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Radiative Magnification of Neutrino Mixings in Split Supersymmetry

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Radiative corrections to neutrino mixings in seesaw models depend on the nature of new physics between the weak and the GUT-seesaw scales and can be taken into account using the renormalization group equations. This new physics effect becomes particularly important for models with quasi-degenerate neutrino masses where small neutrino mixings at the seesaw scale can get magnified by radiative renormalization effects alone to match observations. This mechanism of radiative magnification which provides a simple understanding of why lepton mixings are so different from quark mixings was demonstrated by us for the standard supersymmetry scenario where the particle spectrum becomes supersymmetric above the weak scale. In this paper, we examine this phenomenon in split supersymmetry scenarios and find that the mechanism works also for this scenario provided the SUSY scale is at least 2-3 orders below the GUT-seesaw scale and one has larger values of $\tan\beta$.

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I. INTRODUCTION

An important question in particle physics is the nature of new physics beyond the astoundingly successful standard model. One compelling scenario for this new physics is the weak scale supersymmetry which provides an answer to a number of puzzles of the standard model such as a solution to the gauge hierarchy problem, the origin of electroweak symmetry breaking as well as providing a candidate for dark matter of the universe. An added virtue is that it unifies the disparate strengths of the weak, electromagnetic and strong forces at a scale of about 10^{16} GeV, opening up another rich landscape of new physics around this scale. This high scale new physics can only manifest itself at low energies via large radiative correction effects which depend logarithmically on mass as well as through effects such as proton decay which are suppressed by this new high scale.

There is reason to believe that there might be a manifestation of this new physics effect in the domain of neutrinos. This arises from the consideration of seesaw mechanism[1] which provides a very natural way to understand the extreme smallness of neutrino masses and which requires the existence of massive right handed neutrinos with at least one having mass close to the GUT scale. It is therefore suggestive that both the seesaw scale and the GUT scale are one and the same. This connection becomes particularly plausible in the context of models based on the grand unifying groups such as SO(10)[2] or $SU(2)_L \times SU(2)_R \times SU(4)_c$[3] that predict the existence of the right handed neutrino as part of the common fermion multiplet along with the fermions of the standard model.

In a recent paper, this concept of unification of GUT scale and seesaw scale was taken one step further by making the plausible hypothesis that quark and lepton mixings angles may be same at the seesaw scale due to quark lepton unification whereas the observed large mixings at the weak scale are a consequence of radiative corrections. Using the formulae for renormalization group evolution for neutrino mass[4, 5], it was shown that this possibility can be realized if the neutrino masses are quasi-degenerate and have same CP[6, 7]. Models based on $SU(2)_L \times SU(2)_R \times SU(4)_c$ group were presented where both the quasidegeneracy as well as mixing unification at the seesaw scale arose from the so-called type II seesaw formula[8]. Detailed renormalization group evolution of mixing angles then revealed that one indeed gets the desired bilarge mixing pattern at low energies which are in agreement with the solar and the atmospheric neutrino data. The third angle also undergoes radiative magnification, but remains small and well within the CHOOZ-Palo-Verde limit[9] due to the smallness of the corresponding initial value which is the quark mixing,

$\sin\theta_{13}^0 = 0.0025$.

Furthermore, our hypothesis has the interesting prediction that the common value for the quasi-degenerate neutrino mass is larger than 0.2 eV, which can be tested in the next round of the neutrinoless double beta decay experiments[10] and is in the range claimed in Ref.[11]. It also overlaps with the range accessible to the KATRIN experiment[12] and consistent with the bound obtained from WMAP[13]. Threshold corrections at the low-energy SUSY scale have been found to improve agreement[14] with the most recent data including KamLAND and SNO[15] for a particular choice of the SUSY particle spectrum.

As mentioned, two important ingredients that allow the radiative magnification to work are low-energy supersymmetry(SUSY) and large values of $\tan\beta$ which contribute large logarithmic factors for radiative magnification via amplified values of τ-Yukawa coupling.

Recently, a new unification scenario has been advocated[16] which is based on the observation that one can maintain gauge-coupling unification and neutralino dark matter without necessarily "buying" the entire machinary of weak scale supersymmetry but rather keeping only a subset of SUSY particles i.e. the gauginos and the Higgsino at the weak scale and pushing the rest of the superpartners to high intermediate scales($M_{\rm S}$). This has been called split supersummetry. This approach discards the naturalness requirement for the Higgs mass that motivated the weak scale supersymmetry-thus one has to fine tune the Higgs mass to every loop order. One also has no simple mechanism to understand the electroweak symmetry breaking. Despite these disadvantages, one might consider this as an interesting minimal extension of the standard model that preserves gauge-coupling unification and provides a neutralino dark matter and discuss how neutrinos fit into it.

Clearly, there is no inherent obstacle to implementing the seesaw mechanism and quark-lepton unification in these models. Only thing one has to keep in mind is that since the seesaw physics and quark-lepton unification generally tend to add new contributions to the gauge coupling evolution, if we want to retain the simple unification of three standard model gauge couplings, the seesaw scale must be at or above the GUT scale. In this paper, we work within a framework of this type and see if we can understand the large neutrino mixings via the radiative magnification mechanism.

