Time–of–night variation of solar neutrinos

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(21 February 1997)

We investigate the time–of–night variation of solar neutrino rate which will be of relevance to Super–Kamioka and Sudbury neutrino detectors in the framework of oscillations among the three flavors. An analytical method of computing the regeneration in the earth is presented. If day-night effect is seen, we show how the study of the time–of–night variation will allow the determination of the neutrino parameters.

It has been known for quite some time that an asymmetry between night rate and day rate for the real time solar neutrino detectors is an unambiguous signal for neutrino mixing and oscillations. Conversely the absence of such an effect can put constraints on the neutrino masses and mixing angles. Although no day–night asymmetry outside the error–bars was seen at the Kamioka detector [1,2] the high statistics detectors like Super-Kamioka [3], SNO [4] and Borexino [5] will be much more effective in investigating this effect. If there is a day-night asymmetry, then the profile of the asymmetry as a function of the time during night is a very sensitive function of the neutrino parameters. The counting rates in these detectors are expected to be high enough for the study of this time–of–night variation. The aim of this note is to focus attention on this aspect of the day–night effect [6], in view of the fact that Super–Kamioka has already started functioning and SNO is expected to do so soon.

The neutrino samples different amounts of matter in the earth during a single night and also during a year. The distance d travelled by the neutrino inside the earth during night, as a function of time t, is given by

$$d = 2R(\sin\phi_l \sin\delta + \cos\phi_l \cos\delta \cos(\frac{2\pi t}{T_D})), \qquad (1)$$

$$\sin\delta = \sin 23.5^o \sin(\frac{2\pi t}{T_Y}), \qquad (2)$$

R is the radius of the earth, ϕ_l is the latitude of the location of the detector, T_D is the length of the day, T_Y is the length of the year and zero of t is chosen at midnight on autumnal equinox. Thus assuming that the neutrino parameters are in a suitable range, neutrino data collected as a function of t contain an enormous amount of information on neutrino oscillations, which in principle can be analyzed to yield the neutrino parameters.

The time variation of $x = (\frac{d}{2R})$ during the night and year are illustrated in Fig.1a. If the data collected during successive nights are accumulated at the corresponding x– bins, the calculation of neutrino rates per unit bin will require the function $f(x)$ defined as the time duration per unit interval of x [7]. $f(x)$ for different locations are plotted in Fig.1b, which shows the relative merits of the
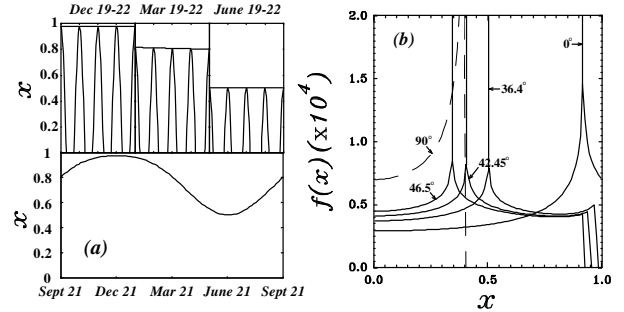

FIG. 1. (a)The fractional distance x travelled by the neutrino inside the earth during night is plotted against time for the latitude (36.4^o) of the Super–Kamioka detector. The lower figure gives the envelope of the 365 maxima during the year . As examples of the actual curves, those for a few nights during three specific seasons of the year are shown in the upper figure. (b) The function $f(x)$ in hours per unit x is plotted for various latitudes: Super–Kamioka (36.4^o), Borexino (42.45^o), SNO (46.5^o) , equator (0.0^o) , pole (90°).

detectors for exposure to regions of x.

