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The electroweak mixing angle in unified gauge theories—the renormalisation effects

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Abstract. A general analysis of the renormalisation corrections to the unification results for the coupling constants of strong and electroweak interactions is attempted. In particular, the effects of introducing an energy scale intermediate between the unification energy and the low-energy regions are studied and found to be important. This analysis is applied to unification schemes of both kinds, namely, unification at superhigh energies, and unification at accessible energies.

Keywords. Electroweak mixing angle; neutral-currents; grand unified gauge theories, renormalisation group equations; energy-dependent coupling constants.

1. Introduction

Unification fixes the ratios of the coupling constants of the strong and electroweak interactions. However, these algebraic results of unification are valid only at the unification energies, i.e., energies which are large as compared to the masses involved in the breaking of the unification group G to the level of the observed group G_0. To obtain results valid at lower energies, from these unification results, one uses renormalisation group equations, which govern the behaviour of the effective coupling constants with respect to the energy scale, μ (Georgi *et al* 1974).

The renormalisation effects depend on the unification energy M. It is useful to distinguish two approaches to unification which differ vastly in the value of M. In the first approach, which seems to be the more popular one (Georgi and Glashow 1973; Georgi *et al* 1974), proton decay occurs in the first order of four-fermion coupling and to keep the decay at a tolerably low level, the mediating bosons have to be made superheavy. This forces the unification energy to be of the order of 10^{16} GeV. According to this scheme, therefore, unification occurs only at fantastically high energies.

In the second approach to unification, in addition to the unified gauge group G, there are unbroken global symmetries and hence there are additional quantum numbers which are exactly conserved. In such models (Pati and Salam 1973; Fritzsch and Minkowski 1975), the proton decay can be either strictly forbidden, or pushed to higher orders in the four-fermion coupling and so unification can be allowed to set in at energies as low as 10^3 GeV. Unification in this scheme is already at hand.

In this paper, we analyse the renormalisation effects for both the above approaches to unification, namely unification at superhigh energies and unification at accessible energies. Renormalisation effects are usually calculated by assuming the existence of two distinct regions in the energy scale, one below M and the other above M, the energy-dependence of the coupling constants suffering a sharp discontinuity at the interface M. This is an unpalatable assumption for more than one reason. This assumption will clearly fail if masses of gauge bosons, Higgs bosons and fermions in the theory are distributed all over the energy region extending from the lowest to the unification energies. Even if the existence of a desert with no particles over a certain energy region is accepted, the assumption of a sharp discontinuity in the energy dependence of the coupling constants remains questionable, especially in the case of unification at accessible energies. In the latter case, even a distribution of particle masses over a region of width about 100 GeV, centred at M, will give rise to a significant transition region. Mass differences of the order of 100 GeV are, of course, the minimum that are expected, this being the mass difference within the Weinberg-Salam gauge multiplet.

Therefore, we attempt a simple generalisation of the usual analysis of renormalisation effects by replacing the sharp discontinuity by an intermediate transition region. This generalisation is, of course, only illustrative, however it allows us to locate the approximations involved in the Georgi-Quinn-Weinberg type of analysis, which is relevant for unification at superhigh energies. Further, we find that for unification at accessible energies omission of the transition region leads to inconsistencies. The introduction of this transition region involves new parameters which cannot be determined until the mass-spectrum of the unified gauge theory is known. This makes all the claimed unification results suspect.

We set up the equations in a sufficiently general fashion so as to cover the wide class of unified models studied earlier (Bajaj and Rajasekaran 1979a, to be referred to as paper I in the text). In particular, we discuss the renormalisation effects for both the standard and the left-right symmetric models.

The paper is organised as follows. The general equations are set up in § 2. These equations are then applied to unification at superhigh energies and at accessible energies in §§ 3 and 4, respectively. The last section is devoted to a discussion of the results.

