Photoelasticity in polycrystalline aggregates

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Abstract. A theory for the photoelastic· behaviour of transparent polycrystalline aggregates consisting of randomly oriented anisotropic crystallites has been developed. Such an aggregate is isotropic but it becomes birefringent under the influence of a uni-axial load. The photoelastic constants of the aggregate are given by the components of the spatial average of the photoelastic tensor of the single crystal, and are worked out by assuming either the strain to be continuous (Voigt approximation) or the stress to be continuous (Reuss approximation). The components of the average photoelastic tensor are very different for these two limits. The elastic and the photoelastic constants of alkali halide aggregates have been evaluated for both the stress continuity and the strain continuity conditions. The maximum variation of the elastic constants in going from the Voigt to the Reuss condition is 50 per cent while the photoelastic birefringence can vary by as much as 300 per cent in alkali halides. In the case of KI and rubidium halides even the sign of the photoelastic birefringence is different for the two limits.

1. Introduction

The elastic behaviour of polycrystalline metals and minerals is of some interest to the materials scientist, and it would be very much dependent on the question of stress or strain continuity across the grains. As early as 1889, Voigt calculated the elastic constants of polycrystalline media assuming strain continuity with the stress disconti-nuous. Reuss (1929) on the other hand computed the elastic constants of the aggre-gate assuming stress to be continuous. In both these procedures crystallites are assumed to be randomly oriented (i.e., with no preferred orientation) and data on single crystals are used to compute those of the polycrystals (see also Bhagavantam 1951). The most general theory of the mechanical behaviour of polycrystals is due to Bishop and Hill (1951).

Experimentally one finds that

$$E_V \geqslant E \geqslant E_R \; ; \; K_V \geqslant K \geqslant K_R \text{ and } G_V \geqslant G \geqslant G_R$$

where E, K and G are the Young's, the bulk and the shear moduli. The subscripts V and R refer to the Voigt and the Reuss conditions while the unsubscripted quan-tities refer to the experimental values. Hill (1952) explained the above relationship as due to the fact that in the Voigt model the forces between the grains would not be

in equilibrium, while in the Reuss model the distorted grains would not fit together. He also suggested that the arithmetic mean between the Voigt and the Reuss values would give a fairly reasonable approximation to the experimental value. Chung (1967) has shown that a similar approach also gives a reasonable approximation to the pressure derivatives of the elastic constants of the aggregate.

Many attempts have been made to improve the Voigt and the Reuss bounds to obtain the elastic constants of the aggregate. Hashin and Shtrikman (1962) employed the variational principle to narrow the bounds while Kroner (1967) used the concept of elastic polarizability. Recently Hearman (1969) reviewed this subject with special reference to attempts made to improve the Voigt and Reuss limits.

One finds that in metals the Voigt values do not differ very much from the Reuss values (Hearman 1969) making it difficult to establish whether strain or stress is continuous in an aggregate that is mechanically deformed. Since the stress optical constants of the aggregate can be measured when the crystallites are transparent it appeared to us that this may be yet another approach to this problem. One has of course to work out both the elastic and the photoelastic tensors under the Voigt and the Reuss limits. One gets the interesting result that the photoelastic constants are very much more sensitive to stress or strain continuity than the elastic constants are.

2. Optical transmission in a stressed aggregate

A polycrystalline aggregate consists of randomly oriented optically anisotropic crystallites. Consider a plane wavefront of linearly polarized light of intensity I_0 falling on such a medium. If the aggregate has a homogeneous packing of crystallites each of the same average size $\bar{\tau}$, then light entering each of the crystallites would have on the average the same intensity. In a unit area of the front face of the plate there will be $N^2(N^{-1}=\bar{\tau})$ crystallites and light entering each has the same intensity I_0/N^2. One such ray has to travel through a stack of N crystallites before emerging from a unit thickness of the medium.

When the incident light enters the first crystallite, it splits into two linear vibrations which travel with different velocities, and the emergent light will be elliptic. The ellipticity and the azimuth of the ellipse depend on the linear phase retardation introduced by the crystallite and the azimuth of its principal planes of vibration. As the light beam passes through the stack, the polarization state continuously changes. The emergent light from the polycrystal, therefore, consists of N^2 light beams polarized in different states; they are also incoherent as they travel through optically uncorrelated paths. Part of the incident intensity would be lost in intercrystalline boundary reflections.

