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Biscalar and Bivector Green's Functions in de Sitter Space Time

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Abstract. Biscalar and bivector Green's functions of wave equations are calculated explicitly in de Sitter space time. The calculation is performed by considering the electromagnetic field generated by the spontaneous creation of an electric charge.

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1. **Introduction.** Wave equations and their Green's functions play an important part in the different branches of theoretical physics. Although most of the investigations are usually made in the background of flat space time with the line element

$$ds^2 = \eta_{ik} dx^i dx^k \tag{1}$$

with η_{ik} = diag. $[-1, -1, -1, +1]$, there is now a growing interest in wave equations in curved space. This interest owes its origin to the general theory of relativity according to which the Newtonian concept of gravitation is replaced by the curvature of space time. Thus instead of the line element (1), we have to consider a more general line element

$$ds^2 = g_{ik} dx^i dx^k, \tag{2}$$

where g_{ik} are functions of space time and are related to the distribution of matter and energy through a set of nonlinear partial differential equations. In such a background the wave operator has the form

$$\Box Q = g^{ik} Q;\, ik, \tag{3}$$

where Q is a tensor quantity of a given rank. The presence of covariant derivatives makes the form (3) much more complicated than the usual flat space form. Although explicit forms for the Green's functions are not known, formal representations have been given by various authors (cf. Courant and Hilbert,[1] DeWitt and Brehme,[2] Lichnerowicz[3, 4]). These representations are useful in understanding the general properties of Green's functions, but they are not of any help in solving an explicit problem.

In this paper explicit forms are given for the scalar and vector Green's functions in de Sitter space. These forms are arrived at by considering a problem in electrodynamics, viz., the electromagnetic potentials and fields associated with charge creation. This problem is of interest in its own right, apart from any cosmological implications that it may have. We begin with the definitions and formal representations of the Green's functions involved.

2. **Notation and Definitions.** We define the biscalar and bivector Green's functions $\bar{G}(X,A)$ and $\bar{G}i_X i_A$ through the wave equations

$$\Box \bar{G}(X,A) = [-\bar{g}(X,A)]^{-1/2}\delta_4(X,A), \tag{4}$$

$$\Box \bar{G}_{i_X i_A} + R_{i_X}{}^{l_X}\, \bar{G}_{l_X i_A} = [-\bar{g}(X,A)]^{-1/2}\bar{g}_{i_X i_A}\delta_4(X,A), \tag{5}$$

and the symmetry conditions

$$\bar{G}(X,A) = \bar{G}(A,X); \quad \bar{G}_{i_X i_A} = \bar{G}_{i_A i_X}. \tag{6}$$

The $\Box$-operator is with respect to the coordinates of X and $\delta_4(X,A)$ is the four-dimensional delta function. R_{ik} is the Ricci tensor. The 4×4 quantities $\bar{g}_{i_X i_A}$ constitute the parallel propagator (cf. Synge[5]) and $\bar{g}(X,A)$ is their determinant.

DeWitt and Brehme[2] have given the following formal representation of $\bar{G}(X,A)$ and $\bar{G}_{i_X i_A}$:

$$\bar{G}(X,A) = \frac{1}{4\pi}\,[\Delta^{1/2}\delta(s^2{}_{XA}) - {}^1/_2 v\theta(s^2{}_{XA})],$$

$$\bar{G}_{i_X i_A} = \frac{1}{4\pi}\,[\Delta^{1/2}\delta(s^2{}_{XA})\bar{g}_{i_X i_A} - {}^1/_2 v_{i_X i_A}\theta(s^2{}_{XA})] \tag{7}$$

where s_{XA} and Δ are given by

$$s_{XA} = \int_{\Gamma_{XA}} ds, \ \Delta = \det \left\|({}^1/_2 s^2{}_{XA})_{;\,i_X i_A}\right\| \cdot [\bar{g}(X,A)]^{-1}. \tag{8}$$

Here we have assumed that Γ_{XA} is a unique geodesic joining A and X. $\theta(s^2{}_{XA})$ is the Heaviside function and its presence in (7) indicates propagation inside the light cone. The part of the Green's functions inside the light cone, represented by the functions v, $v_{i_X i_A}$ is called the tail. In Minkowski space time the tail is absent.