2. General analysis of the energy-dependence of the coupling constants

We divide the energy or mass scale μ into three regions, defined by the way in which the gauge bosons contribute to the μ-dependence of the coupling constants. In the low-μ region called region I, only the gauge bosons of the observed group G_0 contribute. This is followed by a transition region (region II) in which the heavier gauge bosons start contributing, but not with full strength. Then comes the unification region of high-μ, to be called region III, in which all the gauge bosons of the unified group G contribute. For convenience we shall take μ_1 as defining the boundary between regions I and II, while μ_2 defines the boundary between II and III. We define*

*Where undefined, the notation is understood to be the same as in paper I.

$$\alpha_S = g_S^2/4\pi; \quad \alpha = e^2/4\pi; \quad \alpha_L = g_L^2/4\pi; \quad \alpha_Y = g_Y^2/4\pi;$$

$$t = \ln(\mu/10 \text{ GeV}), \quad t_i = \ln(\mu_i/10 \text{ GeV}), \; i = 1, 2.$$

In region I, ($t < t_1$), the renormalisation group equations at the one-loop level (Gross and Wilczek 1973; Politzer 1973) give the following t-dependence for the effective coupling constants:—

$$\frac{1}{\alpha_S(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_C^2} = c_S + 2(b_S + b_F)\, t, \tag{1a}$$

$$\frac{1}{\alpha_L(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_L^2} = c_L + 2(b_L + b_F)\, t, \tag{1b}$$

$$\frac{1}{\alpha_Y(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_Y^2} = c_Y + 2(b_Y + b_F)\, t. \tag{1c}$$

Here c_S, c_L and c_Y are constants to be determined, b_S and b_L are contributions of the gauge bosons of the nonabelian groups $SU(3)_C$ and $SU(2)_L$ given by

$$b_S = \frac{11}{4\pi} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_C^2}, \tag{2}$$

$$b_L = \frac{11}{6\pi} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_L^2}, \tag{3}$$

$b_Y = 0$, and b_F is the contribution of the fermions and Higgs bosons. We assume that the masses of all the fermions and all the Higgs are $\lesssim \mu_1$, so that these contribute uniformly to the three coupling constants. Note b_F is negative.

In region III, ($t > t_2$), we have the unified behaviour:

$$\frac{1}{\alpha_S(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_C^2} = \frac{1}{\alpha_L(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_L^2} = \frac{1}{\alpha_Y(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_Y^2} = c_G + 2(b_G + b_F)\, t, \tag{4}$$

where b_G is the universal contribution from the whole gauge multiplet of the unified group G and c_G is a constant. For a large unification group G, b_G is very large compared to b_S and b_L.

The gauge-boson contribution to the t-dependence in region II is difficult to pin down. The simplest approximation is to take a linear t-dependence for this region too, with coefficients $\tilde{b}_S$, $\tilde{b}_L$ and $\tilde{b}_Y$ whose values lie between their corresponding values in regions I and III. Then, for $t_1 < t < t_2$, we can write

$$\frac{1}{\alpha_S(t)} \frac{\mathrm{Tr}\, T_G^2}{\mathrm{Tr}\, T_C^2} = \tilde{c}_S + 2(\tilde{b}_S + b_F)\, t, \tag{5a}$$

$$\frac{1}{\alpha_L(t)}\frac{\text{Tr } T_G^2}{\text{Tr } T_L^2} = \tilde{c}_L + 2(\tilde{b}_L + b_F)\, t, \tag{5b}$$

$$\frac{1}{\alpha_Y(t)}\frac{\text{Tr } T_G^2}{\text{Tr } T_Y^2} = \tilde{c}_Y + 2(\tilde{b}_Y + b_F)\, t. \tag{5c}$$

