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Solar Neutrinos and the Eclipse Effect

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Abstract

The solar neutrino counting rate in a real time detector like Super–Kamiokanda, SNO, or Borexino is enhanced due to neutrino oscillations in the Moon during a partial or total solar eclipse. The enhancement is calculated as a function of the neutrino parameters in the case of three flavor mixing. This enhancement, if seen, can further help to determine the neutrino parameters.

I. INTRODUCTION

The Sun is a copious source of neutrinos with a wide spectrum of energies and these neutrinos have been detected by terrestrial neutrino detectors, although at a rate lower than expected from theoretical calculations. A new generation of detectors Super-Kamiokanda, SNO and Borexino [1–3] with high counting rates will soon be producing abundant data on solar neutrinos; among these Super-Kamiokanda has already started producing results [4]. Mixing and the consequent oscillations among the neutrinos of different flavors is generally believed to be the cause of the reduced intensity of neutrino flux detected on Earth. However, neutrino–oscillation is a complex phenomenon depending on many unknown parameters (six parameters for three flavors ν_e, ν_μ, ν_τ) and considerable amount of experimental work and ingenuity will be required before the neutrino problem is solved.

Hence it would be desirable, if apart from direct detection, we can subject the solar neutrino beam to further tests by passing it through different amounts of matter, in our attempts to learn more about the neutrinos. Nature has fortunately provided us with such opportunities: (1) Neutrinos detected at night pass through Earth. (2) Neutrinos detected during a solar eclipse pass through the Moon. (3) Neutrinos detected at the far side of Earth during a solar eclipse pass through the Moon and Earth. We shall call this scenario (3) a *double eclipse*. The scenario (1) has been studied in the literature rather extensively [5,6]. The purpose of the present work is to examine the scenarios (2), and (3). Previous works [7–9] have discussed scenario (2), however they are incomplete in many respects.

The plan of the paper is as follows. The relevant astronomy is presented in Sec.II. In Sec.III we give the theory of the passage of the solar neutrinos through the Moon and Earth, taking into account properly the non-adiabatic transitions occuring at the boundaries of the Moon and Earth. In Sec.IV we present the numerical calculations of the neutrino detection rates during the single and double eclipses and the results. Sec.V is devoted to discussion.

II. ECLIPSES AND DOUBLE ECLIPSES

Solar neutrinos are produced within the solar core whose radius is of order 1/10 of the solar radius and we shall approximate this by a point at the center of the Sun. What is required for our purpose is that the lunar disc must cover this point at the center of the Sun and so as far as the neutrino radiation is concerned , the solar eclipse is more like an occultation of a star or a planet by the Moon.

Astronomers characterize the solar eclipse by the optical coverage C which is defined as the ratio of the area of the solar disc covered by the lunar disc to the total area of the solar disc. We first calculate how C varies with time during an eclipse.

There are three parameters which completely characterize the time-dependence of any solar eclipse.

- The first parameter is the ratio, a, of the apparent radii (in radians, say) of the Moon and Sun's discs as seen from Earth, $a = r_m/r_s$. The eclipse is total if $a \geq 1$ and it is annular if $a < 1$.

- The second parameter is the duration of the eclipse, T, measured as the length of time between the initial and final instants where the Moon's and Sun's discs just touch.

- Finally, we have the impact parameter, $p = \ell/r_s$, defined as the apparent distance of closest approach between the center of the Sun's disc and the center of the Moon's disc, in units of the Sun's apparent radius.

Given these quantities, the expression for C as a function of time t during an eclipse is determined by simple geometry as follows. First it is convenient to change the independent variable from t to d, defined as the apparent separation between the centers of the Sun and the Moon (in units of the apparent solar radius). If $t = 0$ represents the point of maximum coverage, which occurs when d equals p and is minimum and $t = \mp T/2$ correspond to the start and end of the eclipse when $d = 1 + a$, then d is given by

$$d(t) = \left\{ p \left[1 - \left(\frac{2t}{T} \right)^2 \right] + (1+a)^2 \left(\frac{2t}{T} \right)^2 \right\}^{1/2}. \tag{1}$$

One then finds the following expressions for $C(d)$ corresponding to four mutually-exclusive (and exhaustive) cases:

1. If $d \geq 1 + a$ then the Sun and Moon's discs are separated and so

$$C(d) \equiv 0. \tag{2}$$

2. If $1 + a > d \geq \sqrt{|1 - a^2|}$ then

$$C(d) = f(u) + a^2 f\left(\frac{u}{a}\right), \tag{3}$$

where

$$f(u) = \frac{1}{\pi}\left(\arcsin u - u\sqrt{1 - u^2}\right), \tag{4}$$

and

$$u = \frac{1}{2d}\left\{\left[(1+a)^2 - d^2\right]\left[d^2 - (1-a)^2\right]\right\}^{1/2}. \tag{5}$$

Notice that these definitions ensure that u is well-defined and climbs monotonically from $u = 0$ as d decreases from $1 + a$, not reaching $u = \min(1, a)$ until $d = \sqrt{|1 - a^2|}$.

3. If $\sqrt{|1 - a^2|} > d \geq |1 - a|$ then the expression for $C(d)$ depends on whether or not the Moon's apparent disc is larger than the Sun's.

   (a) If $a \leq 1$ then

$$C(d) = f(u) + a^2\left[1 - f\left(\frac{u}{a}\right)\right]. \tag{6}$$

   (b) If $a > 1$ then

$$C(d) = 1 - f(u) + a^2 f\left(\frac{u}{a}\right). \tag{7}$$

4. Finally, if $|1-a| > d \geq 0$ then

$$C(d) = 1 \quad \text{if} \quad a \geq 1 \qquad\qquad \text{and} \qquad\qquad C(d) = a^2 \quad \text{if} \quad a < 1. \tag{8}$$

For neutrino physics we require the distance d_M traveled by the neutrino inside the Moon. Defining the fraction $x = \frac{d_M}{2R_M}$ where R_M is the lunar radius, x can be given in terms of C by the following formulae:

$$x = \sqrt{(4z(2-z)-3)} \tag{9}$$

$$C = \frac{2}{\pi}\left(\cos^{-1}(1-z) - (1-z)\sqrt{z(2-z)}\right) \tag{10}$$

Eq.(10) can be inverted to get the parameter z for a given C and this z can be substituted in eq.(9). The relationship between x and C so obtained is plotted in Fig1. When the lunar disc passes through the center of the Sun, C is 0.39 and the neutrino eclipse starts at this value of C.When the optical coverage increases above 39%, x rises sharply from zero and reaches 0.6 and 0.95 for optical coverage of 50% and 80% respectively

For any point of observation of the usual solar eclipse (which we shall call *single eclipse*) there is a corresponding point on the other side of Earth where a double eclipse occurs. With the coordinates labeled as (latitude, longitude), the single eclipse point (α, β) is related to the double eclipse point (λ, σ) by the relations (see Fig.2):

$$\begin{aligned} \lambda &= \alpha + 2\delta \\ \sigma &= \pi - 2\,\Theta_{UT} - \beta \end{aligned} \tag{11}$$

where,

$$\sin\delta = \sin 23.5^o \sin(\frac{2\pi t}{T_Y}), \tag{12}$$

T_Y is the length of the year , zero of time t is chosen at midnight of autumnal equinox *i.e.*Sept. 21, and Θ_{UT} is the angle corresponding to the Universal Time – UT.

During a double eclipse, the neutrinos travel through Earth in addition. The distance d_E traveled by the neutrino inside Earth along the chord between the points (α, β) and (λ, σ) as a function of time t, is given by

$$d_E = 2R_E(\sin\lambda\sin\delta + \cos\lambda\cos\delta\cos(\frac{2\pi t}{T_D})) \qquad (13)$$

where R_E is the radius of Earth and T_D is the length of the day. This is the same distance that needs to be calculated in the study of the day–night effect and a plot of this distance as a function of t is given in our earlier paper [6].

Present and upcoming high statistics neutrino detectors expect to collect utmost a few solar neutrino events every hour. As shown in Sec. IV, single and double eclipse can lead to enhancements of rates by upto two and a half times. Even with such large enhancements during the eclipse the signal may not exceed statistical errors, since each solar eclipse lasts only for a few hours. However they occur fairly often. As many as 32 solar eclipses are listed to occur during the 14 year period 1996 through 2010. Global maps and charts are available on the internet [10], or from computer programs [11], for the location and duration of both the umbral and penumbral coverage. In future planning of neutrino–detector sites, these points may also be kept in mind. It is important to remark here that although partial solar eclipses are not so useful to astronomers they can nevertheless be relevant for neutrino physics, so long as C is above 0.39.

