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# High Scale Mixing Unification and Large Neutrino Mixing Angles

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Starting with the hypothesis that quark and lepton mixings are identical at or near the GUT scale, we show that the large solar and atmospheric neutrino mixing angles together with the small reactor angle $U_{e3}$ can be understood purely as a result of renormalization group evolution provided the three neutrinos are quasi-degenerate and have same CP parity. It predicts the common Majorana mass for the neutrinos larger than 0.1 eV, which falls right in the range reported recently and also the range which will be probed in the planned experiments.

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## I. INTRODUCTION

The idea that disparate physical parameters describing forces and matter at low energies may unify at very short distances (or high mass scales) has been a very helpful tool in seeking a unified understanding of apparently unrelated phenomena [1]. In the context of supersymmetric grand unifed theories, such an approach explains the weak mixing angle $\sin^2\theta_W$ and thereby the different strengths of the weak, electromagnetic and strong forces. One of the key ingredients of the grand unified theories is the unification between quarks and leptons. One may, therefore, hope that in a quark-lepton unified theory, the weak interaction properties of quarks and leptons parameterized by means of the flavor mixing matrices will become identical at high energies.

On the experimental side, recent measurements on atmospheric and solar neutrino fluxes and those at K2K and KamLAND which are a manifestation of the phenomena of neutrino oscillations suggest that two of the neutrino mixings i.e. the mixings between $\nu_e - \nu_\mu$ and $\nu_\mu - \nu_\tau$ (to be denoted by $\theta_{12}$ and $\theta_{23}$, respectively) are large [2, 3, 4, 5, 6] while the third mixing between the $\nu_e - \nu_\tau$ is bounded to be very small by the CHOOZ-Palo Verde reactor experiments i.e. $\sin^2 2\theta_{13} < 0.15$ [7]. On the other hand, it is now quite well established that all observed quark mixing angles are very small. One may therefore ask whether there is any trace of quark lepton unification in the mixing angles as we move to higher scales.

The first question in this connection is whether high scales have anything to do with neutrino masses or it is purely a weak scale phenomenon. One of the simplest ways to understand small neutrino masses is via the seesaw mechanism [8] according to which the neutrino mixing is indeed a high scale phenomenon, the new high scale being that of the right handed neutrino masses ($M_R$) in an appropriate extension of the standard model. Present data put the seesaw scale $M_R$ very close to the conventional GUT scales. It is therefore tempting to speculate whether quark and lepton mixing angles are indeed unified at the GUT-seesaw scale. This would of course imply that all neutrino mixing angles at the high scale $M_R$ are very small whereas at the weak scale two of them are known to be large. In this paper we show that simple radiative correction effects embodied in the renormalization group evolution of parameters from seesaw scale to the weak scale can indeed provide a complete understanding of all neutrino mixings at the weak scale, starting with very small mixings at the GUT-seesaw scale.

The fact that renormalization group evolution from the seesaw scale to the weak scale [9, 10] can lead to drastic changes in the magnitudes of the mixing angles was pointed out in several papers [9, 11, 12, 13, 14, 15]. In particular, it was shown in [11] that this dependence on renormalization group evolution can be exploited in simple seesaw extensions of the minimal supersymmetric standard model (MSSM) to explain the large value of the atmospheric mixing angle starting with a small mixing at the seesaw scale, provided two conditions are satisfied: (i) the two neutrino-mass eigen states have same CP-parity and (ii) they are very nearly degenerate in mass. In general, in gauge models that attempt to explain the large neutrino mixings [16], one needs to make many assumptions to constrain the parameters. In contrast, in this class of "radiative magnification" models [11, 12, 14], there is no need to invoke special constraints on the parameters at high scales beyond those needed to guarantee the quasi-degeneracy. In fact the main content of radiative magnification models is the quasi-degeneracy assumption and since the value of common Majorana mass $m_0$ for all neutrinos is required to be in sub-eV range ($\geq 0.1$ eV,), this assumption is experimentally testable in the ongoing neutrinoless double beta decay searches [12].

It is well known that the radiative magnification technique requires adjustment of initial neutrino mass eigen values at the see-saw scale [9, 11, 12, 13, 14]. In view of the model independence and simplicity of the method involved, and the attractive nature of the results achieved, the question of finetuning has been discussed at lengh by Casas, Espinosa, Ibarra, and Navarro(hereafter called as CEIN) [14] who have also discussed the relevant magnification criteria and shown that, in three flavor case, the existence of infrared stable quasifixed points in the relevant RGEs lead to vanishing mixing matrix elements at low energies. Thus, magnification for mixing angles is expected to occur only if RG-evolutions are stopped before reaching the quasifixed point regime. It has been noted that the radiative magnification mechanism leading to large neutrino mixing can only be achieved if there is substantial cancellation between the initial and the RG-generated mass splittings [11, 12, 13, 14]. In this context we note that similar cancellations are common in well known grand unified theories(GUTs). In the bottom-up approach large differences between low-energy coupling constants of the SM are reduced to vanishing differences due to cancellation with RG-generated contributions. In the well known $b-\tau$ unification scenario, the low-energy mass splitting $\mathrm{m}_b - \mathrm{m}_\tau \simeq 2.5$ GeV is almost cancelled out by RG-generated mass difference leading to $\mathrm{m}_b \simeq \mathrm{m}_\tau$ at the GUT scale. Both these examples apply to nonSUSY as well as SUSY GUTs.

In this paper[17], we show that under the same conditions for radiative magnification as just outlined, if we start with the hypothesis that at the seesaw scale the quark and neutrino mixings are unified to a common set of values, i.e. the known extrapolated values of the well known CKM angles, after renormalization group evolution to the weak scale, we can obtain the solar and the atmospheric mixing angles that are in agreement with observations without contradicting the CHOOZ-Palo Verde bound on $\theta_{13}$. The possibility of achieving two large neutrino mixings by radiative magnification has been reported for the first time in ref.[17].

This result has two important implications: (i) it would provide a very simple and testable way to understand the observed large neutrino mixings and (ii) if confirmed by the neutrinoless double beta decay experiments, it would provide a strong hint of quark lepton unification at high scales. One may wonder why we are addressing the question of unification of the mixing angles for neutrinos with those of quarks and not the unification of neutrino masses with quark masses. The answer is of course the well-known one, namely neutrino masses have their origin (seesaw mechanism) that distinguishes them from the quark masses. Furthermore, within the seesaw mechanism neutrinjos are Majorana fermions whereas the quarks are Dirac fermions. Thus as far as the masses go, we have no reason to expect unification with quarks. We take up the question of neutrino masses in Sec 5.

