnology 17 152

(http://iopscience.iop.org/0957-4484/17/1/024)

INSTITUTE OF PHYSICS PUBLISHING NANOTECHNOLOGY

Nanotechnology **17** (2006) 152–156 doi:10.1088/0957-4484/17/1/024

$1/f$ noise in nanowires

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Received 3 August 2005, in final form 3 October 2005
Published 1 December 2005

Abstract
We have measured the low-frequency resistance fluctuations (1 mHz $< f <$ 10 Hz) in Ag nanowires of diameter 15 nm $\leqslant d \leqslant$ 200 nm at room temperature. The power spectral density (PSD) of the fluctuations has a $1/f^{\alpha}$ character as seen in metallic films and wires of larger dimension. Additionally, the PSD has a significant low-frequency component and the value of α increases from the usual 1 to $\simeq 3/2$ as the diameter d is reduced. The value of the normalized fluctuation $\frac{\langle \Delta R^2 \rangle}{R^2}$ also increases as the diameter d is reduced. We observe that there are new features in the $1/f$ noise as the size of the wire is reduced and they become more prominent as the diameter of the wires approaches 15 nm. It is important to investigate the origin of the new behaviour as $1/f$ noise may become a limiting factor in the use of metal wires of nanometre dimensions as interconnects.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Noise and fluctuation, in general, are enhanced when the system size is reduced. In the case of equilibrium thermal fluctuation of a physical quantity, it is inversely proportional to the system size. In this paper, we would like to investigate the specific issue of electrical noise in metallic nanowires. This is an important issue not only as a basic question but also as an important parameter that should be considered for the use of nanowires as interconnects in nanoelectronics. The equilibrium white thermal noise (the Nyquist noise) of a wire of resistance R at a temperature T can be estimated from the power spectral density $S_{\mathrm{th}} \approx 4k_{\mathrm{B}}TR$ [1], where k_{B} is the Boltzmann constant. This component of the electrical noise depends only on the sample resistance R. The Nyquist noise has no explicit dependence on of the size of the sample; the only size dependence is through the size dependence of R. However, in a current-carrying nanowire a larger contribution to the electrical noise is expected to arise from the 'excess noise' or 'conductance or resistance noise'. This noise has a spectral power density of the type $S(f) \propto 1/f^{\alpha}$, where $\alpha \sim 1$ and is known as '$1/f$ noise'. We note that *a priori* there is no way of estimating the actual value of the $1/f$ noise because $1/f$ can arise from many different sources [2, 3]. As a result, an experimental measurement is an absolute necessity to make even an estimate of this noise component. For metallic films, past studies have shown that an estimate of the value of the $1/f$ noise can be made from the empirical Hooge's formula, given as

$$S_V(f) = \frac{\gamma_{\mathrm{H}} V^2}{N \cdot f} \tag{1}$$

where N is the number of electrons in the sample, V is the bias used for measurements and γ_{H} is an empirical constant which for metallic films lies in the range $\simeq 10^{-3}$–10^{-5}. In this paper we address the issue of $1/f$ electrical noise in nanowires. We have measured the low-frequency resistance fluctuations (1 mHz $< f <$ 10 Hz) in Ag nanowires of diameter 15 nm $\leqslant d \leqslant$ 200 nm at room temperature. The wires were electrochemically grown in templates of anodic alumina or polycarbonate. We observe that the noise indeed has a spectral power density $S(f) \propto 1/f^{\alpha}$ and the magnitude of noise increases as the size is reduced. The observed noise in the metallic nanowires is larger ($\gamma_{\mathrm{H}} \sim 4 \times 10^{-3}$) than one would expect from a simple extrapolation of Hooge's relation to this domain. The exponent α also deviates from 1 as the diameter (d) of the nanowire is reduced. To our knowledge, this is the first report of the investigation of $1/f$ noise in nanowires in this range of diameters.

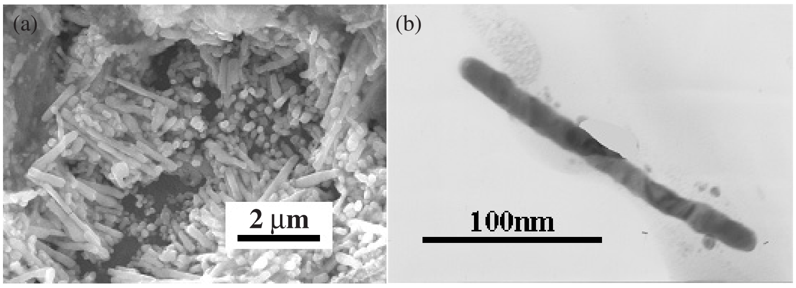

Figure 1. (a) SEM image of 100 nm wires taken after dissolving the template. (b) TEM image of a 15 nm Ag wire.