We now describe an analytical method of calculating the neutrino regeneration effect in the earth. Let a neutrino of flavor α be produced at time $t = t_0$ in the core of the sun. Its state vector is

$$|\Psi_\alpha(t_0)\rangle = |\nu_\alpha\rangle = \sum_i U^C_{\alpha i}|\nu^C_i\rangle. \qquad (3)$$

where $|\nu^C_i\rangle$ are the mass eigenstates with mass eigenvalues μ^C_i and $U^C_{\alpha i}$ are the elements of the mixing matrix in the core of the sun. We use Greek index α to denote the flavors e, μ,τ and Latin index i to denote the mass eigenstates i = 1,2,3. The neutrino propagates in the sun adiabatically upto t_R (the resonance point), makes nonadiabatic transitions at t_R, propagates adiabatically upto t_1 (the edge of the sun) and propagates as a free particle upto t_2 when it makes a nonadiabatic transition as it enters the earth. If it propagates adiabatically upto t_3 (we shall soon correct for nonadiabatic jumps during this propagation), the probability amplitude for detecting a neutrino of flavor β at t_3 is

$$\langle\nu_\beta|\Psi_\alpha(t_3)\rangle = \sum_{k,j,i} U_{\beta k}^{E*} M_{kj}^E M_{ji}^S U_{\alpha i}^C exp\left\{-i\left(\int_{t_2}^{t_3}\varepsilon_k^E(t)dt\right.\right.$$

$$\left.\left. + \varepsilon_j(t_2-t_1)+\int_{t_R}^{t_1}\varepsilon_j^S(t)dt+\int_{t_0}^{t_R}\varepsilon_i^S(t)dt\right)\right\} \qquad (4)$$

$\varepsilon_i(\equiv E+\mu_i^2/2E)$, $\varepsilon_i^S(t)$ and $\varepsilon_i^E(t)$ are the energy eigenvalues in vacuum, sun and earth respectively, M_{ji}^S and M_{ji}^E are the probability amplitudes for nonadiabatic transition $i\to j$ inside the sun and at the vacuum -earth boundary respectively. The latter is due to the abrupt change in density when the neutrino enters the earth and is given by

$$M_{kj}^E=\langle\nu_k^E|\nu_j\rangle=\sum_\sigma\langle\nu_k^E|\nu_\sigma\rangle\langle\nu_\sigma|\nu_j\rangle=\sum_\sigma U_{\sigma k}^E U_{\sigma j}^* \quad (5)$$

where U and U^E are the mixing matrices in vacuum and earth respectively. Averaging the probability $|\langle\nu_\beta|\Psi_\alpha(t_3)\rangle|^2$ over t_R results in the desired incoherent mixture of mass eigenstates of neutrinos reaching the surface of the earth. Calling this averaged probability as $P_{\alpha\beta}^N$ (the probabilty for a neutrino produced in the sun as ν_α to be detected as ν_β in the earth at night), we can write the result as

$$P_{\alpha\beta}^N=\sum_j P_{\alpha j}^S P_{j\beta}^E , \qquad (6)$$

$$P_{\alpha j}^S=\sum_i |M_{ji}^S|^2|U_{\alpha i}^C|^2 , \qquad (7)$$

$$P_{j\beta}^E=\sum_{k,k'} U_{\beta k}^{E*} U_{\beta k'}^E M_{kj}^E M_{k'j}^{E*} exp\left(-2i\Phi_{kk'}\right) , \qquad (8)$$

$$\Phi_{kk'}=\frac{1}{2}\int_{t_2}^{t_3}(\varepsilon_k^E(t)-\varepsilon_{k'}^E(t))dt . \qquad (9)$$

For the daytime, put $t_3=t_2$ so that $P_{j\beta}^E$ becomes $|U_{\beta j}|^2$ and so eq.(6) reduces to the usual [10,11] transition probability in the day:

$$P_{\alpha\beta}^D=\sum_i\sum_j |U_{\beta j}|^2|M_{ji}^S|^2|U_{\alpha i}^S|^2 . \qquad (10)$$

It is important to note that the factorization of probabilities seen in eq.(6) is valid only for mass–eigenstates in the intermediate state. An equivalent statement of this result is that the density matrix is diagonal only in the mass-eigenstate representation and not in the flavor representation.