This paper is organized as follows: in sec. 2, we discuss the RGes for the neutrinos in the mass basis, in sec. 3, we present the main result of our paper i.e. the magnification of mixing angles at the weak scale; in sec. 4, we discuss predictions of our approach for neutrinoless double beta decay and other processes; in sec. 5, we present a gauge model where approximate mixing unification hypothesis is realized and in sec. 6, we present our conclusions.

## II. RENORMALIZATION GROUP EQUATIONS FOR MASSES AND MIXINGS

Our basic assumption will be a seesaw type model which will lead to equal quark and lepton mixing angles at the seesaw scale as well as to a quasi-degenerate neutrino spectrum. In sec. 5, we present a model where at the seesaw scale the neutrinos have this property. We will then follow the "diagonalize and run" procedure for the neutrino parameters and use the RGEs directly for the physical observables, namely, the mass eigenvalues $m_i$ and the mixing angles $\theta_{ij}$ $(i, j = 1, 2, 3)$. We also assume the neutrino mass eigenstates to possess the same CP and ignore CP violating phases in the mixing matrix. Also for simplicity, we adopt the mass ordering among the quasi-degenerate eigenstates to be of type $m_3 \gtrsim m_2 \gtrsim m_1$. The real $3\times 3$ mixing matrix is parametrized as,

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - c_{23}s_{13}s_{12} & c_{13}c_{23} \end{bmatrix}, \tag{1}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij} (i, j = 1, 2, 3)$. $U$ diagonalizes the mass matrix $M$ in the flavor basis with $U^T M U = \mathrm{diag}(m_1, m_2, m_3)$. The RGEs for the mass eigen values can be written as [13, 14]

$$\frac{dm_i}{dt} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u,\ (i = 1, 2, 3)\,. \tag{2}$$

For every $\sin\theta_{ij} = s_{ij}$, the corresponding RGEs are,

$$\frac{ds_{23}}{dt} \;=\; -F_\tau {c_{23}}^2 \left(-s_{12}U_{\tau 1}D_{31} + c_{12}U_{\tau 2}D_{32}\right), \tag{3}$$

$$\frac{ds_{13}}{dt} \;=\; -F_\tau c_{23}{c_{13}}^2 \left(c_{12}U_{\tau 1}D_{31} + s_{12}U_{\tau 2}D_{32}\right), \tag{4}$$

$$\begin{aligned} \frac{ds_{12}}{dt} \;=\; & -F_\tau c_{12} \left(c_{23}s_{13}s_{12}U_{\tau 1}D_{31} - c_{23}s_{13}c_{12}U_{\tau 2}D_{32}\right. \\ & \left. + U_{\tau 1}U_{\tau 2}D_{21}\right). \end{aligned} \tag{5}$$

where $D_{ij} = (m_i + m_j)) / (m_i - m_j)$ and, for MSSM,

$$F_\tau = -h_\tau^2/\left(16\pi^2 \cos^2\beta\right),$$
$$F_u = \left(\frac{1}{16\pi^2}\right)\left(\frac{6}{5}g_1^2 + 6g_2^2 - 6\frac{h_t^2}{\sin^2\beta}\right), \qquad (6)$$

but, for SM,

$$F_\tau = 3h_\tau^2/\left(32\pi^2\right),$$
$$F_u = \left(3g_2^2 - 2\lambda - 6h_t^2 - 2h_\tau^2\right)/\left(16\pi^2\right). \qquad (7)$$

RGEs of mixing angles in the three flavor case have been shown to possess infrared stable quasifixed points leading to vanishing values of mixing matrix elements [14]. Thus as in two-flavor case [11, 12] the radiative magnification of two mixing angles, if at all feasible, could be realizable only if RG-evolution is stopped before reaching the quasifixed point regime. In the CEIN [14] approach this was implemented to magnify the atmospheric mixing angle by adjusting the initial mass eigen values to achieve maximal mixing at $M_{\rm SUSY} \sim M_Z$ such that decrease in mixing angles after reaching the maximum is smaller. In this approach MSSM operates for all scales starting from $M_Z$.

In order to avoid the vanishing matrix elements of the quasifixed point region in this paper we follow a different approach described in [12] which is found to work also for three flavor case in the presence of MSSM with $O$(TeV) SUSY scale.When the mass difference between $m_i$ and $m_j$ tends to vanish, $D_{ij} \to \infty$, and the corresponding term in the RHS of (3)-(5) predominantly drives the RG evolution for $\sin\theta_{ij}$ which might become large or even approach its maximal value anywhere between $\mu = M_{\rm SUSY} - M_R$. Since $m_i$ and $m_j$ are scale dependent, the initial difference existing between them at $\mu = M_R$ is narrowed down during the course of RG evolution as we approach $\mu = M_{\rm SUSY}$. This causes $D_{ij} \to \infty$ and hence large magnification to the mixing angle due to radiative effects. Also $F_\tau$ is enhanced by a factor $\sim 10^3$ in the large $\tan\beta$ region in the case of MSSM as compared to the SM where such effects do not exist. Thus, if the SUSY scale is significantly larger with $M_Z < M_{\rm SUSY} \leq 1$TeV, radiative magnification to large mixings may occur through RG evolution from the seesaw scale down to $M_{\rm SUSY}$. Then the standard model evolution below $M_{\rm SUSY}$ causes negligible contribution to the magnified mixings because of two reasons: (i)absence of $\tan^2\beta$ effects, and (ii)small range of RG evolution from $M_{\rm SUSY}$ to $M_Z$. Consequently, the predicted mixings remains almost flat and very close to $\sin\theta_{ij}(M_{\rm SUSY})$ for all energies below $M_{\rm SUSY}$. This aspect of RG-evolution below the SUSY scale to avoid the approach to infrared stable quasifixed point corresponding to vanishing mixing angle which was demonstrated in [12] is also found to operate in three flavor case. .