2. Experimental techniques

The Ag nanowires have been grown within polycarbonate (PC) templates (etched track membranes) or anodic alumina (AA) templates using electrochemical deposition from $AgNO_3$ solution [4]. The membranes have been procured from a standard commercial source [5]. During the growth one side of the membrane was covered with an evaporated metal and was used as the anode. The other electrode was a micro-tip (radius of curvature $\simeq$ 100 μm) that can be positioned by a micropositioner to a specific area on the other side of the membrane and the growth can thus be localized. The arrangement is shown as inset in figure 2. The growth current used is approximately 10 mA. Generally the growth occurs within the pore by filling it from end to end. As soon as a few wires grow and touch the electrodes, the electrochemical cell becomes short circuited and the electrochemical deposition stops. The membrane containing the nanowire is then cleaned with deionized water and annealed at 400 K for 10–12 h. During annealing a current 1 mA of is passed through the sample. Post-deposition annealing is needed to stabilize the wire. The wires after growth can be retained within the membranes, as was done for the electrical measurements, or can be freed from the membrane by etching the membrane off with a suitable etchant (dichloromethane for PC and KOH for AA).

The wires after growth were characterized by x-ray, SEM and TEM. Figure 1(a) shows an SEM image of the assembly of 100 nm wires (after partially dissolving off the template). A TEM image of a 15 nm wire (taken after removing the membrane) is shown in figure 1(b). From the TEM data we find that the wires are single crystalline with occasional twinning. The selective area diffraction patterns showed that the wires have face-centred cubic (FCC) structure. The diameter of the wire is close (within 10%) to the nominal diameter of the pore of the membrane. This shows that the wires grow by filling the membranes.

Table 1 gives the diameter and lengths of different samples that have been used in the measurements. In figure 2 we show the typical XRD data for a 20 nm wire. The XRD data on the nanowire matches well with the data taken on a bulk silver sample. The observed lines can all be indexed to FCC Ag. We have measured the resistance of the arrays of nanowires down to 4.2 K. This was done to check the residual resistivity ratio and the behaviour of the resistance at low temperatures. The resistance noise measurements were done at room temperature using a digital signal processing (DSP) based ac technique which allows simultaneous measurement of the background noise as well as the bias-dependent noise from the sample [6, 7]. The sample is current biased and the resistance noise appears as a voltage fluctuation. The data are taken by stabilizing the temperature with $\Delta T/T \simeq 4 \times 10^{-3}\%$. The measured background noise (bias independent) was white and was contributed by the $4k_BTR$ Nyquist noise. The apparatus can measure a spectral power down to 10^{-20} V^2 Hz^{-1}. We have used a transformer preamplifier SR554 to couple the sample to the lock-in amplifier. The basic electrical schematic diagram is shown in the inset of figure 3. The details of the data acquisition and the signal processing are given elsewhere [6, 7]. A single set of data is acquired typically for a time period of about 50 minutes or more at a sampling rate of 1024 points s^{-1}. The time series of voltage fluctuations as a function of time consisting

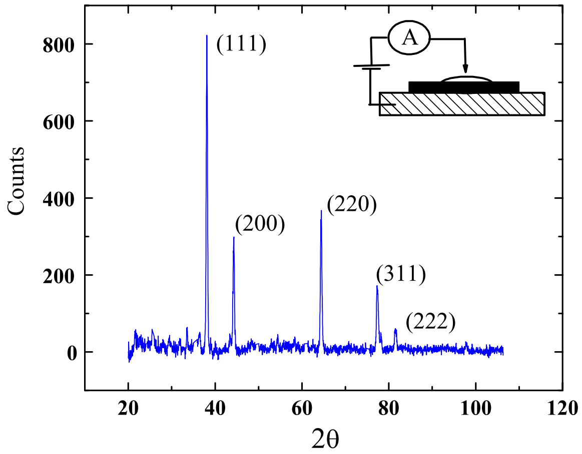

Figure 2. The XRD data for a 20 nm Ag wire. The XRD data on the nanowire matches the data taken on a bulk silver sample and the observed lines can all be indexed to FCC Ag.