By matching at $t=t_2$ and at $t=t_1$, we can determine all the constants, $\tilde{c}_S$, $\tilde{c}_L$, $\tilde{c}_Y$; c_S, c_L and c_Y, in terms of the universal constant c_G. We thus get the complete t-dependence equations for all the regions:

$$\begin{aligned}\frac{1}{\alpha_S(t)}\frac{\text{Tr } T_G^2}{\text{Tr } T_C^2} &= c_G + 2b_G t_2 + 2\tilde{b}_S(t_1 - t_2) + 2b_S(t - t_1) + 2b_F t, \text{ for } t < t_1;\\ &= c_G + 2b_G t_2 + 2\tilde{b}_S(t - t_2) + 2b_F t, \text{ for } t_1 < t < t_2;\\ &= c_G + 2(b_G + b_F)t, \text{ for } t > t_2;\end{aligned} \tag{6a}$$

$$\begin{aligned}\frac{1}{\alpha_L(t)}\frac{\text{Tr } T_G^2}{\text{Tr } T_L^2} &= c_G + 2b_G t_2 + 2\tilde{b}_L(t_1 - t_2) + 2b_L(t - t_1) + 2b_F t, \text{ for } t < t_1;\\ &= c_G + 2b_G t_2 + 2\tilde{b}_L(t - t_2) + 2b_F t, \text{ for } t_1 < t < t_2;\\ &= c_G + 2(b_G + b_F)t, \text{ for } t < t_2;\end{aligned} \tag{6b}$$

$$\begin{aligned}\frac{1}{\alpha_Y(t)}\frac{\text{Tr } T_G^2}{\text{Tr } T_Y^2} &= c_G + 2b_G t_2 + 2\tilde{b}_Y(t_1 - t_2) + 2b_Y(t - t_1) + 2b_F t, \text{ for } t < t_1;\\ &= c_G + 2b_G t_2 + 2\tilde{b}_Y(t - t_2) + 2b_F t, \text{ for } t_1 < t < t_2;\\ &= c_G + 2(b_G + b_F)t, \text{ for } t > t_2.\end{aligned} \tag{6c}$$

By subtraction it is possible to recast these equations into the following equations which are independent of many of the unknown constants:

$$\begin{aligned}\frac{1}{\alpha_S(t)} - \frac{1}{\alpha_L(t)}\frac{\text{Tr } T_C^2}{\text{Tr } T_L^2} &= 2\left(\tilde{d}_S - \tilde{d}_L\frac{\text{Tr } T_C^2}{\text{Tr } T_L^2}\right)(t_1 - t_2)\\ &\quad + 2\left(d_S - d_L\frac{\text{Tr } T_C^2}{\text{Tr } T_L^2}\right)(t - t_1), \text{ for } t < t_1;\\ &= 2\left(\tilde{d}_S - \tilde{d}_L\frac{\text{Tr } T_C^2}{\text{Tr } T_L^2}\right)(t - t_2), \text{ for } t_1 < t < t_2;\\ &= 0, \text{ for } t > t_2;\end{aligned} \tag{7a}$$

$$\frac{1}{\alpha_Y(t)} - \frac{1}{\alpha_L(t)} \frac{\mathrm{Tr}\, T_Y^2}{\mathrm{Tr}\, T_L^2} = 2\left(\tilde{d}_Y - \tilde{d}_L \frac{\mathrm{Tr}\, T_Y^2}{\mathrm{Tr}\, T_L^2}\right)(t_1 - t_2)$$

$$+ 2\left(d_Y - d_L \frac{\mathrm{Tr}\, T_Y^2}{\mathrm{Tr}\, T_L^2}\right)(t - t_1), \text{ for } t < t_1;$$

$$= 2\left(\tilde{d}_Y - \tilde{d}_L \frac{\mathrm{Tr}\, T_Y^2}{\mathrm{Tr}\, T_L^2}\right)(t - t_2), \text{ for } t_1 < t < t_2;$$

$$= 0, \text{ for } t > t_2; \tag{7b}$$

where we have defined the " reduced coefficients "

$$d_S = b_S \frac{\mathrm{Tr}\, T_C^2}{\mathrm{Tr}\, T_G^2}, \quad d_L = b_L \frac{\mathrm{Tr}\, T_L^2}{\mathrm{Tr}\, T_G^2}, \quad d_Y = b_Y \frac{\mathrm{Tr}\, T_Y^2}{\mathrm{Tr}\, T_G^2}; \tag{8}$$

and exactly similar equations for $\tilde{d}_S$, $\tilde{d}_L$ and $\tilde{d}_Y$. We have

$$d_S = \frac{11}{4\pi}, \quad d_L = \frac{11}{6\pi}, \quad d_Y = 0. \tag{9}$$

Equations (7a) and (7b) provide us only with the relative t-dependence between the various coupling constants, but this alone is of interest to us, for the present.

