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Defects in smectic C* liquid crystals

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Abstract. We consider the structure and properties of various topological defects that can occur in smectic C* liquid crystals. The polarization field associated with disclinations, the effect of incommensuration on the structure of dispirations, some interesting situations in the interaction between dispiration and disclination and between dispirations themselves have been discussed in detail. The properties of cholesteric type disclinations and a possible model for the core structure of a wedge disclination have also been dealt with.

Keywords. Chiral smectic C liquid crystals; dislocations; dispirations; disclinations; defects; smectic C.

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1. Introduction

A chiral smectic C which is referred to as smectic C* can be looked upon as an uniformly twisted smectic C with the twist axis normal to the layers. The naturally occurring defects in such systems have attracted the attention of investigators only in recent times. On the experimental side studies of toric domains by Perez *et al* (1981) and focal domains by Bourdon *et al* (1982) are of interest. The theoretical study of defects has only recently been started. Some of the possible defect states have been discussed at some length by Lejcek (1984, 1985). In this paper we consider the various topological defects that can exist in chiral smectic C. Some of the defects have some interesting features not emphasized in the past and some new types of defects that are topologically possible appear to have not been discussed so far. The implications of commensurability have also been considered.

2. Disclinations in C director

Each layer of the smectic C* has a direction of tilt of the molecules whose projection in the X-Y plane gives the local **C** director. As we go along the Z-axis the **C** director rotates in the X-Y plane about the Z-axis. In other words the twisted structure can be described by:

$$\varphi = qz, \qquad q = 2\pi/P. \tag{1}$$

The author felicitates Prof. D S Kothari on his eightieth birthday and dedicates this paper to him on this occasion.

Here φ is the angle which local **C** director makes with X-axis and P is the pitch of the structure. Disclinations are topological defects that occur in the **C** director with the layer system intact. The deviation from (1) involve elastic energies which in the one constant approximation can be given by:

$$F = \frac{K}{2}[(\partial\varphi/\partial x)^2 + (\partial\varphi/\partial y)^2 + [(\partial\varphi/\partial z) - q]^2]. \quad (2)$$

The minimization of F leads to

$$\nabla^2 \varphi = 0. \quad (3)$$

The solution (1) which is allowed by (3) describes an undistorted ideal smectic C*. The allowed defect states are

Wedge disclination along Z axis

$$\varphi = S \tan^{-1}(y/x) + qz. \quad (4)$$

Twist disclination along Y axis

$$\varphi = S \tan^{-1}(z/x) + qz. \quad (5)$$

These are like the disclinations of the cholesteric. But there are differences. These have integral strengths only since **C** and $-$**C** are not equivalent configurations, i.e., S is a positive or a negative integer. Thus for $S = 1$, as we go up the twist axis we go through a sequence source, vortex, sink and anti-vortex structures (figure 1). In another respect also the cholesteric and smectic C* disclinations differ. We know that smectic C* is a ferroelectric. The polarization vector **P** is locally normal to the **C** director. Hence we

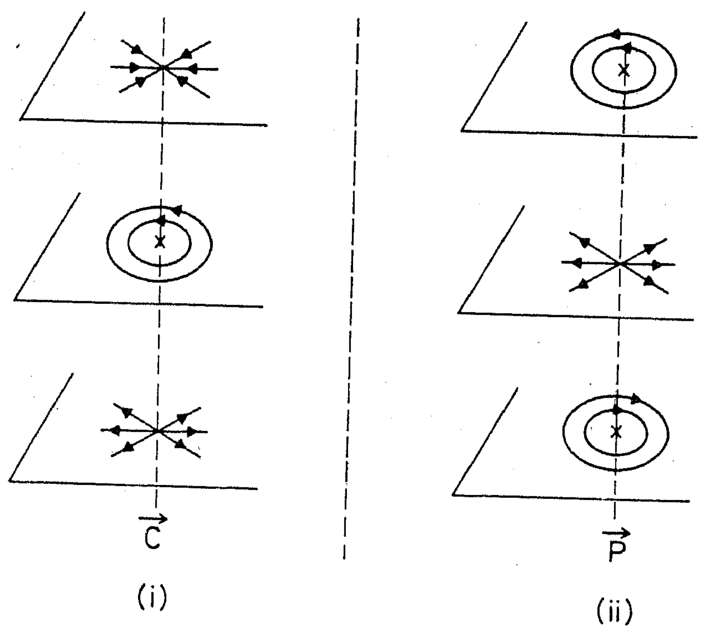

Figure 1. (i) C director structure of $S = 1$ wedge disclination in smectic C*. (ii) Polarization structure associated with this defect.