We next show how to take into account nonadiabatic jumps during the propagation inside the earth. Consider ν propagation through a series of slabs of matter, density varying inside each slab smoothly but changing abruptly at the junction between adjacent slabs. The state vector of the neutrino at the end of the nth slab $|n\rangle$ is related to that at the end of the $(n-1)^{th}$ slab $|n-1\rangle$ by $|n\rangle = F^{(n)}M^{(n)}|n-1\rangle$ where $M^{(n)}$ describes the nonadiabatic

jump occuring at the junction between the $(n-1)^{th}$ and n^{th} slabs while $F^{(n)}$ describes the adiabatic propagation in the nth slab. They are given by

$$M_{ij}^{(n)}=\langle\nu_i^{(n)}|\nu_j^{(n-1)}\rangle=(U^{(n)\dagger}U^{(n-1)})_{ij}^* , \qquad (11)$$

$$F_{ij}^{(n)}=\delta_{ij}exp\left(-i\int_{t_{n-1}}^{t_n}\varepsilon_i(t)dt\right) , \qquad (12)$$

where the indices (n) and $(n-1)$ occuring on ν and U refer respectively to the nth and $(n-1)^{th}$ slabs at the junction between these slabs. Also note that $M^{(1)}$ is the same as M^E defined in eq(5). Defining the density matrix at the end of the nth slab as $\rho^{(n)}=|n\rangle\langle n|$, we have the recursion formula

$$\rho^{(n)}=F^{(n)}M^{(n)}\rho^{(n-1)}M^{(n)\dagger}F^{(n)\dagger}. \qquad (13)$$

Starting with $\rho^{(0)}=|\nu_j\rangle\langle\nu_j|$ (i.e ν_j entering the earth), we can calculate $\rho^{(N)}$ at the end of the Nth slab using (13). The probability of observing ν_β at the end of the Nth slab is

$$P_{j\beta}^E=\langle\nu_\beta|\rho^{(N)}|\nu_\beta\rangle=(U^{(N)}\rho^{(N)*}U^{(N)\dagger})_{\beta\beta} . \qquad (14)$$

This formula (which reduces to eq(8) for $N=1$) can be used for the earth modelled as consisting of $(N+1)/2$ concentric shells, with the density varying gradually within each shell. We shall present numerical results for $N=3$ (mantle and core) [12]. However for $x<0.84$, neutrinos pass only through the mantle and so $N=1$. Accuracy achieved with this model is adequate for the present purposes, but the formalism allows one to improve the accuracy to any desired level, by adding more shells.

Apart from the nonadiabatic jumps occuring at the density–discontinuities, such jumps can occur also at any MSW resonance in the earth. The formalism presented above is capable of handling this. One simply replaces eq.(11) for $M^{(n)}$ for that transition by an appropriate Landau–Zener formula [13].

We parametrize the mixing matrix U in vacuum as $U=U^{23}(\psi)U^{13}(\phi)U^{12}(\omega)$ where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the ith and the jth mass eigenstates with the mixing angle θ_{ij}, neglecting CP violation. In the solar neutrino problem ψ drops out [13,14]. The mass differences in vacuum are defined as $\delta_{21}=\mu_2^2-\mu_1^2$ and $\delta_{31}=\mu_3^2-\mu_1^2$. It has been shown [11,15] that the simultaneous solution of both the solar and the atmospheric neutrino problems requires the mass hierarchy $\delta_{31}\gg\delta_{21}$ and under this condition δ_{31} also drops out. The rediagonalization of the mass matrix in the presence of matter (in the sun or earth) under the hierarchy condition leads to the following results [11]

$$\tan 2\omega_m = \frac{\delta_{21}\sin 2\omega}{\delta_{21}\cos 2\omega - A\cos^2\phi} , \qquad (15)$$

