Neutrino Condensate as Origin of Dark Energy

Jitesh R. Bhatt[a], Bipin R. Desai[b], Ernest Ma[b], G. Rajasekaran[c,d] and Utpal Sarkar[a]

[a] Physical Research Laboratory, Ahmedabad 380 009, India
[b] Physics Department, University of California, Riverside, CA 92521, USA
[c] Institute of Mathematical Sciences, Chennai 600 113, India
dChennai Mathematical Institute, Siruseri, 603103, India

We propose a new solution to the origin of dark energy. We suggest that it was created dynamically from the condensate of a singlet neutrino at a late epoch of the early Universe through its effective self interaction. This singlet neutrino is also the Dirac partner of one of the three observed neutrinos, hence dark energy is related to neutrino mass. The onset of this condensate formation in the early Universe is also related to matter density and offers an explanation of the coincidence problem of why dark energy (70%) and total matter (30%) are comparable at the present time. We demonstrate this idea in a model of neutrino mass with (right-handed) singlet neutrinos and a singlet scalar.

The astrophysical observations that the baryonic and dark matter together account for only about 30% of the total matter while the remaining 70% have the property of producing negative pressure, interpreted as the dark energy, remains a challenging problem in cosmology. A possible explanation that the dark energy is simply the vacuum energy given by the standard model scale fails by many orders of magnitude. It also can not explain the fact that the dark energy has been comparable to the density of ordinary matter during the recent epoch in the evolution of the universe. While solutions to this problem have been provided, notably through the presence of a scalar field called quintessence[1],[2]the observation that the magnitude of the dark energy is comparable to neutrino masses suggests to us that the explanation of the dark energy puzzle may possibly lie in the direction of neutrinos,while the other large effects coming from electroweak breaking and QCD are canceled out by some unknown dynamical effects It is this avenue which we follow in our discussion below.

Neutrinos have been invoked in the past in the dark energy problems. For example, recently it has been argued in a theory with scalar fields, called accelerons, that the dark energy can be obtained dynamically if the neutrino masses are allowed to vary with the scalar field [3],[4]. We propose, however, a somewhat more direct connection through the formation of neutrino condensates [5],[6],[7],[8]. We elaborate below

Our starting point is an extension of the standard model which includes right-handed neutrinos $\nu_{iR},\ i = 1, 2, 3$, in which the neutrino masses have the seesaw structure [9]with both the Dirac and Majorana masses of the order of eV. In this model we introduce a light real singlet scalar S, so that the effective Lagrangian after the electroweak symmetry breaking is given by

$$\mathcal{L} = m_{Di\alpha}\bar{\ell}_i\nu_{\alpha R} + M_\alpha\nu_{\alpha R}\nu_{\alpha R} + f_{\alpha\beta}\nu_{\alpha R}\nu_{\beta R}S \quad (1)$$

where $m_{Di\alpha}$ is the Dirac mass. The Majorana masses of the right-handed neutrinos M_α are assumed to be real and diagonal which is achieved without loss of generality through the choice of our basis. This Lagrangian has a discrete symmetry, $(-1)^L$, where L is the lepton number, so that ℓ_i and $\nu_{\alpha R}$ are odd under this symmetry.

We take the Dirac neutrino masses to be of the order of 0.1 eV, while the Majorana masses of the right-handed, singlet, neutrinos are assumed to be of the order of 0.001 eV giving rise to pseudo-Dirac neutrinos with very small mass differences. In order to be consistent with the solar neutrino data, which does not allow very small mass differences[10], however, two of the neutrinos are allowed to be only either Dirac or Majorana, but not pseudo-Dirac. The Dirac case will correspond to three left-handed and three right-handed neutrinos with the mass matrix given by

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix}$$
$$m_D = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$
$$M_N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M \end{pmatrix}. \quad (2)$$

The two physical neutrino eigenstates (ν_1 and ν_2) are the Dirac neutrinos responsible for the solar neutrino oscillations, and only the state ν_3 is a pseudo-Dirac neutrino.

