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Hierarchical Four-Neutrino Oscillations With a Decay Option

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Abstract

We present a new and novel synthesis of all existing neutrino data regarding the disappearance and appearance of ν_e and ν_μ. We assume four neutrinos: ν_e, ν_μ, ν_τ, as well as a heavier singlet neutrino ν_s of a few eV. The latter may decay into a massless Goldstone boson (the singlet Majoron) and a linear combination of the doublet antineutrinos. We comment on how this scenario may be verified or falsified in future experiments.

Accepting the totality of present experimental evidence for neutrino oscillations[1, 2, 3], it is not unreasonable to entertain the idea that there are four light neutrinos. Since the invisible decay of the Z boson tells us that there are only three light doublet neutrinos, i.e. ν_e, ν_μ, ν_τ, the fourth light neutrino ν_s should be a singlet. Usually, ν_s is assumed to mix with the other neutrinos in a 4×4 mass matrix for a phenomenological understanding[4] of all the data. However, given that ν_s is different from $\nu_{e,\mu,\tau}$, it may have some additional unusual property, such as decay. In fact, as shown below, this is a natural consequence of the spontaneous breakdown of lepton number in the simplest model[5], and it has some very interesting and verifiable predictions in future neutrino experiments.

If only atmospheric[1] and solar[2] neutrino data are considered, then hierarchical three-neutrino oscillations with

$$\nu_1 = \nu_e \cos\theta - \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau)\sin\theta, \tag{1}$$

$$\nu_2 = \nu_e \sin\theta + \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau)\cos\theta, \tag{2}$$

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau), \tag{3}$$

where $m_1 << m_2 << m_3$, would fit the data very well. Here $m_3^2 \sim 10^{-3}$ eV^2, $(\sin^2 2\theta)_{atm} = 1$, and $m_2^2 \sim 10^{-5}$ eV^2 for the matter-enhanced oscillation solution[6] to the solar neutrino deficit with $(\sin^2 2\theta)_{sol} \sim 10^{-3}$ or near 1, or $m_2^2 \sim 10^{-10}$ eV^2 for the vacuum oscillation solution with $(\sin^2 2\theta)_{sol} \sim 1$.

We now add a fourth neutrino ν_s and assume that it mixes a little with ν_e and ν_μ to explain the LSND data[3]. Since the relevant Δm^2 is now about 1 eV^2, it is natural to take $m_4^2 \sim 1$ eV^2, but this hierarchical solution is disfavored[7], because the observed $\bar\nu_\mu \to \bar\nu_e$ probability[3] is contradicted by the $\nu_\mu \to \nu_\mu$ data of CDHSW[8] together with the $\bar\nu_e \to \bar\nu_e$ data of Bugey[9]. However, there are two ways that this conclusion may be evaded. (1) Let $m_4^2 \sim 25$ eV^2, then the constraint due to the CDHSW experiment is not a factor, but now there are three other accelerator $\nu_\mu \to \nu_e$ experiments: BNL-E734[10], BNL-E776[11], and CCFR[12], which have bounds close to but allowed by the LSND 99% likelihood contour. This is a marginal hierarchical four-neutrino oscillation solution to all the data. (2) If ν_4 decays, then the parameter space for an acceptable solution should open up. For example, in the CDHSW experiment, two detectors at different distances compare their respective ν_μ fluxes and the ratio is taken. If the ν_4 component of ν_μ decays away already before reaching the first detector, the ratio remains at unity. In contrast to the case of only oscillations, this experiment is then unable to restrict m_4^2. Not only that, since the argument[7] against the hierarchical four-neutrino spectrum depends crucially on the CDHSW experiment, it is clear that it cannot be valid in general.

The idea of neutrino decay is of course not new. It is naturally related to the spontaneous breakdown of lepton number[5, 13]. The associated massless Nambu-Goldstone boson[14] is called the Majoron and the typical decay $\nu_2 \to \bar{\nu}_1 +$ Majoron occurs if kinematically allowed. The triplet Majoron[13] is ruled out experimentally because the decay $Z \to$ Majoron + partner (imaginary and real parts respectively of the lepton-number carrying scalar field) would have counted as the equivalent of two extra neutrino flavors. The singlet Majoron[5] is unconstrained because it has no gauge interactions. We assign lepton number $L = -1$ to ν_s and assume the existence of a scalar particle χ^0 with $L = 2$. [By convention, ν_s is left-handed. If we use a right-handed singlet neutrino ν_R instead, then it would be assigned $L = +1$.] Hence the relevant terms of the interaction Lagrangian are given by

$$\mathcal{L}_{int} = g_s \nu_s \nu_s \chi^0 + \sum_{\alpha=e,\mu,\tau} h_\alpha \nu_s (\nu_\alpha \phi^0 - l_\alpha \phi^+) + h.c. \tag{4}$$