have a polarization structure exactly like (4) or (5) excepting for an extra $\pm\pi/2$ angle (see figure 1). The magnitude of the polarization is a constant throughout excepting close to the core where it may smoothly go to zero due to C*–A phase transition as in the case of disclinations in smectic C (Ranganath 1983). On the other hand the polarization field of χ disclinations in cholesterics arises from flexoelectric effects and the local polarization is in general at an angle with respect to the local director and the angle depends on the amount of splay and bend. Again the magnitude of the polarization in the cholesteric is not a constant. It is proportional to $|\nabla \cdot \boldsymbol{n}|$ and $|\nabla \times \boldsymbol{n}|$ i.e., it increases as r^{-1} as we go to the core of the disclination. But in spite of these differences both in smectic C* and cholesterics we create these disclinations by the same Volterra process first worked by Friedel and Kleman (1970):

(i) freeze the material into a solid body,

(ii) cut this solid along a plane parallel to twist axis and limited by a line L again parallel to twist axis,

(iii) rotate one side of the cut surface relative to other through $\pm N\pi$, N an integer,

(iv) fill in any void with extra material (or remove overlapping part) so that it is positioned and oriented in register with the cut surfaces.

(v) defreeze the object so that the system can relax.

3. Dispirations

These are new types of defects that have the features of both dislocations and disclinations in them and they occur as topological defects in smectic C* as was shown by Lejcek (1984, 1985). Each layer has a particular orientation of the **C** director. If we create a screw dislocation by the standard process we find the **C** director orientation to mismatch when the two cut surfaces are displaced relative to one another. When the system is allowed to relax this will result in a screw dislocation of Burgers vector b equal to the smectic layer thickness but in addition the **C** director, in order to have a smooth variation, would have developed a disclination (with the singular line parallel to the dislocation line) with the orientation of **C** director given by

$$\varphi = -\frac{b}{P}\tan^{-1}(y/x) + qz.$$

This is quite unlike (4). In fact there is a plane of mismatch which runs through the structure. It is a partial disclination associated with a perfect screw dislocation. It is called a *wedge dispiration* characterized by $[b, -b/P]$. Similarly the creation of an edge dislocation of Burgers vector b again results in a disclination but this time it is of the twist type (5) and, as in the previous case, it is again partial. This is called a *twist dispiration*. It has the features of a perfect edge dislocation and a partial twist disclination. It is characterized by $[b, +b/P]$.

It may be mentioned incidentally that the signs of the disclinations change whenever the sign of the b vector or the handedness of the helix change sign.

In both the examples given above, the dislocation is perfect i.e. it is an integral multiple of d, the smectic layer thickness. Generally $b < P$ and hence the associated disclinations are truly partial.

If we can create defects with $b = \pm P$ then we will get perfect disclinations associated with dislocations of very large b vector. This associated dislocation will also be perfect in commensurate (P/d = integer) structures. But in incommensurate structures ($P/d \neq$ integer) the disclination will be perfect but the dislocation of dispiration is partial. This is different from dispirations considered earlier.

4. Interaction between defects

4.1 *Interaction between disclinations and dispirations*

Dispirations can interact with disclinations through their associated disclination fields. In a simple theory the answers are analogous to what we get for disclination-disclination interaction. For example, a disclination of strength $+1$ attracts or gets attracted to a wedge dispiration $[b, -b/P]$ to result in a wedge dispiration $[b, -(b-P)/P]$.

In the case of a commensurate smectic C* with $b = +P$ we find a $+1$ wedge disclination when attracted by a wedge dispiration $[b, -b/P]$ to completely annihilate the associated disclination and we finally end up with a pure dislocation of Burgers vector $b = P$ in the layered system with a superimposed ideal helical structure of the **C** director. Similarly one can think of pure edge dislocation of strength P. It is as though we decouple the **C** director from the layers.