The symmetric Green's functions defined above can in general be split into retarded and advanced components. For example,

$$\bar{G}(X,A) = {}^1/_2 G^{\text{Ret}}(X,A) + {}^1/_2 \bar{G}^{\text{Adv}}(X,A) \tag{9}$$

where G^{Ret} has support on the future light cone and G^{Adv} has support on the past light cone of A. G^{Ret} and G^{Adv} also satisfy (4) but not (6). Similar results hold for the bivector Green's function. The scalar and bivector Green's functions are related by the following equation

$$\bar{G}^{k_X i_A}{}_{;\,k_X} = -\bar{G}_{;}{}^{i_A}. \tag{10}$$

Similar relations exist for the retarded and advanced components.

The tail functions v, $v_{i_X i_A}$ satisfy the following limiting laws:

$$\lim_{X\to A} v(X,A) = {}^1/_{12}R(A), \ \lim_{X\to A} v_{k_X i_A} = {}^1/_2[R_{ik}(A) - {}^1/_6 g_{ik}(A)R(A)]. \tag{11}$$

We wish to determine the Green's functions in de Sitter space time and, hence, we first proceed to calculate the $\bar{g}_{i_X i_A}$ along null geodesics in this space time. Considerable simplification results if we use a conformally flat line element to describe the space time:

$$ds^2 = \Omega^2(t)[dt^2 - dx^2 - dy^2 - dz^2] = \Omega^2\eta_{ik}dx^i dx^k. \tag{12}$$

Here $\Omega(t)$ is given by

$$\Omega(t) = (-Ht)^{-1}, \qquad -\infty < t < 0, \tag{13}$$

where H is a positive constant. Since null geodesics are conformally invariant, the two points $A:(\mathbf{o},a)$ and $X:(\mathbf{r},t)$ are connected by a null geodesic provided

$$(t - a)^2 = x^2 + y^2 + z^2 = r^2. \tag{14}$$

Here we have taken $\mathbf{r}$ to have three cartesian components (x, y, z).

The $\bar{g}_{i_X i_A}$ are obtained from first principles by propagating a vector from A to X parallel to itself along the null geodesic Γ_{XA}. A simple calculation gives

$$\begin{aligned} \bar{g}_{44} &= \frac{1}{2H^2}\left(\frac{1}{t^2} + \frac{1}{a^2}\right) \\ \bar{g}_{4\mu} &= \frac{1}{2H^2}\left(\frac{t + a}{t^2a^2}\right) r_\mu, \ \bar{g}_{4\mu} = -\frac{1}{2H^2}\left(\frac{t+a}{t^2a^2}\right) r_\mu, \\ \bar{g}_{\mu\nu} &= -\frac{1}{2H^2}\cdot\frac{1}{t^2a^2} r_\mu r_\nu + \frac{1}{H^2at}\eta_{\mu\nu}, \ (\mu,\nu = 1, 2, 3). \end{aligned} \tag{15}$$

For convenience of writing we have suppressed the index suffixes i_A and i_X on the understanding that the first index refers to X and the second to A.

3. **The Electromagnetic Potential Associated with Charge Creation.** We shall calculate the electromagnetic field produced by a particle a of charge e in the de Sitter space given by (12). Let a^{i_A} denote the coordinates of a typical point on the world line of a. Then the retarded 4-potential produced by the charge at a general point X is given by

$$A_i^{(a)}(X) = 4\pi e \int G^{\mathrm{Ret}}{}_{i i_A}\, da^{i_A} \tag{16}$$

where, for convenience, we have dropped the suffix X for the indices at X. This produces no ambiguity in subsequent calculations. The potential (16) satisfies the wave equation

$$\Box A_i^{(a)} + R_i^k A_k^{(a)} = 4\pi j_i^{(a)}, \tag{17}$$

where

$$j_i^{(a)}(X) = e\int \delta_4(X, A)\ [-\bar{g}(X,A)]^{1/2}\bar{g}_{i i_A} da^{i_A} \tag{18}$$

is the current produced by the charge a. The velocity of light is taken to be unity in this context.