The problem of light transmission through a polycrystalline aggregate has been considered by Raman and Viswanathan (1955) and more recently by us (Ranganath and Ramaseshan 1972) and the following are some of the results. When completely polarized light enters a randomly oriented polycrystal, the emergent light is partially polarized. The completely polarized part of the emergent light is in the same state of polarization as the incident light and has an intensity less than that of the incident light. The attenuation coefficient is directly proportional to the crystallite size and the mean square birefringence of the crystallite and is inversely proportional to the square of the wavelength. If the particles of the medium are, however, embedded in a matrix of different refractive index, then part of the incident light would be scattered away from the direct central beam in the form of diffracted light.

If a polycrystal consists of randomly oriented optically isotropic particles, it would be optically isotropic. When this aggregate is stressed by a uniaxial load each of the constituent crystallites becomes optically anisotropic. As the stress induced birefringence is small ($\Delta n = 0.001$ to 0.0001), we can justifiably neglect intensity loss due to intercrystalline reflections. If the particles are made from an isotropic amorphous material like glass, then each of the stressed particle will become optically uniaxial with the axis of symmetry coinciding with the stress direction. Therefore the polycrystal as a whole will behave as an optically uniaxial crystal with the axis of symmetry along the stress direction. The photoelastic birefringence (phase retardation for unit thickness per unit stress or strain) introduced in the medium would be exactly same as that found in a single particle. It is important to point out that in an aggregate consisting of such optically as well as elastically isotropic particles the Voigt and the Reuss modulii coincide.

On the other hand, one finds an entirely different behaviour when particles are produced from a cubic crystal. Such particles though optically isotropic, are elastically anisotropic. A crystal belonging to the cubic symmetry becomes optically uniaxial with the optic axis coinciding with the stress direction only when the uniaxial stress or strain acts along the cube diagonal (111). In the case of crystals belonging to the **O**, **O**$_h$ and **T**$_d$ classes the same behaviour is also found when the uniaxial stress acts along the cube edge (100). In the case of crystals belonging to **T** and **T**$_h$ classes however for a uniaxial stress along the cube edge, the crystal becomes optically biaxial with one of the principal axes coinciding with the stress direction (Bhagavantam 1966). However, when the stress acts along a general direction, we find the principal axes of the index ellipsoid tilted with respect to stress axis.

From these considerations we find that for a general direction of stress or strain, none of the principal axes coincide with the stress direction (Bhagavantam 1966). As the crystallites are randomly oriented in the polycrystal, we find the principal axes of the various crystallites to be symmetrically oriented with respect to the stress direction. Therefore, for a light beam travelling through the medium, normal to the stress direction, the principal vibration planes in the different crystallites would be symmetrically situated with respect to the stress direction. In fact they will be in a range of angles between θ_0 and $-\theta_0$ where θ_0 is the maximum photoelastic elastic tilt with the stress direction.

In the case of crystals of lower symmetry, although the problem is more complicated, the principal axes of the different crystallites would be symmetrically oriented with respect to the stress direction.

The problem of light transmission in a stressed aggregate can be solved by the Poincaré sphere method or the Mueller matrix method (details are given by Ramachandran and Ramaseshan 1961). We present here the solution that makes use of the latter approach.

A light beam of intensity I, ellipticity ω_0 ($= \tan^{-1} b/a$) and azimuth λ_0 is analytically represented by the four component Stokes vector

$$\sigma_0 = \begin{bmatrix} I \\ M_0 \\ C_0 \\ S_0 \end{bmatrix} \qquad \begin{aligned} M_0 &= I_0 \cos 2\omega_0 \cos 2\lambda_0 \\ C_0 &= I_0 \cos 2\omega_0 \sin 2\lambda_0 \\ S_0 &= I_0 \sin 2\omega_0 \end{aligned}$$

where I_0 is the intensity of the completely polarized part. Also $I \geqslant (M_0^2 + C_0^2 + S_0^2)^{\frac{1}{2}}$ depending upon whether the light beam is partially polarized (inequality sign) or

completely polarized (equality sign). When the light beam passes through an optical system σ_0 changes to σ', the two being related by the equation

$$\sigma' = [\mathbf{m}] \, \sigma_0 \tag{1}$$

where $[\mathbf{m}]$ is the 4×4 Mueller matrix. For an optically anisotropic crystal having a phase retardation δ with one of its principal planes at an azimuth θ, one has

$$[\mathbf{m}] = \mathsf{S} \, \mathsf{M} \, \mathsf{S}^{-1} \tag{2}$$

where S^{-1} is the inverse of S and

$$\mathsf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathsf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \delta & -\sin \delta \\ 0 & 0 & \sin \delta & \cos \delta \end{bmatrix}$$

