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Light Sterile Neutrinos from Large Extra Dimensions

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Abstract

An experimentally verifiable Higgs-triplet model of neutrino masses from large extra dimensions was recently proposed. We extend it to accomodate a light sterile neutrino which also mixes with the three active neutrinos. A previously proposed phenomenological model of four neutrinos (the only viable such model now left, in view of the latest atmospheric and solar neutrino-oscillation data) is specifically realized.

In the standard model of particle interactions, neutrinos ν_i belong in left-handed doublets (ν_i, l_i) under the electroweak $SU(2)_L \times U(1)_Y$ gauge group. Without right-handed singlet partners, they are massless but could obtain small Majorana masses through the effective dimension-5 operator [1]

$$\frac{f_{ij}}{\Lambda}(\nu_i \phi^0 - l_i \phi^+)(\nu_j \phi^0 - l_j \phi^+), \tag{1}$$

as the Higgs doublet (ϕ^+, ϕ^0) acquires a nonzero vacuum expectation value (vev). The parameter Λ is an effective large mass scale.

Different models of neutrino mass are merely specific realizations [2] of this operator in different extensions of the standard model. The usual approach is to identify the lepton-number violation with a very large scale, i.e. $\langle \phi^0 \rangle = v << \Lambda$. However, it is also possible that $1/\Lambda$ is actually of the form m/M^2, so that the scale of lepton-number violation may be associated with m which happens to be very small. Hence M does not have to be very large, in which case the origin of neutrino mass may be tested directly in future high-energy colliders. In a recently proposed model [3] in the context of large extra dimensions [4], exactly this very desirable feature is accomplished by adding [5] a Higgs triplet (ξ^{++}, ξ^+, ξ^0) to the standard model, as well as a scalar singlet χ which also exists in the bulk [6].

We now extend this proposal to include a light sterile neutrino ν_s. (This is to be distinguished from other proposals [6, 7] where singlet fermions exist in the bulk which pair up with ν_i to become massive Dirac particles. We note that strong bounds on the fundamental scale in such theories have also been obtained [8].) The totality of present experimental evidence for neutrino oscillations [9, 10, 11] calls for 1 sterile neutrino in addition to the 3 active ones. There are already a number of phenomenological scenarios for fitting all such data, but the latest results from the Super-Kamiokande Collaboration rule out the $\nu_\mu \to \nu_s$ explanation of the atmospheric data [12] at the 99% confidence level, and also rule out the $\nu_e \to \nu_s$ explanation of the solar data [13] at the 95% confidence level. This means that ν_s

must be used in explaining the LSND result [11, 14] because a third mass-squared difference is required, and as far as we know, there is only one such phenomenological model that has been previously proposed [15]. The salient feature of this model is the fast decay of ν_s into an antineutrino $\bar{\nu}_i$ and a massless Goldstone boson (the Majoron) corresponding to the spontaneous breaking of lepton number. We show in the following how this may naturally occur in the context of large extra dimensions.

The scalar singlet χ carries lepton number $L = -2$ [3] and its *vev* is the source of all lepton-number violation in our four-dimensional world (called a 3-brane). The smallness of $\langle\chi\rangle$ is due to the distance of our brane in the extra space dimensions from the source of the lepton-number violation located in another brane [16]. Let y denote a point in the extra dimensions. Our brane is located at $y = 0$, whereas another 3-brane (the one providing the lepton-number violation) is at $y = y_*$. We assume that they are separated by $|y_*| = r$, where r is the radius of compactification of the extra space dimensions, which is only a few μm in magnitude. The fundamental scale M_* in this theory is then related to the usual Planck scale $M_P = 2.4 \times 10^{18}$ GeV by the relation

$$r^n M_*^{n+2} \sim M_P^2, \tag{2}$$

where n is the number of extra space dimensions. The scalar singlet χ exists in the bulk and could thus communicate between the 2 branes. For the source of lepton-number violation, one possibility is that a scalar field σ, carrying lepton number $L = 2$, acquires a large *vev* in the other brane. Assuming all mass parameters are of order M_*, the field σ has an interaction with χ given by

$$\mathcal{L} = \alpha M_*^2 \int d^4x' \sigma(x') \chi(x', y = y_*), \tag{3}$$

where α is a parameter of order unity.

