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Bi-large neutrino mixings by radiative magnification

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Abstract. Starting with the unification hypothesis of mixings of quarks and leptons and small quark-like mixings at the see-saw scale, we find that two large mixings for $\nu_e - \nu_\mu$ and $\nu_\mu - \nu_\tau$ at the weak scale are obtained as a result of renormalization group evolution and radiative magnification if the three neutrinos are quasi degenerate in masses and possess the same CP parity. We also find that U_{e3} remains small and well within the CHOOZ-Palo Verde bound since the corresponding V_{ub} for CKM mixing is very small. Several testable pedictions are pointed out.

Keywords. Unification of mixings; radiative magnification.

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Recent experimental measurements suggest that the mixings between $\nu_e - \nu_\mu$ and $\nu_\mu - \nu_\tau$ are large while the mixing angle between $\nu_e - \nu_\tau$ is bounded to be small with $\sin^2 2\theta_{13} < 0.15$ [1]. The existence of two large neutrino mixings is one of the biggest mysteries in particle physics since the corresponding mixings among the quarks are small. If quarks and leptons are unified [2], it is expected that the neutrino mixings would also be small and may be unified with quark mixings at high scales. Enhancement of a mixing angle between two neutrinos through RG evolution has been noted earlier [3]. It has been shown recently that if two Majorana neutrinos are quasidegenerate in masses and possess the same CP parity, radiative magnification of their small mixing at high scale leading to nearly maximal value at the weak scale is caused purely due to RG evolution [4].

In this paper, using MSSM, we show that simple radiative effects through RG evolutions can indeed provide a complete understanding of all the three neutrino mixings starting with small quark-like mixings at the see-saw scale. As a result diverse values of quark and neutrino mixings unify at the GUT see-saw scale.

We follow diagonalise and run procedure and use RGEs directly for the three neutrino mass eigenvalues m_i $(i = 1, 2, 3)$ and the sines of mixing angles s_{12}, s_{23} and s_{13} defined through the 3×3 real mixing matrix [5],

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}s_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{bmatrix}, \qquad (1)$$

where $c_{ij} = \cos\theta_{ij}$ $(i, j = 1, 2, 3)$, $D_{ij} = (m_i + m_j)/(m_i - m_j)$ $(i \neq j)$,

$$\frac{\mathrm{d}m_i}{\mathrm{d}t} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u \ (i = 1, 2, 3), \qquad (2)$$

$$\frac{\mathrm{d}s_{23}}{\mathrm{d}t} = -F_\tau c_{23}^2 \left(-s_{12}U_{\tau 1}D_{31} + c_{12}U_{\tau 2}D_{32}\right), \qquad (3)$$

$$\frac{\mathrm{d}s_{13}}{\mathrm{d}t} = -F_\tau C_{23}c_{13}^2 \left(c_{12}U_{\tau 1}D_{31} + s_{12}U_{\tau 2}D_{32}\right), \qquad (4)$$

$$\frac{\mathrm{d}s_{12}}{\mathrm{d}t} = -F_\tau C_{12} \left(c_{23}s_{13}s_{12}U_{\tau 1}D_{31} - c_{23}s_{13}c_{12}U_{\tau 2}D_{32} + U_{\tau 1}U_{\tau 2}D_{21}\right), \qquad (5)$$

$$F_\tau = \frac{-h_\tau^2}{(16\pi^2\cos^2\beta)}, \quad F_u = \left(\frac{1}{16\pi^2}\right)\left(\frac{6}{5}g_1^2 + 6g_2^2 - \frac{6h_t^2}{\sin^2\beta}\right). \qquad (6)$$

Since we are interested in radiative magnifications of s_{12} and s_{23} we assume the three neutrinos to be quasidegenerate with same CP parity at the see-saw scale. The effect of a Dirac phase in (1) will be discussed in a separate paper.

The hypothesis of unification of mixings at the see-saw scale ($M_R = 10^{13}$ GeV) demands $s_{12} \simeq \lambda_0 = 0.2$ as in Wolfenstein parametrization, $s_{23} \simeq O(\lambda_0^2)$ and $s_{13} \simeq O(\lambda_0^3)$. Using $|D_{31}| \simeq |D_{32}| \ll |D_{21}|$, we note from (3)–(5) that the dominant contribution to RG evolution in $s_{12}(\mu)$ is due to the term $\sim\lambda_0^2 F_\tau D_{32}$ and to $s_{13}(\mu)$ is due to $\sim\lambda_0^3 F_\tau D_{32}$ or $\sim\lambda_0^3 F_\tau D_{31}$. The corresponding dominant contribution to $s_{12}(\mu)$ is $\sim\lambda_0^5 F_\tau D_{21}$ where the large enhancement due to D_{21} is damped out by the higher power in λ_0^5. Since the mixing angles change substantially around M_{SUSY}, such dominance to RG evolutions hold approximately at all other scales below M_R.

Assuming the neutrino mixings at M_R to be similar to quark mixings, we expect the initial conditions at $\mu = M_R$ to be $\sin\theta_{12}^0 \simeq 0.20$, $\sin\theta_{23}^0 \simeq 0.035$, and $\sin\theta_{13}^0 \simeq 0.0025$. The mass eigenvalues m_i^0 at M_R are treated as unknown parameters and are determined in such a way that the top-down approach to the RG evolutions matches with the experimental data within 90% CL with $\Delta m_{12}^2 = (2\text{–}50) \times 10^{-5}$ eV^2, $\Delta m_{23}^2 = (1.2\text{–}5) \times 10^{-3}$ eV^2, $s_{23} = 0.54\text{–}0.83$, $s_{12} = 0.40\text{–}0.7$, and $s_{13} < 0.16$.

