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THE QUASI–STEADY STATE COSMOLOGY: A PROBLEM OF STABILITY

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ABSTRACT

This paper examines the gravitational stability against small perturbations of the quasi–steady state cosmological model. This model was first introduced by Hoyle et al., who in subsequent papers looked at its various theoretical and observational implications. Here we carry out a perturbation analysis of the exact solution of the field equations obtained by Sachs et al. in the noncreative mode which describes the oscillatory feature of this model. We show that the perturbations grow only to a limited amount and then fall off, thus confirming the stability of the solution. We discuss the implications of this result for structure formation in this cosmology.

Subject headings: cosmology: theory

1. INTRODUCTION

The quasi–steady state theory was proposed and explored by Hoyle, Burbidge, & Narlikar in a series of papers (1993, 1994a, 1994b, 1995a, 1995b). The theory is based on a Machian approach to gravitation, and it automatically incorporates a negative cosmological constant and additional terms denoting creation of matter in a set of Einstein-like equations for general relativity. The rationale for this approach is given in detail in Hoyle et al. (1995a). In recent years several cosmological solutions were also obtained (Sachs, Narlikar, & Hoyle 1996) which we collectively call the quasi–steady state cosmological models. Following standard simplifying assumptions used in cosmology, these models are based on the usual Robertson-Walker metric given by

$$ds^2 = c^2\,dt^2 - S^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right], \quad (1)$$

where (r, θ, ϕ) are the comoving coordinates of the fundamental Weyl observer, t is the cosmic time, and $k = 0, +1$, and -1 represent the three possible curvature signatures for the spaces $t =$ constant. An approximate oscillatory scale factor describing a quasi–steady state cosmology (QSSC) is then given by

$$S(t) = \exp\left(\frac{t}{P}\right)\left(1 + \eta\cos\frac{2\pi t}{Q}\right). \quad (2)$$

A more exact solution, obtained by Sachs et al. (1996), will be used later. The exponential expansion represents the "steady state" solution driven by creation of matter, which takes place preferentially near collapsed massive objects. The oscillatory part of the solution represents the "on-off" nature of creation. It is assumed for this solution that creation takes place when density is maximum and that there is no creation between two consecutive maxima of density. In the above expression P is the longer timescale for the steady state expansion and Q is the period of a typically oscillatory fluctuation from steady-state; η is a parameter restricted between the values 0 and 1. P and Q are closely related to the observed quantities in the universe, which include the Hubble's constant at the present epoch and the counts of radio sources. The QSSC also enables us to describe the cosmic microwave background and the observed abundances of the light nuclei. Hence the quasi–steady state cosmology can be put forward as a viable alternative cosmology to the standard classical big bang cosmology.

In Sachs et al. (1996), a homogeneous exact periodic solution is given for the Robertson-Walker metric using the QSSC in which these authors started with usual Robertson-Walker metric for $c = 1$. The field equations to be solved are

$$R^{ik} - \tfrac{1}{2}Rg^{ik} + \lambda g^{ik} = -8\pi G[T^{ik} - f(c^i c^k - \tfrac{1}{4}g^{ik}c^l c_l)], \quad (3)$$

where λ is the (negative) cosmological constant and c is a scalar field representing creation of matter. The expression c_i denotes $\partial c/\partial x^i$, x^i being the spacetime coordinates. T^{ik} is the matter tensor. In a dust medium the field equations for the Robertson-Walker line element are

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2 + k}{S^2} = 3\lambda + 2\pi G f \dot{c}^2. \quad (4)$$

$$\frac{3(\dot{S}^2 + k)}{S^2} = 3\lambda + 8\pi G\rho - 6\pi G f \dot{c}^2. \quad (5)$$