Once σ acquires a *vev*, lepton number will be broken in the other brane. This will then act as a point source in the extra space dimensions for our world, so that the profile of χ is given by the Yukawa potential in the transverse dimensions [16]:

$$\langle\chi(y=0)\rangle = \langle\sigma(y=y_*)\rangle\Delta_n(r), \tag{4}$$

where

$$\Delta_n(r) = \frac{1}{(2\pi)^{\frac{n}{2}} M_*^{n-2}} \left(\frac{m_\chi}{r}\right)^{\frac{n-2}{2}} K_{\frac{n-2}{2}}\left(m_\chi r\right), \tag{5}$$

K being the modified Bessel function. Assuming that $\langle\sigma\rangle = M_*$, we then have the *shining* of χ everywhere in our world corresponding to

$$\langle\chi\rangle \approx \frac{\Gamma(\frac{n-2}{2})}{4\pi^{\frac{n}{2}}} \frac{M_*}{(M_* r)^{n-2}} \approx \frac{\Gamma(\frac{n-2}{2})}{4\pi^{\frac{n}{2}}} M_* \left(\frac{M_*}{M_P}\right)^{2-\frac{4}{n}}, \tag{6}$$

where $n > 2$ and $m_\chi r << 1$ have also been assumed. This *shined* value of χ now appears as a boundary condition for our brane. In other words, the localized fields in our world must interact with χ in such a way that $\langle\chi\rangle$ is unaffected.

Consider now the addition of a singlet left-handed sterile neutrino ν_s. We propose the following 2 simple scenarios: (A) ν_s has $L = +1$, and (B) ν_s has $L = -1$. In (A), the interaction Lagrangian involving ν_i and ν_s is given by

$$\mathcal{L} = f_{ij}[\xi^0\nu_i\nu_j + \xi^+(\nu_i l_j + l_i\nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + f_i\nu_s(\nu_i\eta^0 - l_i\eta^+) + h.c., \tag{7}$$

where (ξ^{++}, ξ^+, ξ^0) is a scalar triplet and (η^+, η^0) is a scalar doublet, both carrying $L = -2$. We define $\langle\chi\rangle \equiv z$ and express the bulk field as

$$\chi = \frac{1}{\sqrt{2}}(\rho + z\sqrt{2})e^{i\varphi}. \tag{8}$$

We assume that its behavior is not altered by the parameters in different branes. All such effects are already included in the boundary condition of Eq. (6). The lepton-number conserving interactions of χ with the other scalar fields in our world are then contained in

$$\begin{aligned} L \;=\; & \int d^4x \Big[hz(y=0)e^{i\varphi(x)} \left(\bar{\xi}^0(x)\phi^0(x)\phi^0(x) - \sqrt{2}\xi^-(x)\phi^+(x)\phi^0(x) + \xi^{--}(x)\phi^+(x)\phi^+(x)\right) \\ & + \mu z(y=0)e^{i\varphi(x)} \left(\bar{\eta}^0(x)\phi^0(x) + \eta^-(x)\phi^+(x)\right) + h.c. \Big], \end{aligned} \tag{9}$$

where μ is a mass parameter which could be of order M_* or the electroweak symmetry breaking scale. The self-interaction terms for the bulk scalar are now given by

$$V(\chi) = \lambda z(y)^2 \rho(x,y)^2 + \frac{1}{\sqrt{2}} \lambda z(y) \rho(x,y)^3 + \frac{1}{8} \lambda \rho(x,y)^4. \tag{10}$$

This formulation has the virtue of universality, i.e. λ is unchanged, but z can change depending on where our brane is from the distant brane. It is also invariant under $U(1)_L$: $\nu_i \to e^{i\theta}\nu_i$, $\nu_s \to e^{i\theta}\nu_s$, $\xi \to e^{-2i\theta}\xi$, $\eta \to e^{-2i\theta}\eta$, $\rho \to \rho$, and $\varphi \to \varphi - 2\theta$. The form of the potential $V(\chi)$ is that of the usual spontaneously broken U(1) theory and is independent of parameters in our brane.

The *shining* of the field χ in our world induces a very weak lepton-number violating trilinear coupling of the Higgs triplet ξ with the standard Higgs doublet Φ [3], as well as the mixing of the Higgs doublets η and Φ. In addition, the *shined* value of χ supplies a Majorana mass term to the sterile neutrino through the interaction

$$L = f_s \int d^4x \; z(y=0) e^{i\varphi(x)} \nu_s(x) \nu_s(x) + h.c. \tag{11}$$

From the Lagrangian of Eq. (9), it can easily be shown that ξ^0 and η^0 will have small *vevs* (say u and w respectively) which are proportional to z. Although the *shined* value of χ, i.e. z, comes from lepton-number violation in a distant brane and may not be determined in terms of the parameters entering in our world, the *vevs* of the other fields will be obtained in the usual way by minimizing the appropriate Higgs potential.