As $\langle \chi^0 \rangle$ and $\langle \phi^0 \rangle$ become nonzero, ν_s becomes massive and also mixes with $\nu_{e,\mu,\tau}$ to form the mass eigenstates $\nu_{1,2,3,4}$. At the same time, $\sqrt{2} Im \chi^0$ becomes the massless Majoron M and the decay

$$\nu_4 \to \bar{\nu}_{1,2,3} + M \tag{5}$$

is now possible. Neutrino decay involving only $\nu_{e,\mu,\tau}$ was recently proposed[15] to explain the atmospheric data[1], but that becomes a poor fit after the inclusion of the upward going muons[16]. More recently, it was shown[17] that combining oscillation and decay (at the expense of also adding ν_s) gives again a good fit. In contrast, the effects we envisage here of ν_4 decay in atmospheric and solar neutrino data are both small and do not change the usual oscillation interpretation appreciably, as shown below.

Let $\nu_{e,\mu,\tau,s}$ be related to the mass eigenstates $m_{1,2,3,4}$ through the unitary matrix $U_{\alpha i}$, which will be assumed real in the following for simplicity. Let $m_4 >> m_3 >> m_2 >> m_1$ with ν_4 having the decay lifetime τ_4. Then for solar and atmospheric neutrino oscillations with $m_4^2 L/4E >> 1$, the probability of $\nu_\alpha \to \nu_\beta$ is given by

$$P_{\alpha\beta} = \delta_{\alpha\beta}(1 - 2U_{\alpha 4}^2) + U_{\alpha 4}^2 U_{\beta 4}^2 (1 + x^2) - 4 \sum_{i<j<4} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \frac{\Delta m_{ij}^2 L}{4E}, \tag{6}$$

where

$$x = e^{-m_4 L/2E\tau_4}. \tag{7}$$

In the case of laboratory experiments where $\Delta m_{ij}^2 L/4E << 1$ for $i < j < 4$ but $m_4^2 L/4E$ is not necessarily large or small, the corresponding formula is

$$P_{\alpha\beta} = \delta_{\alpha\beta} \left[1 - 2U_{\alpha 4}^2 \left(1 - x \cos \frac{m_4^2 L}{2E} \right) \right] + U_{\alpha 4}^2 U_{\beta 4}^2 \left[1 - 2x \cos \frac{m_4^2 L}{2E} + x^2 \right]. \tag{8}$$

Note that the above expression simplifies to a function of $U_{\alpha 4}$, $U_{\beta 4}$, and x if m_4 is large, and to a function of $U_{\alpha 4}$ and $U_{\beta 4}$ alone if $x = 0$ whatever the value of m_4. In those circumstances, the corresponding laboratory experiment has no sensitivity to oscillations, but does measure one fixed number. Specifically, if m_4 is large, then

$$P_{\mu e} = U_{e4}^2 U_{\mu 4}^2 (1 + x^2), \quad P_{ee} = (1 - U_{e4}^2)^2 + x^2 U_{e4}^4, \quad P_{\mu\mu} = (1 - U_{\mu 4}^2)^2 + x^2 U_{\mu 4}^4. \tag{9}$$

If $x = 0$, then regardless of m_4, Eq. (8) reduces to Eq. (9) but with x set equal to zero. The

LSND experiment obtains[3]

$$P_{\mu e} = 3.1 \begin{array}{c} +1.1 \\ -1.0 \end{array} \pm 0.5 \times 10^{-3}, \tag{10}$$

whereas BNL-E734 has[10] $P_{\mu e} < 1.7 \times 10^{-3}$ and BNL-E776 has[11] $P_{\mu e} < 1.5 \times 10^{-3}$. Using the LSND 90% confidence-level limit of $P_{\mu e} > 1.3 \times 10^{-3}$, we find therefore reasonable consistency among these experiments. [The most recent result of the ongoing KARMEN II experiment[18] is $P_{\mu e} < 2.1 \times 10^{-3}$, which will eventually have the sensitivity to test Eq. (10).] The recent CCFR experiment[12] measures $P_{\mu e} < 0.9 \times 10^{-3}$, but its average L/E is one to two orders of magnitude smaller than those of the other experiments, hence its x-value may be taken to be close to one and the usual oscillation interpretation of the data holds. This constraint implies that $m_4^2 < 30$ eV^2.