The mixing unification hypothesis implies that we choose all neutrino mixings at the seesaw scale equal to the corresponding quark mixings, which in the Wolfenstein parameterization are dictated by the parameter $\lambda_0 = .2$. We then have, at the seesaw scale, $s_{12} \simeq \lambda_0$, $s_{23} \simeq O(\lambda_0^2)$ and $s_{13} \simeq O(\lambda_0^3)$. These values get substantially magnified in the region around $M_{\rm SUSY}$. Using $|D_{31}| \simeq |D_{32}| \ll |D_{21}|$, we see from (3)-(5), that the dominant contribution to RG evolution of $s_{23}(\mu)$ is due to the term $\sim \lambda_0^2 F_\tau D_{32}$. Similarly the terms contributing to the evolution of $s_{13}(\mu)$ are $\sim \lambda_0^3 F_\tau D_{32}$ or $\sim \lambda_0^3 F_\tau D_{31}$. On the other hand the evolution of $s_{12}$ is dominated by the term $\sim \lambda_0^5 F_\tau D_{21}$ where the large enhancement likely to be caused by the largeness in $|D_{21}|$ is damped out due to higher power of $\lambda_0^5$. Since the mixing angles change substantially only around $M_{\rm SUSY}$, such dominance to RG evolutions holds approximately at all other lower scales below $M_R$.

If the neutrino mixing angles are to be compatible with experimental observations at low energies, we need at most the magnification factors: $(\sin\theta_{23}/\sin\theta_{23}^0) \simeq 20$, $(\sin\theta_{13}/\sin\theta_{13}^0) \leq 60$, and $(\sin\theta_{12}/\sin\theta_{12}^0) \simeq 4$, where we have used the experimental neutrino mixings for $\theta_{ij}$ [2, 3, 4, 5, 6, 7] and quark mixings for $\theta_{ij}^0$ [18]. That the CHOOZ-Palo Verde bound can tolerate a magnification factor as large as 60 is crucial to achieve bi-large mixings by radiative magnification while keeping the magnified angle $\theta_{13}$ at low energies well below the upper bound. This is of course because of the smallness of $(\lambda_0)^3$, which is the starting value (order of magnitude) of the reactor angle. One can also observe that it is the smallness of the reactor angle that provides the "hidden" signal for the unification!

## III. BI-LARGE NEUTRINO MIXINGS BY RG EVOLUTION

Starting from known values of gauge couplings, masses of quarks and charged leptons, and CKM mixings in the quark sector at low energies, at first we use the bottom-up approach and all the relevant RGEs to obtain the corresponding quantities at higher scales, $10^{11}$ GeV-$2 \times 10^{18}$ GeV. Assuming the neutrino mixing at $\mu = M_R$ to be small and similar to quark mixings, we then expect the initial conditions at $\mu = M_R$ to be $\sin\theta_{23}^0 \simeq 0.038$, $\sin\theta_{13}^0 \simeq 0.0025$ and $\sin\theta_{12}^0 \simeq 0.22$ [18]. Using these as input and the mass eigenvalues $m_i^0$ as unknown parameters at the high scale, we then follow the top-down approach though (2)-(5) and other standard RGEs. The unknown parameters $m_i^0$ are determined in such a way that the solutions obtained at low energies agree with mass squared differences and the mixing angles given by the experimental data within 90% C.L. [2, 3, 4, 5, 6, 7]

$$\Delta m_{12}^2 = (2 - 50) \times 10^{-5}{\rm eV}^2,$$
$$\Delta m_{23}^2 = (1.2 - 5) \times 10^{-3}{\rm eV}^2,$$
$$\sin\theta_{23} = 0.54 - 0.83, \sin\theta_{12} = 0.40 - 0.70,$$

$$\sin\theta_{13} \;\leq\; 0.16 \;. \tag{8}$$

Our model described in sec.5 is consistent with quasidegenerate mass eigen values over a wider range of the see-saw scale: $M_R = 10^{11}$ GeV-$10^{15}$ GeV. However, in view of the phenomenological importance of the rersults, we have explored the RG-evolutions to bi-large mixings including higher scales upto the reduced Planck scale($2 \times 10^{18}$ GeV). In Table I we prersent input mass eigenvalues at the see-saw scale $M_R = 10^{13}$ GeV and our solutions at $M_Z$ in the large $\tan\beta(= 55)$ region. The solutions clearly exhibit radiative magnification of both the mixing angles, $\theta_{23}$ and $\theta_{12}$ for a wide range of input values of $m_i^0$. We find that although enhancement due to RG evolution occurs in the $\nu_e - \nu_\tau$ sector also, $\sin\theta_{13}$ remains well within the CHOOZ-Palo Verde bound [7].

In Table II we present three sets of initial mass eigenvalues and our solutions for three different high-scale values, $M_R = 10^{11}, 10^{15}$ and $2 \times 10^{18}$ GeV. We find that for the same value of $\tan\beta = 55$, the predicted lowest mass eigenvalue at $M_Z$ decreases slowly with increase of the see-saw scale. For example, the lowest mass eigenvalues predicted at $\mu = M_Z$ are 0.27 eV, 0.22 eV , 0.209 eV, and 0.17 eV for $M_R$= $10^{11}$ GeV, $10^{13}$ GeV, $10^{15}$ GeV and $2 \times 10^{18}$ GeV , respectively.

A magnification formula has been derived by CEIN [14] for the product of the mixing matrix elements,

$$F_{\rm m} \;=\; \frac{U_{\tau i}U_{\tau j}(\mu)}{U_{\tau i}U_{\tau j}(M_R)} \simeq \left[1 + \frac{h_\tau^2}{32\pi^2} D_{ij}(M_R) \ln\frac{M_R}{\mu}\right]^{-1} . \tag{9}$$

Using the values given in Tables I-II, we find that the magnification predicted by the formula matches reasonably well with our estimations for mixing between the second and the third generations(i, j=2, 3).