Below the high SUSY scale($M_{\rm S}$) in split supersymmetry, apart from the presence of gauginos and Higgsinos, the effective theory is governed by nonSUSY Standard Model with one light Higgs doublet. As the two important ingredients in the radiative magnification scenario i.e. the long intervals of running in the presence of weak scale SUSY and the enhancement factor due to $\tan\beta$ are missing in the split supersymmetry below M_S, a natural apprehension emerges about the validity of this mechanism[6, 7] for large neutrino mixings. Working within a model with quark-lepton unification based on the group $SU(2)_L \times SU(2)_R \times SU(4)_c$ and degenerate masses for neutrinos obtained via the type II seesaw mechanism[8, 19] and the usual split SUSY spectrum, we show that this model does indeed lead to bilarge mixing pattern at low energies. The $SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry guarantees mixing unification between the quark and the lepton sectors at the seesaw scale (i.e. $\theta_{ij}^q(M_R) = \theta_{ij}^\ell(M_R)$).

The new point which is different from the work of Ref.[7] is that running of neutrino masses is now different. Furthermore, in order to maintain gauge coupling unification, we must have the seesaw scale equal to the GUT scale, as already noted. We carry out this running effect and show that despite the high split SUSY scale, the radiative magnification mechanism operates successfully to give the bilarge mixings at the weak scale provided that, in addition to the quasi-degenerate Majorana neutrino masses with same CP, we have the scalar SUSY partners with masses starting at least 2-3 orders below the seesaw scale. Whereas with low-energy SUSY, the radiative magnification occurs at $\mu \leq 1$ TeV, in split supersymmetry it occurs at the high value of the SUSY scale which may be any where between 10^5 GeV-10^{15} GeV. Below this scale, the RGE running does not affect the neutrino mixings significantly. What is interesting is that radiative magnification occurs for the scalar susy partner masses close to even the scale 10^{15} GeV so that a major part of the running is nonsupersymmetric. Including small threshold effects needed to fit the details like the solar mass-squared difference, the extrapolated low enery values of Δm_{32}^2 , Δm_{21}^2 , and the mixing angles are in excellent agreement with the current solar , atmospheric, and the reactor neutrino data.

This paper is organized as follows: in sec. 2 we discuss the relevant RGEs in the context of split supersymmetry. In sec. 3, we present allowed perturbative region of $\tan\beta$, radiative magnification of mixing angles at high SUSY scales and low energy extrapolation of masses and mixings. In sec. 4, we discuss threshold effects and their estimations. Discussion of the results and conclusions are presented in sec.5.

II. RENORMALIZATION GROUP EQUATIONS IN SPLIT SUPERSYMMETRY

In this section while discussing renormalization group equations(RGEs) for neutrino masses and mixings we point out some special features of split supersymmetry that contribute to radiative magnification of the mixings at high SUSY scales. Since we will follow the bottom-up approach to determine initial conditions at the seesaw scales for RG-evolutions of neutrino parameters in the top-down approach, we use the stadard RGEs for the SUSY, split supersymmetry, and nonSUSY cases as applicable in appropriate domains[16, 17, 18]. In the context of split supersymmetry where the scale(M_S) of the scalar superpartner masses could be much larger than the weak scale[16], the seesaw scale(M_R) is clearly always larger than the SUSY scale(M_S) i,e $M_R >> M_S$. We assume the neutrino mass eigenstates to be quasi-degenerate and possess the same CP. We also ignore all CP violating phases in the mixing matrix, and adopt the mass ordering to be of type $m_3 \gtrsim m_2 \gtrsim m_1$. Parametrizing the 3×3 mixing matrix as

$$U = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}-c_{12}s_{13}s_{23} & c_{12}c_{23}-s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}s_{13}c_{23} & -c_{12}s_{23}-c_{23}s_{13}s_{12} & c_{13}c_{23} \end{bmatrix}, \tag{1}$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}(i,j=1,2,3)$, the RGEs for the mass eigen values and mixing angles can be written as [4, 5]

$$\frac{dm_i}{dt} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u = b_i^{(m)} m_i,\ (i=1,2,3)\,. \tag{2}$$

$$\begin{aligned}
\frac{ds_{23}}{dt} &= -F_\tau {c_{23}}^2\left(-s_{12}U_{\tau 1}D_{31}+c_{12}U_{\tau 2}D_{32}\right), && (3)\\
\frac{ds_{13}}{dt} &= -F_\tau c_{23}{c_{13}}^2\left(c_{12}U_{\tau 1}D_{31}+s_{12}U_{\tau 2}D_{32}\right), && (4)\\
\frac{ds_{12}}{dt} &= -F_\tau c_{12}\left(c_{23}s_{13}s_{12}U_{\tau 1}D_{31}-c_{23}s_{13}c_{12}U_{\tau 2}D_{32}\right.\\
&\quad \left.+U_{\tau 1}U_{\tau 2}D_{21}\right). && (5)
\end{aligned}$$