Table 1. The diameter and length of the various samples used in this work.

Sample	Diameter of pore in membrane (nm)	Length of wire (μm)
1	15	6
2	20	60
3	30	6
4	50	6
5	100	60
6	200	6

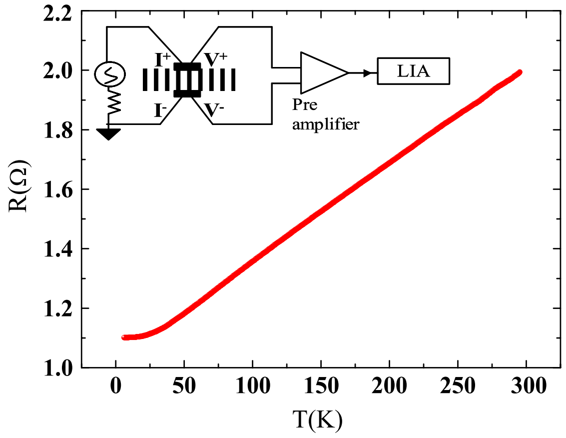

Figure 3. The resistance of Ag wires of 15 nm diameter as a function of temperature. The inset shows a schematic diagram of the measurement electronics.

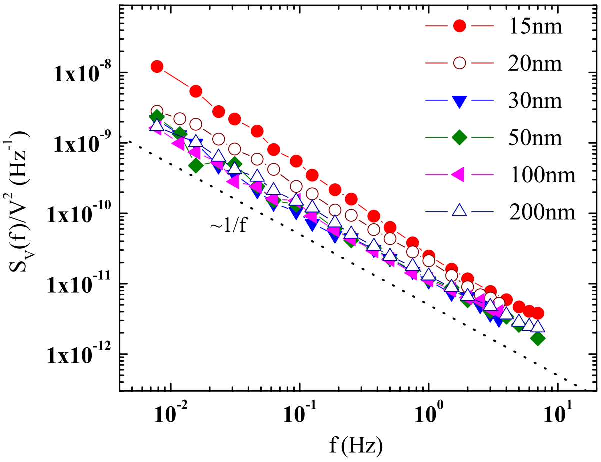

Figure 4. Low-frequency power spectrum of the voltage noise arising from the resistance fluctuation in the current-biased nanowires. The dotted line is a plot for which $\alpha = 1$.

of nearly 3 million points was decimated to about 0.1 million points before the power spectral density $S_V(f)$ is determined numerically. The frequency range probed by us ranges from 1 mHz to 10 Hz. The frequency range is determined mostly by practical considerations. The lower frequency limit is determined by the quality of the temperature control. The sample resistance and the bridge output may also show a long time drift (in general such a long time drift is subtracted out by a least-square fit to the data). Taking these factors into consideration, the lower spectral limit in our experiment has been kept at 1 mHz. The high-frequency limit of 10 Hz is decided by the spectral window of the lock-in amplifier output filter which is operated at 24 dB/octave with a time constant of 3 ms.

The resistance and noise measurements were carried out by retaining the wires within the polymeric or alumina membranes. The sample, as stated before, consists of an array of nanowires. On each of the two sides of the membrane two electrical leads were attached using silver epoxy. Though the measurements were made with the wires retained within the membrane, the system is an array of parallel nanowires where the individual wires are well separated by the insulating membrane. A very important issue in this measurement is the contribution of the contacts to the measurement. We have paid attention to this aspect and measured both resistance and noise by making the contact in different ways. We discuss this issue in more detail at a later stage when we discuss the data. We find that the contact contribution, if any, is negligibly small both in the measured resistance as well as the noise. The resistance measurements were done using the same electrodes that were used for the noise measurements.