We consider here the second, Majorana, possibility where there is only one sterile neutrino, which we assume forms a condensate. The corresponding neutrino mass matrix will be

$$M_\nu = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_3 \\ 0 & 0 & m_3 & M \end{pmatrix}$$

Here ν_1 and ν_2 are Majorana neutrinos, needed to explain the solar neutrino data, while ν_3 is now a pseudo-Dirac neutrino. For the rest of our analysis, we will restrict our discussions to only the third eigenstate, the pseudo-Dirac neutrino with a mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_3 \\ m_3 & M \end{pmatrix}$$

and represent the left-handed and right-handed states as ν_L and ν_R. Below we outline the dynamics behind the condensate formation which we argue is the source for the dark energy.

The exchange of the real scalar, S, is found to give rise to attraction between the right handed neutrinos ν_R. Thus, in the context of cosmic evolution, as the universe cools down to a temperature below the mass of the neutrinos, this attractive interaction then causes the right-handed neutrinos to form condensates, the candidates for the dark energy.

The four-Fermion effective contact interaction generated through S exchange is given by

$$H_I = -\mathcal{C}\,(\overline{\nu_M}\,\nu_M)\,(\overline{\nu_M}\,\nu_M)\,. \quad (3)$$

where the right-handed Majorana neutrinos ν_M have been defined as

$$\nu_M = \lambda\nu_R + {\nu^c}_L\,, \quad (4)$$

where ${\nu^c}_{\alpha L}$ is the CP conjugate of $\nu_{\alpha R}$ and λ is the Majorana phase, so that the Majorana neutrino satisfies the Majorana condition $\nu_M^c = \lambda^*\nu_M$ .A number operator for Majorana particles can not be defined, but in cosmology one can define the number density per comoving volume of any Majorana particle, when the interaction rate of the particle is slower than the expansion rate of the universe. This allows us to define the chemical potential for the particles. Thus, for the neutrino condensation formation the decoupling temperature may be considered as the cut off scale for the neutrinos. Below this temperature, the four-fermi form will be valid with the coupling strength $\mathcal{C}$ given by

$$\mathcal{C} = \frac{f^2}{m_S^2} \quad (5)$$

where m_S is the mass of the scalar field and the generation index of the coupling constant f has been suppressed .

Below we describe the formalism for the ν_R condensation formation. It is convenient to work in the Weyl representation, in which the γ_5 matrix is diagonal. The left-handed and right-handed neutrinos can be written as

$$\begin{aligned}
\nu_L &= \left(1-\gamma^5\right)\nu = \begin{bmatrix}\psi\\0\end{bmatrix}\\
\nu_R &= \left(1+\gamma^5\right)\nu = \begin{bmatrix}0\\\overline{\chi}\end{bmatrix}\\
{\nu^c}_R &= {\nu_L}^c = \begin{bmatrix}0\\\overline{\psi}\end{bmatrix}\\
{\nu^c}_L &= {\nu_R}^c = \begin{bmatrix}\chi\\0\end{bmatrix} \qquad (6)
\end{aligned}$$

so that

$$\nu_M = \begin{bmatrix}\chi\\\lambda\overline{\chi}\end{bmatrix};\;\; \nu^c_M = \begin{bmatrix}\lambda^*\chi\\\overline{\chi}\end{bmatrix};\;\; \overline{\nu_M}^T = \begin{bmatrix}\lambda^*\chi\\\overline{\chi}^\dagger\end{bmatrix};$$

The four-fermion Hamiltonian can then be expressed in terms of the component fields as

$$\begin{aligned}
H_I &= -\mathcal{C}\left[\lambda^{*2}\;\overline{\chi}_a^\dagger\;\chi_a\;\overline{\chi}_b^\dagger\;\chi_b + \overline{\chi}_a^\dagger\;\chi_a\;\chi_b^\dagger\;\overline{\chi}_b\right.\\
&+ \left.\chi_a^\dagger\;\overline{\chi}_a\;\overline{\chi}_b^\dagger\;\chi_b + \lambda^2\;\chi_a^\dagger\;\overline{\chi}_a\;\chi_b^\dagger\;\overline{\chi}_b\;\right], \qquad (7)
\end{aligned}$$

and the condensate will correspond to a spin-0 pairing:

$$\langle\chi_a\;\overline{\chi}_b^\dagger\rangle = \epsilon_{ab}\;D\,, \qquad (8)$$

giving us the interaction Hamiltonian in the mean-field approximation as:

$$H_1^{MF} = -2\,\mathcal{C}\left[\lambda^{*2}\;\overline{\chi}_a^\dagger\;\chi_b\;D + \lambda^2\;\chi_a^\dagger\;\overline{\chi}_b\;D^*\right]\;\epsilon_{ab}\,. \qquad (9)$$