The conservation equation in the general case is given by

$$T^{ik}_{;k} = f(c^i c^k_{;k} + \tfrac{1}{2}c^i_{;k}c^k). \quad (6)$$

The above equation could be understood as representing two different modes of evolution: first, when both sides of the equation are individually zero which we call the *noncreative mode* and second, when they are both equal and nonzero, which we term as the *creative mode*. Sachs et al. have studied the noncreative mode as well as the creative mode. Here we shall concentrate only on the noncreative mode solution, for which we have

$$T^{ik}_{;k} = 0, \quad (7)$$

and

$$c^i c^k_{;k} + \tfrac{1}{2}c^i_{;k}c^k = 0. \quad (8)$$

The above equations, together with the field equations (4) and (5), lead to

$$\frac{\dot{S}^2 + k}{S^2} = \lambda + \frac{A}{S^3} - \frac{B}{S^4}. \quad (9)$$

For $k = 0$ and suitably chosen A and B an exact solution for the above equation is given by

$$S = \bar{S}[1 + \eta \cos \theta(t)] , \tag{10}$$

where $\bar{S}$ is a constant, η is a parameter, and the function $\theta(t)$ has the following implicit form

$$\dot{\theta}^2 = -\lambda(1 + \eta \cos \theta)^{-2}[6 + 4\eta \cos \theta + \eta^2(1 + \cos^2 \theta)] , \tag{11}$$

with

$$A = -4\lambda \bar{S}^3(1 + \eta^2) , \tag{12}$$

$$B = -\lambda \bar{S}^4(1 - \eta^2)(3 + \eta^2) . \tag{13}$$

This model oscillates between the finite scale limits

$$S_{\min} \equiv \bar{S}(1 - \eta) \le S \le \bar{S}(1 + \eta) \equiv S_{\max} . \tag{14}$$

The matter density ($\bar{\rho}$) and the c-field energy density ($f\dot{c}^2$) at $S = \bar{S}$ then take the forms

$$\bar{\rho} = -\frac{3\lambda}{2\pi G}(1 + \eta^2) , \tag{15}$$

$$f\dot{c}^2 = -\frac{\lambda}{2\pi G}(1 - \eta^2)(3 + \eta^2) . \tag{16}$$

The period of oscillation is given by

$$Q = \frac{1}{(-\lambda)^{1/2}} \int_0^{2\pi} \frac{(1 + \eta \cos \theta)d\theta}{6 + 4\eta \cos \theta + \eta^2(1 + \cos^2 \theta)} . \tag{17}$$

In the present paper we shall consider a small perturbation of this solution. Thus the metric is geometrically perturbed, while the matter density and the c-field energy density are also supposed to have small fluctuations. We shall explore the dynamical behavior of the perturbed system to see if it is stable or otherwise. In particular, we will be interested in seeing how the density fluctuations evolve.

2. FIELD EQUATIONS

We begin by imposing a small perturbation on the Robertson-Walker line element, for $k = 0$, for which we redefine the metric tensor as

$$g_{\mu\nu} = -S^2(\eta_{\mu\nu} + h_{\mu\nu}); \quad g_{0\mu} = h_{0\mu}; \quad g_{00} = 1 + h_{00} , \tag{18}$$

where $\eta_{\mu\nu} = 1$, $\mu = \nu$, and $\eta_{\mu\nu} = 0$, $\mu \ne \nu$; μ, $\nu = 1, 2, 3$, and also $S \equiv S(t)$ and $h_{ij} = h_{ij}(t, x^\mu)$. We have considered a comoving preferred observer for which all of the off-diagonal components of the metric tensor are assumed to be zero. In the present coordinate system the matter flow vector need not be comoving. We define the first-order perturbation of flow vector as u_1^i, the zeroth-order being $u_0^i \equiv (1, 0, 0, 0)$. Thus,

$$T^{ik} = \rho_0 u_0^i u_0^k + \rho_1 u_0^i u_0^k + \rho_0(u_0^i u_1^k + u_1^i u_0^k) , \tag{19}$$

where

$$\rho \equiv \rho_0 + \rho_1; \quad \rho_0 = \frac{\bar{\rho}\bar{S}^3}{S^3} , \tag{20}$$