We make the following comments concerning the neutrino possibilities for the eclipses (single or double) that will be visible from Kamiokanda, Sudbury or Gran Sasso (the site of Borexino) between 1997 and 2002:

March 9, 1997 – This eclipse was a single eclipse at Kamiokanda, with an approximate duration of two and a half hours and a maximum optical coverage of just over 50%. This eclipse was a double eclipse at the Gran–Sasso, for much the same duration, also with roughly 50% optical coverage.

February 26, 1998 – For this eclipse only Gran Sasso saw more than 39% coverage, with the Moon and Sun's discs barely touching as seen from Sudbury and 23% coverage seen in Japan. The eclipse was double in Gran–Sasso with a maximum coverage of 60%.

August 11, 1999 – *This eclipse will provide one of the best opportunities*, being seen as a single eclipse with about 80% optical coverage and almost 3 hour duration at Gran-Sasso. *It is also visible from Sudbury*, with 70% coverage, for just short of two hours close to the horizon near Sunrise. Unfortunately the Super–Kamiokanda site will get the smallest coverage, utmost about 30%, for a double eclipse lasting just over an hour.

December 25, 2000 – This eclipse lasts for three hours around noon (and so is a single eclipse) on Christmas day as seen from Sudbury, but the Moon's disc does not intersect the Sun's as seen from the other two sites. 50% is the maximum coverage presented to SNO.

July 31, 2000 – This eclipse is not visible at all from Japan, and presents less than 10% coverage in Italy, but is close to our threshold of interest as seen from Sudbury, being a double eclipse seen from there, with maximum coverage of 32%.

December 14, 2001 – Kamiokanda and Gran Sasso see respectably large coverage this day (70% and 85%, respectively), with both seeing a double eclipse whose duration is about 100 minutes. By contrast, Sudbury sees only a grazing eclipse with coverage not reaching 10%.

June 10, 2002 – This two-hour-long (single) eclipse falls just below threshold — 32% maximum coverage — seen from Kamiokanda. Sudbury again sees less than 10% coverage, while no eclipse occurs at all from Gran Sasso's perspective.

Tables I through III list the times, durations, maximum coverage and direction of all these eclipses. Plots of the optical coverage C and the fractional distance x traveled by the neutrino as a function of time t for the same eclipses are given in Fig.3 to 5

The lesson to be learned from these tables and figures is that there is, on the average, about one eclipse per year for which the maximum coverage exceeds 39%, and once several detectors become available it will not be uncommon for any of these eclipses to be seen by more than one detector at a time.

III. THEORY

A. Regeneration in the Moon

We now describe a straightforward way of obtaining the neutrino regeneration effect in the Moon. The Moon has a radius of 1738 km and has an approximately uniform density of 3.33gms/cc, except for a core of radius 238 km, where the density jumps to 7.55 gm/cc [12]. The effect of the core can be treated by the method in Sec.III B, but this is required only for $x > 0.99$ which occurs only very rarely, as seen from Figs.(3–5). Hence we restrict ourselves to a model of Moon of constant density.

Let a neutrino of flavor α be produced at time $t = t_0$ in the core of the Sun. Its state vector is

$$|\Psi_\alpha(t_0)\rangle = |\nu_\alpha\rangle = \sum_i U^S_{\alpha i}|\nu^S_i\rangle. \tag{14}$$

where $|\nu^S_i\rangle$ are the matter dependent mass eigenstates with mass eigenvalues μ^S_i and $U^S_{\alpha i}$ are the matrix elements of the matter dependent mixing matrix in the core of the Sun. We use Greek index α to denote the three flavors e, μ,τ and Latin index i to denote the mass eigenstates i = 1,2,3. The neutrino propagates in the Sun adiabatically upto t_R (the resonance point), makes non-adiabatic transitions at t_R, propagates adiabatically upto t_1 (the edge of the Sun) and propagates as a free particle upto t_2 when it enters the Moon. So the state vector at t_2 is

$$|\Psi_\alpha(t_2)\rangle = \sum_{j,i} |\nu_j\rangle exp\left(-i\varepsilon_j(t_2 - t_1)\right) exp\left(-i\int_{t_R}^{t_1} \varepsilon^S_j(t)dt\right) M^S_{ji} exp\left(-i\int_{t_0}^{t_R} \varepsilon^S_i(t)dt\right) U^S_{\alpha i}. \tag{15}$$

where $\varepsilon^S_i(t)(\equiv E + (\mu^S_i(t))^2/2E)$ are the matter dependent energy eigenvalues in the Sun, ε_i and $|\nu_i\rangle$ are the energy eigenvalues and the corresponding eigenstates in vacuum and M^S_{ji} is the probability amplitude for the non-adiabatic transition $i \to j$. We multiply the right hand side of eq.(15) by $\sum_k |\nu^M_k\rangle\langle\nu^M_k|$ ($=1$)where $|\nu^M_k\rangle$ (i =1,2,3) is the complete set of matter dependent mass eigenstates inside the Moon. The neutrino propagates upto the the other end of the Moon at t_3, and the state vector at t_3 is

$$\begin{aligned}
|\Psi_\alpha(t_3)\rangle &= \sum_{k,j,i} |\nu_k^M\rangle exp\left(-i\varepsilon_k^M(t_3-t_2)\right)\langle\nu_k^M|\nu_j\rangle exp\left(-i\varepsilon_j(t_2-t_1) - i\int_{t_R}^{t_1}\varepsilon_j^S(t)dt\right) \\
&\quad \times M_{ji}^S exp\left(-i\int_{t_0}^{t_R}\varepsilon_i^S(t)dt\right)U_{\alpha i}^S \\
&= \sum_{k,j,i} |\nu_k^M\rangle exp\left(-i\varepsilon_k^M(t_3-t_2)\right) M_{kj}^M exp\left(-i\varepsilon_j(t_2-t_1) - i\int_{t_R}^{t_1}\varepsilon_j^S(t)dt\right) \\
&\quad \times M_{ji}^S exp\left(-i\int_{t_0}^{t_R}\varepsilon_i^S(t)dt\right)U_{\alpha i}^S
\end{aligned} \tag{16}$$

We have introduced the probability amplitude M_{kj}^M for non-adiabatic transitions $j \to k$ due to the abrupt change in density when the neutrino enters the Moon . It is given by

$$M_{kj}^M = \langle\nu_k^M|\nu_j\rangle = \sum_\gamma \langle\nu_k^M|\nu_\gamma\rangle\langle\nu_\gamma|\nu_j\rangle = \sum_\gamma U_{\gamma k}^M U_{\gamma j}^* \tag{17}$$

where $U_{\gamma j}$ is the mixing matrix in vacuum . We multiply the right hand side of eq.(16) by $\sum_l |\nu_l\rangle\langle\nu_l|$ where $|\nu_l\rangle$ (l =1,2,3) is the complete set of vacuum mass eigenstates. The neutrino leaves the other end of the Moon at $t = t_3$ and propagates upto the surface of Earth ,which it reaches at t_4 .So the state vector at t_4 is

$$\begin{aligned}
|\Psi_\alpha(t_4)\rangle &= \sum_{k,j,i,l} |\nu_l\rangle exp\left(-i\varepsilon_k^M(t_3-t_2)\right)\langle\nu_l|\nu_k^M\rangle M_{kj}^M exp\left(-i\varepsilon_j(t_2-t_1) - i\int_{t_R}^{t_1}\varepsilon_j^S(t)dt\right) \\
&\quad \times M_{ji}^S exp\left(-i\int_{t_0}^{t_R}\varepsilon_i^S(t)dt\right)U_{\alpha i}^S exp\left(-i\varepsilon_l(t_4-t_3)\right) \\
&= \sum_{k,j,i,l} |\nu_l\rangle M_{kj}^M M_{kl}^{M*} M_{ji}^S U_{\alpha i}^S exp\left(-i\Phi_{ijkl}\right)
\end{aligned} \tag{18}$$

where

$$\Phi_{ijkl} = \varepsilon_k^M(t_3-t_2) + \varepsilon_l(t_4-t_3) + \varepsilon_j(t_2-t_1) + \int_{t_R}^{t_1}\varepsilon_j^S(t)dt + \int_{t_0}^{t_R}\varepsilon_i^S(t)dt \tag{19}$$