Here $D_{ij}=(m_i+m_j))/(m_i-m_j)$ and, for MSSM with $\mu \geq M_{\rm S}$,

$$\begin{aligned}
F_\tau &= -h_\tau^2/\left(16\pi^2\cos^2\beta\right),\\
F_u &= \left(\frac{1}{16\pi^2}\right)\left(\frac{6}{5}g_1^2+6g_2^2-6\frac{h_t^2}{\sin^2\beta}\right),
\end{aligned} \tag{6}$$

but, for $\mu \leq M_{\rm S}$,

$$\begin{aligned}
F_\tau &= 3h_\tau^2/\left(32\pi^2\right),\\
F_u &= \left(3g_2^2-2\lambda-6h_t^2-2T\right)/\left(16\pi^2\right).
\end{aligned} \tag{7}$$

with the definitions of the couplings at $M_{\rm S}$,

$$\begin{aligned}
g_u\left(M_{\rm S}\right) &= g_2\left(M_{\rm S}\right)\sin\beta,\quad g_d\left(M_{\rm S}\right)=g_2\left(M_{\rm S}\right)\cos\beta,\\
\tilde{g}_u\left(M_{\rm S}\right) &= (3/5)^{1/2}g_1\left(M_{\rm S}\right)\sin\beta,\quad \tilde{g}_d\left(M_{\rm S}\right)=(3/5)^{1/2}g_1\left(M_{\rm S}\right)\cos\beta.
\end{aligned} \tag{8}$$

The additional term present in eq.(7) which is specific to the split SUSY below $M_{\rm S}$ and the Higgs quartic coupling are,

$$\begin{aligned}
T &= \frac{3}{2}\left({g_u}^2+{g_d}^2\right)+\frac{1}{2}\left(\tilde{g}_u^2+\tilde{g}_d^2\right),\\
\lambda(M_{\rm S}) &= \frac{\left[g_2^2(M_{\rm S})+(3/5)g_1^2(M_{\rm S})\right]}{4}\cos^2 2\beta.
\end{aligned} \tag{9}$$

The basic mechanism responsible for radiative magnification of mixings of quasi-degenerate neutrinos with same CP in MSSM which has been pointed out earlier[6, 7] is also applicable with split SUSY but with high value of $M_{\rm S}$. Since m_i and m_j are scale dependent, the initial difference existing between them at $\mu = M_R$ is narrowed down during the course of RG evolution as we approach the SUSY scale. This causes $D_{ij} \to$ large, and hence large magnification to the mixing angle due to radiative effects through eqs.(3)-(5). Another major factor contributing to radiative magnification is due to amplified negative value of F_τ in eqs.(3)-(5) in the presence of SUSY by a factor $\simeq \tan^2\beta$ in the large $\tan\beta$ region.

An approximate estimation of values of $\tan\beta$ needed for radative magnification in split SUSY is possible by noting that the magnification has been realized in MSSM with weak-scale SUSY for $\tan\beta \approx 50-55$ and in that case the major factor controlling magnification is due to amplification of τ-Yukawa coupling[6, 7]. Expecting similar amplification in split SUSY gives the appoximate relation,

$$\left(h_\tau^0 \tan\beta^0\right)^2 \log(M_R^0/M_S^0) \;=\; \left(h_\tau \tan\beta\right)^2 \log(M_R/M_S). \tag{10}$$

where quantities with zero superscript(without superscript) refer to weak-scale(split) supersymmetry. Noting that the ratio of the two τ-Yukawa coplings in eq.(10) is $\approx O(1)$, and using $\tan\beta^0 = 50-55$[6, 7], and $M_R^0 = M_R = M_U = 2\times 10^{16}$ GeV, the above relation gives the approximate requirements for split supersymmetry case as $\tan\beta = 75$, and 110 for $M_{\rm S} = 10^9$ GeV, and 10^{13} GeV, respectively. In fact, it has been noted that split SUSY permits larger values of $\tan\beta$ than the MSSM with weak scale SUSY[20].

It would be shown in Sec.3 that the allowed perturbative upper limit of $\tan\beta$ increases with increase of the SUSY scale. Thus even though the supersymmetric part of the running of neutrino parameters is much less than in the weak scale supersymmetry case, larger values of tan beta increases the value of F_τ and this in turn leads to the desired enhancement of the radiative magnification effect on the mixing angles.

In addition to the above, here we point out that there is a specific mechanism which operates in the allowed parameter space of split SUSY that causes mass eigenvalues to approach one another faster and thus, drives radiative magnification even if the SUSY scale is many orders larger than the weak scale. Noting that the β-function coefficient for the evolution of mass eigenvalue $m_i(\mu)$ in eq.(2) is $b_i^{(m)} = -2F_\tau U_{\tau i}^2 - F_u$, near the seesaw scale $U_{\tau i}^2 \sim 0$, for $i = 1, 2$ because of high-scale mixing unification constraint, but $U_{\tau 3}^2 \sim 1$. Therefore, the evolution of the third mass eigen value would be different from the first two. Further, since supersymmetric F_τ is negative for $\mu > M_{\rm S}$ and, depending upon the allowed values of the gauge and the Yukawa coupling constants contributing to F_u in eq.(6) above the high SUSY scale, the β-function coefficient may be positive or negative thus resulting in the decreasing or increasing behavior of the mass-eigen value below the seesaw scale down to $M_{\rm S}$. In particular with split supersymmetry $F_{\rm u}$ may be positive due to dominance of gauge couplings over the top-Yukawa coupling resulting in negative values of $b_{1,2}^{(m)}$ for $\mu > M_{\rm S}$. But since F_τ is negative, the positive value of the first term, $-2F_\tau U_{\tau i}^2$, may partly or largely cancel with the second term leading to a large or small positive value of $b_3^{(m)}$. As a result, the first two mass eigen values may be expected to increase but the third may decrease, rapidly or slowly, from their input values at $M_{\rm R}$. This increasing behavior of some mass eigen values(m_1, m_2) in the region $M_{\rm S} < \mu < M_{\rm R}$ would be in sharp contrast to that in MSSM with weak-scale SUSY where the allowed high scale values of the gauge and the Yukawa couplings and the mixing unification constraint have been found to make all the three β-function coefficients positive resulting in the decrasing behavior of the three mass eigen values below the seesaw scale although with different rates[7]. Thus, with split SUSY the allowed values of the parameter space may cause three mass eigen values to approach sufficiently closer to one another even after much shorter interval of running. This may occur at high values of $M_{\rm S}$ which may even be only $2-3$ orders smaller than $M_{\rm R}$. Then the functions $D_{ij} \to$ large near $M_{\rm S}$ causing magnification of the mixing angles.