Our result on the approximate unification of quark and neutrino mixings at the high scale $M_R = 10^{13}$ GeV is exhibited in Fig.1 where we present the RG evolutions of the sines of the three neutrino mixing angles starting from $M_R = 10^{13}$ GeV down to $M_Z$ for one set of input masses given in Table I: $m_1^0 = 0.2983$ eV, $m_2^0 = 0.2997$ eV, and $m_3^0 = 0.3383$ eV. The flatness of the curves below $M_{SUSY}$ is due to negligible renormalization effect from SM which evades the approach to the quasifixed points. The corresponding low-energy solutions are $m_1 = 0.2201$ eV, $m_2 = 0.2223$ eV, $m_3 = 0.2244$ eV, $\Delta m^2_{12} = 1.6 \times 10^{-4}$ eV$^2$, $\Delta m^2_{23} = 1.0 \times 10^{-3}$ eV$^2$, $\sin\theta_{23} = 0.667$, $\sin\theta_{13} = 0.09$, and $\sin\theta_{12} = 0.606$. Almost horoizontal lines in the figure represent the sines of the CKM mixings, $\sin\theta^q_{ij}$, having negligible one-loop radiative corrections [18]. Unification of the neutrino mixings with the corresponding quark mixings are clearly demonstrated at the high scale.

The evolution of mass eigen values corresponding to mixings given in Fig. 1 are shown in Fig.2 for $M_R = 10^{13}$ GeV. In cntrast to sines of mixing angles which have negligible RG-corrections below the SUSY scale, the mass eigen values are found to decrease till the lowest scale $M_Z$. The rate of decrease of the third eigen value is the highest, but the rates of decrease of the first and the second eigen values are similar. The initial mass splittings at the higest scale are narrowed down to match the experimental values at low energies due to cancellations caused by RG-generated splittings.

In Fig.3-Fig.5 we present evolutions of neutrino mixing angles for $M_R = 10^{11}$ GeV, $10^{15} GeV$ and $2 \times 10^{18}$ GeV with input mass eigen values given in Table II. In Fig.6-Fig.8 the RG-evolution of corresponding mass eigen values with input parameters given in Table II for $M_R = 10^{11}$ GeV, $10^{15}$ GeV, and $2 \times 10^{18}$ GeV are presented. It is quite clear that radiative magnification to bi-large mixings is possible over a wide range of choices of $M_R$ and input mass eigenvalues.

In addition to the solutions of the type shown in Fig. 1 which are valid for $M_{\rm SUSY} > M_Z$, we have also found solutions corresponding to two large and one small mixings for $M_{\rm SUSY} = M_Z$ with somewhat different mass eigen values in agreement with the experimental data at low energies. We have also noted that the radiative magnification mechanism leading to bilarge mixings works more easily if we take all other initial values same as mentioned above but $\sin\theta^0_{13} = 0.0$ which could be relevant to certain neutrino mass textures. In this case the CHOOZ-Palo Verde bound is always protected.

It is worth re-emphasizing that since we determine 3 input parameters (the 3 mass eigenvalues at high scale) to fit 5 experimentally known numbers as output parameters it is a over-determined problem and there may be no solution. So there is a possibility of not being able to obtain correct mixing angles at the weak scale. But we have found that it is possible, thus showing that there is perhaps an element of truth in the unification hypothesis. It is also significant that the scale of 0.16 - 0.65 eV comes out as the range of allowed mass eigenvalues although such a scale was not put in at all a priori.

## IV. PREDICTIONS FOR BETA-DECAY, DOUBLE BETA DECAY, $U_{e3}$ AND WMAP

Very recently the possibility of testifying our mixing unification hypothesis through lepton-flavor violating processes like $\mu \to e\gamma$ and $\tau \to \mu\gamma$ has been investigated [19]. We discuss here other possible experimental tests of the specific mechanism proposed here for radiative magnification.

**Double beta and tritium beta decays:** Our RG solutions permit radiative magnification consistent with experimental data on $\Delta m^2_{21}$, $\Delta m^2_{32}$ and the mixing angles, if the input mass eigenvalues for $M_R = 10^{11} - 2 \times 10^{18}$ GeV are in the range 0.35 eV - 1.0 eV. This corresponds to the low energy limits $0.16\ eV < m_i(M_Z) < 0.65\ eV$. Then,our choice of phases leads to the prediction

$$|< M_{ee} >| = |\sigma_i m_i U_{ei}^2| = 0.16\ eV - 0.65\ eV. \quad (10)$$

Recent searches for neutrinoless double beta decay have obtained the upper limit: $| < M_{ee} > | < (0.33 - 1.35)$ eV [20]. The range in Eq (8) overlaps the one reported in [21], or the ones that will be covered in [22]. Thus a clear and testable prediction of the bi-large radiative magnification mechanism is that neutrinoless double beta decay should be observed in the next round of experiments.

Further, our low-energy limit on the quasi-degenerate $m_i(M_Z)$ can be directly measured in Tritium beta decay experiment. Although the present experimental bound on the mass is $< 2.2$ eV, mass value as low as 0.35 eV can be reached by KATRIN experiment [23].

**Prediction for $U_{e3}$:** Starting from the allowed range of high-scale input values of the CKM mixing angle with $V_{ub} \simeq U_{e3}^0 \simeq 0.0025 - 0.004$, the RG-evolutions predict enhancement of $\sin\theta_{13}$ at low energies

$$U_{e3} = \sin\theta_{13} = 0.08 - 0.10. \quad (11)$$

Although this prediction is well below the present experimental upper bound [6], it is accessible to several planned long-baseline neutrino experiments in future such as NUMI-Off-Axis or JHF proposals.

**WMAP constraints on neutrino masses:** Recently the Wilkinson Microwave Anisotropy Probe (WMAP) observations have provided very interesting constraints on the sum of neutrino masses [24, 25]. The analysis depends on a number of cosmological parameters such as $H_0$, the bias parameter $b(k)$, $\Omega_m$ from SN-Ia observations etc. Depending on what values one chooses for the "priors", the constraint on the the sum all neutrino masses varies from 2.1 eV to 0.7 eV. Since we are proposing that the neutrino masses are degenerate, each individual mass will have an upper limit of 0.23 eV to 0.7 eV. Thus the radiative magnification hypothesis is consistent with WMAP observations [24] and also with the combined analysis of WMAP+2dF GRS data [25].

We have found that with $\tan\beta = 55$ and due to RG-effects alone the lowest allowed value of the neutrino-mass eigen value at $M_Z$ decreses slowly with increase in the see-saw scale . We obtain the lower bound to be 0.27 eV - 0.16 eV for $M_R = 10^{11} - 2 \times 10^{18}$ GeV.