At $m_4 \sim 5$ eV, we are below the CCFR exclusion and in a marginal region of the parameter space for pure neutrino oscillations consistent with the LSND evidence and the exclusion from BNL-E734 and BNL-E776. Between $m_4 \sim 5$ eV and $m_4 \sim 3$ eV, the BNL-E734 data exclude a solution if $x = 1$ and because that experiment has an average L/E an order of magnitude smaller than that of BNL-E776, LSND, or CDHSW, the decay factor goes against having a consistent solution here even if $x < 1$. Below $m_4 \sim 3$ eV, the oscillation + decay interpretation of the latter 3 experiments becomes important, as shown below.

Ideally, one should reanalyze the results of all the laboratory experiments using Eq. (8) and verify whether the positive LSND signal can coexist with the exclusion limits from the other laboratory experiments by extending the usual parameter space of m_4, U_{e4}, and $U_{\mu 4}$ to include τ_4 as well. This can be done only by using the full data set of each of the experiments and is best performed by the experimenters themselves. In the absence of such a calculation, we point out here the crucial fact that the CDHSW experiment[8] would see no difference in its two detectors at distances of 130 m and 885 m, if the effective values of the quantity $\exp(-m_4 L/2E\tau_4)\cos(m_4^2 L/2E)$ is the same. In Table I, we show $\Gamma_4/m_4 (= 1/\tau_4 m_4)$ as a function of m_4^2 near 6 eV^2 for which this happens, using as our very crude approximation the fixed values of $L_1/E = 0.065$ m/MeV and $L_2/E = 0.442$ m/MeV. This illustrates the possibility that the decrease from x_1 to x_2 due to decay may be compensated by the increase in the value of the cosine from L_1 to L_2 due to oscillations. Note also that there is a range of m_4^2 for which a null solution exists with varying Γ_4/m_4, whereas if the latter is zero, then m_4^2 has only discrete solutions (at 4.8 and 6.6 eV^2 for example). In the realistic case of integrating over the experimental energy spectrum, both solutions will be smeared out, but the possibility of decay should result in a larger range of acceptable values of m_4^2. For consistency, we also show in Table I the values of $f \equiv P_{\mu e}/U_{e4}^2 U_{\mu 4}^2 = 1 - 2x\cos(m_4^2 L/2E) + x^2$ for the LSND and BNL-E776 experiments, using the fixed values of $L/E = 0.75$ and 0.5 m/MeV respectively. This shows that the value of $P_{\mu e}$ as seen by the LSND experiment can be larger than that of BNL-E776 for $4.8 < m_4^2 < 5.8$ eV^2.

To discuss solar and atmospheric neutrino oscillations, let us focus on the following specific model. Let $\cos\theta = \sqrt{2/3}$ and $\sin\theta = \sqrt{1/3}$ in Eqs. (1) and (2), and let ν_s mix with ν_2 only, then $U_{\alpha i}$ is given by

$$U = \begin{bmatrix} \sqrt{2/3} & c\sqrt{1/3} & 0 & s\sqrt{1/3} \\ -\sqrt{1/6} & c\sqrt{1/3} & -\sqrt{1/2} & s\sqrt{1/3} \\ -\sqrt{1/6} & c\sqrt{1/3} & \sqrt{1/2} & s\sqrt{1/3} \\ 0 & -s & 0 & c \end{bmatrix}, \tag{11}$$

where c and s are respectively the cosine and sine of the $\nu_s - \nu_2$ mixing angle. For solar neutrino oscillations, we have

$$P_{ee} = \left(1 - \frac{s^2}{3}\right)^2 - \frac{4}{9}(1 - s^2)\left(1 - \cos\frac{\Delta m_{12}^2 L}{2E}\right) + \frac{x^2 s^4}{9}. \tag{12}$$

In the limit $s = 0$, this reduces to the usual two-neutrino formula with $\sin^2 2\theta = 8/9$ which is a good fit to the data[2], either as the large-angle matter-enhanced solution or the vacuum oscillation solution. With a small $s^2/3$ of order a few percent [between 0.026 ($x = 1$) and

0.037 ($x = 0$) for $P_{\mu e}(\text{LSND}) = 1.35 \times 10^{-3}$], this is definitely still allowed. Note that this result is not sensitive at all to the last term because $s^4/9$ is of order 10^{-3}.