On the otherhand for $\mu < M_{\rm S}$, the τ-Yukawa coupling has negligible contribution and the β-function coefficients are nearly the same for all the three eigen values and approximately equal to $-2F_u$. From eq.(7) is clear that in the presence of nonSUSY SM, F_u is negative below $M_{\rm S}$ resulting in positive beta-function coefficients for all mass-eigen values causing them to decrease approximately in the same manner down to $M_{\rm Z}$. It is easy to check, using boundary conditions given in eqs.(8)-(9), that F_u is negative at the starting point $M_{\rm S}$ of the nonSUSY theory, with $16\pi^2 F_u(M_{\rm S}) = -\frac{3}{5}g_1^2(M_{\rm S}) - 2\lambda(M_{\rm S}) - 6h_t^2(M_{\rm S})$. Although this feature is also common to MSSM with the weak-scale SUSY, in split SUSY the running interval for nonsupersymmetric theory is much larger.

Our numerical solutions to masses and mixing angles reported in Sec.3 with split SUSY are found to corroborate these properties of the RG-evolution.

III. RADIATIVE MAGNIFICATION AT HIGH SUSY SCALES

In this section we discuss the realization of radiative magnification of neutrino mixing angles at high SUSY scales and low energy extrapolations of mass eigen values and mixing angles for comparison with the available experimental data.

III.1. Perturbative limit on $\tan\beta$

The fact that grand unification with split supersymmetry allows larger values of $\tan\beta$ than those with weak-scale SUSY has been also noted in ref.[20]. Since the value of $\tan\beta$ is an important ingredient, at first we estimate the maximum allowed values of $\tan\beta$ for every value of $M_{\rm S}$=10^3 GeV - 10^{15} GeV. For this purpose we use the standard PDG values of masses and couplings[23] at $\mu = M_Z$ and follow the bottom-up approach scanning the values of the three gauge couplings and the third generation Yukawa couplings while varying input values of $\tan\beta$ over a wide range at all values of μ up to the GUT-scale using the RGEs in the appropriate regions[16, 17, 18]. The lowest allowed value of $\tan\beta$ is determined when the top-quark Yukawa coupling reaches the perturbative limit($h_t = 3.54$). Similarly the upper limit is determined by noting that the τ-lepton Yukawa coupling attains its perturbative limit at the GUT-scale. These are shown in Fig.1. It is clear that the upper limit on $\tan\beta$ is larger compared to the weak-scale SUSY model and this limit also increases quite significantly with increasing value of $M_{\rm S}$ in split supersymmetry. For examle wth $M_{\rm S}$=10^7 GeV, 10^9 GeV, 10^{13} GeV, and 10^{15} GeV, , the upper limits are found to be $\tan\beta = 85, 100, 132,$ and 160, respectively. Through this bottom-up approach we also noted the values of different coupling constants and CKM mixings at different values of the seesaw scales starting from their low-energy values. Some of the coupling constants at the seesaw scales are presented in Table.1 where the factors like $\sin\beta$ or $\cos\beta$ are included in the values of SUSY Yukawa couplings. With large values of $\tan\beta$ allowed near the upper limit, the Higgs mass prediction in split SUSY tend to be independent of this parameter and the two Yukawa couplings, g_d and $\tilde{g}_d$, have negligible effects below M_S.

III.2.Radiative Mgnification of Mixing Angles

The values of gauge and Yukawa couplings , the CKM mixing angles obtained at the seesaw scale from the bottom-up approach and finetuned values of light neutrino masses($m_i^0, i = 1, 2, 3$) are used as inputs to obtain solutions from the neutrino RGEs in the top-down approach. These input parameters are given in Table 1. The unknown parameters m_i^0 are determined in such a way that the magnified values of mixing angles are at first obtained at $M_{\rm S}$ along with the mass eigen-values. These are further extrapolated down to $\mu = M_Z$ through the corresponding RGEs to give closest agreement with the experimental data:

$$\begin{aligned}\Delta m^2_{21} &= (5-8)\times 10^{-5}{\rm eV}^2,\ \Delta m^2_{32} = (1.2-3)\times 10^{-3}{\rm eV}^2,\\ \sin\theta_{23} &= 0.67-0.707,\ \sin\theta_{12} = 0.5-0.6,\ \sin\theta_{13} \le 0.16\ .\end{aligned} \tag{11}$$