We have used the fact that the the probability amplitude for non-adiabatic transitions $k \to l$ due to the abrupt change in density when the neutrino leaves the Moon is

$$\langle\nu_l|\nu_k^M\rangle = M_{kl}^{M*} \tag{20}$$

The probability of detecting a neutrino of flavor β at t_4 is

$$|\langle\nu_\beta|\Psi_\alpha(t_4)\rangle|^2 = \sum U^*_{\beta l} U_{\beta l'} M^M_{kj} M^{M*}_{k'j'} M^{M*}_{kl} M^M_{k'l'} M^S_{ji} M^{S*}_{j'i'} U^S_{\alpha i} U^{S*}_{\alpha i'} exp\left(-i(\Phi_{ijkl} - \Phi_{i'j'k'l'}\right) \quad (21)$$

where the summation is over the set of indices $i, j, k, l, i', j', k', l'$ Averaging over t_R leads to $\delta_{ii'}\delta_{jj'}$ and this results in the desired incoherent mixture of mass eigenstates of neutrinos reaching the surface of the Moon at t_2. Calling this averaged probability as $P^M_{\alpha\beta}$ (the probability for a neutrino produced in the Sun as ν_α to be detected as ν_β in Earth after passing through the Moon), we can write the result as

$$P^M_{\alpha\beta} = \sum_j P^S(\alpha \to j) P^M(j \to \beta) \quad (22)$$

where

$$P^S(\alpha \to j) = \sum_i |M^S_{ji}|^2 |U^S_{\alpha i}|^2 \quad (23)$$

$$P^M(j \to \beta) = \sum_{l,k,l',k'} U^*_{\beta l} U_{\beta l'} M^M_{kj} M^{M*}_{k'j} M^{M*}_{kl} M^M_{k'l'} exp\left(-i(\varepsilon^M_k - \varepsilon^M_{k'}) d_M - i(\varepsilon_l - \varepsilon_{l'}) r\right) \quad (24)$$

where we have replaced $(t_3 - t_2)$ by d_M, the distance traveled by the neutrino inside the Moon, and $(t_4 - t_3)$ by r the distance traveled by the neutrino from the Moon to Earth. If there is no Moon , we put $d_M = 0$, so that $P^M(j \to \beta)$ becomes $|U_{\beta j}|^2$ and so eq.(22) reduces to the usual [13,16] averaged probability for ν_α produced in the Sun to be detected as ν_β in Earth :

$$P^O_{\alpha\beta} = \sum_{i,j} |U_{\beta j}|^2 |M^S_{ji}|^2 |U^S_{\alpha i}|^2. \quad (25)$$

B. Regeneration during double eclipse.

We start with $\Psi_\alpha(t_4)$ given by eq.(18) and multiply the right hand side by $\sum_p |\nu^E_p\rangle\langle\nu^E_p|$ $(= 1)$ where $|\nu^E_p\rangle$ $(p = 1, 2, 3)$ is a complete set of mass eigenstates just below the surface of Earth. If the neutrino that enters Earth at time $t = t_4$ propagates adiabatically upto t_5 (non-adiabatic jumps during the propagation will be considered subsequently), its state vector at time $t = t_5$ is

$$|\Psi_\alpha(t_5)\rangle = \sum_{k,j,i,l,p} |\nu_p^E\rangle M_{pl}^E M_{kj}^M M_{kl}^{M*} M_{ji}^S U_{\alpha i}^S \exp\left(-i\Phi_{ijklp}\right) \tag{26}$$

where we have introduced the probability amplitude M_{pl}^E for non adiabatic transitions $l \rightarrow p$ due to the abrupt change in density when the neutrino enters the Earth . It is given by

$$M_{pl}^E = \langle \nu_p^E | \nu_l \rangle = \sum_\sigma U_{\sigma p}^E U_{\sigma l}^* \tag{27}$$

and

$$\Phi_{ijklp} = \int_{t_4}^{t_5} \varepsilon_p^E dt + \varepsilon_l (t_4 - t_3) + \varepsilon_k^M (t_3 - t_2) + \varepsilon_j (t_2 - t_1) + \int_{t_R}^{t_1} \varepsilon_j^S(t) dt + \int_{t_0}^{t_R} \varepsilon_i^S(t) dt \tag{28}$$

The probability of detecting a neutrino of flavor β at t_5 is

$$\begin{aligned} |\langle \nu_\beta | \Psi_\alpha(t_5)\rangle|^2 = & \sum U_{\beta p}^{E*} U_{\beta p'}^E M_{pl}^E M_{p'l'}^{E*} M_{kj}^M M_{k'j'}^{M*} M_{kl}^{M*} M_{k'l'}^M M_{ji}^S M_{j'i'}^{S*} U_{\alpha i}^S U_{\alpha i'}^{S*} \\ & \times \exp\left(-i(\Phi_{ijklp} - \Phi_{i'j'k'l'p'}\right) \end{aligned} \tag{29}$$

where the summation is over the set of indices $i, j, k, l, p, i', j', k', l'p'$ Again averaging over t_R and calling this averaged probability as $P_{\alpha\beta}^{ME}$ (the probability for a neutrino produced in the Sun as ν_α to be detected as ν_β in Earth after passing through the Moon and Earth), we can write the result as

$$P_{\alpha\beta}^{ME} = \sum_j P^S(\alpha \rightarrow j) P^{ME}(j \rightarrow \beta) \tag{30}$$

where

$$\begin{aligned} P^{ME}(j \rightarrow \beta) = & \sum_{l,k,p,l',k',p'} U_{\beta p}^{E*} U_{\beta p'}^E M_{pl}^E M_{p'l'}^{E*} M_{kj}^M M_{k'j}^{M*} M_{kl}^{M*} M_{k'l'}^M \\ & \times exp\left(-i \int_{t_4}^{t_5} (\varepsilon_p^E(t) - \varepsilon_{p'}^E(t)) dt - i(\varepsilon_k^M - \varepsilon_{k'}^M) d_M - i(\varepsilon_l - \varepsilon_{l'}) r\right) \end{aligned} \tag{31}$$

where we have replaced $(t_3 - t_2)$ by d_M, the distance traveled by the neutrino inside the Moon and $(t_4 - t_3)$ by r the distance traveled by the neutrino from the Moon to Earth.

We next show how to take into account non-adiabatic jumps during the propagation inside Earth. Consider ν propagation through a series of slabs of matter, density varying inside each slab smoothly but changing abruptly at the junction between adjacent slabs.

The state vector of the neutrino at the end of the n^{th} slab $|n\rangle$ is related to that at the end of the $(n-1)^{th}$ slab $|n-1\rangle$ by $|n\rangle = F^{(n)}M^{(n)}|n-1\rangle$ where $M^{(n)}$ describes the non-adiabatic jump occuring at the junction between the $(n-1)^{th}$ and n^{th} slabs while $F^{(n)}$ describes the adiabatic propagation in the n^{th} slab. They are given by

$$M_{ij}^{(n)} = \langle \nu_i^{(n)} | \nu_j^{(n-1)} \rangle = (U^{(n)^\dagger} U^{(n-1)})^*_{ij}\,, \tag{32}$$

$$F_{ij}^{(n)} = \delta_{ij} exp \left(-i \int_{t_{n-1}}^{t_n} \varepsilon_i(t) dt \right)\,, \tag{33}$$

where the indices (n) and $(n-1)$ occuring on ν and U refer respectively to the n^{th} and $(n-1)^{th}$ slabs at the junction between these slabs. Also note that $M^{(1)}$ is the same as M^E defined in eq.(27). Defining the density matrix at the end of the n^{th} slab as $\rho^{(n)} = |n\rangle\langle n|$, we have the recursion formula

$$\rho^{(n)} = F^{(n)} M^{(n)} \rho^{(n-1)} M^{(n)\dagger} F^{(n)\dagger}. \tag{34}$$

Corresponding to the state vector $|\Psi_\alpha(t_4)\rangle$ (eq.18) of the ν entering Earth, the density matrix is $|\Psi_\alpha(t_4)\rangle\langle\Psi_\alpha(t_4)|$. After averaging over t_R, we call it $\rho^{(0)}$ and calculate $\rho^{(N)}$ recursively at the end of the N^{th} slab using (34). The probability of observing ν_β at the end of the N^{th} slab is

$$P_{\alpha\beta}^{ME} = \langle \nu_\beta | \rho^{(N)} | \nu_\beta \rangle = (U^{(N)} \rho^{(N)^*} U^{(N)^\dagger})_{\beta\beta}\,. \tag{35}$$

This formula (which reduces to eq.(30) and (31) for $N = 1$) can be used for Earth modeled as consisting of $(N+1)/2$ concentric shells, with the density varying gradually within each shell. For the Earth, the major discontinuity in the density occurs at the boundary between mantle and core and adequate accuracy can be achieved with $N = 3$ (mantle and core). However, for $(d_E/2R_E) < 0.84$ neutrinos pass only through mantle and so, $N = 1$.

For the sake of completeness, we state that if we put $d_M = 0$ in eq.(31) we get the regeneration in Earth alone:

$$P_{\alpha\beta}^{E} = \sum_j P^S(\alpha \to j) P^E(j \to \beta) \tag{36}$$

where

$$P^E(j \to \beta) = \sum_{k,k'} U^{E*}_{\beta k} U^{E}_{\beta k'} M^{E}_{kj} M^{E*}_{k'j} exp\left(-i \int_{t_4}^{t_5} (\varepsilon^E_p(t) - \varepsilon^E_{p\prime}(t))dt\right) \tag{37}$$

Eqs.(36) and (37) have been used to study the day–night effect [6].