ρ_1 being the density perturbation given as $\rho_1 = \rho_0 \zeta$. The perturbation vector u_1^i need not be along the four-vector $u_0^i = \delta_0^i$ but will have small departures from the normal direction. A perturbation is similarly assumed for the c-field as

$$c_i \equiv c_{0i} + \xi_i ; \quad c_0 \equiv c_0(t) ; \quad \xi_i \equiv \xi_i(t, x^\mu) . \tag{21}$$

h_{ij}, ζ, ξ_i, and u_1^i are quantities of the first order of smallness. In the noncreative mode the field equations for the Robertson-Walker metric ($k = 0$, $c = 1$) are then given by the following three groups of equations:

1.—*Conservation equations for the matter field:*

$$\dot{\zeta} + \tfrac{1}{2}(\dot{h}_{11} + \dot{h}_{22} + \dot{h}_{33}) + u^\mu_{1,\mu} = 0 , \tag{22}$$

$$\frac{\partial(S^2 u_1^1)}{\partial t} = -\frac{1}{2} h_{00,1} , \tag{23}$$

$$\frac{\partial(S^2 u_1^2)}{\partial t} = -\frac{1}{2} h_{00,2} , \tag{24}$$

$$\frac{\partial(S^2 u_1^3)}{\partial t} = -\frac{1}{2} h_{00,3} . \tag{25}$$

2.—*Conservation equations for the creation field:*

$$\nabla^2 \xi = \frac{3}{2}\frac{\partial(S^2 \dot{\xi})}{\partial t} + \frac{D}{2}\left(\dot{h}_{11} + \dot{h}_{22} + \dot{h}_{33} - \frac{3}{2}\dot{h}_{00}\right) , \tag{26}$$

$$\frac{\partial(S^2 \xi_1)}{\partial t} = \frac{D}{2} h_{00,1} , \tag{27}$$

$$\frac{\partial(S^2 \xi_2)}{\partial t} = \frac{D}{2} h_{00,2} , \tag{28}$$

$$\frac{\partial(S^2 \xi_3)}{\partial t} = \frac{D}{2} h_{00,3} , \tag{29}$$

where $\nabla^2 \xi \equiv \xi_{11} + \xi_{22} + \xi_{33}$ and $D = [B/(2\pi G f)]^{1/2}$.

3.—*Einstein's gravitational field equations:*

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{3\dot{S}}{2S}(\dot{h}_{22} + \dot{h}_{33}) - \frac{\dot{S}}{S}\dot{h}_{00} + \frac{1}{2}(\ddot{h}_{22} + \ddot{h}_{33}) - \frac{1}{2S^2}(h_{22,33} + h_{33,22}) - \frac{1}{2S^2}(h_{00,22} + h_{00,33}) = 3\lambda(1 + h_{00}) + 2\pi G f \frac{D}{S^2}\left(\frac{D}{S^2} + 2\dot{\xi}\right) , \tag{30}$$

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{3\dot{S}}{2S}(\dot{h}_{11} + \dot{h}_{33}) - \frac{\dot{S}}{S}\dot{h}_{00} + \frac{1}{2}(\ddot{h}_{11} + \ddot{h}_{33}) - \frac{1}{2S^2}(h_{11,33} + h_{33,11}) - \frac{1}{2S^2}(h_{00,11} + h_{00,33}) = 3\lambda(1 + h_{00}) + 2\pi G f \frac{D}{S^2}\left(\frac{D}{S^2} + 2\dot{\xi}\right) , \tag{31}$$