4.2 *Interaction between dispirations*

In smectic A and smectic C liquid crystals screw dislocations have no interaction between them in a linear one constant model (Pershan 1974; Kleman and Williams 1974; Kleman and Lejcek 1980). But in smectic C*, as a rule, screw dislocations have disclinations associated with them. Hence two screw dislocations [i.e. wedge dispirations] interact like disclinations.

Again in the same model we find (Pershan 1974) a pair of like edge dislocations placed with their burgers vectors perpendicular to the line joining them (wedge geometry) have no interaction at all between them. But in smectic C* they have a repulsion between them through their associated like twist disclinations.

On the other hand Pershan (1974) showed that an unlike pair of edge dislocations in a similar geometry is unstable. But in smectic C* they have an attraction between them through their twist disclinations and this may stabilize the configuration.

5. Disclinations and dislocations in the helical lattice

We shall now consider the topological defects that can occur in the helical lattice (we assume the helical structure to be commensurate with the smectic C layer thickness).

5.1 Λ^- disclinations

The Volterra process in this case is:

(a) The material is cut by a half plane which is perpendicular to the twist axis and ending in a straight line L (disclination line) parallel (or antiparallel) to the local **C** director.

(b) The two lips of the cut surface are rotated by a relative angle of π.

(c) The empty space left on the right side is filled with matter which is in register with the structure of the two surfaces.

(d) The structure is allowed to relax.

We get two situations, Λ_U^- and Λ_D^- from this operation. In one case the disclination line has its **C** director towards the observer in the other case it is away from the observer. Figure 2 depicts the situation. It is similar to what one gets for cholesterics.

5.2 Λ^+ *disclinations*

We repeat the same process excepting that the two lips are rotated to have a relative angle of $-\pi$. We remove materials from the overlapping regions.

Again we end up with two possible structures with the disclination line L being parallel or antiparallel to the **C** director. This is shown in figure 3. These are again similar to their counterparts in cholesterics.

5.3 τ^+, τ^- *disclinations*

In cholesterics one can get perfect disclinations called τ disclinations, with the disclination line perpendicular to the local director. But it is easy to see that with L perpendicular **C** director the disclinations are imperfect as we cannot have perfect structural registry everywhere around the defect. Figure 4 brings out this feature clearly. Thus the only perfect disclinations topologically permitted are the Λ's.

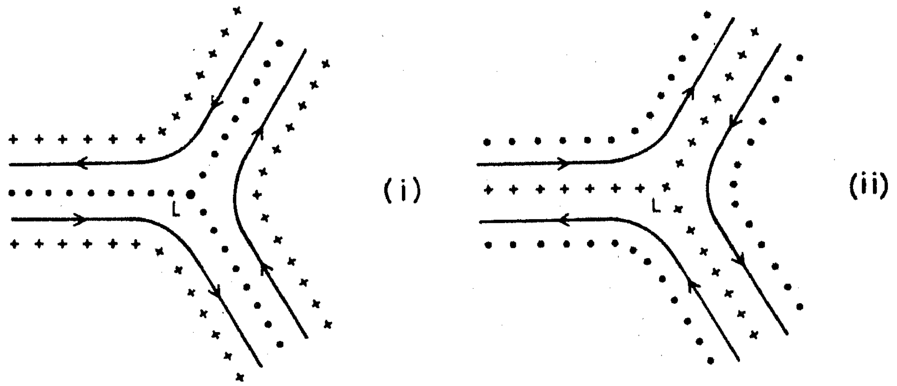

Figure 2. (i) Structure of Λ_U^- disclination. (ii) Structure of Λ_D^- disclination.