In terms of the creation and annihilation operators of any Majorana field:

$$\psi_M(x) = \sum_{p,s}\sqrt{\frac{M_\alpha}{2\epsilon}}\left(f_{ps}u_{ps}e^{-ipx} + \lambda^* f^\dagger_{ps}v_{ps}e^{ipx}\right)\,, \qquad (10)$$

where $\epsilon^2 = p^2 + m^2_{i\alpha}$ is square of the energy of the physical pseudo-Dirac neutrinos. The interaction Hamiltonian can now be written as:

$$\begin{aligned}
H_1^{MF} &= -\mathcal{C}\sum_p\frac{M_\alpha}{\epsilon}\\
&\left[D\;\lambda^{*2}\;e^{-2i\epsilon t}\left(f_{p\uparrow}f_{-p\downarrow} - f_{p\downarrow}f_{-p\uparrow}\right)\right.\\
&\left.+D^*\;\lambda^2\;e^{2i\epsilon t}\left(f^\dagger_{p\uparrow}f^\dagger_{-p\downarrow} - f^\dagger_{p\downarrow}f^\dagger_{-p\uparrow}\right)\right]. \qquad (11)
\end{aligned}$$

The complete Hamiltonian will be a sum of the free Hamiltonian (H_0) and the interaction Hamiltonian (H_1^{MF}).It can be transformed to a canonical form

$$\mathcal{H} = \sum_p E\;\left(b^\dagger_{p\uparrow}b_{p\uparrow} + b^\dagger_{p\downarrow}b_{p\downarrow}\right)\,, \qquad (12)$$

by a time-dependent transformation. Following the standard condensed matter formalism one finds that a consistent solution, that allows a non-vanishing condensate $D \neq 0$, gives

$$\frac{\mathcal{C}}{2}\int\frac{d^3p}{(2\pi)^3}\frac{M_\alpha^2}{\epsilon^2}\frac{1}{[(\epsilon-\mu)^2+\kappa^2]^{1/2}} = 1\,, \qquad (13)$$

where κ is related to the gap parameter, Δ and μ is the chemical potential. The lower limit on the energy integral is taken to be $(M_\alpha - \mu)$, and the upper limit is the cut off scale Λ, which is the decoupling scale for the neutrinos. This limit is determined by the requirement that above this temperature neutrino interactions are in equilibrium and the number density and the chemical potential are not defined.

The solution of (13) gives us the magnitude of the gap:

$$\Delta = 2\sqrt{\frac{2\Lambda}{M_\alpha}}\left(3\pi^2 n_\nu\right)^{1/3}e^{-x}\,, \qquad (14)$$

where $x = 2\pi^2/[\mathcal{C}M_\alpha(3\pi^2 n_\nu)^{1/3}]$ and the critical temperature and the Pippard coherent length are given by

$$\begin{aligned}
T_c &= \frac{e^\gamma}{\pi} \approx 0.57\Delta \qquad (15)\\
\xi &= \frac{e^x}{\pi\sqrt{2\Lambda M_\alpha}}\,. \qquad (16)
\end{aligned}$$

In the present example, the coupling of the neutrinos to S becomes strong in the non-relativistic limit, which will imply that the condition $M_\alpha \ll m_S$ is required to be satisfied for equation (3) to remain valid. However, this condition may be relaxed considerably in a relativistic treatment of superconductivity. A numerical study of fermions interacting with a scalar field in a strong coupling regime i.e. $f \sim 1$, shows that the scalar-field propagator can be scaled as $1/m_S^2$ even for $M_\alpha \geq m_S$ [11]. In this strong coupling regime, the right-handed neutrino condensates thus formed start dominating the universe when the size of Cooper pair becomes comparable to the inter-particle spacing. For $\xi = 0.1$ cm $\sim (2\times10^{-4}\text{eV})^{-1}$ and $n_\nu \sim 110$, we get $x \sim 13.5$, $m_S \sim 4.6\times10^{-4}$ eV and $\Delta \sim 4\times10^{-5}$ eV. The existence of a finite, non zero, gap provides the evidence for a condensate [1]