For atmospheric neutrino oscillations, we have

$$P_{ee} = \left(1 - \frac{s^2}{3}\right)^2 + \frac{x^2 s^4}{9}, \quad P_{e\mu} = P_{\mu e} = (1 + x^2)\frac{s^4}{9}, \tag{13}$$

$$P_{\mu\mu} = \left(1 - \frac{s^2}{3}\right)^2 - \frac{1}{2}\left(1 - \frac{2s^2}{3}\right)\left(1 - \cos\frac{\Delta m_{23}^2 L}{2E}\right) + \frac{x^2 s^4}{9}. \tag{14}$$

Here the limit $s = 0$ corresponds to the canonical $\nu_\mu \to \nu_\tau$ solution with $\sin^2 2\theta = 1$. As it is, the prediction of $\nu_e \to \nu_e$ is still a fixed number, but smaller than unity (0.93 for $s^2/3 = 0.037$). Given that there is an uncertainty of about 20% in the absolute flux normalization, we should consider instead the ratio

$$\frac{2P_{\mu\mu} + P_{e\mu}}{P_{ee} + 2P_{\mu e}} \simeq 2\left[1 - \frac{s^4}{6} - \frac{1}{2}\left(1 - \frac{2s^4}{9}\right)\left(1 - \cos\frac{\Delta m_{23}^2 L}{2E}\right)\right], \tag{15}$$

where we have made an expansion in powers of s^2 and assumed that the ratio of ν_μ to ν_e produced in the atmosphere is two. It is clear that this is numerically indistinguishable from the case $s = 0$.

In this model, the decay $\nu_4 \to \bar{\nu}_2 + M$ has some very interesting experimental consequences. For example, ν_e from the sun decays through its ν_4 component into $\bar{\nu}_2 = (c/\sqrt{3})(\bar{\nu}_e + \bar{\nu}_\mu + \bar{\nu}_\tau) - s\bar{\nu}_s$. Hence

$$P(\nu_e \to \bar{\nu}_e) = P(\nu_e \to \bar{\nu}_\mu) = P(\nu_e \to \bar{\nu}_\tau) = \frac{s^2 c^2}{9} \sim 10^{-2}, \tag{16}$$

where the energy of $\bar{\nu}_\alpha$ is only 1/2 that of ν_e and $x = 0$ has been assumed. This is in principle detectable especially since the $\bar{\nu}_e p$ capture cross section is about 100 times that of $\nu_e e$ scattering at a few MeV. Unfortunately, the Super-Kamiokande experiment has an energy threshold of 6.5 MeV for the recoil electron and taking into account the additional 1.8 MeV threshold for the $\bar{\nu}_e p \to e^+ n$ reaction, this would require the original ν_e energy to be above 16.6 MeV, placing it outside the solar neutrino spectrum. With the recently lowered Super-Kamiokande energy threshold of 5.5 MeV, the fraction of solar ν_e above 14.6 MeV is 1.6×10^{-4}. Given the small probability of $P(\nu_e \to \bar{\nu}_e)$, this will not change appreciably the total number of observed e-like events. Regardless of energy threshold, the inability of Super-Kamiokande to distinguish e^+ from e^- or to detect the 2.2 MeV photon from neutron capture on free protons makes it difficult to pin down this possibility in any case.

In the Sudbury (SNO) neutrino experiment[19], the energy threshold for detecting recoil electrons is 5 MeV, but since there is also a threshold of about 4 MeV for breaking up the deuterium nucleus into two neutrons and a positron, the neutrino energy required is more than about 18 MeV. This again places it outside the solar neutrino spectrum. On the other hand, if the experimental energy threshold can be significantly lowered, then SNO may be able to see this effect because the $\bar{\nu}_e$ signature ($\bar{\nu}_e + d \to n + n + e^+$) is distinct from that of ν_e.