Therefore, the light beam emerging from the ijth row of crystallites is described by the Stokes vector

$$\sigma_{ij} = \prod_{k=1}^{N} [\mathbf{m}_{ijk}] \sigma \tag{3}$$

where $\sigma = \sigma_0/N^2$ and $[\mathbf{m}_{ijk}]$ is the Mueller matrix of the ijkth crystallite which has the same form as (2). As the N^2 emergent beams are incoherent, light emerging from the polycrystal is given by the vector sum of the σ_{ij}'s (Ramachandran and Ramaseshan 1961)

$$\sigma' = \sum_{i,j=1}^{N} \sigma_{ij} = \frac{1}{N^2} \sum_{i,j=1}^{N} \prod_{k=1}^{N} [\mathbf{m}_{ijk}] \, \sigma_0 = [\mathbf{m}]\sigma_0 \tag{4}$$

$$[\mathbf{m}] = \frac{1}{N^2} \sum_{i,j=1}^{N} \prod_{k=1}^{N} [\mathbf{m}_{ijk}] \tag{5}$$

is the Mueller matrix for the polycrystal. This relationship is exact; however, in the case of weakly birefringent crystallites it can be further simplified. If the phase retardation δ_{ijk} of the ijkth crystallite is very small, then

$$[\mathbf{m}_{ijk}] \approx \mathsf{E} + (Q_{ijk})$$

i.e.,

$$[\mathbf{m}] = \mathsf{E} + \frac{\displaystyle\sum_{i,j,k=1}^{N} [Q_{ijk}]}{N^3} \tag{6}$$

where E is the 4×4 unit matrix, and

$$[Q_{ijk}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\tfrac{1}{2}\delta_{ijk}^{2}\sin^2 2\theta_{ijk} & \tfrac{1}{2}\delta_{ijk}^{2}\sin 2\theta_{ijk}\cos 2\theta_{ijk} & \delta_{ijk}\sin 2\theta_{ijk} \\ 0 & \tfrac{1}{2}\delta_{ijk}^{2}\sin 2\theta_{ijk}\cos 2\theta_{ijk} & -\tfrac{1}{2}\delta_{ijk}^{2}\cos^2 2\theta_{ijk} & -\delta_{ijk}\cos 2\theta_{ijk} \\ 0 & -\delta_{ijk}\sin 2\theta_{ijk} & \delta_{ijk}\cos 2\theta_{ijk} & -\tfrac{1}{2}\delta_{ijk}^{2} \end{bmatrix}$$

As the second term in (6) is the spatial average of $[Q_{ijk}]$ between angles θ_0 and $-\theta_0$ we find to a first order in δ_{ijk}

$$[\mathbf{m}] \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -N\bar{\delta} \\ \end{bmatrix} \tag{7}$$

where $\bar{\delta}$ is the spatial average δ_{ijk}. Therefore the polycrystal is linearly birefringent with a phase retardation φ given by

$$\varphi = \mathcal{N}\bar{\delta} = \mathcal{N}\frac{2\pi}{\lambda}\overline{(\Delta n_{ijk})}\,\bar{\tau}_{ijk} = \frac{2\pi}{\lambda}\overline{(\Delta n_{ijk})} \text{ as } \mathcal{N}\bar{\tau} = 1.$$

In other words the photoelastic birefringence of the medium is the average of the stress induced birefringences of the various crystallites. If we take higher order terms in δ, we still get the same result, but the light will be slightly depolarized.