These equations, so far, are common to both the standard as well as the left-right symmetric models. However, when they are converted into equations for $\sin^2\theta(t)$ and $\alpha(t)/\alpha_S(t)$, we shall get different results for the two models.

We call the values of $\sin^2\theta$ and α/α_S in the unification limit, given by equation (10) of paper I, as S and R respectively. For the standard model, we have

$$1/\alpha_L(t) = [1/\alpha(t)]\,[\sin^2\theta(t)], \tag{10}$$

$$1/\alpha_Y(t) = [1/\alpha(t)]\,[\cos^2\theta(t)]; \tag{11}$$

$$\mathrm{Tr}\, T_Y^2/\mathrm{Tr}\, T_L^2 = (1-S)/S, \quad \mathrm{Tr}\, T_C^2/\mathrm{Tr}\, T_L^2 = R/S. \tag{12}$$

We then get, from simple manipulation of equation (7a) and (7b),

$$\sin^2\theta(t) = S + 2\,\alpha(t)\,K(t), \tag{13}$$

$$\alpha(t)/\alpha_S(t) = R + 2\,\alpha(t)\,L(t), \tag{14}$$

where
$$K(t) = \{\tilde{d}_L - S(\tilde{d}_L + \tilde{d}_Y)\}(t_1 - t_2)$$

$$+ \{d_L - S(d_L + d_Y)\}(t - t_1), \quad \text{for } t < t_1;$$

$$= \{\tilde{d}_L - S(\tilde{d}_L + \tilde{d}_Y)\}(t - t_2), \quad \text{for } t_1 < t < t_2;$$

$$= 0, \quad \text{for } t > t_2; \tag{15}$$

$$\begin{aligned} L(t) &= \{\tilde{d}_S - R(\tilde{d}_L + \tilde{d}_Y)\}\,(t_1 - t_2) \\ &\quad + \{d_S - R(d_L + d_Y)\}\,(t - t_1), \text{ for } t < t_1\,; \\ &= \{\tilde{d}_S - R(\tilde{d}_L + \tilde{d}_Y)\}\,(t - t_2), \text{ for } t_1 < t < t_2\,; \\ &= 0, \text{ for } t > t_2\,. \end{aligned} \tag{16}$$

For the L-R model, we have

$$1/\alpha'_L(t) = [1/\alpha'(t)]\,[\sin^2\theta'(t)], \tag{17}$$

$$1/\alpha'_Y(t) = [1/\alpha'(t)]\,[1-2\sin^2\theta'(t)]; \tag{18}$$

$$\mathrm{Tr}\,T'^2_Y/\mathrm{Tr}\,T'^2_L = (1-2S)/S\,;\ \mathrm{Tr}\,T'^2_C/\mathrm{Tr}\,T'^2_L = R/S. \tag{19}$$

The values of $\sin^2\theta'$ and α'/α'_S in the unification limit are same as in W-S model and hence, are still denoted by S and R only. Using these, we now get

$$\sin^2\theta'(t) = S + 2\,\alpha'(t)\,K'(t), \tag{20}$$

$$\alpha'(t)/\alpha'_S(t) = R + 2\,\alpha'(t)\,L'(t), \tag{21}$$

where

$$\begin{aligned} K'(t) &= \{\tilde{d}_L - S\,(2\tilde{d}_L + \tilde{d}_Y)\}\,(t_1 - t_2) \\ &\quad + \{d_L - S\,(2d_L + d_Y)\}\,(t - t_1), \text{ for } t < t_1; \\ &= \{\tilde{d}_L - S\,(2\tilde{d}_L + \tilde{d}_Y)\}\,(t - t_1), \text{ for } t_1 < t < t_2; \\ &= 0, \quad \text{for } t > t_2; \end{aligned} \tag{22}$$