This condensate, we would like to argue, is a dark energy candidate. Since the Cooper pairs are formed around the scale of neutrino masses, the amount of dark energy density becomes comparable to the matter density in this scenario. The amount of dark energy is determined by the Majorana mass of the neutrinos, which is $M \sim 10^{-3}$ eV, and we get the correct order of magnitude.Thus, without invoking any dynamical field like the quintessence or accelerons, we have found a natural explanation as to why the scales of dark energy and neutrino masses are comparable and why dark energy dominates in this epoch. Furthermore, our model is also consistent with the fact that the amount of dark energy is the same as the matter density.

We conclude with a short discussion of the basic dynamics of the condensates, which we call ξ_s.The Lagrangian for ξ_s is given by

$$\mathcal{L} = (\partial_0 + i\mu_s)\,\xi_s^\dagger\,(\partial_0 - i\mu_s)\,\xi_s - \partial_i\xi_s^\dagger\partial_i\xi_s - V(\xi_s), \tag{17}$$

where m and μ_s represent the mass and the chemical potential of the condensate respectively, and

$$V(\xi_s) = m^2|\xi_s|^2 + g|\xi_s|^4\,.$$

For the present case we assume, $m \simeq 2m_\nu$ and $\mu_s \simeq \mu$. We note that the interaction lagrangian can be written as

$$V(\xi_s) = m_s^2|\xi_s\xi_s^\dagger| + \mathcal{C}(\nu_R^c\nu_R + hc), \tag{18}$$

[1] It is interesting to note what happens if we consider the neutrinos in isolation and discuss their bound state in a non-relativistic framework. Because of the miniscule mass of the scalar, S, one can approximate the interaction to be Coulomb-like . Since $f \sim 1$, the Bohr radius of the resulting atom will be $\sim M_\alpha^{-1}$ and the binding energy $\sim M_\alpha$ which is essentially of the order of the dark energy

which gives $V(\xi_s)$ after taking into account the quartic self-interactions.

In absence of chemical potential, the equation of state for the scalar-field becomes

$$\omega = \frac{p}{\rho} = \frac{\text{KE} - V(\xi_s)}{\text{KE} + V(\xi_s)}\,.$$

For KE $\ll V(\xi_s)$, the scalar field then behaves as dark energy with the desired value $\omega \sim -1$. This would imply that $g|\xi_s|^2 \ll 0$ or $g < 0$. The coupling constant g can be calculated from the knowledge of scattering between the condensates from an analogy with atomic physics. One can write $g = \frac{4\pi a}{m}$ for an attractive interaction with $a < 0$. Under this condition the condensates can represent the dark energy [12]. Another possibility is the case when there is a finite chemical potential and there is spontaneous symmetry breaking [8, 13]. In this case one can use the arguments of Ref. [8] to estimate the parameters m and g as

$$\begin{aligned} m^2 &= m_S^2 - \frac{N_f\mathcal{C}}{8\pi^2}\left(\Lambda^2 - \mu^2\right) \\ \text{and}\quad g &= \frac{N_f\mathcal{C}}{8\pi^2} ln\left(\frac{\Lambda^2}{\mu^2}\right). \end{aligned} \tag{19}$$

In this case the equation of state can be written as $\rho = 3\,p + 4(1 - \frac{\mu}{m}\sqrt{1+\sqrt{p}})\sqrt{p}$ [14] which gives $\omega \sim -1$ if $p = (\frac{\mu^2}{m^2} - 1)^2$ and $\mu^2 > m^2$ which is consistent with the condensate formation condition [13]. p and ρ are made dimensionless by factoring out with $m^4/4g$.

In summary we have proposed a new solution of the dark energy problem where the dark energy is the condensate formed by self interaction of right handed (singlet) neutrinos generated through the exchange of a singlet scalar. Since this neutrino is the Dirac partner of one of the three observed neutrinos, the dark energy is related to the neutrino mass. The fact that the matter and dark energy are comparable follows naturally from our model.

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