$$\sin\phi_m=\sin\phi , \qquad (16)$$

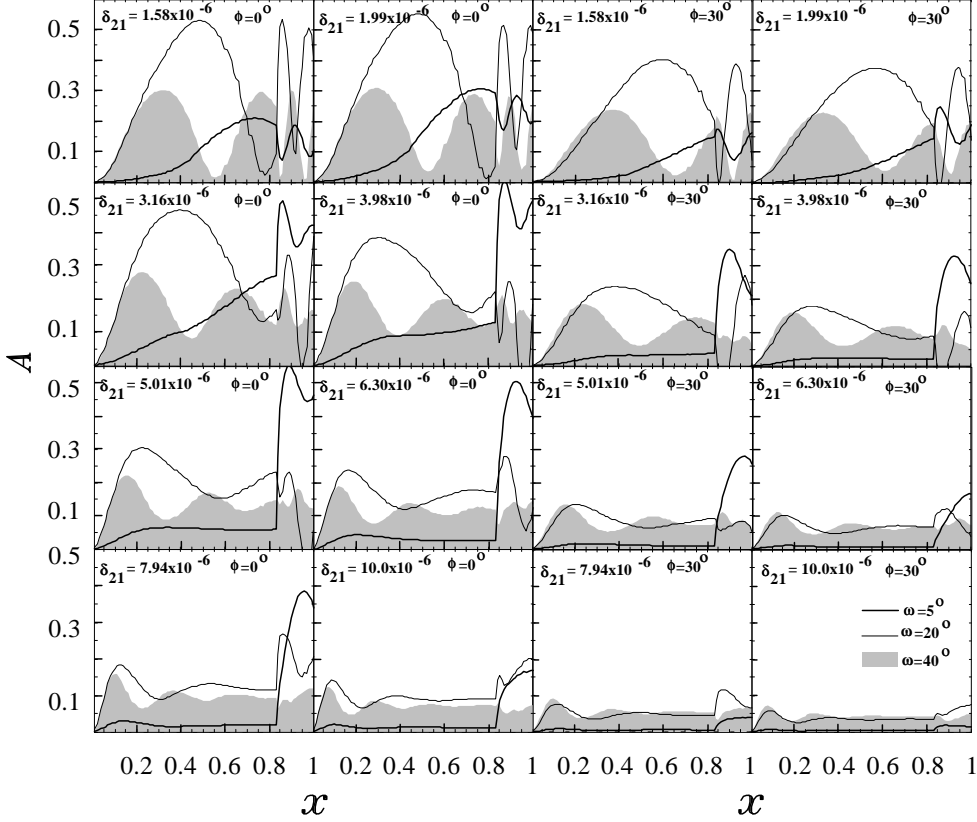$$\delta_{21}^m=\delta_{21}\cos 2(\omega-\omega_m)-A\cos^2\phi\cos 2\omega_m , \qquad (17)$$

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FIG. 2. A as a function of x, the fractional distance travelled by the neutrino, for stated values of ω, δ_{21} (in eV^2) and ϕ.

where A is the Wolfenstein term $A = 2\sqrt{2}\ G_F\ N_e\ E$ (N_e is the number density of electrons and E is the neutrino energy) . We note that $\delta_{31} \gg A$, for A evaluated at any point in the sun or the earth. In eqs (15) - (17), the "m" stands for matter and in using these equations, one must use the appropriate density of matter that is required at the various points along the trajectory of the neutrino.

All the probabilities $P_{\alpha\beta}^N$, $P_{\alpha j}^S$ and $P_{j\beta}^E$ satisfy the normalization conditions, as for instance, $\sum_j P_{j\beta}^E = 1$. For three flavors, use of these conditions allows us to express P_{ee}^N in terms of P_{ee}^D, P_{1e}^E, P_{2e}^E and P_{e3}^S as

$$P_{ee}^N = [P_{ee}^D(P_{1e}^E - P_{2e}^E) - \cos^2\phi(\sin^2\omega P_{1e}^E - \cos^2\omega P_{2e}^E)$$
$$-P_{e3}^S((3\cos^2\phi\cos^2\omega - 1)P_{2e}^E - (3\cos^2\phi\sin^2\omega - 1)P_{1e}^E$$
$$-\cos^2\phi\cos^2\omega)]/(\cos^2\phi\cos 2\omega). \quad (18)$$