The best chance for detecting antineutrinos from the decay of ν_4 is offered by the BOREXINO experiment[20] with a very low energy threshold of 0.25 MeV. Taking into account the 1.8 MeV needed for inverse beta decay, i.e. $\bar{\nu}_e p \to e^+ n$, this means that solar neutrinos with energy above 4.1 MeV can be detected as antineutrinos. The idea of looking for antineutrinos from the sun was motivated by the possibility of a large neutrino magnetic moment which may convert ν_e into $\bar{\nu}_e$ in the sun's magnetic field. The capability of BOREXINO for detecting this has been discussed earlier[21]. For our new distinctive effect of ν_4 decay, the observed antineutrino energy spectrum is predicted to go from $f(E)$ to $f(E/2)$, where E is the energy of the original neutrino.

For atmospheric neutrinos, since $\bar{\nu}_\mu$ and $\bar{\nu}_e$ are produced together with ν_μ and ν_e in about equal amounts, it is not possible to tell if a given event comes from the primary neutrino or its decay product, even if the detector could measure the charge of the observed lepton.

To search for the $\nu_\mu \to \bar{\nu}_e$ transition in the LSND and KARMEN experiments, one would use the monoenergetic (29.8 MeV) ν_μ from π^+ decay at rest, which has the signature of a monoenergetic positron of 13.1 MeV from inverse beta decay, i.e. $\bar{\nu}_e p \to e^+ n$, in coincidence with a 2.2 MeV photon from the subsequent capture of the neutron by a free proton. However, this signal is overwhelmed by the neutral-current reaction $\nu\ ^{12}C \to \nu\ ^{12}C^*$, with the subsequent emission of a 15.1 MeV photon.

In proposed long-baseline $\nu_\mu \to \nu_\tau$ appearance experiments, the oscillation probability is given by

$$P_{\mu\tau} = \left(1 - \frac{s^2}{3}\right)^2 - \frac{1}{2}\left(1 - \frac{2s^2}{3}\right)\left(1 + \cos\frac{\Delta m_{23}^2 L}{2E}\right) + \frac{x^2 s^4}{9}, \tag{17}$$

which is not easily distinguished from the $s = 0$ case. However, the decay products of ν_4, i.e. $\bar{\nu}_e$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$, may be observable with their own unique signatures, depending on the capabilities of the proposed detectors.

In the case of four-neutrino oscillations, the effective number of neutrinos N_ν in Big Bang Nucleosynthesis is an important constraint[22]. In this model, with $m_4 \sim$ few eV and $s^2 \sim$ few percent, the presence of a stable ν_s would have counted as an extra neutrino species, making $N_\nu = 4$. This may not be acceptable if $N_\nu < 4$, as indicated from the observed primordial ^{4}He abundance[23]. The decay of ν_4 changes N_ν to 3 + the contribution of the Majoron (i.e. 4/7).

With ν_4 as a component of ν_e, neutrinoless double decay has an effective ν_e mass of $(s^2/3)m_4 \sim 0.2$ eV if $m_4 \sim 5$ eV. This value is just at the edge of the most recent experimental upper bound[24].

Finally a comment on the neutrino contribution to dark matter may be in order. With ν_4 decaying and m_1, m_2, and m_3 being too small, there is no neutrino dark matter. However, it is possible that $m_1 \simeq m_2 \simeq m_3 \simeq$ few eV, while m_4 is higher by another few eV, in which case ν_1, ν_2, and ν_3 will contribute to dark matter. Our discussion goes through almost unchanged, except that m_4^2 in Eq. (8) will be replaced by $m_4^2 - m_{1,2,3}^2$.

In conclusion, we have shown in this paper that a hierarchical four-neutrino scenario is acceptable as a solution to all present neutrino data regarding the disappearance and appearance of ν_e and ν_μ. The assumed singlet neutrino of a few eV may decay into a linear combination of the three known doublet neutrinos with half of the energy. This new feature allows our proposal to be tested in future solar neutrino experiments such as BOREXINO (and perhaps SNO), and should be considered in forthcoming long-baseline accelerator neutrino experiments.

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$m_4^2(\mathrm{eV}^2)$	Γ_4/m_4	$f(\mathrm{LSND})$	$f(\mathrm{E776})$
4.8	0	3.92	0.04
5.0	0.030	3.04	0.04
5.2	0.065	2.21	0.19
5.4	0.085	1.72	0.38
5.6	0.095	1.37	0.57
5.8	0.095	1.09	0.78
6.0	0.086	0.54	1.03
6.2	0.068	0.55	1.37
6.4	0.038	0.22	1.94
6.6	0	0.0	3.0

Table 1: Null solution for oscillation and decay at the two CDHSW detector distances.