To know the magnitude of photoelastic birefringence, we must know the orientation of the crystal with respect to the direction of stress or strain and the direction of light propagation. In the present problem the crystallites are randomly oriented with respect to the external stress or strain. Therefore we transform in each crystal, the photoelastic tensor (which refers to the crystallographic axes) to the external coordinate system, namely, that of stress or strain, and then work out the birefringence induced. This procedure is applied to each one of the crystallites and then the birefringence is averaged. It is clear that this is equivalent to averaging the photoelastic tensor itself.

3. The average photoelastic tensor

The elastic and the photoelastic properties of a single crystal are described by the following set of equations.

$$\left.\begin{array}{l} \sigma_{ij} = c_{ijkl}\,e_{kl} \\ e_{ij} = s_{ijkl}\,\sigma_{kl} \end{array}\right\} \text{ Elastic behaviour} \qquad \left.\begin{array}{l} \Delta a_{ij} = p_{ijkl}\,e_{kl} \\ \Delta a_{ij} = q_{ijkl}\,\sigma_{kl} \end{array}\right\} \text{ Photoelastic behaviour}$$

Here (σ) and (e) are the stress and strain tensors; (s) and (c) are the elastic tensors, and (p) and (q) the photoelastic tensors. (Δa) is the change in the index tensor (a) (Bhagavantam 1966).

For the Voigt condition e_{kl} is assumed to be same for all crystallites and one has to average c_{ijkl} to get the average stress. $(\bar{p}_{ijkl})$ gives the average birefringence in this case. On the other hand for the Reuss condition σ_{kl} is assumed to be the same for all crystallites so that the average strain and birefringence are obtained from the averages of (s_{ijkl}) and (q_{ijkl}).

In the present work we are justified in assuming that the rotational effects on photoelasticity considered by Nelson and Lax (1970) do not arise.

Let the external coordinate system (i.e., of stress or strain) be $Ox'y'z'$. Let the local coordinate system of the crystallite with respect to which we know the tensors (c_{ijkl}), (s_{ijkl}), (p_{ijkl}) and (q_{ijkl}) be $Oxyz$. We transform the tensor from the local coordinate system (which varies from crystallite to crystallite) to the outside coordinate system and then find the average. If (A_{ijk}) be the tensor, then its average is given by

$$\bar{A}_{ijk} = \frac{(A'_{ijkl})_1 + (A'_{ijkl})_2 + \cdots + (A'_{ijkl})_N}{N}$$

$$= \frac{1}{N}[(a^1_{ip}a^1_{jq}a^1_{kr}a^1_{ls}) + (a^2_{ip}a^2_{jq}a^2_{kr}a^2_{ls}) + \cdots (a^N_{ip}a^N_{jq}a^N_{kr}a^N_{ls})]\,A_{pqrs}$$

where $N =$ number of crystallites.

Hence $\bar{A}_{ijkl} = \overline{a_{ip}a_{jq}a_{kr}a_{ls}}\,A_{pqrs}$

where we have used the transformation law for the tensor A_{ijkl}, i.e.,

$$A'_{ijkl} = a^n_{ip}a^n_{jq}a^n_{kr}a^n_{ls}\,A_{pqrs}$$

where (a_{ij}^n) are the components of the matrix that transforms the $Oxyz$ coordinate system to $Ox'y'z'$ in the nth crystallite.

We can use a single index 'i' in place of two indices 'pq', (Nye 1955) and abbreviate A_{pqrs} by A_{ij}. The forms of $\bar{c}_{ij}=\bar{c}_{pqrs}$ were first obtained by Voigt (1889).

$$\bar{c}_{11} = \tfrac{1}{5}(3A+2B+4C); \quad \bar{c}_{12} = \tfrac{1}{5}(A+4B-2C); \quad \bar{c}_{44} = \tfrac{1}{2}(\bar{c}_{11}-\bar{c}_{12}) \tag{8}$$

the last equation shows that the aggregate is elastically isotropic. In the above equations

$$A = \tfrac{1}{3}(c_{11}+c_{22}+c_{33}); \quad B = \tfrac{1}{3}(c_{12}+c_{23}+c_{31}); \quad C = \tfrac{1}{3}(c_{44}+c_{55}+c_{66})$$

The forms of $\bar{s}_{ij} = \bar{s}_{pqrs}$ were obtained by Reuss (1929). They are given by

$$\bar{s}_{11} = \tfrac{1}{5}(3A'+2B'+C'); \quad \bar{s}_{12} = \tfrac{1}{5}(A'+4B'-\tfrac{1}{2}C'); \quad \bar{s}_{44} = 2(\bar{s}_{11}-\bar{s}_{12}) \tag{9}$$