3. Results and discussion

In figure 3 we show the temperature dependence of the resistance of Ag nanowire arrays of diameter 15 nm. The resistivity data for the wires of other diameters are qualitatively similar and are therefore not shown. The arrays contain different numbers of wires. It is possible to make a rough estimate of the number of wires by a procedure described before [8] to within a factor of 2. Due to the uncertainty in the number of wires in the arrays we cannot fix the exact resistivities of the nanowires even if their diameter and the length are known to a reasonable degree of certainty. For all the samples the resistance has a fairly linear temperature dependence down to 75 K, and R reaches a residual value below 30 K. The residual resistivity ratio (RRR) $\frac{\rho_{300\ \mathrm{K}}}{\rho_{4.2\ \mathrm{K}}}$ for all the wires lies between 3 and 6. We find that the wires do not show any upturn in resistivity at low temperatures. This shows that the conduction of electrons in these wires is not controlled by disorder-induced effects like localization [9]. This ensures that the product $k_F l_e$ is much larger than 1. (Here k_F is the Fermi wavevector and l_e is the elastic mean free path.) The temperature coefficient of resistivity ($\beta = \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}T}$) can be obtained from the resistance data for the nanowires. β is in the range $\approx 2.5\text{–}3.5 \times 10^{-4}\ \mathrm{K}^{-1}$. This is smaller than but comparable to that of bulk silver, which is $\approx 3.8 \times 10^{-4}\ \mathrm{K}^{-1}$. The temperature dependence of the resistivity thus shows predominantly metallic behaviour, which is in agreement with the TEM observation that the wires are single crystalline. For bulk silver at room temperature the mean free path is $\approx$60 nm. For the wires with diameter $<$50 nm, the mean free path will have significant contribution from the surface as well as from the grain boundaries within the wire [10, 11]. This is an issue of considerable current interest due to diminishing size of the metallic interconnects in integrated circuits [12, 13]. Given the scope of the paper we do not elaborate on this particular issue in detail but note that these factors which limit the electron mean free path can also contribute to the resistance noise.

In figure 4 we show the typical low-frequency power spectrum of the voltage noise arising from the resistance fluctuation in the current-biased nanowires. The spectral power $S_V(f)$ is $\propto$ to V^2, where V is the bias voltage (typically a few tens of μV) and in figure 4 the spectral power is shown as $S_V(f)/V^2$. The power spectra have a predominant $1/f^\alpha$ nature. The observed noise appears to be independent of the nature of the membrane (polymer or alumina) as the observed noise data in wires of similar diameter are qualitatively the same. We note that contact can be a serious issue in noise measurements and we carried out a number of checks to rule out any predominant contribution of the contact to the

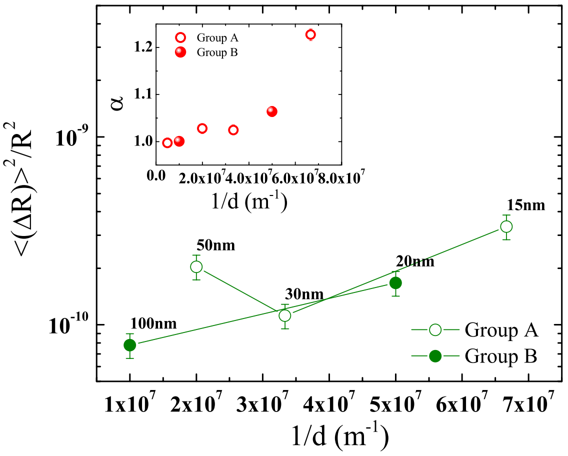

Figure 5. The normalized resistance fluctuation $\frac{\langle \Delta R^2 \rangle}{R^2}$ at 298 K as a function of the inverse wire diameter $1/d$. The inset shows the variation of α with $1/d$.

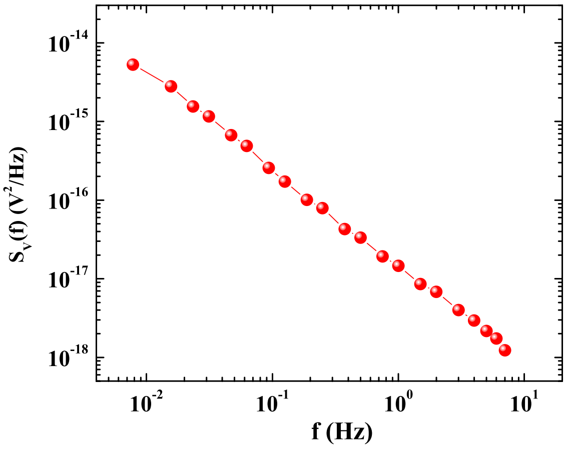

Figure 6. Relative resistance fluctuations in Cu nanowires of diameter 15 nm grown by an electrochemical method in PC membranes.

measured noise. In addition to silver epoxy contacts we also used evaporated silver films and Pb–Sn solder to make contact. We find that they give similar results to within ±15%. In the case of the Pb–Sn contact the change in R of the sample as we go below the superconducting transition temperature (∼7 K) of the solder is negligibly small (<2–3%), implying a very small contribution of the contact to the total R measured. We have also measured the noise in different samples with varying numbers of wires in them. The resistance of the array, due to different number of wires in them, varies. Also, such samples will have significantly different contact areas and hence different contact noise, if any. Yet we find that the normalized noise $\frac{\langle (\Delta R)^2 \rangle}{R^2}$ in different arrays of the same diameter wire lie within ±15%. All these tests rule out any predominant contribution from the contacts.