As noted in Sec.2 the signs and values of the three β-functions for the mass eigen values at the respective seesaw scales are easily checked from the initial values of couplings given in Table 1 using eq.(2). For example for the case with $M_{\rm R} = 2\times 10^{16}$ GeV, $M_{\rm S} = 10^9$ GeV, $\tan\beta = 90$, the values of coupling constants given in Table.1 yield the β-function coefficients at M_R to be $b_3^{(m)} = 0.0016$, $b_1^{(m)} = b_2^{(m)} = -0.0115$ which, as per the predictions of Sec.2, are responsible for the slow decrease of m_3 and rather faster increase of m_1 and m_2 below the seesaw scale. These features are clearly displayed in Fig.2. Similarly for $M_{\rm R} = 2\times 10^{16}$ GeV, $M_{\rm S} = 10^{13}$ GeV, $\tan\beta = 90$, these coefficients are $b_3^{(m)} \simeq 0.0015$, $b_1^{(m)} = b_2^{(m)} = -0.01165$ at the GUT-seesaw scale. It is quite interesting to note that the mutual differences are narrowed down by the increase of the first and the second mass eigenvalues and simultaneous decrease of the third eigen value leading to large D_{ij} and the magnification at $M_{\rm S}$. This may be contrasted with the low-energy SUSY case where all the three masses showed decreasing behavior although with different rates[7]. This increasing behavior of neutrino masses $m_{1,2}$ in contrast to the decrasing behavior in low-energy SUSY is due to different relative values of the gauge and Yukawa couplings at higher scales in the large $\tan\beta$-region in split SUSY some of which are given in Table.1. Below $M_{\rm S}$ the decrease is almost uniform due to approximate equality of the corresponding β-function coefficients of the three mass eigenvalues in this region as discussed in Sec.2. The solutions shown in the Fig.4 and Fig.5 clearly exhibit radiative magnification to bilarge mixings at the high values of $M_{\rm S}$=10^9 GeV, and $M_{\rm S}$=10^{13} GeV corresponding to the evolutions of mass eigen values as shown in Fig.2, and Fig.3, respectively.

For the seesaw scale at $M_{\rm R} = M_{\rm Pl} = 2 \times 10^{18}$ GeV the magnification of mixings takes place even at such high SUSY-scale as $M_{\rm S} = 10^{15}$ GeV.

It is clear that due to larger allowed values of $\tan\beta$ and also due to opposite signs of β-function coefficients of mass eigen values at high scales, the radiative magnification of neutrino mixings is possible in split SUSY at high SUSY scales even if the SUSY-scale is onle $2-3$ orders smaller than the seesaw scale. We find that although enhancement due to RG evolution occurs in the $\nu_e - \nu_\tau$ sector also, the predicted low energy value remains at $\sin\theta_{13} = 0.08 - 0.1$ which is well within the CHOOZ-Palo Verde bound[9] and can be tested in the planned θ_{13} search experiments.

IV. THRESHOLD EFFECTS

In the previous section we considered the mass squared differences obtained at lower energies purely by RG-evolution and these, as shown in Table 1, are in agreement with gross features of the experimental data on atmospheric neutrinos but falls somewhat on the higher side of solar neutrino data. Since threshold effects have been shown to make significant contribution on quasidegenerate neutrinos[14, 21, 22] in this section we estimate them to spplement our RG-solutions of Sec.3. In particular we note that the loop facotr at the high-SUSY-scale threshold($M_{,\rm S}$), where the heavy superpartners are located, assumes a very simple form. The threshold loop factor for neutrino mass at the electroweak scale is similar to the SM[22]

In contrast to the weak-scale SUSY where the superpartners contribute predominantly near $M_{\rm Z}$[21, 22], in split SUSY the dominant threshold corrections to neutrino masses occur near high SUSY-scale threshold $\mu_0 \sim M_{\rm S}$. Since the mixing angles in the radiative magnification scenario are very nearly the same at both the thresholds, it is quite convenient to use the approximation for the PMNS matix $U_{\alpha\rm i}(M_{\rm S}) \sim U_{\alpha\rm i}(M_{\rm Z})$ and evaluate them in the limit $\theta_{13} \to 0$ and $\theta_{23} \to \pi/4$ which are very closely cosistent with our solutions[14].

Using the loop factors T_{ij} in the mass basis with $(i,j = 1,2,3)$ the threshold effects on mass eigen values at the threshold μ_0 are expressed as[21, 22],

$$m_{ij}(\mu_0) = m_i(\mu_0)\delta_{ij} + m_i(\mu_0)T_{ij}(\mu_0) + m_j(\mu_0)T_{ji}(\mu_0), (\mu_0 = M_{\rm Z}, M_{\rm S}) \tag{12}$$

Transforming the loop factors in terms of those in the flavor basis, $T_{\alpha\beta}(\alpha,\beta = e,\mu,\tau)$, it is straightforward to evaluate the effects in the limiting case and for quasidegenerate neutrinos with slightly different values of the common mass $m(\mu_0)$ at $\mu_0 = M_{\rm S}$, $M_{\rm Z}$ leading to the formula[14]

$$(\Delta m^2_{21})_{th}(\mu_0) = 4{\rm m}^2(\mu_0)\cos 2\theta_{12}[-{\rm T_e}(\mu_0) + ({\rm T}_\mu(\mu_0) + {\rm T}_\tau(\mu_0))/2], \tag{13}$$
$$(\Delta m^2_{32})_{th}(\mu_0) = 4{\rm m}^2(\mu_0)\sin^2\theta_{12}[-{\rm T_e}(\mu_0) + ({\rm T}_\mu(\mu_0) + {\rm T}_\tau(\mu_0))/2], \tag{14}$$