$$\begin{aligned} L'(t) &= \{\tilde{d}_S - R\,(2\tilde{d}_L + \tilde{d}_Y)\}\,(t_1 - t_2) \\ &\quad + \{d_S - R\,(2d_L + d_Y)\}\,(t - t_1), \text{ for } t < t_1\,; \\ &= \{\tilde{d}_S - R\,(2\tilde{d}_L + \tilde{d}_Y)\}\,(t - t_2), \text{ for } t_1 < t < t_2\,; \\ &= 0, \quad \text{for } t > t_2\,. \end{aligned} \tag{23}$$

We see that the only difference between the t-dependence equations for the two models is that $d_L + d_Y$ in the standard model equations is replaced by $2d_L + d_Y$ for the left-right symmetric model equations. This arises from the difference in the formulae for the fine-structure constant in the two models:

$$1/\alpha = (1/\alpha_L) + (1/\alpha_Y) \quad \text{(standard model)}, \tag{24}$$

$$1/\alpha' = (2/\alpha'_L) + (1/\alpha'_Y) \quad \text{(left-right symmetric model)}. \tag{25}$$

We shall now apply the general equations derived here to both unification schemes, namely unification at superhigh energies as well as unification at accessible energies.

3. Unification at superhigh energies

For unification at superhigh energies, we may take both μ_1 and μ_2 to be of the order of 10^{16} GeV, so that

$$t_1 \approx t_2 \approx t_M = \ln (M/10 \text{ GeV}),$$

where $M \approx 10^{16}$ GeV. So, among the 3 regions which we had defined, region I ($t < t_1$) alone is of physical importance at present and we have to write down the formulae only for this region. Further, if we ignore the terms $\tilde{d}_S\,(t_2-t_1)$, $\tilde{d}_L\,(t_2-t_1)$ and $\tilde{d}_Y\,(t_2-t_1)$ in these equations, the analysis simplifies considerably.

Thus, the standard-model equations (13) – (16) become

$$\sin^2\theta\,(t) = S - \frac{11}{3\pi}\,\alpha\,(t)\,(1-S)\,(t_M - t), \tag{26}$$

$$\frac{\alpha(t)}{\alpha_S(t)} = R - \frac{11}{6\pi}\,\alpha(t)\,(3-2R)\,(t_M - t)\,, \tag{27}$$

where we have used the numerical values of d_S, d_L and d_Y given by (9). Since $(t_M - t)$ is large, the renormalisation corrections to both (26) and (27) are sizable. It is also convenient to write down the relation between the observables at t obtained by elimination of the unknown (t_M-t) between the two equations (26) and (27). Then we get:

$$\sin^2\theta\,(t) = \frac{3S-2R}{3-2R} + \frac{2(1-S)}{3-2R}\,\frac{\alpha(t)}{\alpha_S(t)}. \tag{28}$$

For low values of t where the strong coupling constant $\alpha_S(t)$ reaches such large values that the second term can be ignored, one has

$$\sin^2\theta\,(0) \approx (3S-2R)/(3-2R). \tag{29}$$

The corresponding equations for the left-right symmetric model (obtained from (20)–(23)) are:

$$\sin^2\theta'\,(t) = S - \frac{11}{3\pi}\,\alpha'\,(t)\,(1-2S)\,(t_M - t), \tag{30}$$

$$\alpha'(t)/\alpha'_S\,(t) = R - \frac{11}{6\pi}\,\alpha'\,(t)\,(3-4R)\,(t_M - t), \tag{31}$$

and

$$\sin^2\theta'\,(t) = \frac{3S-2R}{3-4R} + \frac{2\,(1-2S)}{3-4R}\,\frac{\alpha'\,(t)}{\alpha'_S\,(t)}, \tag{32}$$