It is important to note that the factorization of probabilities seen in eqs(22),(30) and (36) is valid only for mass eigenstates in the intermediate state. An equivalent statement of this result is that the density matrix is diagonal only in the mass-eigenstate representation and not in the flavour representation.

C. Three flavor mixing parameters

We parameterize the mixing matrix U in vacuum as $U = U^{23}(\psi)U^{13}(\phi)U^{12}(\omega)$ where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the i^{th} and the j^{th} mass eigenstates with the mixing angle θ_{ij}, neglecting CP violation. In the solar neutrino problem ψ drops out [14,15] The mass differences in vacuum are defined as $\delta_{21} = \mu_2^2 - \mu_1^2$ and $\delta_{31} = \mu_3^2 - \mu_1^2$. It has been shown [16,17] that the simultaneous solution of both the solar and the atmospheric neutrino problems requires

$$\delta_{31} \gg \delta_{21} \tag{38}$$

and under this condition δ_{31} also drops out. The rediagonalization of the mass matrix in the presence of matter (in solar core, Moon or Earth) under condition (38) leads to the following results [16]

$$\tan 2\omega_m = \frac{\delta_{21} \sin 2\omega}{\delta_{21} \cos 2\omega - A \cos^2 \phi} \tag{39}$$

$$\sin \phi_m = \sin \phi \tag{40}$$

$$\delta^m_{21} = \delta_{21} \cos 2(\omega - \omega_m) - A \cos^2 \phi \cos 2\omega_m \tag{41}$$

where A is the Wolfenstein term $A = 2\sqrt{2}\ G_F\ N_e\, E$ (N_e is the number density of electrons and E is the neutrino energy) . We note that $\delta_{31} \gg A^S, A^M, A^E$. In eqs.(39)–(41) the subscript "m" stands for matter. Under the condition $\delta_{31} \gg A \approx \delta_{21}$ we need the non adiabatic transition probability $|M^S_{ij}|^2$ for i,j =1,2 only and this is taken to be the modified Landau–Zener jump probability for an exponentially varying solar density [15].

IV. CALCULATIONS AND RESULTS

The neutrino detection rates for a Super–Kamoika type of detector is given by

$$R = \int \phi(E)\, \sigma(E) P_{ee} dE + \frac{1}{6}(\int (\phi(E)\sigma(E)(1 - P_{ee})dE) \tag{42}$$

where the second term is the neutral current contribution for $\nu_\alpha(\alpha \neq e)$ and $\phi(E)$ is the solar neutrino flux as a function of the neutrino energy E and $\sigma(E)$ is the cross section from neutrino electron scattering and we integrate from $5MeV$ onwards. The cross section is taken from [18] and the flux from [19]. The rates for a single eclipse, double eclipse and without eclipse (at day–time) R_M, R_{ME} and R_O are calculated using P^M_{ee}, P^{ME}_{ee} and P^O_{ee} from eqns.(22),(30) and (25) respectively. We define the enhancement factors F and G for a single and double eclipse respectively:

$$F = \frac{R_M - R_O}{R_O} \tag{43}$$

$$G = \frac{R_{ME} - R_O}{R_O}. \tag{44}$$

It is easy to see that F and G have to be less than 5 and this theoretical maximum value occurs when $P^O_{ee} = 0$ and P^M_{ee} and P^{ME}_{ee} are put 1. If one imposes the constraint that the observed [20] neutrino rate is 0.51 ± 0.07 of the prediction of the standard solar model, the maximum possible enhancement is reduced to about 1.40 (at 90% C.L.).

Although we have given the theory for double eclipse in Sec.III B, we shall restrict the detailed calculations to the single eclipse in the present paper. For the double eclipse we present results only for the model of Earth with constant density (5.52 gms/cc). This can be taken as a rough guide and detailed calculations for the double eclipse are reserved for the future.

We calculate the enhancement factors F and G for various values of the neutrino parameters, ω, δ_{21}, and ϕ. We show the results as contour plots in the δ_{21}–ω plane for different values of ϕ. Figs.6 and 7 show the F–contours for $\phi = 0^o$ and $\phi = 30^o$ respectively. For each ϕ we show the contours for different distances of travel of the neutrino through the Moon. Fig. 8 shows the G–contours for $\phi = 0$ for the maximum distance of travel of the neutrino inside the Moon and Earth. The main features of the results are as follows:

- As the distance traveled by the neutrino inside the Moon increases one can see an appreciable increase in the enhancement factor F. It increases from less than 10% to about almost 100% when the neutrino travels the whole diameter of the Moon in the case of two flavor mixing i.e $\phi = 0$.

- Large ($> 40\%$) values of F occur for ω between 20^o and 30^o and $\delta_{21} \sim 10^{-6} eV^2$ This is true even for nonzero ϕ.

- It may be noted from Fig.6 that the maximum enhancement region for $\phi = 0$ is not far away from the present (2–generation) best fit parameters of Super Kamiokanda [21]($\delta_{21} = 1.4 \times 10^{-7} eV^2; \omega = 22^o$), which may be encouraging for the observation of the eclipse effect.

- The effect of a non zero "13" mixing angle ϕ, is to dilute the enhancement factor F for all values of distance traveled through the Moon.(In fact for $\phi \approx 45^o$, F is practically zero and so we have not plotted this case.) This is because a non zero ϕ means $\nu_e \leftrightarrow \nu_\tau$ oscillations, and matter cannot reconvert ν_τ back to ν_e, because the "13" mixing angle ϕ is not affected by matter.

- If large enhancement F is seen for values of $x \leq 0.6$, it immediately signals a very small value of ϕ. On the other hand, if no enhancement is seen for small x but there is enhancement only for $x \geq 0.8$ it signals an appreciable value of ϕ.

- For a double eclipse there are considerable enhancements even for small values of ω. There is enhancement throughout the range of ω from small angles till about

40^o. In fact the regions of largest enhancement ($> 100\%$) are for ω between 5^o to 20^o. Although the region of maximum enhancement factor G is centered around $\delta_{21} \sim 3 \times 10^{-6} eV^2$,sizeable enhancement occurs over a wide range of δ_{21}. However, the details are likely to change when calculations are done for a more realistic density distribution inside Earth.

- If enhancement is not seen, then certain regions of the neutrino parameter space can be excluded.If no enhancement is seen for single eclipse, a panel of ω between $5 - 25^o$ and $\delta_{21} \approx 2 \times 10^{-7} - 2 \times 10^{-6} eV^2$ for $\phi = 0$ can be ruled out. If it is not seen for a double eclipse, a larger region can be ruled out.

Having calculated the enhancements as a function of the distance traveled, we may now estimate roughly the enhancement in the event rate for an eclipse at a particular site. For this purpose we choose an eclipse with a rather long duration to evaluate the enhancement. One such eclipse occurs rather soon at Gran Sasso on August 11, 1999 with 84 minutes of duration with nonzero x. A simple estimate based on the enhancements presented in Fig.6 integrated using the time–dependence presented in Fig.5 for this eclipse, leads one to conclude that the total neutrino count during this 84 minutes could be enhanced by a factor of 1.5 for $\phi = 0$, $\omega = 22^o$, $\delta_{21} = 10^{-6} eV^2$ and it is less for other values of the neutrino parameters.

Our calculations thus indicate that the observation of the eclipse effect is *not* possible at the present detectors as the counting rates currently are no more than about one per hour. However, the calculated enhancements are not too small to be discarded completely. We envisage that the eclipse effect can become detectable in the near future in two or three ways. Experimental neutrino physics will continue to cross new frontiers with innovative techniques leading to counting rates that are larger by an order of magnitude. One such proposal that has been already made is the Borex detector [22] where the counting rate can be as large as 40 per hour. With such a detector, our calculations of enhancement factors show that the eclipse effect would be clearly observable. Another possibility is the accumulation of data during a large number of eclipses to obtain a statistically significant sample, which may enable one to observe an enhancement or to rule out a given parameter space. Yet another interesting possibility is to exploit the fact that some eclipses occur simultaneously at two sites. Correlation between the data collected at the two sites can enhance the statistical significance of scanty data.

It must be remembered that the counting rates at the present–day detectors such as Super-Kamiokanda could hardly have been anticipated 20 years back. One of the strengths of neutrino physics is that even when counting rates have been small the accumulated effect has stood the test of time over a 30 year period. The eclipse effect is perhaps an effect which may be observable only in a prolonged study and ours is only an effort to initiate such a study. It is also pertinent to remark here that although the original proposal to observe the shadow of the Moon [23] in high-energy cosmic rays was made in 1957 [24], it took more than 30 years to observe it [25].

V. DISCUSSION

We have studied the effect on the solar neutrinos of their passage through the Moon as well as the Moon together with Earth. Although the numerical results presented in the paper cover only a representative sample of the set of various parameters, our analytical expressions can be used for more extensive calculations depending on the requirement. Also one can go beyond the hierarchy : $\delta_{31} >> \delta_{21}$.