When corrections at both the thresholds are included along with the contribution due to RG-evolution, the analytical formula for the mass squared differences is expressed as,

$$(\Delta m^2_{ij})(M_{\rm Z}) = (\Delta m^2_{ij})_{\rm RG} + (\Delta m^2_{ij})_{\rm th}(M_{\rm Z}) + (\Delta m^2_{ij})_{\rm th}(M_{\rm S}), (i > j = 1,2,3), \tag{15}$$

where the first term in the RHS of eq.(15) has been already estimated from RG-evolution and the second and the third terms are estimated through eqs.(13)-(14). As the RG-evolution effects estimated in Sec.3 have already given approximately the correct values of Δm^2_{32} for atmospheric neutrino data and Δm^2_{21} only $4\sigma - 5\sigma$ larger than the KamLAND and SNO data, small and simpler threshold effects might be sufficient to account for these deviations. In split supersymmetry such contributions can easily arise from the high-scale SUSY-threshold effects at $\mu_0 = M_{\rm S}$.

Since $M_{\rm S} \gg M_{\rm Z}$ in split supersymmetry, left-handed charged sleptons and sneutrinos have almost identical masses. Denoting the masses of the left-handed sleptons by $M_{\tilde\ell^L_\alpha}$ and the right-handed charged sleptons mass by $M_{\tilde\ell^R_\alpha}$ we obtain a simple formula for the loop factors at the high SUSY scale($\mu_0 = M_{\rm S}$),

$$\begin{aligned} 16\pi^2 T_\alpha(M_S) &= (3/8)(g_1^2(M_S)/5 + g_2^2(M_S))\left(ln(M^2_{\tilde\ell^L_\alpha}/M_S^2) - 1/2\right) \\ &\quad + \delta_{\alpha\tau}(1/4)h_\tau^2(M_S)(1+\tan^2\beta)\left(ln(M^2_{\tilde\ell^R_\alpha}/M_S^2) - 1/2\right). \end{aligned} \tag{16}$$

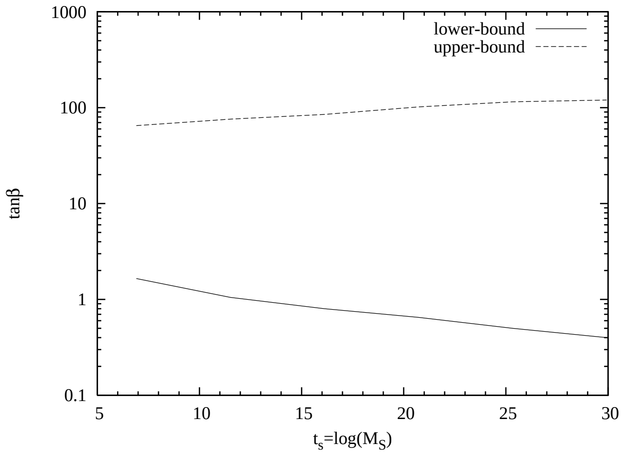

FIG. 1: Perturbative upper and lower limits on the allowed values of $\tan\beta$ as a function of SUSY scale $M_{\rm S}$, taken in unit of GeV, in split supersymmetry.

Ignoring the low-energy threshold effect at $M_{\rm Z}$ we find that with $M_{\tilde{\ell}_\alpha}$, or $M_{\tilde{\ell}^R_\alpha}$ few times lighter than $M_{\rm S}$ gives the desired threshold corrections with negative sign. For, example, using a simple and plausible choice of approximate degeneracy in the scalar superpartner spectrum around $M_{\rm S}$ with $M_{\tilde{e}_L} = M_{\tilde{\mu}_L} = M_{\tilde{\tau}_L} = M_{\tilde{e}_R} = M_{\tilde{\mu}_R}$, we have $[-T_e(M_S)+(T_\mu(M_S)+T_\tau(M_S))/2] \simeq (h^2_\tau(M_S)\tan^2\beta)[ln(M^2_{\tilde{\tau}_R}/M^2_S)-1/2]/(128\pi^2)$. Then with $M_{\tilde{\tau}_R}/M_S = 0.5-1.1$, numerical values of threshold corrections are obtained as shown in Table.1. Such threshold contributions, when added to the RG-effects, bring the theoretical predictions in concordance with all the available neutrino data including those from KamLAND and SNO.

V. DISCUSSION AND CONCLUSION

In this section we discuss the implications of our results briefly and state our conclusions. Our solutions require quasi-degenerate neutrino mass eigen values in the range $0.15\ eV < m_i(M_Z) < 0.5\ eV$ leading to the effective mass in neutrinoless double beta decay given by $| < M_{ee} > | = |\Sigma_i m_i U^2_{ei}| = 0.15$ eV -0.5 eV. This is accessible to all the experiments being planned to search for the neutrinoless double beta decay. Furthermore, this range of neutrino masses also overlaps with the range accessible to the KATRIN[12] Tritium beta decay experiment. The prediction $U_{e3} \equiv \sin\theta_{13}$=$0.08-0.10$ is also accessible to several planned long-baseline neutrino experiments[25] as well as the planned reactor experiments[24]. As discussed in ref.[7] the range of eigenvalues of neutrino masses is consistent with WMAP observations and also with the combined analysis of WMAP+2dF GRS data[13].