$$\sin^2\theta'\,(0) \approx (3S-2R)/(3-4R). \tag{33}$$

Formulae of this type were first written down by Georgi *et al* (1974) and most of the recent papers on grand unified models are based on such formulae. Of course, these could have been derived much more directly, but our purpose in arriving at them this way is to draw attention to the approximations involved in their derivation. Basically, the approximation is to ignore the existence of the transition region. By referring to (15) and (16), we can see that the validity of this approximation requires

$$\{\tilde{d}_L-S(\tilde{d}_L+\tilde{d}_Y)\}\ (t_1-t_2) \ll \{d_L-S(d_L+d_Y)\}\ (t-t_1),$$

and $$\{\tilde{d}_S-R(\tilde{d}_L+\tilde{d}_Y)\}\ (t_1-t_2) \ll \{d_S-R(d_L+d_Y)\}\ (t-t_1);$$

and similar inequalities for the left-right symmetric model. Although (t_1-t_2) may be expected to be small as compared to $(t-t_1)$, how does one know that $\tilde{d}_S$, $\tilde{d}_L$, etc, are not large as compared to d_S, d_L, etc.? We do not have an answer to this question. So, it is good to keep this rather dubious approximation in mind, when assessing the accuracy of the numerical parameters derived from the above formulae.

Now, we may remark on the physical content of the above formulae. The most important message of the above equations is that although we start with the same unification value of $\sin^2\theta$ (namely S) for the standard and left-right symmetric models, the value of $\sin^2\theta(t)$ relevant at low energies are different in the two models. The same is true of α/α_S also.

We should also point out that the formulae (26)–(29) are valid for any unified group G which reduces to $G_0=SU(3)_C\times SU(2)_L\times U(1)$ at low energies while (30)–(33) are valid for any G which leads to $G_0'=SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)$. The corresponding formulae given by Georgi *et al* (1974) and Chanowitz *et al* (1977) are valid only if the two parameters R and S are equal. It turns out that R and S are equal for $G=SU(5)$ or $SO(10)$, but this is not true in general.

On using (29) and (33) for the sequential doublet scheme of paper I for which $S=R=3/8$, we get

$$\sin^2\theta\,(0) \approx \tfrac{1}{6} \text{ and } \sin^2\theta'(0) \approx \tfrac{1}{4}.$$

These numbers are in reasonable agreement with the respective phenomenologically determined values: $\sin^2\theta=0\cdot23\pm0\cdot01$ (Musset 1979) and $\sin^2\theta'=0\cdot28\pm0\cdot09$* (Bajaj and Rajasekaran 1979b). We may also remark that application of (29) and (33) to the sequential triplet scheme of paper I for which $S=\frac{1}{4}$ and $R=\frac{3}{8}$ will lead to

$$\sin^2\theta\,(0)\approx 0 \text{ and } \sin^2\theta'\,(0)\approx 0.$$

Clearly, this scheme is not viable for unification at superhigh energies.

4. Unification at accessible energies

For definiteness, we may take

$$\mu_1 = 100 \text{ GeV},\quad \mu_2 = 1000 \text{ GeV},$$

*A more careful analyis leads to $\sin^2\theta'=0\cdot25\pm0\cdot01$ (Bajaj and Rajasekaran, 1980).

so that $\quad t_1 = 2{\cdot}3, \quad t_2 = 4{\cdot}6.$ (34)

Now the analysis of the renormalisation effects is more complex, since the complete set of equations has to be used but the parameters $\tilde{d}_S$, $\tilde{d}_L$ and $\tilde{d}_Y$ are unknown.

It has been argued by Fritzsch and Minkowski (1975) that since the strong coupling constant $\alpha_S(t)$ reaches the unification value rather quickly, there are no sizable renormalisation corrections to the algebraic relations derived from unification. In fact, they claim

$$\sin^2 \theta\,(0) \approx S + O\,(\alpha).$$

It is easy to see from our formulae that this claim is unjustified. We get from (13) and (20)

$$\sin^2 \theta\,(0) = S + 2\alpha\,(0)\ \{(\tilde{d}_L - S\tilde{d})\,(t_1 - t_2) - (d_L - Sd)\,t_1\}, \tag{35}$$