If we assume charge conservation and solve Maxwell's equations in terms of retarded potentials we would arrive at (16). However, we will now consider the situation where the charge a is created at a world point A_0, without the creation of a compensating opposite charge. This situation cannot be dealt with by Maxwell's equations which require charge conservation. A different approach is necessary.

The method we follow here is inspired by the electrodynamics of direct inter-particle action (cf. Schwarzschild,[6] Tetrode,[7] Fokker,[8–10] Wheeler and Feynman[11]). In this approach, fields do not exist on their own; the charged particles interact directly with each other. The interaction between a pair of charges a, b is described by a term in the action of the form

$$4\pi e_a e_b \int\int \bar{G}_{i_A i_B} da^{i_A} db^{i_B}. \tag{19}$$

The presence of the time symmetric Green's function results in the appearance of advanced as well as retarded interactions. However, if we have a suitable universe, the interactions between all charges in it can result in the cancellation of the advanced effects (cf. Hogarth,[12] Hoyle and Narlikar[13]). The steady-state universe is one such case. Hence the direct particle field produced by the charge in such a universe can be described by the potential (16). The form (16) can be easily generalized to cover the case of charge creation at A_0. For, in that case the world line of the charge begins at A_0 and the line integral (16) starts from that point. This leads to a modification of the corresponding Maxwell-like equations satisfied by the charge in the following way. From (10) we get

$$A^{(a)i}{}_{;i} = 4\pi e \int G^{\mathrm{Ret}\ i}{}_{i_A;\ i} da^{i_A} = 4\pi e G^{\mathrm{Ret}}(X, A_0). \tag{20}$$

Thus the gauge condition is violated by charge creation. The field

$$F_{ik}{}^{(a)} = A_{k;\ i}{}^{(a)} - A_{i;\ k}{}^{(a)} \tag{21}$$

satisfies the equation

$$F^{(a)ik}{}_{;k} + [A^{(a)l}{}_{;l}]^{;i} = 4\pi j^{(a)i}. \tag{22}$$

The presence of the extra term in the left-hand side indicates the departure from Maxwell's equations.

We now take advantage of the fact that the underlying metric is $\Omega^2\eta_{ik}$ and rewrite the relations (17), (20), and (22) in terms of η_{ik}. To avoid confusion we shall underline the indices which refer to the flat space metric. Also we drop the superscript a on $A_i{}^{(a)}$ and $F_{ik}{}^{(a)}$. Write

$$A_i = \hat{A}_{\underline{i}} + \phi_{;\underline{i}} \tag{23}$$

where $\hat{A}_i$ is a 4-vector in flat space and ϕ is a scalar. Then

$$F_{ik} = A_{k;\ i} - A_{i;\ k} = \hat{A}_{\underline{k};\ \underline{i}} - \hat{A}_{\underline{i};\ \underline{k}} = \hat{F}_{\underline{ik}} \tag{24}$$

where $\hat{F}_{ik}$ is an antisymmetric tensor in flat space. We have

$$F_{;k}{}^{ik} = \frac{1}{\Omega^4} \hat{F}_{;\underline{k}}{}^{\underline{ik}}, \tag{25}$$

and we choose ϕ such that

$$A_{;i}{}^{i} = \frac{1}{\Omega^2} \hat{A}_{;\underline{i}}{}^{\underline{i}}. \tag{26}$$