This is a general formula that goes over to the one given in Ref. [16] for two flavors. Simplifications arise from the mass heirarchy condition. First, M_{ij}^S is nonzero for $i,j = 1,2$ only and hence we can replace P_{e3}^S in eq(18) by $\sin^2\phi_C$. $|M_{12}^S|^2$ is taken to be the modified Landau-Zener jump probability for an exponentially varying solar density [13]. Further, $M^{(n)}$ also are reduced to 2×2 matrices. In fact eq(11) gives $M^{(n)} = U^{12}(\theta)$ where $U^{12}(\theta)$ is the 2–flavor mixing matrix, with mixing angle

$\theta = \omega_n - \omega_{n-1}$. As a result, we get the simple formulae from eq(8), valid for $x < 0.84$:

$$P_{1e}^E = \cos^2\phi[\cos^2\omega_E - \sin 2\omega_E \sin 2(\omega_E - \omega)\sin^2\Phi_{12}] \quad (19)$$
$$P_{2e}^E = \cos^2\phi[\sin^2\omega_E + \sin 2\omega_E \sin 2(\omega_E - \omega)\sin^2\Phi_{12}] \quad (20)$$

where ω_E is the mixing angle, just below the surface of the earth.

The neutrino detection rates for a Super–Kamioka type of detector is given by

$$R = \int \phi\sigma P_{ee}dE + \frac{1}{6}\int \phi\sigma(1 - P_{ee})dE \quad (21)$$

where the second term is the neutral current contribution and $\phi(E)$ is the solar neutrino flux as a function of the neutrino energy E and $\sigma(E)$ is the cross section from neutrino electron scattering and we integrate from $5MeV$ onwards. The cross section is taken from [17] and the flux from [18]. The rates for the night and day R_N and R_D are calculated using P_{ee}^N and P_{ee}^D respectively. We define the day–night asymmetry ratio as $A = (R_N - R_D)/(R_N + R_D)$. We can multiply R_N and R_D by the function $f(x)$ displayed in Fig.1 to get the rates per unit interval in x. Note that $f(x)$ cancels in the asymmetry ratio calculated theoretically. However,

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the experimentalists have to weight the day rate R_D with $f(x)$ before comparing their data with our theoretical curves.

In Fig.2 and 3 we have plotted A as a function of x, the fractional distance travelled by the neutrino inside the earth for various values of the neutrino parameters δ_{21}, ω and ϕ. Different values of these parameters have distinguishable characteristics. Some gross features which may enable us to specify their approximate domains are the following:

- For small angle ω there is a gradual increase of the asymmetry with x, whereas for large ω the oscillations in x start showing up. For $x < 0.84$ (*i.e.* trajectories through mantle only) there is a very clear discrimination between the small ω and large ω, irrespective of δ_{21} and ϕ.

- As ϕ increases, the asymmetry at any x decreases.

- The amplitude of the oscillatory pattern is largest for small δ_{21} and decreases steadily as δ_{21} increases.

- For small ω and large δ_{21}, asymmetry is appreciable only in the core and is a sensitive function of δ_{21}.

For non–zero ϕ a fraction of the solar neutrinos come out of the sun as ν_τ and these ν_τ cannot reconvert back to ν_e inside earth because ϕ is not affected by matter (see eqn.(16)). So a non zero ϕ dilutes the asymmetry. Our numerical results include the effect of any adiabatic MSW resonances that may occur inside the earth. For $\phi = 0$, as pointed out recently [8,9], MSW resonances do occur in the earth's core. However, for large ϕ they disappear and this is another reason for the regeneration efect to be smaller for large ϕ, in the core.

In Fig. 3 we have chosen a few parameter sets for which A is very small (< 0.15) since they are possible solutions to the solar and atmospheric neutrino problems [11,15,19]. But we cannot rigorously exclude other values of the neutrino parameters at the present stage of knowledge. Day–night effect must be studied in an unbiased manner, especially because the ratio A is relatively independent of the uncertainties of the solar models.

After this work was completed, we came to know of a related work: E. Lisi and D. Montanino, hep-ph/9702343. They have also stressed the importance of an analytic approach, however their method is different from ours and they have ignored the third flavor.