## V. DEGENERATE NEUTRINOS FROM TYPE II SEESAW AND A MODEL FOR APPROXIMATE MIXING UNIFICATION

In this section, we address the question of how a quasi-degenerate neutrino spectrum can arise within a gauge model that employs the seesaw mechanism for understanding neutrino masses [26].

To begin the discussion, let us present the different forms of the seesaw mechanism that provide a natural way to understand the small neutrino masses. Following literature, we will call the two types of seesaw mechanism as type I and type II. In the type I seesaw mechanism the neutrino mass matrix is given by the formula,

$$M_\nu \;=\; -M_D(fv_R)^{-1}M_D^T \quad (12)$$

where $f$ is the Majorana Yukawa coupling of the RH neutrinos, $v_R$ is the $B-L$ symmetry breaking scale, and $M_D$ is the Dirac neutrino-mass matrix. In models where information about the B-L symmetry is not given explicitly, $fv_R$ is replaced by the mass matrix of the right handed (RH) neutrinos $M_N = fv_R$. Since one expects the pattern of $M_D$ to be similar to the quark and lepton mass matrices, one expects the eigenvalues of $M_N$ to be hierarchical and mixing angles to be small. The Eq.(12) then tells us that the neutrino masses are hierarchical. Clearly in such models the radiative magnification of mixing angles does not occur via the renormalization group evolution as is clear from Eqs.(3)-(5) in the previous section.

The type I seesaw formula is generic to models which do not have any connection between the left and right handed fermions such as in models where one extends the standard model by adding a right handed neutrino and mass terms for the RH neutrinos. Things however undergo a drastic change in models that have asymptotic parity invariance. In such models there are always Higgs fields that are parity partners of the RH Higgs fields which give mass to the RH neutrinos. Thus there are operators which give direct mass to the left handed neutrinos at the same time that the right handed neutrinos get mass. It turns out also that the direct neutrino mass term is seesaw suppressed i.e. as the $v_R$ scale goes to infinity, this contribution, like the right handed neutrino contribution, vanishes. This direct mass contribution leads to a modification of the seesaw formula to the following form (type II seesaw formula [27])

$$M \;=\; fv_L - M_D(fv_R)^{-1}M_D^T \quad (13)$$

Examples of models where type II seesaw formula arises are left-right symmetric models or SO(10) models with either $B-L=2$ triplet Higgs fields or $B-L=1$ doublet Higgs fields breaking the B-L symmetry. Below we give an example of a model with triplet Higgs fields. It is important to note that the renormalization group equations hold for both the type I as well type II seesaw formula.

The Yukawa coupling matrix $f$ in Eq.(11) that contributes to the first term in the seesaw formula as well as the right handed neutrino mass matrix depends on high scale physics and is therefore unconstrained by by standard model results. We could therefore choose $f$ to be close to the unit matrix. In this case, quark-lepton unification requires that the lepton mixing angles be very close to the quark mixing angles but the neutrino mass spectrum dominated by the first term in Eq.(11) in combination with second term can easily lead to a quasi-degenerate spectrum of Majorana neutrinos as well as approximate mixing unification. In such schemes, radiative magnification works to provide an understanding of the large neutrino mixings. The question is whether there is some underlying symmetry of the theory for which one can write down a natural gauge model where $f = \mathbf{1} f_0$ as well as the near unification of quark and lepton mixings. Below we provide an example of this kind of model. An important point is that the renormalization group equations hold for this type II seesaw formula as long as we assume that the $SU(2)_L$ triplet Higgs whose vev responsible for the first term in Eq.(11) is heavier than the seesaw scale. This is true in models realizing the type II seesaw.

We consider a nonsupersymmetric $SU(2)_L \times SU(2)_R \times SU(4)_{PS}$ gauge model with an $S_4$ global symmetry [28]. Before describing the model, a few words about $S_4$ symmetry may be helpful. This is a nonabelian discrete symmetry group with 24 elements and has the irreducible representations $\mathbf{3}, \mathbf{3}', \mathbf{2}, \mathbf{1}', \mathbf{1}$. We will assign fundamental fermions to the $\mathbf{3}$ dimensional representation of $S_4$ and the Higgs fields $\phi_a$ and B-L=2 triplet fields to representations of $S_4$ as follows:

| Fields | $S_4$ rep. |
|---|---|
| $\Psi_{L,R}(2,1,4) + (1,2,\bar{4})$ | $\mathbf{3}$ |
| $\phi_0(2,2,1)$ | $\mathbf{1}$ |
| $\phi_{1,2}(2,2,1)$ | $\mathbf{2}$ |
| $\phi'_{1,2,3}(2,2,1)$ | $\mathbf{3}$ |
| $\Delta_{L,R}(3,1,10) + (1,3,\overline{10})$ | $\mathbf{1}$ |

$$\Psi = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}.$$

Let us now write down the $S_4$ invariant Yukawa couplings:

$$\begin{aligned} \mathcal{L}_Y &= f_0(\sum_a \psi^T_{L,a}\Psi_{L,a}\Delta_L + L \leftrightarrow R) \\ &+ h_0\phi_0(\sum_a \bar{\psi}_{L,a}\Psi_{R,a}) + h_2[(\bar{\psi}_{L,3}\Psi_{R,2} \\ &+ \bar{\psi}_{L,2}\Psi_{R,3})\phi_1 + (\bar{\psi}_{L,3}\Psi_{R,3} + \bar{\psi}_{L,2}\Psi_{R,2} \\ &- 2\bar{\psi}_{L,1}\Psi_{R,1})\phi_2] + h_3[(\bar{\psi}_{L,1}\Psi_{R,3} + \bar{\psi}_{L,3}\Psi_{R,1}\phi'_1 \\ &+ (\bar{\psi}_{L,2}\Psi_{R,1} + \bar{\psi}_{L,1}\Psi_{R,2})\phi'_2 + (\bar{\psi}_{L,3}\Psi_{R,3} \\ &- \bar{\psi}_{L,2}\Psi_{R,2})\phi'_3] + h.c. \end{aligned} \tag{14}$$

To get the desired form of the seesaw formula, first note that $< \Delta^0_L >= v_L \equiv v^2_{wk}/v_R$, $\Delta^0_R = v_R$, the bidoublet vevs are of the form

$$< \phi_i >= \begin{pmatrix} \kappa_i & 0 \\ 0 & \kappa'_i \end{pmatrix},$$

and that $f_0$ is the identity matrix.