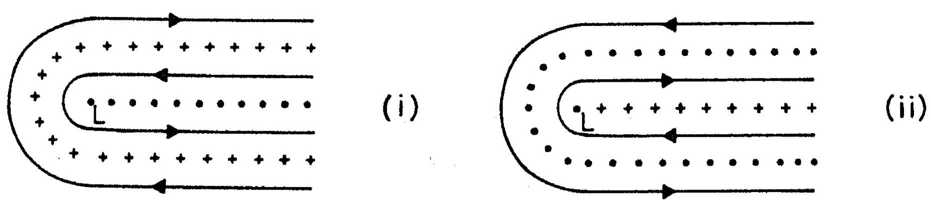

Figure 3. (i) Λ_U^+ disclination. (ii) Λ_D^+ disclination.

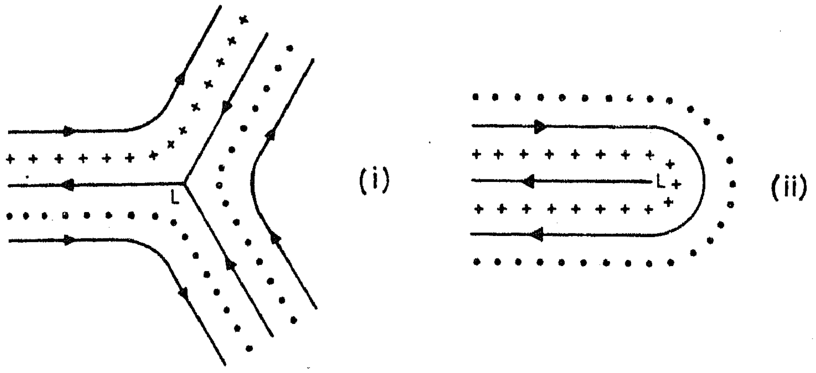

Figure 4. (i) Imperfect τ^- disclination. (ii) Imperfect τ^+ disclination.

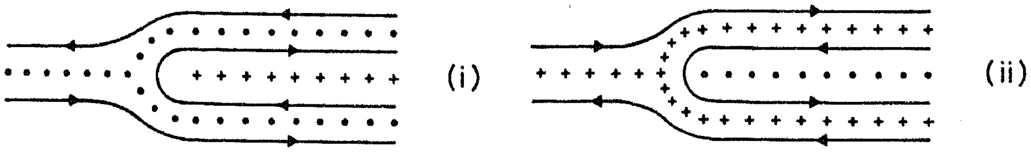

Figure 5. (i) Edge dislocation with Λ_U^- and Λ_D^+ disclination pair. (ii) Edge dislocation with Λ_D^- and Λ_U^+ disclination pair.

5.4. *Dislocations*

With Λ_U^+ and Λ_D^- or Λ_D^+ and Λ_U^- we can construct edge dislocations that are again neat topological solutions. They have been shown in figure 5.

6. Consequences of commensurability

At the level of smectic layering the creation of Λ^+ and Λ^- disclinations in the helical structure lead to Ω disclinations in smectic layer structure. But the nature of these disclinations depends on the nature of commensuration. An odd number of layers in half a pitch leads to one type of disclination while an even number of layers leads to another type of disclination. Figures 6 and 7 depict these differences.

On the other hand if the helical pitch and the smectic layer thickness are incommensurate we find a perfect disclination in one lattice leading to an imperfect one in the other.

7. Core structure of the defect

The structure of the defect core is not always easy to work out. But in certain situations we can get a reasonable model. For example in the case of smectic C disclinations, we get a Ginzburg-Pitaevskii type of core where the order parameter (in this case the tilt angle) smoothly and continuously goes to zero (Ranganath 1983). It is reasonable to