Here

$$A' = \tfrac{1}{3}(s_{11}+s_{22}+s_{33}); \quad B' = \tfrac{1}{3}(s_{12}+s_{23}+s_{31}); \quad C' = \tfrac{1}{3}(s_{44}+s_{55}+s_{66})$$

Equations (8) and (9) hold for any crystal symmetry. In a single crystal we know that

$$c_{ij} = (s_{ij})^{-1}$$

However, for a polycrystal we find that

$$(\bar{c}_{ij}) \neq (\bar{s}_{ij})^{-1}$$

Hence the elastic constants as obtained from Voigt and Reuss models are different. If $(\bar{s}_{ij})^{-1}=\bar{c}'_{ij}$, then $\bar{c}_{ij}$ and $\bar{c}'_{ij}$ correspond elastic constants of the aggregate for strain and stress continuity in the polycrystal.

Though, the tensor transformation laws are the same for elasticity and photoelasticity there are two important differences.

(1) The elastic tensors are symmetric, i.e., $c_{ij}=c_{ji}$ and $s_{ij}=s_{ji}$ while the photoelastic tensors are not symmetric, i.e., $p_{ij} \neq p_{ji}$ and $q_{ij} \neq q_{ji}$. However, crystal symmetry may result in $p_{ij}=p_{ji}$ and $q_{ij}=q_{ji}$.

(2) The transformations from the two index symbol to the one index symbol are different for the elastic and the photoelastic tensors (Nye 1955).

In elasticity

$$c_{pqrs} = c_{ij} \quad \text{for all } i \text{ and } j$$
$$s_{pqrs} = s_{ij} \quad \text{for } i, j = 1, 2 \text{ or } 3$$
$$2s_{pqrs} = s_{ij} \quad \text{for } i = 1, 2 \text{ or } 3$$
$$4s_{pqrs} = s_{ij} \quad \text{for } i, j = 4, 5 \text{ or } 6.$$

and in photoelasticity

$$p_{pqrs} = p_{ij} \quad \text{for all } i \text{ and } j$$
$$q_{pqrs} = q_{ij} \quad \text{for} \quad \begin{cases} i=1, 2, 3, 4, 5 \text{ or } 6 \\ j=1, 2 \text{ or } 3 \end{cases}$$
$$2p_{pqrs} = q_{ij} \quad \text{for} \quad \begin{cases} i=1, 2, 3, 4, 5 \text{ or } 6 \\ j=4, 5 \text{ or } 6. \end{cases}$$

Therefore we find that $(\bar{p}_{ij})$ and $(\bar{c}_{ij})$ have the same form, excepting that $p_{ij} \neq p_{ij}$. Hence

$$\bar{p}_{11} = \tfrac{1}{5}(3R+2S+4T); \quad \bar{p}_{12} = \tfrac{1}{5}(R+4S-2T); \quad \bar{p}_{44} = \tfrac{1}{2}(\bar{p}_{11}-\bar{p}_{12}) \tag{10}$$

Here
$$R = \tfrac{1}{3}(p_{11}+p_{22}+p_{33})$$
$$S = \tfrac{1}{6}(p_{12}+p_{21}+p_{23}+p_{32}+p_{31}+p_{13})$$
$$T = \tfrac{1}{3}(p_{44}+p_{55}+p_{66}).$$

On the other hand (q_{ij}) are different from those of $(\bar{s}_{ij})$.

In this case we get

$$\bar{q}_{11} = \tfrac{1}{5}(3R' + 2S' + 2T'); \quad \bar{q}_{12} = \tfrac{1}{5}(R' + 4S' - T'); \quad \bar{q}_{44} = \bar{q}_{11} - \bar{q}_{12} \quad (11)$$

Here

$$R' = \tfrac{1}{3}(q_{11} + q_{22} + q_{33})$$
$$S' = \tfrac{1}{6}(q_{12} + q_{21} + q_{23} + q_{32} + q_{31} + q_{13})$$
$$T' = \tfrac{1}{3}(q_{44} + q_{55} + q_{66})$$