Consider the following Higgs potential containing the fields ξ, η, and Φ with interaction terms involving χ in our brane:

$$\begin{aligned} V &= m^2 \Phi^\dagger \Phi + m_\xi^2 \xi^\dagger \xi + m_\eta^2 \eta^\dagger \eta + \frac{1}{2}\lambda_1 (\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2 (\xi^\dagger\xi)^2 + \frac{1}{2}\lambda_3(\eta^\dagger\eta)^2 + \lambda_4(\Phi^\dagger\Phi)(\xi^\dagger\xi) \\ &\quad + \lambda_5(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_6(\xi^\dagger\xi)(\eta^\dagger\eta) + \left(hze^{i\varphi}\xi^\dagger\Phi\Phi + \mu z e^{i\varphi}\eta^\dagger\Phi + h.c.\right) \end{aligned} \tag{12}$$

Let $\langle\Phi\rangle = v$, $\langle\eta\rangle = w$ and $\langle\xi\rangle = u$, then the conditions for the minimum of V are given by

$$v(m^2 + \lambda_1 v^2 + \lambda_4 u^2 + \lambda_5 w^2 + 2hzu) + \mu z w = 0, \tag{13}$$

$$u(m_\xi^2 + \lambda_4 v^2 + \lambda_2 u^2 + \lambda_6 w^2) + hv^2 z = 0, \tag{14}$$

$$w(m_\eta^2 + \lambda_5 v^2 + \lambda_6 u^2 + \lambda_3 w^2) + \mu v z = 0. \tag{15}$$

These equations tell us that the *vevs* of the fields ξ and η are small, and are given by

$$u \simeq -\frac{hv^2 z}{m_\xi^2 + \lambda_4 v^2}, \qquad w \simeq -\frac{\mu v z}{m_\eta^2 + \lambda_5 v^2}, \tag{16}$$

where $v \simeq \sqrt{-m^2/\lambda_1} \simeq 174$ GeV as usual. [In the above, we have neglected the term $\xi^\dagger \Phi \eta$ for simplicity. Its presence would only change the expressions for u and w, not their proportionality to z. It also does not affect the composition of the Majoron in Eq. (21) to be given below.] The 4×4 neutrino mass matrix including the sterile neutrino in the basis (ν_i, ν_s) is then of the form

$$\mathcal{M}_\nu = \begin{pmatrix} 2f_{ij}u & f_i w \\ f_i w & 2f_s z \end{pmatrix}, \tag{17}$$

where all the mass terms are naturally small, say of order 1 eV or less.

The kinetic-energy term of χ is

$$|\partial_\mu \frac{1}{\sqrt{2}}(\rho + z\sqrt{2})e^{i\varphi}|^2 = \frac{1}{2}(\partial_\mu \rho)^2 + z^2(\partial_\mu \varphi)^2 + ..., \tag{18}$$

which implies that $\sqrt{2}z\varphi$ is the properly normalized massless Goldstone boson (the Majoron) from the spontaneous breaking of lepton number in the bulk. In the presence of the interaction terms of Eq. (9), other fields also participate in the spontaneous breaking of lepton number, hence the Majoron in our world becomes a combination of all these fields. The 4×4 mass matrix in the basis $(\mathrm{Im}\phi^0, \mathrm{Im}\xi^0, \mathrm{Im}\eta^0, z\varphi)$ is given by

$$\begin{pmatrix} -4hzu - \mu z w/v & 2hzv & \mu z & -2huv - \mu w \\ 2hzv & -hzv^2/u & 0 & hv^2 \\ \mu z & 0 & -\mu z v/w & \mu v \\ -2huv - \mu w & hv^2 & \mu v & -huv^2/z - \mu w v/z \end{pmatrix}. \tag{19}$$

Diagonalizing this matrix, we get one massless eigenstate

$$\frac{v\mathrm{Im}\phi^0 + 2u\mathrm{Im}\xi^0 + w\mathrm{Im}\eta^0}{\sqrt{v^2 + 4u^2 + w^2}}, \tag{20}$$

which becomes the longitudinal component of the Z boson, and another one which is the physical Majoron field:

$$N^{-\frac{1}{2}}\left[-v(w^2+2u^2)\mathrm{Im}\phi^0+u(v^2-w^2)\mathrm{Im}\xi^0+w(v^2+2u^2)\mathrm{Im}\eta^0+z(v^2+w^2+4u^2)z\varphi\right], \tag{21}$$

where N is a normalization constant. The Majoron coupling matrix now becomes

$$\frac{1}{\sqrt{N}}\begin{pmatrix} 2f_{ij}u(v^2-w^2) & f_iw(v^2+2u^2) \\ f_iw(v^2+2u^2) & 2f_sz(v^2+w^2+4u^2) \end{pmatrix} \tag{22}$$

In the limit $v\to\infty$, the Majoron coupling matrix and the neutrino mass matrix are simultaneously diagonalized, in which case the Majoron will have only diagonal couplings. For finite v, the off-diagonal Majoron couplings to the neutrino mass eigenstates are suppressed by the factor $(w^2+2u^2)/v^2$, hence neutrino decay rates are very small and phenomenologically unimportant.