Thus ϕ satisfies the equation

$$\underline{\Box}\phi + \frac{2\Omega_{;i}}{\Omega}(\phi^{;\underline{i}} + \hat{A}^{\underline{i}}) = 0 \tag{27}$$

where $\Box$ denotes the wave operator in flat space. From (18) we get

$$j^{(a)i}(X) = \frac{1}{\Omega^4} j^{(a)\underline{i}}(X). \tag{28}$$

Finally, using the above relations we get for $\hat{A}_i$,

$$\underline{\Box}\hat{A}_{\underline{i}} - \frac{2\Omega_{;\underline{i}}}{\Omega}\hat{A}_{;\underline{l}}{}^{\underline{l}} = 4\pi j_{\underline{i}} \tag{29}$$

Thus we arrive at a wave equation for $\hat{A}_i$ in flat space. Its solution enables us to determine G^{Ret} from (20) and (26). We use this G^{Ret} to determine ϕ from (27) and thus complete the problem of determining A_i. We shall carry through this procedure in the following section.

4. $\bar{G}(X,A)$ **in de Sitter Space.** Consider the electromagnetic potential generated by a charge a created at A_0: $(\mathbf{o},a)$, and subsequently at rest in the (x,y,z,t) frame. For such a charge

$$j_{\underline{i}} = e\delta_3(\mathbf{r})\theta(t - a)[\mathbf{o},1]. \tag{30}$$

The symmetry of source and space time suggests a solution of the form

$$\hat{A}^{\underline{i}} = \hat{A}_{\underline{i}} = (\mathbf{o},\chi) \tag{31}$$

where χ is a function of r and t. From (29), we get

$$\frac{\partial^2\chi}{\partial t^2} + \frac{2}{t}\frac{\partial\chi}{\partial t} - \frac{\partial^2\chi}{\partial r^2} - \frac{2}{r}\frac{\partial\chi}{\partial r} = 4\pi e\delta_3(\mathbf{r})\theta(t - a). \tag{32}$$

The retarded solution of this is given by

$$\chi = e\left(\frac{t - r}{tr}\right)\theta(t - r - a). \tag{33}$$

Thus $\hat{A}_{\underline{i}}$ is fully determined. Next from (20) and (26) we get

$$G^{\text{Ret}}[\mathbf{r},t;\,\mathbf{o},a] = \frac{H^2}{4\pi}\left\{at\,\frac{\delta(t - a - r)}{r} + \theta(t - a - r)\right\}. \tag{34}$$

The corresponding time symmetric Green's function is given by

$$\bar{G}[\mathbf{r},t;\,\mathbf{o},a] = \frac{H^2}{4\pi}\left\{at\,\delta[(t - a)^2 - r^2] + {}^1/_2\theta[(t - a)^2 - r^2]\right\}. \tag{35}$$

Comparison with (7) and (11) shows that v is constant and equal to its limiting value $R/12 = H^2$.

Since (27) is essentially a wave equation in de Sitter space, its solution can be written by using (34) or (35). The retarded solution for ϕ is therefore given by the integral

$$\phi(\mathbf{r},t) = e\iiiint G^{\mathrm{Ret}}[\mathbf{r},t;\, r',t']\cdot \frac{2\dot{\Omega}(t')}{[\Omega(t')]^3}\cdot\left(\frac{t'-r'}{t'r'}\right)\theta(t'-r'-a)d^3\mathbf{r}'dt'$$

$$= -\frac{e}{2\pi}\iiiint\left[\frac{tt'}{|\mathbf{r}-\mathbf{r}'|}\,\delta(t-t'-|\mathbf{r}-\mathbf{r}'|)\right.$$

$$\left. + \theta(t-t'-|\mathbf{r}-\mathbf{r}'|)\right]\left(\frac{t'-r'}{t'r'}\right)\theta(t'-r'-a)\,\frac{1}{t'^2}\,d^3\mathbf{r}'dt', \quad (36)$$

where $\mathbf{r}' = (x',y',z')$ and $|\mathbf{r}'| = r'$. The integration is over the entire space time. The delta and Heaviside functions, however, restrict the integration to finite regions. The evaluation of the integral is straightforward and leads to the result

$$\phi = \frac{e}{3}\left[\ln\frac{16t^2(t-r)^3}{a[(t+a)^2-r^2]} - \frac{(t-a)^2-r^2}{(t+a)^2-r^2}\right]\theta(t-a-r). \quad (37)$$