In figure 1 we present RG evolutions of the three mixing angles from $M_R = 10^{13}$ to M_Z with $M_{\mathrm{SUSY}} = 1$ TeV, $\tan\beta \simeq 55$, $m_1^0 = 0.2983$ eV, $m_2^0 = 0.2997$ eV, and $m_3^0 = 0.3383$ eV leading to their weak scale values $m_1 = 0.2410$ eV, $m_2 = 0.2411$ eV, and $m_3 = 0.2435$ eV, $\Delta m_{12}^2 = 4.8 \times 10^{-5}$ eV^2, $\Delta m_{23}^2 = 1.1 \times 10^{-3}$ eV^2, $s_{23} = 0.680$, $s_{12} = 0.568$, and $s_{13} = 0.080$. We note that radiative magnification to bi-large mixings takes place over a wider range of mass eigenvalues while keeping $s_{13} = 0.08\text{–}0.10$ establishing our hypothesis of high-scale unification of quark and neutrino mixings provided the three neutrinos are quasidegenerate in masses and possess the same CP parity. Although here the flatness of curves below M_{SUSY} is due to negligible RG corrections in the presence of SM, we have also found that similar magnifications takes place with nearly maximal values of s_{12} and s_{23} at M_Z in the presence of non-SUSY 2HDM below M_{SUSY}.

It is interesting to note that the radiative magnification to bi-large mixings takes place in agreement with low-energy data provided 0.15 eV$\leq m_i(M_Z) \leq 0.65$ eV

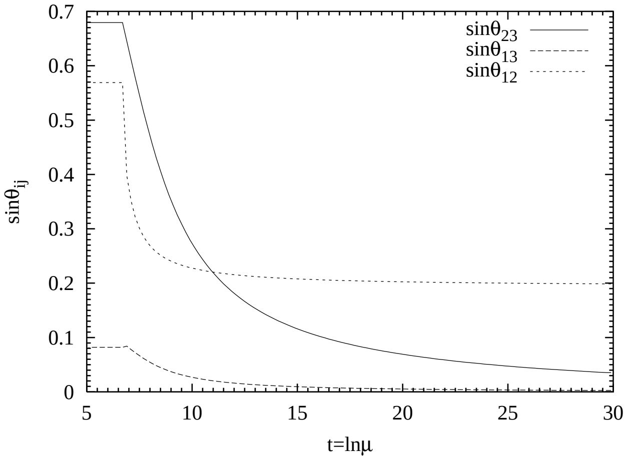

Figure 1. RG evolutions of neutrino mixings from small quark-like mixings at the see-saw scale (10^{13} GeV) leading to bi-large $\nu_e - \nu_\mu$ and $\nu_\mu - \nu_\tau$ with $U_{e3} \simeq 0.08$ at the weak scale.

leading to $|\langle M_{ee}\rangle| = 0.15$–$0.65$ eV. Thus the mixing unification hypothesis predicts observation of neutrinoless double beta decay in the next round of experiments and neutrino mass measurement through tritium beta decay by KATRIN experiment [6]. Although s_{13} is also magnified, it remains small at the weak scale. The high scale input in the range $V_{ub}^0 \simeq U_{e3}^0 \simeq 0.0025$–$0.004$ gives the low-energy prediction $U_{e3} = 0.08$–0.10 which is well within the CHOOZ-Palo Verde bound but accessible to several long baseline neutrino experiments. The allowed range of neutrino mass eigenvalues partly overlap with WMAP and also combined analyses of the WMAP and 2dF GRS data [7]. Such quasidegenerate neutrino masses [8] could be generated through type-II see-saw mechanism [9].

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References

[1] J N Bahcall *et al*, hep-ph/0212146
S Goswami, hep-ph/0303075
[2] J C Pati and A Salam, *Phys. Rev.* **D10**, 275 (1974)
[3] K S Babu *et al*, *Phys. Lett.* **B319**, 191 (1993)
P H Chankowshi *et al*, *Phys. Lett.* **B316**, 312 (1993)
[4] K R S Balaji, A S Dighe, R N Mohapatra and M K Parida, *Phys. Lett.* **B481**, 33 (2000); *Phys. Rev. Lett.* **84**, 5034 (2000)
K R S Balaji, R N Mohapatra, M K Parida and E A Paschos, *Phys. Rev.* **D63**, 113002 (2001)

[5] P H Chankowski *et al*, hep-ph/9910231
J A Casas *et al*, hep-ph/9910420
[6] H V Klapdor-Kleingrothaus *et al*, hep-ph/0201231
O Cremonesi, hep-ex/0210007
A Osipowicz *et al*, hep-ex/0109033
[7] C L Bennet *et al*, astro-ph/0302207
D N Spergel *et al*, astro-ph/0302209
S Hannestad, astro-ph/0303076
A Pierce and H Murayama, hep-ph/0302131
[8] For some earlier models of degenerate neutrinos, see D O Caldwell and R N Mohapatra, *Phys. Rev.* **D48**, 3259 (1993)
D G Lee and R N Mohapatra, *Phys. Lett.* **B329**, 463 (1994)
P Bamert and C Burgess, *Phys. Lett.* **B329**, 109 (1994)
E Ma and G Rajasekaran, hep-ph/0106291
A S Joshipura, *Z. Phys.* **C 64**, 31 (1994)
K S Babu, E Ma and J W F Valle, hep-ph/0206292
[9] R N Mohapatra, M K Parida and G Rajasekaran, hep-ph/0301234