We now offer a few concluding remarks:

- Together with the day-night effect, the eclipse effects provide us with the tools for studying solar neutrinos, in a way independent of the uncertainties of the solar models.

- If the neutrino mass differences are really very small ($\delta_{21} < 10^{-5} eV^2$) there is no way of pinning down the neutrino parameters other than using the astronomical objects such as the Moon or Earth for the "long-base-line experiments".

- It is important to stress that even the demonstrated absence of any eclipse effect would provide us with definitive information on neutrino physics.

- Accumulation of data over many eclipses may be needed for good statistics. In any future planning of detector sites, this may be kept in mind.

- It appears that Nature has chosen the neutrino parameters in such a way that the Sun affects the propagation of solar neutrinos. It may be hoped that Nature has similarly chosen "lucky" parameters so that the Moon and the Earth too can affect the neutrinos!

- Finally, we stress the novelty of the whole phenomenon, and urge the experimentalists to look for and study the eclipse effects in an unbiased manner. They may even discover some surprises, not predicted by our calculations!

Acknowledgments: We thank KVL Sarma for bringing ref [9] to our attention, N.D. Hari Dass for raising the possibility of detecting the neutrinos on the opposite side of Earth during the eclipse, and M.C Sinha for much help with the astronomical aspects of the problem and Sandip Pakvasa for useful discussions.

FIGURES

FIG. 1. The fractional distance traveled by the neutrino inside the Moon $x(=\frac{d_M}{2R_M})$ is plotted against the optical coverage C of the solar eclipse.

FIG. 2. Geometry relating the double eclipse point (λ, σ) to the single eclipse point (α, β). (a) Section of Earth passing through (α, β) and perpendicular to the ecliptic. (b) Section passing through (α, β) and parallel to the equator.

FIG. 3. The fractional distance travelled by the neutrino inside the Moon (x) and the optical coverage (C) are plotted as a function of time t (in minutes), for five eclipses visible from Kamiokanda.

FIG. 4. Same as Fig. 3, for six eclipses visible from Sudbury.

FIG. 5. Same as Fig. 3, for five eclipses visible from Gran Sasso.

FIG. 6. Contour plots of the enhancement factor for single eclipse $F(=\frac{R_M - R_O}{R_O})$ in the $\omega - \delta_{21}$ plane for $\phi = 0^o$ and for four values of x (x = 0.4, 0.6, 0.8 and 1.0). The enhancement factor (regarded as a percentage) increases by 10% for every adjacent ring , as we move inwards towards the center of the plot.

FIG. 7. Contour plots of the enhancement factor for single eclipse $F(=\frac{R_M - R_O}{R_O})$ in the $\omega - \delta_{21}$ plane for $\phi = 30^o$ and x = 0.6, 0.8 and 1.0. The enhancement factor (regarded as a percentage) increases by 10% for every adjacent ring , as we move inwards towards the center of the plot.

FIG. 8. Contour plot of the enhancement factor for double eclipse $G(=\frac{R_{ME} - R_O}{R_O})$ for $\phi =$ 0 and x= 1.0. The distance travelled by the neutrino inside the Earth is also taken to be the full Earth diameter. The enhancement factor increases by 20% as we move inwards.

TABLES

Date	UT	Duration (T)	Solar Position		C (at max)
			altitude	azimuth	
Mar. 9, 1997	1:06	143 min	$42^o16'$	$143^o28'$	0.53
Feb. 26, 1998	16:57	67 min	$-50^o25'$	$53^o07'$	0.23
Aug. 11, 1999	11:46	74 min	$-23^o11'$	$312^o17'$	0.30
Dec. 14, 2001	19:16	108 min	$-28^o56'$	$98^o47'$	0.71
Jun. 10, 2002	22:37	122 min	$36^o33'$	$86^o37'$	0.32

TABLE I. Eclipse parameters for eclipses visible from Super Kamiokanda (latitude: 36^o 24' N, longitude: 140^o 0' E) between 1997 and 2002. Date, Universal Time, the Sun's position (given as altitude and azimuth from the observer's position, with negative altitude indicating a double eclipse since the direction is below the horizon) and Optical Coverage (C) are all given for the instant of maximum coverage. Eclipse Duration is given in minutes.

Date	UT	Duration (T)	Solar Position		C (at max)
			altitude	azimuth	
Mar. 9, 1997	2:36	58 min	-33^{o}04'	303^{o}19'	0.13
Feb. 26, 1998	18:08	31 min	34^{o}28'	189^{o}02'	0.003
Aug. 11, 1999	9:40	100 min	-6^{o}28'	59^{o}34'	0.72
Dec. 25, 2000	17:29	189 min	20^{o}07'	181^{o}07'	0.51
Jul. 31, 2000	2:45	72 min	-15^{o}19'	319^{o}24'	0.32
Dec. 14, 2001	21:57	89 min	-3^{o}43'	239^{o}35'	0.09
Jun. 11, 2002	1:05	59 min	0^{o}43'	303^{o}45'	0.08

TABLE II. Eclipse parameters for eclipses visible from Sudbury (SNO) (latitude: 46^o 29' N, longitude: 81^o 0' W) between 1997 and 2002. Date, Universal Time, the Sun's position (given as altitude and azimuth from the observer's position, with negative altitude indicating a double eclipse since the direction is below the horizon) and Optical Coverage (C) are all given for the instant of maximum coverage. Eclipse Duration is given in minutes.

Date	UT	Duration (T)	Solar Position		C (at max)
			altitude	azimuth	
Mar. 9, 1997	1:09	78 min	-44^{o}34'	41^{o}15'	0.49
Feb. 26, 1998	18:57	93 min	-23^{o}47'	280^{o}32'	0.59
Aug. 11, 1999	10:44	171 min	62^{o}12'	165^{o}46'	0.83
Jul. 31, 2000	1:14	45 min	-23^{o}07'	31^{o}03'	0.07
Dec. 14, 2001	21:20	96 min	-61^{o}34'	304^{o}37'	0.85

TABLE III. Eclipse parameters for eclipses visible from Gran Sasso (latitude: 42^o 29' N, longitude: 13^o 30' E) between 1997 and 2002. Date, Universal Time, the Sun's position (given as altitude and azimuth from the observer's position, with negative altitude indicating a double eclipse since the direction is below the horizon) and Optical Coverage (C) are all given for the instant of maximum coverage. Eclipse Duration is given in minutes.