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{3\dot{S}}{2S}(\dot{h}_{11} + \dot{h}_{22}) - \frac{\dot{S}}{S}\dot{h}_{00} + \frac{1}{2}(\ddot{h}_{11} + \ddot{h}_{22}) - \frac{1}{2S^2}(h_{11,22} + h_{22,11}) - \frac{1}{2S^2}(h_{00,11} + h_{00,22}) = 3\lambda(1 + h_{00}) + 2\pi G f \frac{D}{S^2}\left(\frac{D}{S^2} + 2\dot{\xi}\right) . \tag{32}$$

$$\frac{3\dot{S}^2}{S^2}+\frac{\dot{S}}{S}(\dot{h}_{11}+\dot{h}_{22}+\dot{h}_{33})-\frac{1}{2S^2}\times(h_{11,22}+h_{11,33}+h_{22,11}+h_{22,33})-\frac{1}{2S^2}(h_{33,11}+h_{33,22})=3\lambda(1+h_{00})+8\pi G\rho_0(1+\zeta+h_{00})-6\pi Gf\frac{D}{S^2}\left(\frac{D}{S^2}+2\dot{\xi}\right), \quad (33)$$

$$\frac{\dot{S}}{S}h_{00,1}-\frac{1}{2}(\dot{h}_{22,1}+\dot{h}_{33,1})=-8\pi GS^2\rho_0u_1^1-8\pi Gf\frac{D}{S^2}\xi_1\,, \quad (34)$$

$$\frac{\dot{S}}{S}h_{00,2}-\frac{1}{2}(\dot{h}_{11,2}+\dot{h}_{33,2})=-8\pi GS^2\rho_0u_2^2-8\pi Gf\frac{D}{S^2}\xi_2\,, \quad (35)$$

$$\frac{\dot{S}}{S}h_{00,3}-\frac{1}{2}(\dot{h}_{11,3}+\dot{h}_{22,3})=-8\pi GS^2\rho_0u_3^3-8\pi Gf\frac{D}{S^2}\xi_3\,, \quad (36)$$

$$h_{00,12}+h_{33,12}=0\,, \quad (37)$$

$$h_{00,13}+h_{22,13}=0\,, \quad (38)$$

$$h_{00,23}+h_{11,23}=0\,. \quad (39)$$

These equations represent the evolution of an interrelated set of perturbations in the metric tensor, the density of matter, and the intensity of the c-field. Here we are primarily interested in the physical problem of how the perturbations in the matter energy tensor and the c-field tensor grow and drive the solution of these equations. *No extra constraining condition has been used in this calculation.* Algebraic simplifications and necessary eliminations of terms between the equations and retaining only those relating to the density perturbation could produce a second-order linear differential equation for ζ as a function of the time-dependent scale factor S:

$$\frac{d^2\zeta}{dS^2}+\left(\frac{1}{2F}\frac{dF}{dS}+\frac{4}{S}\right)\frac{d\zeta}{dS}+\frac{4\pi G\bar{\rho}\bar{S}^3}{S^3F}\zeta=\frac{-2q}{S^3\sqrt{F}}\,. \quad (40)$$

Here q is a function of space variables $x^{\mu}(\mu=1,2,3)$ and is a quantity of first order of smallness. It arises as a constant of integration from the combination of equations (23)–(25) and (37)–(39). The function $F(S)$ is given by

$$F(S)=\lambda S^2+\frac{A}{S}-\frac{B}{S^2}\,. \quad (41)$$

The above equation could be solved numerically in the framework of the present solution, i.e.,

$$P=20Q\,,\quad Q=4\times10^{10}\ \text{yr}\,,$$
$$\lambda=-0.4\times10^{-56}\ \text{cm}^{-2}\,,\quad \eta=0.75\,. \quad (42)$$