The normalized resistance fluctuation $\frac{\langle \Delta R^2 \rangle}{R^2}$ for a nanowire array of resistance R is obtained by integrating the power spectral density $S_V(f)/V^2$ over the band width of our measurement. This equation, while giving correctly the relative resistance fluctuation of a single wire, is also applicable when there is more than one wire in parallel because of the following reasoning. If R is the resistance of **n** identical parallel wires each of resistance R_1, then $1/R = n/R_1$. From this we get $\frac{\langle \Delta R^2 \rangle}{R^2} = \frac{\langle \Delta R_1^2 \rangle}{R_1^2}$, thus implying that the measured noise is independent of the number of wires in parallel.

We make the interesting observation that resistance fluctuation has a dependence on the diameter of the wire. For this purpose we separate the wires into two groups according to their length. Group A contains wires grown in the PC membranes, and the have a length of 6 μm. Group B contains wires of length 60 μm that were grown in alumina templates. Thus a nanowire array belonging to a given group differs only in their diameters. The data are shown in figure 5. One can clearly see that for nanowires of a given length (group A or B) there is an increase in the normalized noise as the diameter d is reduced.

The exponent α has an interesting dependence on the size of the wire as well. This is shown in the inset of figure 5, where α is plotted as a function of $1/d$. For wires with diameter $d \geqslant 100$ nm the exponent α is ≈1. The value of the exponent is similar to what one finds in wires of larger diameter and in metallic films. This is generally expected to arise from activated defect dynamics as envisioned in the Dutta–Horn model [3]. However, there is a perceptible increase in α once the wire diameter reaches below 50 nm, and it increases further when the diameter is reduced to 15 nm. Such a change in α is rather interesting and would indicate the presence of an additional noise mechanism in the nanowires which is not present in wires of larger diameter. We are currently investigating the origin of such an extra component as well as the physical reason for the enhancement of the normalized noise on reducing the diameter d of the nanowire [14].

To check that such a size dependence of the $1/f$ noise is indeed observed in other metallic nanowires we have also carried out noise measurements in Cu nanowires of diameter 15 nm grown by a similar electrochemical method in PC membranes. The spectral power density is shown in figure 6. The behaviour of the spectral power is rather similar to the Ag nanowire of the same diameter and the magnitude of the resistance fluctuation $\frac{\langle \Delta R^2 \rangle}{R^2} \approx 1 \times 10^{-9}$ at 300 K, which is comparable to that seen in 15 nm Ag nanowire. This strongly suggests that the observed behaviour may be a general behaviour of metallic nanowires.

It is tempting to express the observed results in the form of a Hooge relation (equation (1)). However, due to the uncertainty in the exact numbers of wires in the arrays we cannot estimate γ_H. Also, the Hooge relation is valid for $\alpha = 1$ and any deviation of α from 1 makes γ_H dependent on the exact frequency of measurement.

To summarize, we find that in metallic Ag (and Cu) nanowires the low-frequency resistance fluctuation is enhanced when the diameter is reduced. The power spectral density of the fluctuation has a $1/f^{\alpha}$ character as seen in metallic films and wires of larger dimension. The fluctuation has a significant low-frequency component and α increases from the usual 1 as the diameter d is reduced. The reduction of the diameter d also leads to an enhancement of the normalized fluctuation $\frac{\langle \Delta R^2 \rangle}{R^2}$. Our results show that there are new features in the $1/f$ noise as the size is reduced, and they become more prominent when the diameter of the wires falls to 15 nm. It will be important to investigate the origin of the new behaviour.

This is needed because $1/f$ noise will play an important role and may even become a limiting factor in use of such nanometre-size metal wires as interconnects.

Acknowledgments

The authors would like to thank Dr N Ravishankar (MRC, IISc) for the TEM data. This work is supported by DST, Government of India, CSIR, Government of India and DIT, Government of India. A Bid would like to thank CSIR for a fellowship and A Bora would like to thank UGC for a fellowship.

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