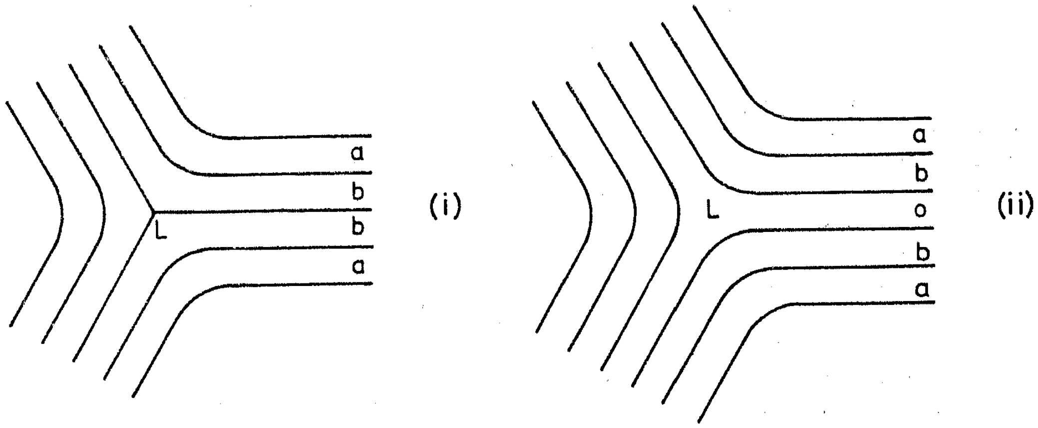

Figure 6. $\Omega = +\pi$ disclination with the disclination line L passing (i) between two molecular layers, (ii) through the centre of molecular layer.

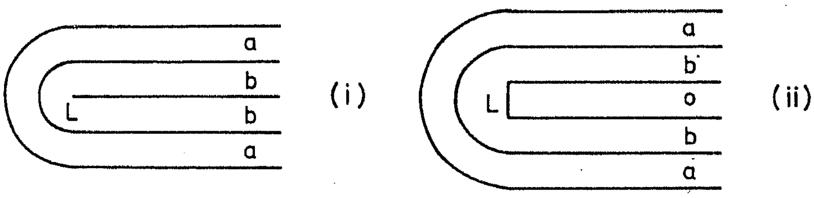

Figure 7. $\Omega = -\pi$ disclination with the disclination line L passing (i) between two molecular layers, (ii) through the centre of molecular layer.

expect a very similar solution for smectic C* disclinations also. But we have an additional complication. It has been well established experimentally by Martinot-Lagarde (1976), Ostroviskii *et al* (1978) and very recently by Kondo *et al* (1982, 1985) that near C*–A transition the pitch P behaves anomalously. It increases to large values reaching a peak and then close to the transition point it rapidly falls to zero. We have to take this factor into account while speculating on the structure of the cores of defects. We discuss below one example.

In the case of wedge disclinations (disclinations along Z-axis) we get an interesting core model. As said earlier the tilt angle will smoothly go to zero as we reach the centre of the disclination. But we will also have to accommodate the large rapid changes in P. It is easy to see that for any doubling (or halving) of pitch we have to have a system of periodically stacked (along the Z direction) twist disclination loops encircling the main wedge disclination. These disclination loops are in the horizontal plane. This process may repeat itself many times as we move towards the centre. The vertical spacing between the rings increases or decreases as we go in depending upon the pitch variation. In short the wedge disclination line has a coaxial system of stacked twist disclination loops circling it.

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References

Bourdon L, Sommeria J and Kleman M 1982 *J. Phys. (Paris)* **43** 77

Friedel J and Kleman M 1970 *Fundamental aspects of dislocation theory* (ed.) J A Simmons, R de Wit and R Bullough *Nat. Bur. Stand. (U. S.)* Special Publication (317) Vol. 1, p. 607

Kleman M and Lejcek L 1980 *Philos. Mag.* **A42** 671

Kleman M and Williams C E 1974 *J. Phys. Lett. (Paris)* **35** 49

Kondo K, Takezoe H, Fukuda A and Kuze E 1982 *Jpn. J. Appl. Phys.* **21** 224

Kondo K, Era S, Isogai M and Mukoh A 1985 *Jpn. J. Appl. Phys.* **24** 1389

Lejcek L 1984 *Czech. J. Phys.* **B34** 563

Lejcek L 1985 *Czech. J. Phys.* **B35** 726

Martinot-Lagarde Ph 1976 *J. Phys. (Paris)* **36** C3–129

Ostrovskii B I, Rabinovich A Z, Sonin A S and Strukov B A 1978 *Sov. Phys. JETP* **47** 912

Perez A, Brunet M and Parodi O 1981 *J. Phys. (Paris)* **42** 1559

Pershan P S 1974 *J. Appl. Phys.* **45** 1590

Ranganath G S 1983 *Mol. Cryst. Liq. Cryst. Lett.* **92** 201