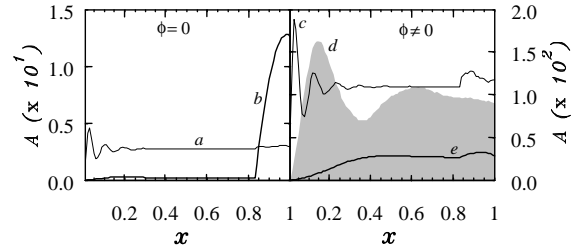We thank M. C. Sinha, M. V. N. Murthy and S. Uma Sankar for discussions.

FIG. 3. A as a function of x, for (a) $\delta_{21} = 3 \times 10^{-5} \ eV^2$, $\omega = 28^o$, $\phi = 0$; (b) $\delta_{21} = 6 \times 10^{-6}$, $\omega = 2.5^o$, $\phi = 0$; (c) $\delta_{21} = 2.5 \times 10^{-5}$, $\omega = 20^o$, $\phi = 32^o$; (d) $\delta_{21} = 6.3 \times 10^{-5}$, $\omega = 18^o$, $\phi = 45^o$; (e) $\delta_{21} = 2.5 \times 10^{-6}$, $\omega = 6^o$, $\phi = 50^o$.

[2] Kamiokande Collaboration: Y.Fukuda *et al*,Phys. Rev. Lett. **77**, 1683 (1996).

[3] Y.Totsuka, Nucl. Phys.(Proc. Suppl.) **B48**, 547 (1996); A. Suzuki, in *Physics and Astrophysics of Neutrinos*, edited by M. Fukugita and A. Suzuki (Springer–Verlag, Tokyo, 1994), page 414.

[4] G.T.Ewan *et al*,SNO Collabaration, "Sudbury Neutrino Observatory Proposal", Report No. SNO-87-12, 1987 (unpublished).

[5] R.S. Raghavan, in *Proceedings of the 25th International Conference on High Energy Physics*, Singapore, 1990, edited by K.K. Phua and Y. Yamaguchi (World Scientific, Singapore, (1991), p. 698.

[6] For earlier work, see [7–9] and references therein.

[7] M. L. Cherry and K .Lande, Phys. Rev, **D36**, 3571 (1987)

[8] A. J. Baltz and J. Weneser, Phys. Rev, **D35**, 528 (1987); *ibid*,**D50**,5971 (1994); *ibid*, **D51**, 3960 (1994).

[9] J. M. Gelb, W. Kwong, and S. P. Rosen, hep-ph/9612332.

[10] Stephen Parke, Phys. Rev. Lett. **57**, 1275 (1986).

[11] M. Narayan, M. V. N. Murthy, G. Rajasekaran and S. Uma Sankar, Phys. Rev. **D53**, 2809 (1996),*ibid.* **D56**, 437 (1997).

[12] We have taken the densities from F. D. Stacey, *Physics Of the Earth*, John Wiley & Sons, Inc., New York. (1977).

[13] T. K. Kuo and J. Pantaleone, Rev. Mod. Phys. **61**, 937 (1989).

[14] A. S. Joshipura and M. V. N. Murthy, Phys. Rev. **D37**, 1374 (1988)

[15] G.L.Fogli, E. Lisi and D. Montanino, Phys. Rev. **D49**, 3626 (1994);*ibid.* **D54**, 2048 (1996).

[16] S.P.Mikheyev and A.Yu. Smirnov, in *New and Exotic Phenomena*, Proceedings of the Moriond Workshop, Les Arc Savoie, France, 1987, edited by O.Fackler and J. Tran Thanh Van (Editions Frontie´res, Gif-Sur-Yvette, France,1987),p. 405.

[17] J.Bahcall, *Neutrino Astrophysics*, Cambridge University Press,New York, (1989)

[18] J. N. Bahcall and M. H. Pinsonneault, Rev. Mod.Phys. **64**,885 (1992).

[19] N. Hata and P. Langacker, Phys. Rev. **D52**, 420 (1995).

[1] Kamiokande Collaboration: K.S.Hirata *et al*,Phys. Rev. **D 44**, 2241 (1991).