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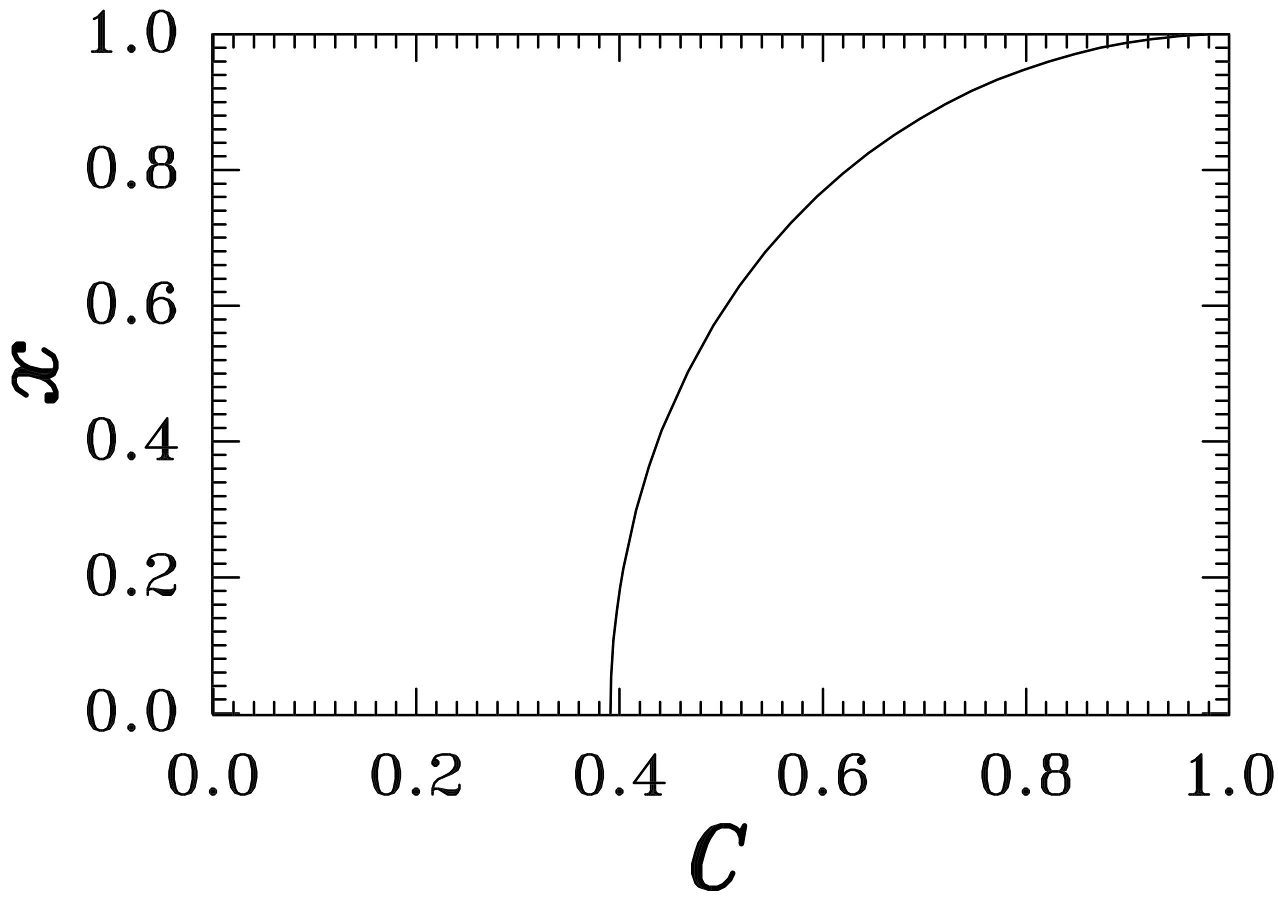
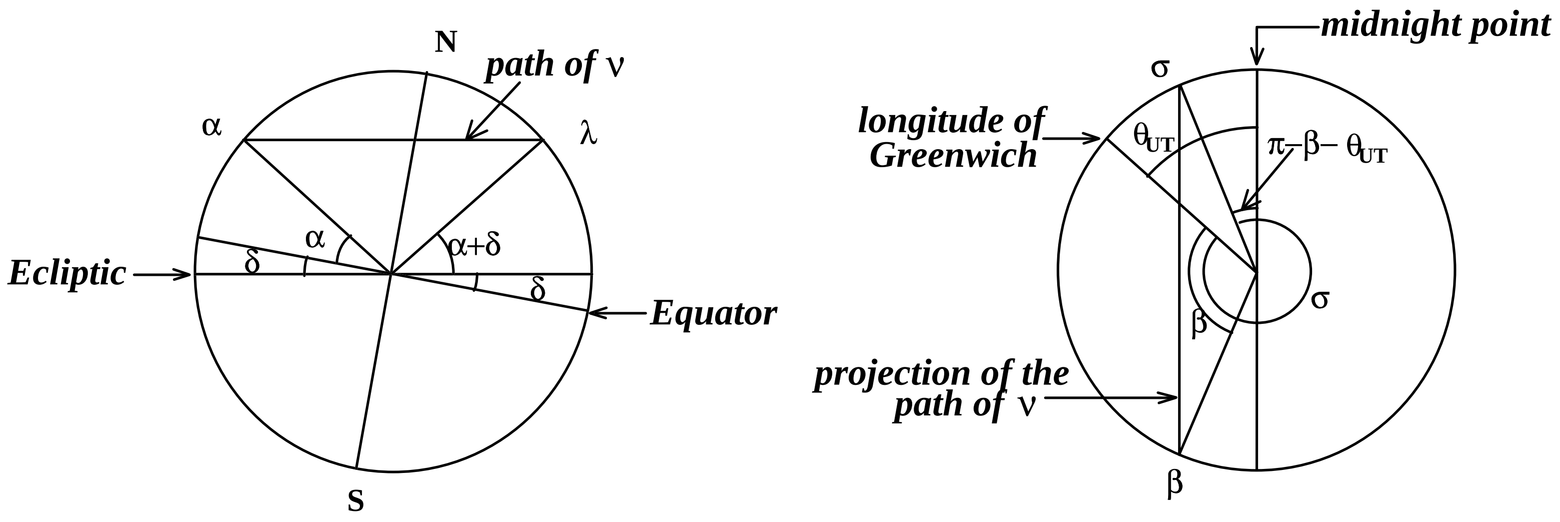
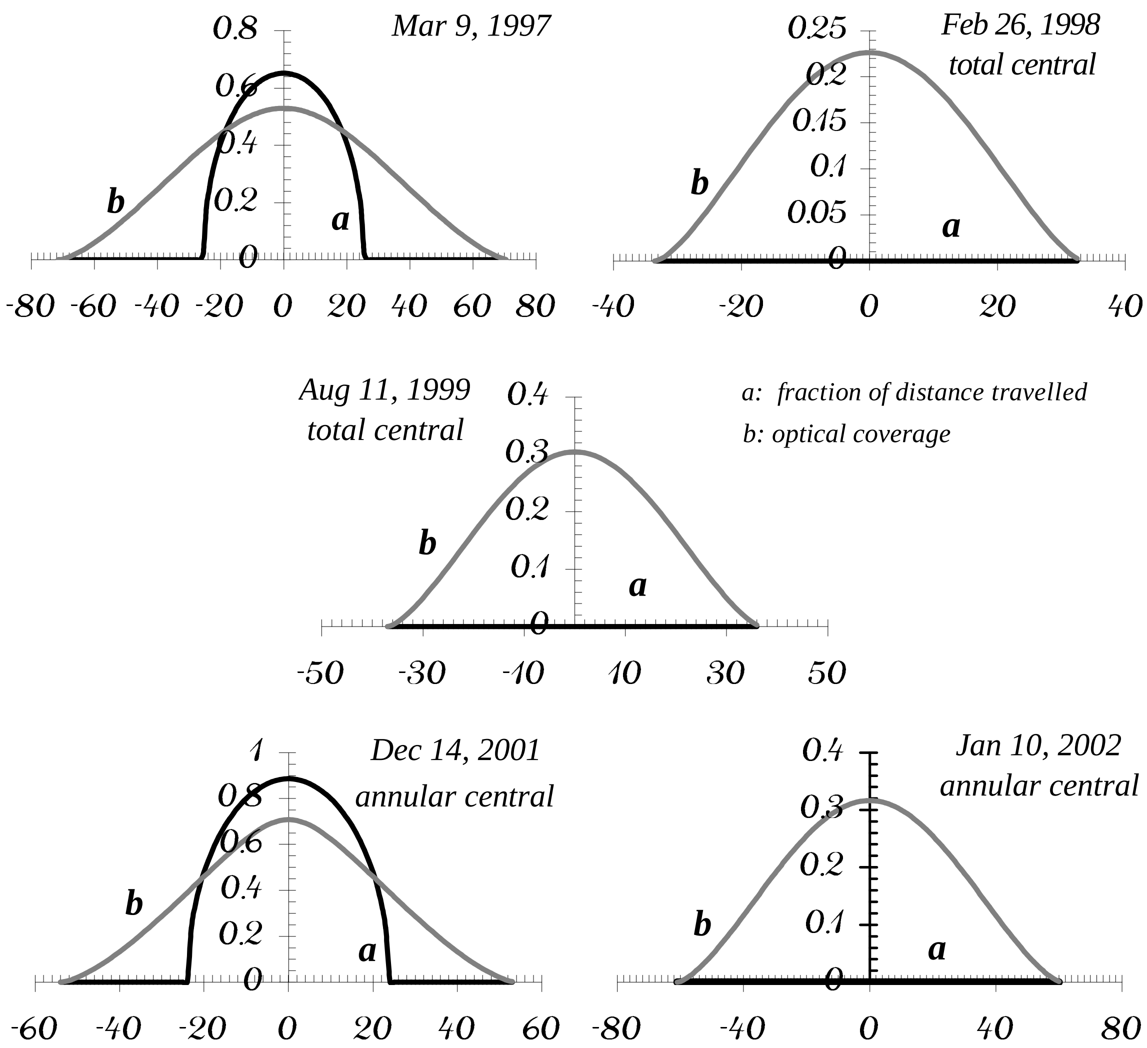


Fig 1


N
path of ν
α
λ
α
α+δ
Ecliptic
δ
δ
Equator
S
midnight point
σ
longitude of
Greenwich
θUT
π–β– θUT
σ
β
projection of the
path of ν
β