In summary, we had shown previously that in the MSSM with weak-scale SUSY[7], the hypothesis of quark- lepton mixing unification and quasi-degenerate neutrino spectrum at the seesaw scale successfully explains the observed mixing pattern for neutrinos i.e. two large mixings needed for $\nu_e-\nu_\mu$ and $\nu_\mu-\nu_\tau$ and small mixing for U_{e3} at low energies. In this paper we have extended this discussion to the case of split supersymmetry where we find that, despite the absence of low-energy SUSY and the corresponding absence of RG-running with an amplified value of τ-Yukawa coupling over a considerable mass interval (M_Z-$M_{\rm S}$), radiative magnification of neutrino mixings does occur. These deficits seem to be adequately compensated by the larger values of $\tan\beta$ allowed in this case and the positive and negative values of β-function coefficients for the running of mass eigen values at high scales. The key tests of the model still remain the common mass of neutrinos above 0.15 eV and U_{e3} between 0.08-0.1, as in [7].

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TABLE I: Radiative magnification to bilarge neutrino mixings for input values of $m_i^0(i=1,2,3)$, $\sin\theta_{23}^0=0.038$, $\sin\theta_{13}^0=0.0025$, and $\sin\theta_{12}^0=0.22$ at the high scale M_R. The initial values of SUSY Yukawa couplings at seesaw scales include factors $\sin\beta$ or $\cos\beta$ as applicable.

M_R(GeV)	2×10^{16}	2×10^{16}	2×10^{16}	2×10^{18}
M_S(GeV)	10^{13}	10^{13}	10^{9}	10^{15}
$\tan\beta$	90	130	90	140
g_1^0	0.6206	0.6200	0.6518	0.6540
g_2^0	0.6203	0.6198	0.6522	0.6125
g_3^0	0.6262	0.6260	0.6565	0.5935
h_t^0	0.3943	0.4035	0.4450	0.3625
h_b^0	0.5325	0.9627	0.8052	0.9399
h_τ^0	1.0181	2.2676	1.7592	2.5684
m_1^0(eV)	0.4483	0.2965	0.2267	0.3648
m_2^0(eV)	0.4500	0.30	0.2300	0.3700
m_3^0(eV)	0.4911	0.2965	0.3188	0.5060
m_1(eV)	0.2934	0.1938	0.1766	0.2301
m_2(eV)	0.2937	0.1944	0.1773	0.2310
m_3(eV)	0.2956	0.1983	0.1816	0.2364
$(\Delta m_{21}^2)_{\rm RG}$(eV^2)	1.693×10^{-4}	2.28×10^{-4}	2.31×10^{-4}	3.96×10^{-4}
$(\Delta m_{32}^2)_{\rm RG}$(eV^2)	1.25×10^{-3}	1.53×10^{-3}	1.55×10^{-3}	2.53×10^{-3}
$M_{\tilde\tau_R}/M_S$	1.1	0.92	0.50	1.03
$(\Delta m_{21}^2)_{\rm th}$(eV^2)	-0.84×10^{-4}	-1.48×10^{-4}	-1.51×10^{-4}	-3.16×10^{-4}
$(\Delta m_{32}^2)_{\rm th}$(eV^2)	-0.61×10^{-4}	-0.11×10^{-3}	-0.10×10^{-3}	-0.19×10^{-3}
$(\Delta m^2)_{\rm sol}$(eV^2)	8.5×10^{-5}	8.0×10^{-5}	8.0×10^{-5}	8.0×10^{-5}
$(\Delta m^2)_{\rm atm}$(eV^2)	1.19×10^{-3}	1.42×10^{-3}	1.45×10^{-3}	2.34×10^{-3}
$\sin\theta_{12}$	0.545	0.5474	0.549	0.526
$\sin\theta_{23}$	0.701	0.703	0.707	0.707
$\sin\theta_{13}$	0.101	0.101	0.103	0.104

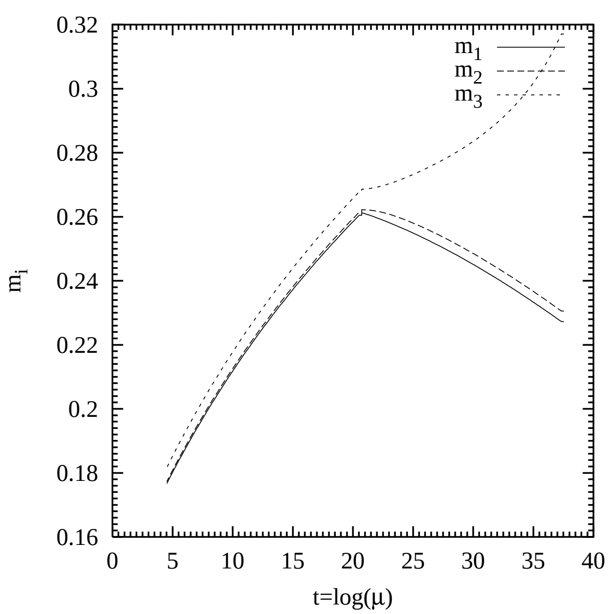

FIG. 2: Renormalization group evolution of light Majorana neutrino mass eigenvalues showing both the increasing and decreasing behaviors between SUSY scale $M_S=10^9$ GeV and the GUT-seesaw scale $M_R=2\times10^{16}$ GeV in split supersymmetry where $t=\log(\mu)$ and μ is in unit of GeV. The input values are given in the fourth column of Table 1.