One can break the $S_4$ symmetry softly so that all the the $\phi$'s have different vevs. Also note that $h_i$'s can be complex. Thus six $\phi$'s with independent vevs give us 12 parameters which is enough to fit the quark mixings and will predict all lepton mixings equal to quark mixings at the GUT scale. At the GUT scale, this would predict $m_b = m_\tau$ and $m_s = m_\mu$. For the b-quark, this is the well known $b - \tau$ unification. Using the PDG values for $m_{b,s}$, we can run it upto the GUT scale to get $m_b(M_R) \simeq 0.98 - 1.10$ GeV whereas the corresponding value of $m_\tau \sim 1.18$. However we have for $m_s(M_R) \simeq 0.03$ GeV if we use the PDG values. This is about 3 times smaller than the muon mass at the seesaw scale [18]. So we have to add some terms that break quark lepton symmetry.

To cure the $m_s - m_\mu$ problem, we invoke higher dimensional terms and add a new Higgs multiplet $\Sigma(1,1,15)$ that transforms as (1,1,15) under $G_{224}$. Also let us assume that $\Sigma(1,1,15)$ transforms like a $\mathbf{3}$ dimensional representation of $S_4$ with only $< \Sigma_3 > \neq 0$.The higher dimensional operators that involve $\Sigma$ have the form $\frac{\phi_0}{M}[(\bar{\psi}_{L,1}\Psi_{R,3} + \bar{\psi}_{L,3}\Psi_{R,1}\Sigma_1 + (\bar{\psi}_{L,2}\Psi_{R,1} + \bar{\psi}_{L,1}\Psi_{R,2})\Sigma_2 + (\bar{\psi}_{L,3}\Psi_{R,3} - \bar{\psi}_{L,2}\Psi_{R,2})\Sigma_3]$. Since $\Sigma(1,1,15)$ has a vev that breaks only $SU(4)_c$ symmetry, it gives different masses to quarks and leptons. For $< \Sigma_3 > /M \simeq 10^{-3}$, this has the right order of magnitude to lead to the difference between $m_s$ and $m_\mu$ and not effect the off diagonal elements that are responsible for mixings. Since the mixing angles go roughly like $\frac{M_{23}}{M_{33}-M_{22}}$, they do not deviate too much from the symmetric values (since $M_{22} \ll M_{33}$).

As far as the $m_e$ and $m_d$ goes, we can again add non-renormalizable Yukawa couplings such as $\bar{\Psi}_L\Psi_R\Delta^\dagger_R\Delta_R\phi$ type terms which will only modify the first generation masses since their magnitude is of order $v^2_R/M^2_{P\ell}$ down compared to the renormalizable terms. Again this contribution being a purely diagonal contribution will change the mixing angles only slightly. Therefore, we can get a a model of the type we are considering with degenerate neutrinos and with quark and neutrino mixing angles approximately equal at the seesaw scale. This model can easily be supersymmetrized and all our conclusions go through.

Coming to the neutrino sector, we will first show how type II seesaw emerges in this model. The complete Higgs content of this model for the supersymmetric case is: $\Psi(2,1,4); \Psi^c(1,2,\bar{4}), \phi_0(2,2,1), \phi_{1,2}(2,2,1), \phi'_{1,2,3}(2,2,1)$, $\Delta(3,1,10) \oplus \bar{\Delta}(3,1,\overline{10})$ and $\Delta^c(1,3,\overline{10}) \oplus \overline{\Delta}^c(1,3,10)$ as shown in Table in this section. In addition we add a Higgs field transforming as $\Omega(3,3,1)$. The Higgs part of the superpotential can be written as

$$W' = \lambda\Omega(\Delta\Delta^c + \overline{\Delta\Delta^c} + Tr\phi^2_0 + \cdots) \tag{15}$$

where $\cdots$ denote the $S_4$ singlet bilinears involving the other $\phi$ fields. Clearly, when we set $F_\Omega = 0$ to maintain supersymmetry down to the weak scale, we find that $< \Delta^0 > \neq 0$. This leads to the type II seesaw which is the cornerstone of our discussion.

The gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C (= G_{224})$ is a subgroup of a numbetr of GUTs like $SO(10)$, $SO(18)$, and $E_6$ etc. It also contains the subgroups like $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (= G_{2213})$ and the standard model. Thus the model worked out with $S4 \times G_{224}$ is equivalent to a number of underlying high-scale models such as $S4 \times SO(10)$, $S4 \times SO(18)$, $S4 \times E_6$ etc. It also suggests the possibility of having $S4 \times G_{2213}$ as an approximate symmetry for quasi-degeneracy.

In the absence of such symmetries as dicussed in this section where a non-abelian discrete symmetry $S4$ occurs along with the gauge symmetry $G_{224}$, high-scale unification of quark and neutrino mixings with quasi-degenerate neutrinos but with hierachial quark masses would have been accidental. But the type II seesaw mechanism in the presence of $S4 \times G_{224}$ and its spontaneous breaking guarantees quasi-degenerate neutrinos with almost equal mixings in the quark and lepton sectors at the high scale while the model fits all the masses and mixings at low energies.

## VI. CONCLUSION

In summary, we have shown that in the MSSM, the hypothesis of quark- lepton mixing unification at the see-saw scale seems to generate the correct observed mixing pattern for neutrinos i.e. two large mixings needed for $\nu_e - \nu_\mu$ and $\nu_\mu - \nu_\tau$ and small mixing for $U_{e3}$ at low energies. Quasi-degenerate neutrino spectrum with a common mass for neutrinos $\geq 0.1$ eV is a testable prediction of the model. Important new result of our analysis is that although magnification occurs for the $U_{e3}$ parameter, it remains small due to the fact that $V_{ub}$ is very small. The prediction for $U_{e3}$ also provides another test of the model.

Througout this paper we have treated all phases(Majorana and Dirac) to be vanishingly small in the MNS matrix. It would be interesting to investigate the effect of phases [29] on the implications of our mixing unification hypothesis.