where $\quad d = d_L + d_Y$ for Weinberg-Salam model,

$\quad\quad = 2d_L + d_Y$ for left-right symmetric model. (36)

and similarly for $\tilde{d}$. The coefficient of α (0) may not be small, since $\tilde{d}_L - S\tilde{d}$ can be quite large. The corresponding formula for $\alpha\,(0)/\alpha_S\,(0)$ is, from (14) and (22):

$$\alpha\,(0)/\alpha_S\,(0) = R + 2\alpha\,(0)\ \{(\tilde{d}_S - R\tilde{d})\,(t_1 - t_2) - (d_S - Rd)\,t_1\}. \tag{37}$$

It is clear that, all the coefficients $\tilde{d}_L$, $\tilde{d}$ and $\tilde{d}_S$ cannot be taken to be small. Otherwise, $\alpha(0)/\alpha_S(0)$ will not differ much from R, which will be in conflict with the empirically known result

$$\alpha\,(0)/\alpha_S\,(0) \approx 5/137. \tag{38}$$

In fact, equation (37) can be rewritten in the form:

$$2\{(\tilde{d}_S - R\tilde{d})\,(t_2 - t_1) + (d_S - Rd)\,t_1\} = \frac{R}{\alpha\,(0)} - \frac{1}{\alpha_S(0)}. \tag{39}$$

Since all the quantities in this equation are known except $\tilde{d}_S$ and $\tilde{d}$, it can be regarded as a constraint on $\tilde{d}_S$ and $\tilde{d}$. To be specific, let us put some representative numerical values (For R, we take the sequential-doublets result 3/8, (paper I)):

$$\frac{1}{\alpha_S(0)} = 5;\ \frac{R}{\alpha(0)} = \frac{3}{8}\Big/\left(\frac{1}{137}\right) \approx 50,$$

$$d_S = 1{\cdot}75;\ Rd = \tfrac{3}{8} \times 1{\cdot}18.$$

Then, (39) becomes

$$2\{(\tilde{d}_S - R\tilde{d})(t_2 - t_1) + (1{\cdot}75 - \tfrac{3}{8}\times 1{\cdot}18)\, t_1\} = 45. \tag{40}$$

For unification at superhigh energies, the first term can be ignored if we so desire; for, t_1 is large enough. But, for unification at accessible energies, the first term can *never* be ignored. Using the values of t_1 and t_2 given in (34) we get

$$\tilde{d}_S - \tfrac{3}{8}\, \tilde{d} \approx 10. \tag{41}$$

Thus, it is clear that region II can neither be identified with region I where this slope differential is of the order of unity, nor with region III where it should be zero. Further, since this slope-differential in the intermediate region is large, the slopes $\tilde{d}_S$ etc. should be individually large, thus implying that the unification group must be large. So, unification at accessible energies is possible only for rather large unification groups.

We do not claim that $\sin^2\theta$ (0) necessarily has a large deviation from S; for, we cannot prove that $\tilde{d}_L - S\tilde{d}$ is large. We can only prove that $\tilde{d}_S - R\tilde{d}$ has to be large. To emphasise the uncertainty in the renormalised value of $\sin^2\theta$, we plot the t-dependence of the inverses of the coupling constants for two arbitrary choices of parameters consistent with all the constraints discussed above (see figure 1). The first case (case (a)), corresponds to $\tilde{d}_L - S\tilde{d} \approx 10 \approx \tilde{d}_S - R\tilde{d}$. This choice leads to large