In scenario (B), ν_s has $L=-1$. Hence the scalar doublet (η^+,η^0) in Eq. (7) now carries no lepton number, i.e. $L=0$. To distinguish it from the usual Higgs doublet (ϕ^+,ϕ^0), we add a discrete Z_2 symmetry, such that ν_s and η are odd and all other fields are even. Instead of the $e^{i\varphi}\nu_s\nu_s$ term in Eq. (11), we now have $e^{-i\varphi}\nu_s\nu_s$. Instead of the $\mu z e^{i\varphi}\eta^\dagger\Phi$ term in Eq. (12), we now have $\mu^2\eta^\dagger\Phi$ which breaks the Z_2 discrete symmetry softly, and as such, the parameter μ can be small. [We have neglected the term $h' z e^{i\varphi}\xi^\dagger\eta\eta$ for simplicity. It does not affect the composition of the Majoron in Eq. (25) to be given below.]

The equation of constraint for u remains the same as Eq.(14), whereas the $\mu z w$ term in Eq. (13) is replaced by $\mu^2 w$, and the $\mu v z$ term in Eq. (15) is replaced by $\mu^2 v$. Hence u is the same as in Eq. (16), and

$$w\simeq-\frac{\mu^2 v}{m_\eta^2+\lambda_5 v^2}. \tag{23}$$

To obtain $w\sim 1$ eV, we need $\mu\sim 1$ MeV for $m_\eta\sim 1$ TeV.

The 4×4 mass matrix in the basis $(\mathrm{Im}\Phi^0, \mathrm{Im}\xi^0, \mathrm{Im}\eta^0, z\varphi)$ now becomes

$$\begin{pmatrix} -4hzu - \mu^2 w/v & 2hzv & \mu^2 & -2huv - \mu w \\ 2hzv & -hzv^2/u & 0 & hv^2 \\ \mu^2 & 0 & -\mu^2 v/w & 0 \\ -2huv - \mu w & hv^2 & 0 & -huv^2/z \end{pmatrix}, \tag{24}$$

and the physical Majoron is

$$\begin{aligned} & N^{-\frac{1}{2}} \left[-2u^2 v \mathrm{Im}\Phi^0 + u(v^2 + w^2)\mathrm{Im}\xi^0 - 2u^2 w \mathrm{Im}\eta^0 + z(v^2 + w^2 + 4u^2) z\varphi \right] \\ & \simeq (u\mathrm{Im}\xi^0 + z^2\varphi)/\sqrt{u^2 + z^2}. \end{aligned} \tag{25}$$

Because ν_s has $L = -1$ instead of $L = +1$, the Majoron coupling matrix is no longer proportional to the neutrino mass matrix even in the limit of $v \to \infty$. The latter remains the same as given by Eq. (17), but the former now differs from Eq. (22), i.e.

$$\frac{1}{\sqrt{u^2 + z^2}} \begin{pmatrix} 2f_{ij}u & 0 \\ 0 & -2f_s z \end{pmatrix}. \tag{26}$$

When expressed in the basis of neutrino mass eigenstates, there are now unsuppressed off-diagonal terms. Hence neutrino decay is fast and the phenomenological model of Ref. [15] is realized.

We advocate thus the 4×4 neutrino mass matrix of Eq. (17), where the diagonal ν_s entry is of order a few eV. The 3×3 submatrix spanning the ν_e, ν_μ, and ν_τ neutrinos is used to accommodate the atmospheric [9, 12] and solar [10, 13] neutrino data, and the mixing between ν_s and ν_i is used to fit the LSND data [11, 14]. Fast decay of ν_s into $\bar{\nu}_i$ and the Majoron allows us to evade the constraints of the CDHSW accelerator experiment [17], as explained in Ref. [15]. This model can be tested in present and future solar-neutrino experiments by the observation of antineutrinos (instead of neutrinos) from the sun. Details are already given in Ref. [15].

In conclusion, we have shown how a light sterile neutrino may be naturally accommodated in the context of a theory of large extra space dimensions where a scalar field χ in the bulk carries lepton number which is spontaneously broken in a distant brane. The effects on our world are a small *vev* for χ and its associated massless Goldstone boson (the Majoron). This allows us to have a naturally light 4×4 mass matrix including the 1 sterile and the 3 active neutrinos. Two simple scenarios are discussed, one of which allows the fast decay of a heavier neutrino into a lighter antineutrino and the Majoron, as previously proposed [15]. In view of the latest experimental results which exclude $\nu_\mu \to \nu_s$ in atmospheric [12] and $\nu_e \to \nu_s$ in solar [13] neutrino-oscillation data, this is the only viable such model now left.

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