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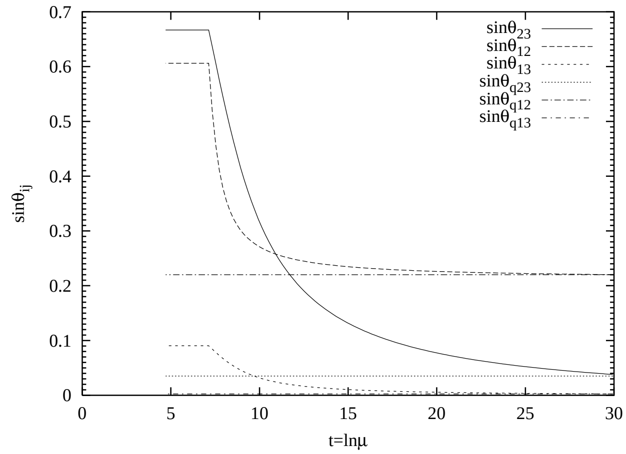


FIG. 1: Evolution of small quark-like mixings at the see-saw scale to bilarge neutrino mixings at low energies for the see-saw scale $M_R = 10^{13}$ GeV with input and output mass-eigen values and mixing angles given in the first column of Table I. The solid, long-dashed and short-dashed lines represent $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively, as defined in the text. Almost horizontal lines represent the corresponding sines of the CKM mixing angles in the quark sector

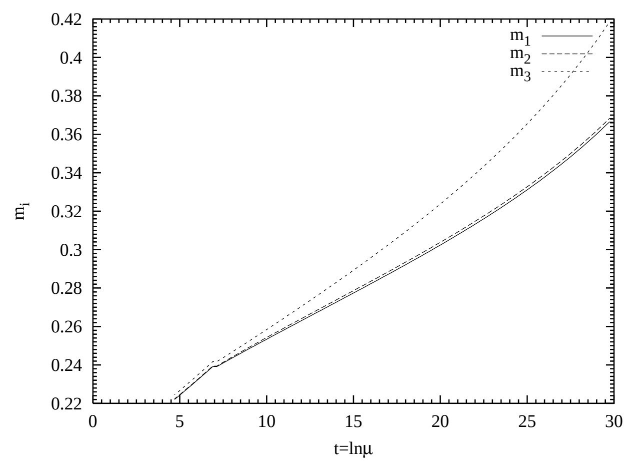


FIG. 2: Evolution of neutrino mass eigenvalues from the see-saw scale $M_R = 10^{13}$ GeV corresponding to initial values $m_1^0 = 0.3682$ eV, $m_2^0 = 0.37$ eV, and $m_3^0 = 0.4210$ eV, leading to the predictions $m_1 = 0.2201$ eV, $m_2^0 = 0.2223$ eV, and $m_3^0 = 0.2244$ eV at $M_Z$ and bi-largemixings as shown in Fig.1 and Table I

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TABLE I: Radiative magnification to bilarge mixings at low energies for input values of $\sin\theta^0_{23} = 0.038$, $\sin\theta^0_{13} = 0.0025$, and $\sin\theta^0_{12} = 0.22$ at the see-saw scale $M_R = 10^{13}$ GeV.

| | | | | | |
|---|---|---|---|---|---|
| $m^0_1$(eV) | 0.3682 | 0.5170 | 0.6168 | 0.7160 | 0.8160 |
| $m^0_2$(eV) | 0.3700 | 0.5200 | 0.6200 | 0.7200 | 0.8200 |
| $m^0_3$(eV) | 0.4210 | 0.5910 | 0.7050 | 0.8190 | 0.9330 |
| $m_1$(eV) | 0.2201 | 0.3107 | 0.3719 | 0.4317 | 0.4920 |
| $m_2$(eV) | 0.2223 | 0.3122 | 0.3723 | 0.4324 | 0.4926 |
| $m_3$(eV) | 0.2244 | 0.3152 | 0.3759 | 0.0.4366 | 0.4973 |
| $\Delta m^2_{12}$(eV$^2$) | $1.2 \times 10^{-4}$ | $3.0 \times 10^{-4}$ | $3.5 \times 10^{-4}$ | $6.0 \times 10^{-4}$ | $5.9 \times 10^{-4}$ |
| $\Delta m^2_{23}$(eV$^2$) | $1.0 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $2.6 \times 10^{-3}$ | $3.6 \times 10^{-3}$ | $4.6 \times 10^{-3}$ |
| $\sin\theta_{23}$ | 0.667 | 0.708 | 0.690 | 0.677 | 0.668 |
| $\sin\theta_{13}$ | 0.090 | 0.104 | 0.097 | 0.096 | 0.090 |
| $\sin\theta_{12}$ | 0.606 | 0.520 | 0.604 | 0.486 | 0.606 |

TABLE II: Same as Table I but for different mass scales

| $M_R$(**GeV**) | $10^{11}$ | $10^{15}$ | $2 \times 10^{18}$ |
|---|---|---|---|
| $m^0_1$(eV) | 0.4083 | 0.3970 | 0.5150 |
| $m^0_2$(eV) | 0.4100 | 0.400 | 0.5200 |
| $m^0_3$(eV) | 0.4510 | 0.4730 | 0.668 |
| $m_1$(eV) | 0.2723 | 0.2093 | 0.1714 |
| $m_2$(eV) | 0.2726 | 0.2098 | 0.1718 |
| $m_3$(eV) | 0.2745 | 0.2124 | 0.1750 |
| $\Delta m^2_{12}$(eV$^2$) | $1.6 \times 10^{-4}$ | $2.0 \times 10^{-4}$ | $1.36 \times 10^{-4}$ |
| $\Delta m^2_{23}$(eV$^2$) | $1.0 \times 10^{-3}$ | $1.1 \times 10^{-3}$ | $1.1 \times 10^{-3}$ |
| $\sin\theta_{23}$ | 0.711 | 0.682 | 0.684 |
| $\sin\theta_{13}$ | 0.103 | 0.098 | 0.094 |
| $\sin\theta_{12}$ | 0.571 | 0.463 | 0.422 |