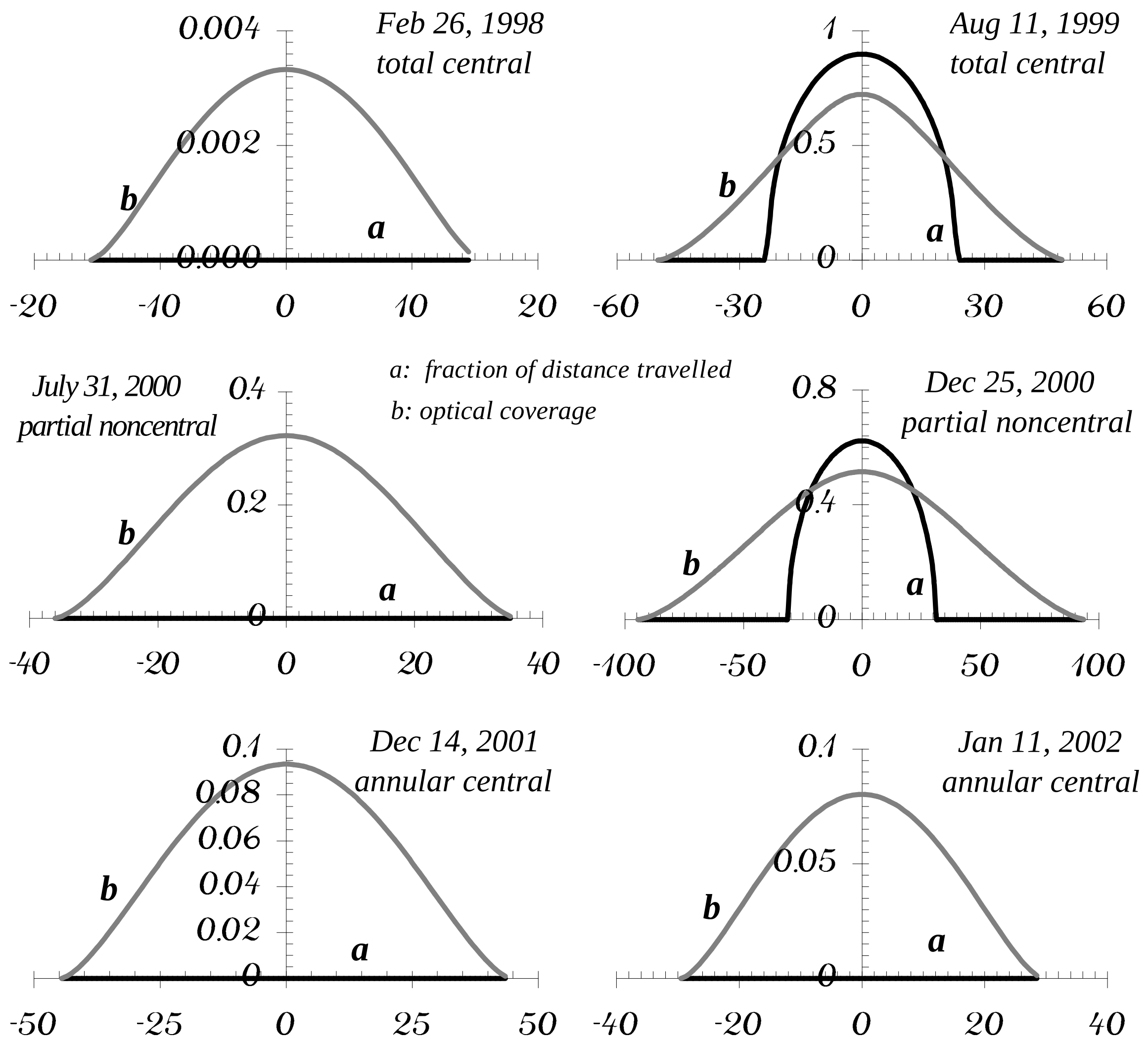
(a) (b)