In equations (10) and (11) the last relation shows that the medium is photoelastically isotropic. These equations hold for a crystal belonging to any symmetry class. For a single crystal we have

$$(p_{ij}) = (q_{im})(c_{mj}) \quad \text{and} \quad (q_{ij}) = (p_{im})(s_{mj}).$$

For a polycrystal, however, we have

$$(\bar{p}_{ij}) \neq (\bar{q}_{im})(\bar{c}_{mj})$$
$$(\bar{q}_{ij}) \neq (\bar{p}_{im})(\bar{s}_{mj})$$

We know that $(\bar{q}_{ij})$ and $(\bar{c}_{ij})[=(\bar{s}_{ij})^{-1}]$ correspond to stress continuity in the medium while $(\bar{p}_{ij})$ and $(\bar{s}'_{ij})[=(\bar{c}'_{ij})^{-1}]$ correspond to strain continuity in the medium. Hence $\bar{q}_{im}\bar{c}'_{mj} = \bar{p}'_{ij}$ are the strain optical constants for stress continuity and $\bar{p}_{im}\bar{s}'_{mj} = \bar{q}'_{ij}$ are the stress optical constants for strain continuity.

4. Results

We shall apply the results of the previous sections to calculate the elastic and photoelastic constant of randomly oriented polycrystalline aggregate of alkali halides from the experimental values for single crystals. As these are cubic crystals, the equations get further simplified, since

$$p_{ij} = p_{ji}; \quad q_{ij} = q_{ji}; \quad c_{ij} = c_{ji}; \quad s_{ij} = s_{ji}$$
$$p_{44} = p_{55} = p_{66}; \quad q_{44} = q_{55} = q_{66}; \quad c_{44} = c_{55} = c_{66}$$
$$p_{11} = p_{22} = p_{33}; \quad q_{11} = q_{22} = q_{33}; \quad c_{11} = c_{22} = c_{33}$$
$$\text{and} \quad s_{44} = s_{55} = s_{66}; \quad s_{11} = s_{22} = s_{33}$$

Anderson (1965) has worked out the mean of the Voigt and Reuss elastic constants

$$\left[\text{i.e.,} \quad \frac{\bar{c}_{ij} + \bar{c}'_{ij}}{2} \quad \text{and} \quad \frac{\bar{s}_{ij} + \bar{s}'_{ij}}{2} \right]$$ for different substances including alkali halide aggre-

gates. However, he has not given the explicit values. Using the elastic constants of alkali halide crystals reported recently (1969) these have been calculated. We require explicit values of $\bar{c}_{ij}$ (or $\bar{s}_{ij}$) and $\bar{c}'_{ij}$ (or $\bar{s}'_{ij}$) to work out the photoelastic constants of the aggregate for Voigt and Reuss conditions. The values of $\bar{c}_{ij}$, $\bar{c}'_{ij}$, $\bar{s}_{ij}$ and $\bar{s}'_{ij}$ are given in table 1 along with the single crystal values. It is seen that

$$\frac{1}{\bar{s}'_{11}} > \frac{1}{\bar{s}_{11}}; \quad \frac{1}{\bar{s}'_{11} + 2\bar{s}'_{12}} > \frac{1}{\bar{s}_{11} + 2\bar{s}_{12}} \quad \text{and} \quad \frac{1}{\bar{s}'_{44}} > \frac{1}{\bar{s}_{44}}$$

or

$$E_V > E_R; \quad K_V > K_R \quad \text{and} \quad G_V > G_R \quad (12)$$

Lithium halide aggregates seem to behave very differently from those of other alkali halides, as is seen in table 2.

In table 3 values of $\bar{p}_{ij}$, $\bar{p}'_{ij}$, q_{ij} and $\bar{q}'_{ij}$ are given along with the single crystal value. We find

$$\bar{p}_{11} > \bar{p}'_{112}; \quad \bar{p}_{12} < \bar{p}'_{1} \quad \text{and} \quad \bar{p}_{44} > \bar{p}'_{44} \quad (13)$$

Table 1. Elastic constants

c_{ij}, s_{ij} — Single crystal values
$\bar{c}_{ij}, \bar{s}_{ij}$ — Polycrystal values for strain continuity
$\bar{c}'_{ij}, \bar{s}'_{ij}$ — Polycrystal values for stress continuity
c_{ij} in units of 10^{11} dyne cm^{-2}; s_{ij} in units of 10^{-11} cm^2 dyne^{-1}