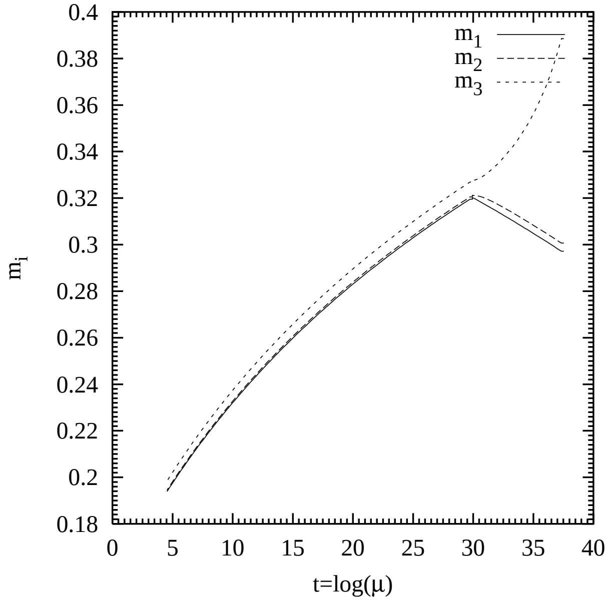

FIG. 3: Same as Fig.2 but with $M_R = 2 \times 10^{16}$ GeV and SUSY scale $M_{\rm S} = 10^{13}$ GeV. The input values and low energy extrapolations are given in the third column of Table 1.

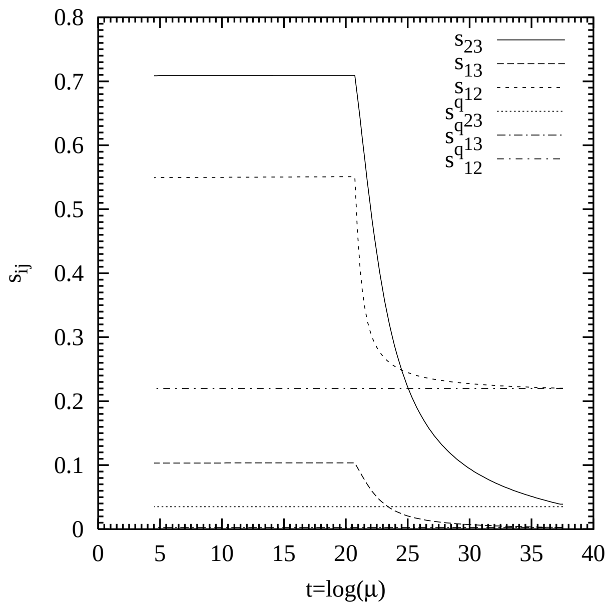

FIG. 4: Evolution of small quark-like mixings at the GUT-seesaw scale, $M_R = 2 \times 10^{16}$ GeV to bilarge neutrino mixings at the SUSY scale $M_S = 10^9$ GeV, and extrapolation to low energies for the input and output mass-eigen values and mixing angles given in the fourth column of Table 1. The solid, long-dashed, and short-dashed lines represent the sines of neutrino mixing angles $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively, as defined in the text. Almost horizontal lines originating from the seesaw scale represent the sines of corresponding CKM mixing angles in the quark sector. The mass scale μ is in GeV

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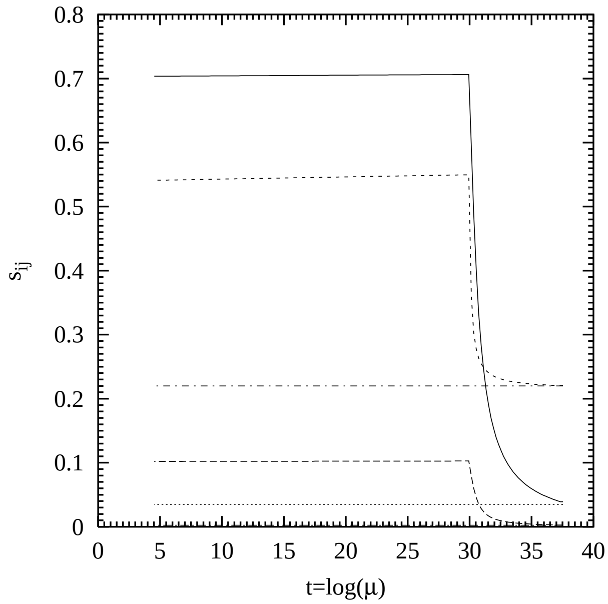

FIG. 5: Same as Fig.4 but with $M_R = 2 \times 10^{16}$ GeV and M_S=10^{13} GeV with input and output masses and mixing angles as given in the third column of Table 1.

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