From (23) we then get the components of $A_i \equiv (\mathbf{A},A_4)$:

$$A_4 = e\left\{\frac{1}{r} - \frac{1}{3t} + \frac{1}{t-r} - \frac{4t}{3}\left(\frac{1}{D} + \frac{2a(t+a)}{D^2}\right)\right\}\theta(t-a-r), \quad (38)$$

$$\mathbf{A} = e\frac{\mathbf{r}}{r}\left\{\frac{1}{r-t} + \frac{4r}{3}\left(\frac{1}{D} + \frac{2at}{D^2}\right)\right\}\theta(t-a-r), \quad (39)$$

where

$$D = (t+a)^2 - r^2. \quad (40)$$

The equations (17) and (20) provide a direct check on these calculations. We now turn to the determination of $G^{\mathrm{Ret}}{}_{i_X i_A}$.

5. **$\bar{G}_{i_X i_A}$ in de Sitter Space.** A look at equation (7) shows that the leading term in $\bar{G}_{i_X i_A}$ is $\bar{g}_{i_X i_A}$ times the leading term in $\bar{G}(X, A)$. We have already calculated $\bar{g}_{i_X i_A}$ along the null geodesics. It therefore remains to determine $v_{i_X i_A}$. For the charge considered in the previous section we have

$$A_i(X) = 4\pi e\int G^{\mathrm{Ret}}{}_{ii_A}da^{i_A} = 4\pi e\int_a^0 G^{\mathrm{Ret}}{}_{i4_A}da. \quad (41)$$

Hence differentiation with respect to a gives

$$\frac{\partial A_i}{\partial a} = -4\pi e G^{\mathrm{Ret}}{}_{i4_A}. \quad (42)$$

Using (38) and (39) we therefore get

$$G^{\mathrm{Ret}}{}_{44_A} = \frac{(t^2+a^2)\,\delta(t-a-r)}{2at\qquad r} - \left(1-\frac{4r^2}{3D}\right)\frac{8at}{D^2}\,\theta(t-a-r), \quad (43)$$

$$G^{\mathrm{Ret}}{}_{14_A} = -\frac{x(t+a)}{2at}\,\frac{\delta(t-a-r)}{r} + \frac{8ax}{3D^2}\left[1 + \frac{4t(t+a)}{D}\right]\theta(t-a-r). \quad (44)$$

with similar expressions for $G^{\text{Ret}}{}_{24_A}$, $G^{\text{Ret}}{}_{34_A}$. The corresponding components of $\bar{G}_{t4_A}$ can be easily written down. Then from (6) we also get $\bar{G}_{4t_A}$ and hence $G^{\text{Ret}}{}_{4t_A}$. A simple calculation along these lines leads to

$$G^{\text{Ret}}{}_{41_A} = \frac{x(t+a)}{2at}\frac{\delta(t-a-r)}{r} + \tau\theta(t-a-r), \tag{45}$$

and similar expressions for $G^{\text{Ret}}{}_{42_A}$, $G^{\text{Ret}}{}_{43_A}$ where

$$\tau = -\frac{8tx}{3D^2}\left[1 + \frac{4a(t+a)}{D}\right]. \tag{46}$$

It now remains to determine the purely spacelike components, $\bar{G}^{\text{Ret}}{}_{\mu\mu_A}$, $\mu,\mu_A = 1, 2, 3$. To this end we use the relation (10) for $\mu_A = 1, 2, 3$. Dropping the superscript Ret on G, we get for $\mu_A = 1$,

$$G^{i1}{}_{;i} = H^2t^2\left[\dot{G}_{41} - \frac{2}{t}G_{41} - \frac{\partial G_{11}}{\partial x} - \frac{\partial G_{21}}{\partial y} - \frac{\partial G_{31}}{\partial z}\right]. \tag{47}$$