Fig 2



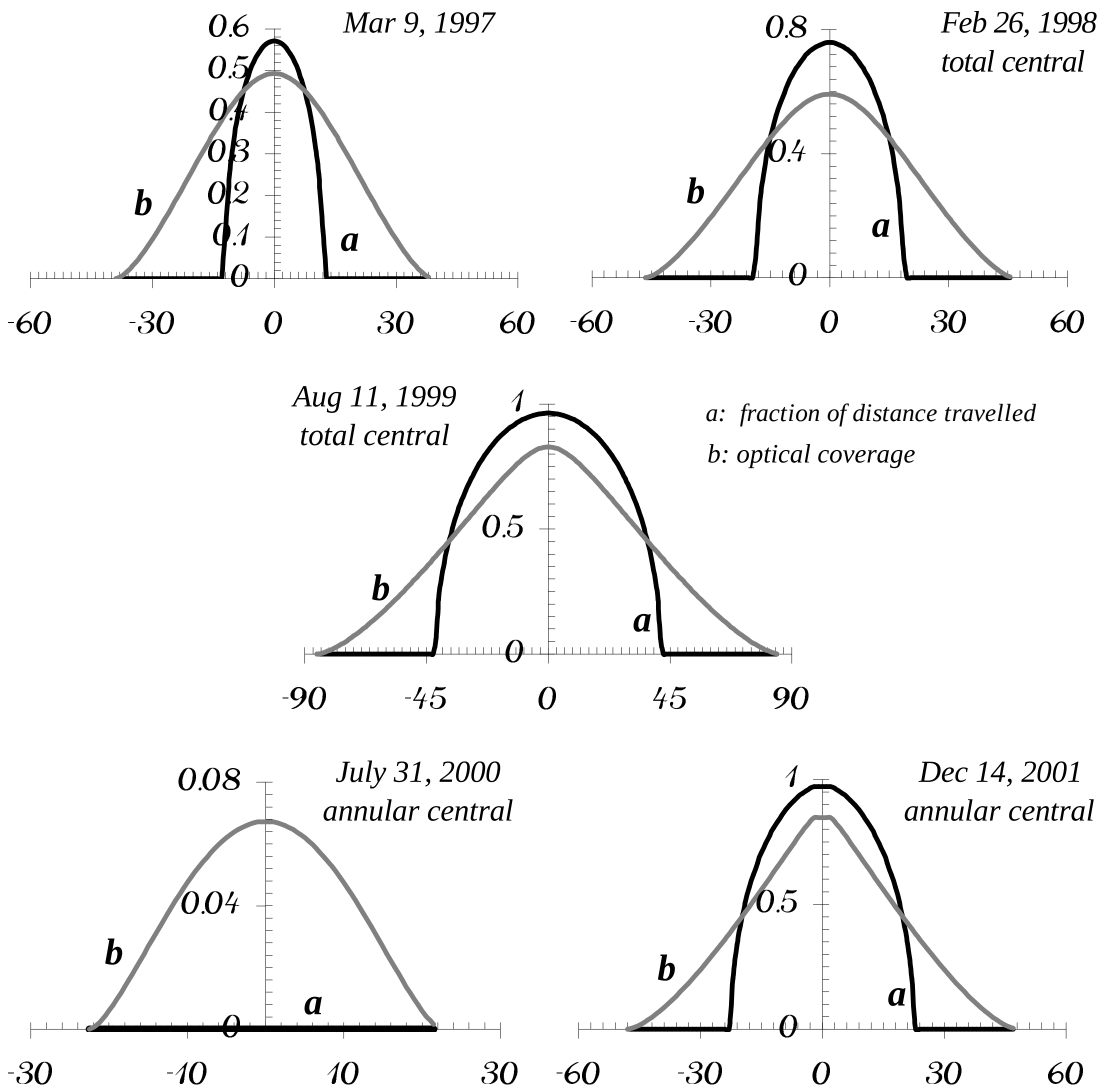
Kamioka

Fig 3



Sudbury

Fig 4



Gran Sasso

Fig 5

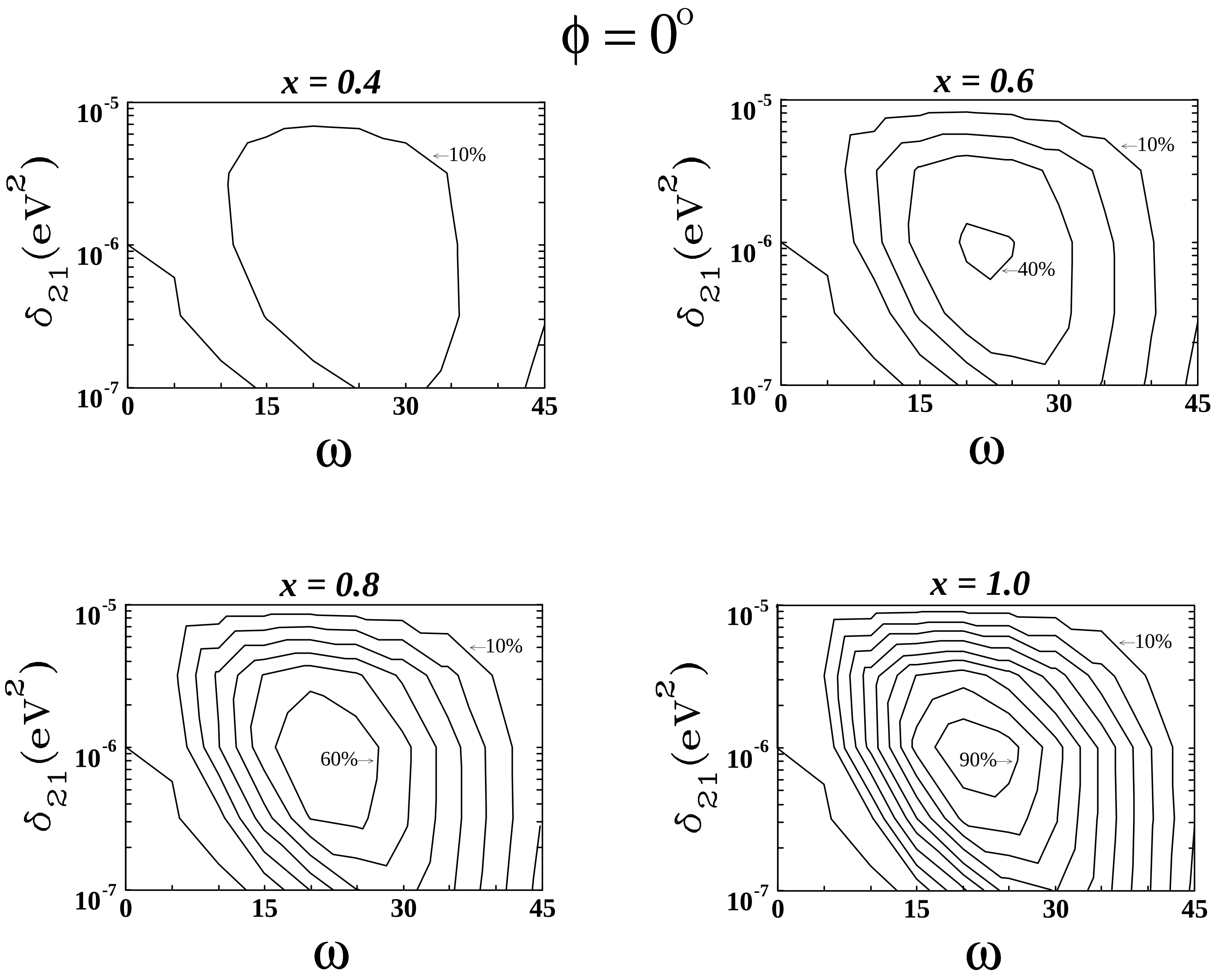


Fig 6

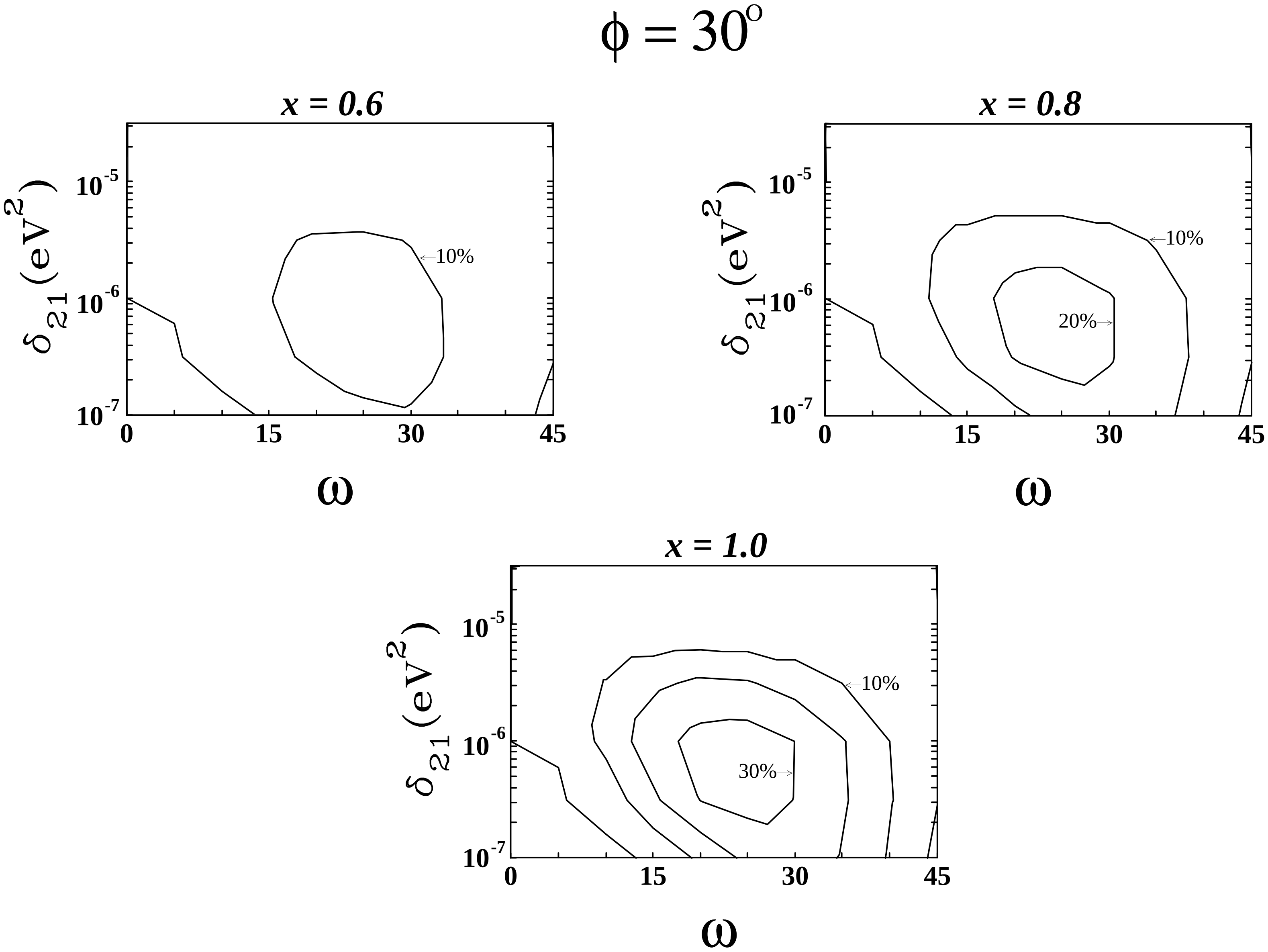


Fig 7

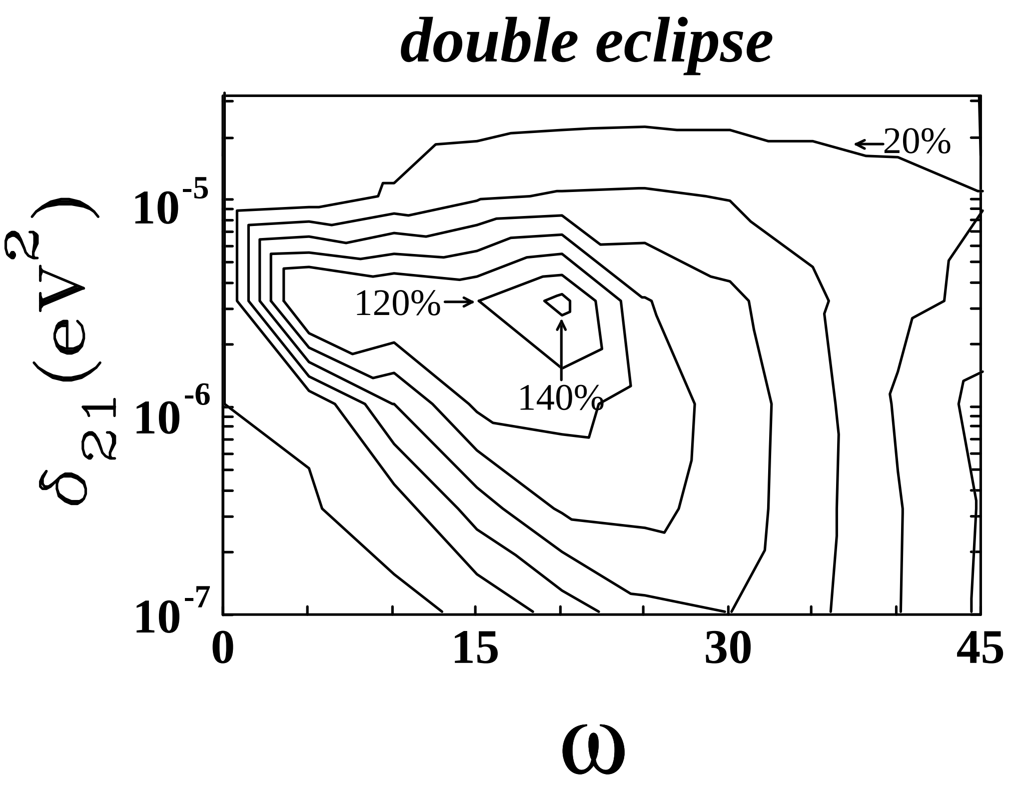


Fig 8