For the right-hand side of equation (40) we have taken $q=10^{-4}$. The value chosen for q is arbitrary but small in magnitude. In a generic solution ζ is plotted against t for a complete cycle ($0\le\theta\le2\pi$) along with S (Fig. 1). The starting value of ζ has been taken as 0.0004, at $\theta=0$. In this figure the values of ζ have been magnified by a factor ~100 for a clear representation. In the first half-cycle the scale factor (S) continuously decreases with time starting from its maximum value $S=1.75$ and at the same time the density (ζ) goes up from its minimum point. One can therefore easily see in Figure 1 that as the universe goes on contracting with time the perturbation in the density of the universe increases within a finite level and reaches its maximum limit at 0.01 when $S=0.25$. Again for the next half-cycle the figure shows that the scale factor starts increasing from its minimum position and goes up to the maximum where the density fluctuation has the minimum value: i.e., as universe expands with time, the density fluctuation of the universe goes down. Although Figure 1 shows a specific case, it is typical of the general solution, with other initial conditions.

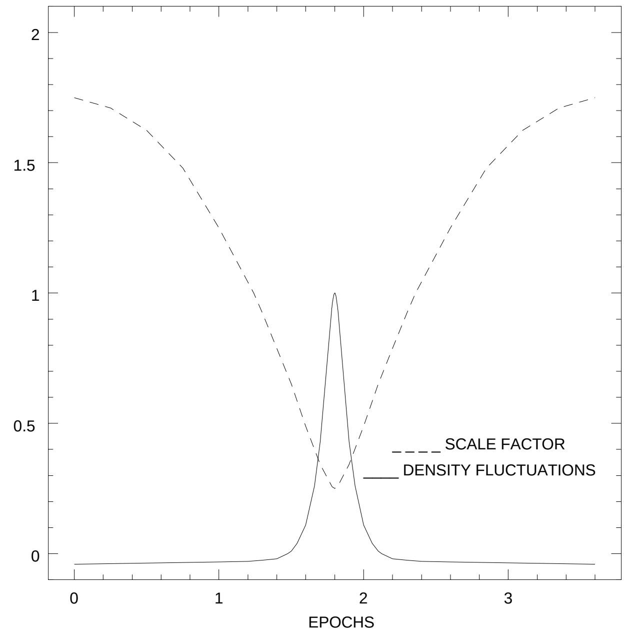

FIG. 1.—Plot of the scale factor and density fluctuations over 1 cycle. The density fluctuations are too small to be adequately represented on the vertical scale of this graph. They have been magnified by a factor ~100 for easy comparison.

The other equations of the set follow a similar periodic pattern with the perturbed quantities $h_{\mu\nu}$, ξ_{μ}, etc., always staying small. This analysis demonstrates that the basic cycle of QSSC is stable as a cosmological solution and may be used as a robust model for testing cosmological predictions.

3. STRUCTURE FORMATION

This example shows that typically density perturbations grow by a modest factor ($\lesssim10^2$) during the contracting phase whereas they decline during the expanding phase of the universe. This result is not unexpected since, in a purely gravitational scenario, expansion is expected to smoothen out inhomogeneities while contraction would make them more significant.

However, the relatively modest growth of $\delta\rho/\rho$ suggests that other nongravitational forces must play a role in the creation of large-scale structure. The clue to these lies in the creation events which operate near $S = S_{min}$. The creative mode which leads to the exp t/P factor in the expansion of the universe has not been included in our analysis here.

The reason for this is, as pointed out by Hoyle et al. (1993, 1994a), the creation of matter takes place in strong gravitational fields near collapsed massive objects. In such situations the smoothed-out solution used here and its perturbation would not apply. An altogether different approach will be needed to understand how creation of new units of matter take place and how they are ejected as coherent objects as these authors have claimed. Preliminary work along these lines using computer simulations shows that a filamentary structure interspersed with voids may emerge through successive minicreation events. This approach will be described in a future paper.

4. CONCLUSION

The exact solution given by Sachs et al. (1996) is found to be stable against small gravitational perturbations. The physical behavior of the models for $k = \pm 1$, although not explicitly demonstrated here, will remain the same as for $k = 0$. This stability ensures that the model is robust as a viable theoretical model. It also shows that for inhomogeneous structures to emerge, the model must look to its creative mode, not discussed here.

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