Here we have also dropped the label A on the second vector index of G. In G_{41} for example, 1 refers to A and 4 to X. Using our knowledge of the leading part of the bivector we have

$$G_{11} = -\left(1 + \frac{x^2}{2at}\right)\frac{\delta(t-a-r)}{r} + \xi\theta(t-a-r),$$

$$G_{21} = -\frac{xy}{2at}\frac{\delta(t-a-r)}{r} + \eta\theta(t-a-r);$$

$$G_{31} = -\frac{xz}{2at}\frac{\delta(t-a-r)}{r} + \zeta\theta(t-a-r), \tag{48}$$

where ξ, η, ζ are to be determined. From equations (45)–(48) we have

$$\frac{\partial\xi}{\partial x} + \frac{\partial\eta}{\partial y} + \frac{\partial\zeta}{\partial z} = \dot{\tau} - \frac{2\tau}{t}, \tag{49}$$

together with the condition to be satisfied on the light cone $t = a + r$,

$$\frac{x\xi + y\eta + z\zeta}{r} = -\tau + \frac{x}{2rt}\left(\frac{3}{t} + \frac{2a}{r^2}\right) - \frac{x}{r}\left(\frac{1}{t^2} + \frac{a^2}{tr^2}\right). \tag{50}$$

Closer inspection of (49) and (50) suggests a solution of the form

$$\xi = \left[1 - \frac{x^2}{(a+t)^2}\right]f(r,t),\ \eta = \frac{xy}{(a+t)^2}\,g(r,t),\ \zeta = \frac{xz}{(a+t)^2}\,g(r,t). \tag{51}$$

Substitution into (49) gives $g = -f$ and an equation to be satisfied by f:

$$\frac{\partial}{\partial D}[fD^2] = -\frac{4(a+t)^2}{3}\left[\frac{1}{D} + \frac{4(a^2+at+t^2)}{D^2} + \frac{24at(a+t)^2}{D^3}\right], \tag{52}$$

where $\partial/\partial D$ denotes differentiation with respect to D at a constant t. The integral of (52) therefore includes an arbitrary function of t, which can be completely determined from the boundary condition (50). We finally arrive at the result

$$f = -g = \frac{4(a+t)^2}{3D^2}\left[\ln\frac{4at}{D} + \frac{2(a^2+t^2+r^2)}{D} + \frac{12at(t+a)^2}{D^2} - \frac{3(a^2+t^2)}{4at} - 1\right]. \quad (53)$$

Thus G_{11}, G_{21}, G_{31} are fully determined. The remaining components of G_{ik} can now be written down by cyclic symmetry of x, y, z. We give below all the components of G_{ik} for completeness. We suppress the labels A, X, and Ret for convenience of writing. From these it is easy to construct the components of $\bar{G}_{iXiA}$. Uniqueness theorem guarantees that these expressions are unique.

$$\begin{aligned}
G_{11} &= -\left(1+\frac{x^2}{2at}\right)\frac{\delta(t-a-r)}{r} + \left[1-\frac{x^2}{(a+t)^2}\right]f\theta(t-a-r),\\
G_{21} &= -\frac{xy}{2at}\,\frac{(t-a-r)}{r} - \frac{xy}{(a+t)^2}f\theta(t-a-r),\\
G_{41} &= \frac{(a+t)x}{2at}\,\frac{\delta(t-a-r)}{r} - \frac{8xt}{3}\left[\frac{1}{D^2}+\frac{4a(t+a)}{D^3}\right]\theta(t-a-r),\\
G_{14} &= -\frac{(a+t)x}{2at}\,\frac{\delta(t-a-r)}{r} + \frac{8ax}{3}\left[\frac{1}{D^2}+\frac{4t(t+a)}{D^3}\right]\theta(t-a-r),\\
G_{44} &= \frac{t^2+a^2}{2at}\,\frac{\delta(t-a-r)}{r} - \frac{8at}{3}\left[\frac{3}{D^2}-\frac{4r^2}{D^3}\right]\theta(t-a-r),
\end{aligned} \quad (54)$$

and the rest by symmetry.

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