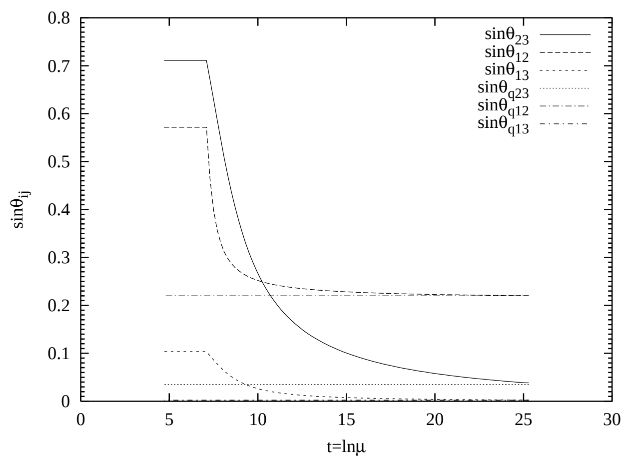


FIG. 3: Same as Fig.1 but for $M_R = 10^{11}$ GeV and inputs given in Table II

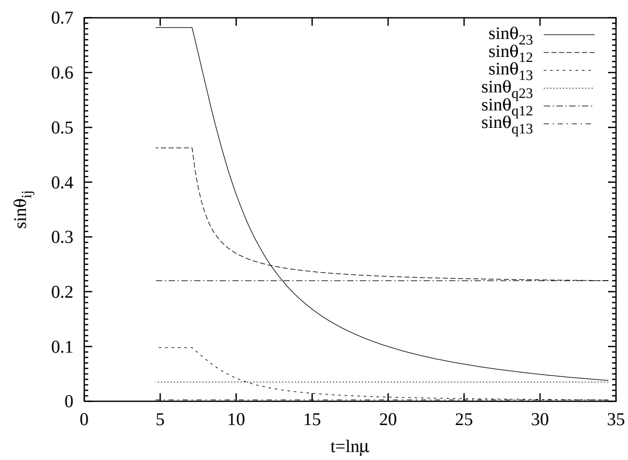


FIG. 4: Same as Fig.1 but for $M_R = 10^{15}$ GeV and input given in Table II


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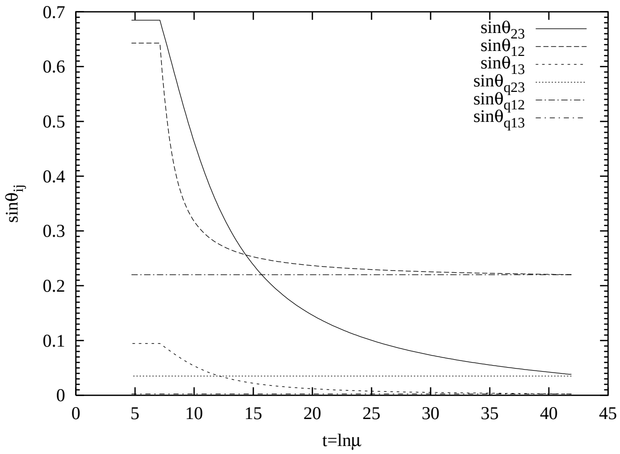


FIG. 5: Same as Fig.1 but for $M_R = 2\times10^{18}$ GeV and input given in Table II

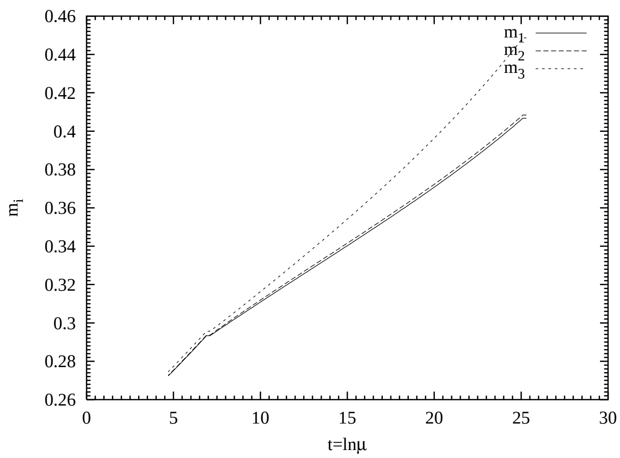


FIG. 6: Same as Fig.2 but for $M_R = 10^{11}$ GeV and input given in Table II

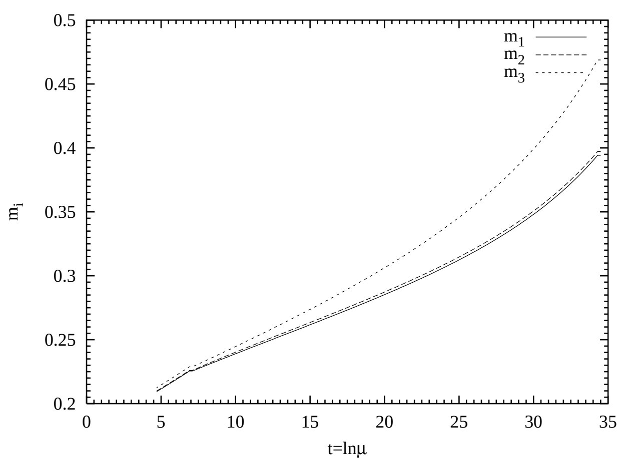


FIG. 7: Same as Fig.2 but for $M_R = 10^{15}$ GeV and input values given in Table II

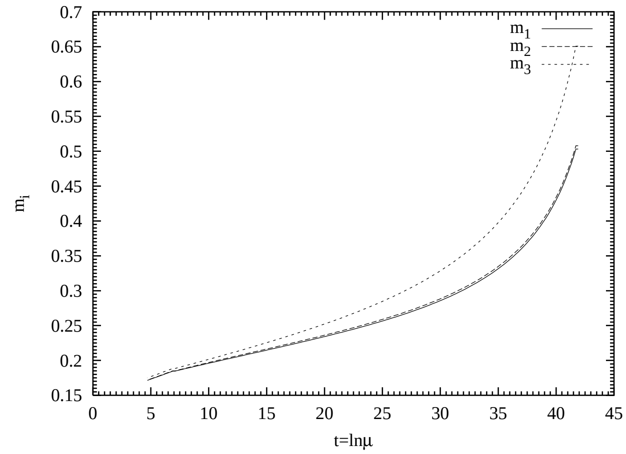


FIG. 8: Same as Fig.2 but for $M_R = 2\times10^{18}$ GeV and injput values given in Table II

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