Crystal	c_{11} / $\bar{c}_{11}$ / $\bar{c}'_{11}$	c_{12} / $\bar{c}_{12}$ / $\bar{c}'_{12}$	c_{44} / $\bar{c}_{44}$ / $\bar{c}'_{44}$	s_{11} / $\bar{s}_{11}$ / $\bar{s}'_{11}$	s_{12} / $\bar{s}_{12}$ / $\bar{s}'_{12}$	s_{44} / $\bar{s}_{44}$ / $\bar{s}'_{44}$
LiF	12·460	4·240	6·490	0·097	−0·025	0·154
	14·364	3·288	5·538	0·076	−0·014	0·181
	14·006	3·467	5·269	0·079	−0·016	0·190
LiCl	6·074	2·270	2·692	0·207	−0·056	0·371
	6·706	1·954	2·376	0·172	−0·039	0·421
	6·616	1·999	2·308	0·176	−0·041	0·433
LiBr	3·940	1·880	1·910	0·367	−0·119	0·524
	4·644	1·528	1·558	0·257	−0·064	0·642
	4·465	1·618	1·424	0·277	−0·074	0·702
LiI	2·850	1·400	1·350	0·519	−0·171	0·741
	3·350	1·150	1·100	0·362	−0·093	0·909
	1·222	1·214	1·004	0·391	−0·107	0·996
NaF	10·850	2·290	2·899	0·099	−0·017	0·345
	9·745	2·842	3·451	0·118	−0·027	0·290
	9·581	2·924	3·329	0·122	−0·028	0·300
NaCl	5·733	1·123	1·331	0·186	−0·031	0·751
	4·954	1·513	1·721	0·236	−0·055	0·581
	4·795	1·592	1·602	0·250	−0·062	0·624
NaBr	4·800	0·986	1·070	0·224	−0·038	0·935
	4·130	1·321	1·405	0·287	−0·069	0·712
	3·988	1·392	1·298	0·306	−0·079	0·771
NaI	3·761	0·798	0·781	0·287	−0·050	1·280
	3·201	1·078	1·061	0·376	−0·095	0·942
	3·070	1·144	0·963	0·408	−0·111	1·038
KF	7·570	1·350	1·336	0·140	−0·021	0·749
	6·151	2·060	2·046	0·195	−0·049	0·489
	5·731	2·269	1·731	0·225	−0·064	0·578
KCl	3·980	0·620	0·625	0·262	−0·035	1·600
	3·136	1·042	1·047	0·382	−0·095	0·955
	2·853	1·184	0·835	0·463	−0·136	1·198
KBr	4·175	0·560	0·520	0·247	−0·029	1·923
	3·145	1·075	1·035	0·385	−0·098	0·966
	2·735	1·280	0·727	0·521	−0·166	1·375
KI	3·380	0·220	0·368	0·298	−0·018	2·717
	2·410	0·705	0·853	0·478	−0·108	1·173
	1·981	0·919	0·531	0·715	−0·227	1·884
RbF	5·700	1·250	0·910	0·190	−0·034	1·099
	4·648	1·776	1·436	0·273	−0·075	0·696
	4·322	1·939	1·192	0·320	−0·099	0·839
RbCl	4·297	0·649	0·493	0·242	−0·032	2·028
	3·232	1·181	1·025	0·385	−0·103	0·975
	2·793	1·401	0·696	0·538	−0·180	1·436
RbBr	3·863	0·474	0·409	0·266	−0·029	2·445
	2·835	0·988	0·923	0·430	−0·111	1·083
	2·387	1·212	0·587	0·637	−0·215	1·703
RbI	3·210	0·360	0·292	0·319	−0·032	3·425
	2·304	0·813	0·745	0·532	−0·139	1·342
	1·881	1·025	0·428	0·863	−0·304	2·335