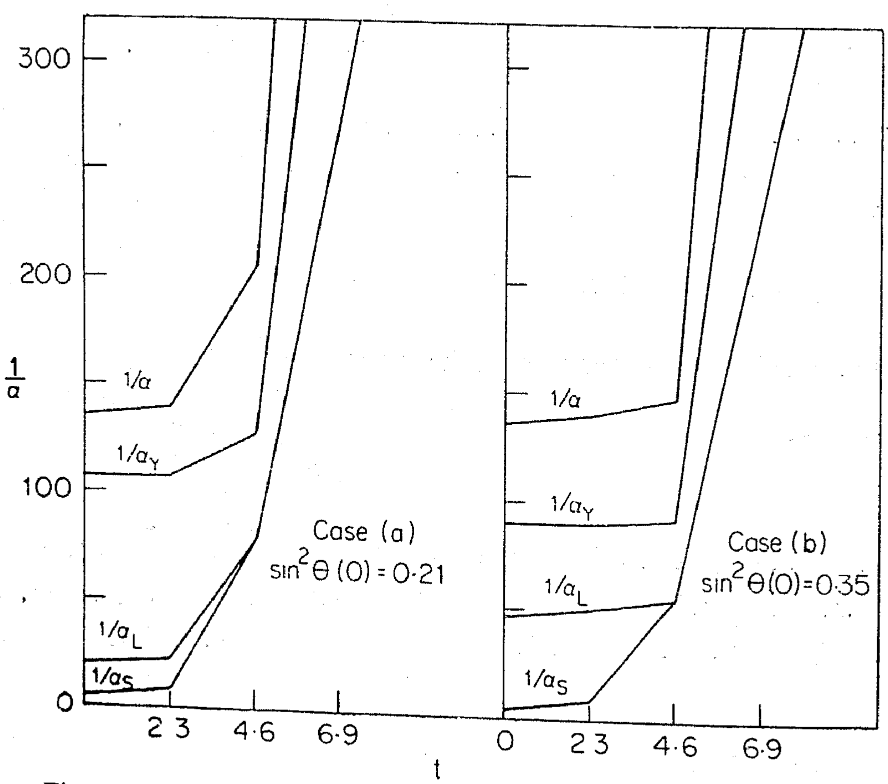

Figure 1. The t-dependence of the inverses of the coupling constants for unification at accessible energies in the doublets scheme. Case (a): Large renormalisation corrections to $\sin^2\theta$, Case (b): Small renormalisation corrections to $\sin^2\theta$.

renormalisation corrections for $\sin^2\theta$ and gives $\sin^2\theta(0)=0\cdot21$. In the second case (case (b)), $\tilde{d}_L - S\tilde{d} \approx 2 \ll \tilde{d}_S - R\tilde{d}$. In this case, the strong coupling constant suffers large renormalisation, but $\sin^2\theta$ does not, and we get $\sin^2\theta(0)=0\cdot35$. A similar numerical example could be worked out for the sequential triplets scheme also, where $S=\frac{1}{4}$. Because of the possibility of $\sin^2\theta$ suffering no large renormalisation effects, the triplets scheme may be viable in unified models in which unification sets in at accessible energies.

At the present stage of our ignorance of the actual mass-spectrum of the particles involved in the unified gauge theory, it is clearly impossible to decide in favour of either of the two possibilities (case (a) and (b)) discussed above. Nevertheless, one may note from the shape of the graphs in figure 1, that the second possibility (case (b)) seems a little unnatural in view of the abrupt change in the slopes of $1/\alpha_L$ and $1/\alpha_Y$ at the unification energy.

5. Conclusions

We have generalised the analysis of the renormalisation effects on the coupling constants of unified gauge theories by including a transition region between the low-energy and the unification regions. We find that even this simple correction spoils the results discussed in most of the recent literature on unified models.

We consider both unification at superhigh energies as well as unification at accessible energies. For unification at superhigh energies, one may ignore the transition region if one so wishes. In that case, the results on renormalistion corrections are generalised to cover a wide class of models. In particular, for the sequential doublets scheme the renormalised value of $\sin^2\theta$ is not much different for the standard and the left-right symmetric models and for both models, the value is compatible with the empirical value obtained from neutral-current experiments.

For unification at accessible energies, the transition region turns out to be essential. Its omission leads to a definite inconsistency. Because of our ignorance of the parameters in the transition region, no definite results can be derived, but the possibility of small renormalisation correction for $\sin^2\theta$ remains. So, the sequential-triplets scheme of paper I may be viable, for unification at accessible energies.

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