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Table 2. Comparison of single crystal elastic constants with those of polycrystals

| Lithium halides | $\tilde{s}_{11} < s_{11}$ | $\tilde{s}_{12} > s_{12}$ | $\tilde{s}_{44} < s_{44}$ |
| Other alkali halides | $\tilde{s}_{11} > s_{11}$ | $\tilde{s}_{12} < s_{12}$ | $\tilde{s}_{44} > s_{44}$ |

$$\tilde{s}_{ij} = \tfrac{1}{2}(\tilde{s}_{ij} + \tilde{s}'_{ij})$$

Table 3. Photoelastic constants

p_{ij}, q_{ij} — Single crystal values; p_{ij} in units of 10^{-2}; q_{ij} in units of 10^{-13} cm^2 dyne^{-1}

$\bar{p}_{ij}, \bar{q}'_{ij}$ — Polycrystal values for strain continuity

$\bar{q}_{ij}, \bar{p}'_{ij}$ — Polycrystal values for stress continuity

Crystal	p_{11} $\bar{p}_{11}$ $\bar{p}'_{11}$	p_{12} $\bar{p}_{12}$ $\bar{p}'_{12}$	p_{44} $\bar{p}_{44}$ $\bar{p}'_{44}$	q_{11} $\bar{q}'_{11}$ $\bar{q}_{11}$	q_{12} $\bar{q}'_{12}$ $\bar{q}_{12}$	q_{44} $\bar{q}'_{44}$ $\bar{q}'_{44}$
LiF	4·514	17·008	−5·387	−0·400	1·120	−0·830
	5·202	16·664	−5·731	−0·077	0·958	−1·035
	5·073	16·729	−5·828	−0·124	0·982	−1·106
NaF	7·138	19·036	−2·464	0·050	1·440	−0·850
	9·926	17·642	−3·858	0·231	1·349	−1·118
	10·339	17·435	−3·548	0·266	1·332	−1·066
NaCl	13·076	19·115	−1·118	1·270	2·580	−0·840
	14·597	18·354	−1·879	1·415	2·507	−1·092
	14·906	18·119	−1·647	1·458	2·486	−1·028
KCl	22·464	16·147	−2·700	4·750	2·870	−4·320
	17·777	18·490	−0·357	3·270	3·610	−0·341
	16·205	19·276	−1·536	2·270	4·110	−1·840
KBr	22·570	16·461	−2·049	4·620	2·930	−3·940
	18·487	18·502	−0·007	3·489	3·496	−0·007
	16·861	19·316	−1·228	2·368	4·056	−1·688
KI	22·864	17·524	−0·979	6·180	4·490	−2·660
	19·945	18·983	0·481	5·429	4·865	−0·564
	18·652	19·629	−0·488	4·440	5·360	−0·920
RbCl	34·132	19·357	−3·875	7·040	2·990	−7·860
	25·122	23·862	0·630	4·750	4·135	0·614
	21·408	25·719	−2·155	2·276	5·372	−3·096
RbBr	31·430	16·519	−3·509	7·400	3·000	−8·580
	22·658	20·905	0·877	5·100	4·150	0·950
	18·837	22·815	−1·989	2·208	5·596	−3·388
RbI	33·221	21·137	−2·313	9·230	4·990	−7·920
	26·537	24·479	1·029	7·324	5·943	1·381
	23·420	26·037	−1·309	4·366	7·422	−3·056

This result does not follow from equation (12) which has been theoretically established by Hill (1952). It would therefore be interesting to find a theoretical basis for this relation.

Experimentally, the quantity $(\bar{p}_{11}-\bar{p}_{12})$ or $(\bar{q}_{11}-\bar{q}_{12})$ [i.e., $\bar{p}_{44}/2$ or $\bar{q}_{44}$ as the medium is isotropic] can be easily obtained rather than the individual components. To determine the sensitivity of the elastic and the photoelastic constants to stress or strain continuity, we define a parameter f given by

$$f = 2\,|\,(\bar{A}_{ij}-\bar{A}_{ij}')/(\bar{A}_{ij}+\bar{A}_{ij}')\,| \tag{14}$$

where $\bar{A}_{ij}$ and $\bar{A}_{ij}'$ are the ijth components of elastic or photoelastic matrix for strain and stress continuity. For the elastic constant $[\bar{c}_{44}]$ the maximum value of f is $0\cdot5$. However for the photoelastic constant $[\bar{p}_{44}]$ the highest value of f is $3\cdot0$. Therefore we conclude that the photoelastic birefringence is very much more sensitive to stress or strain continuity than the elastic constants are.

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