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Hexagonal SU(3) Unification and its Manifestation at the TeV Scale

Qing-Hong Cao, Shao-Long Chen, Ernest Ma

Physics Department, University of California, Riverside, California 92521, USA

G. Rajasekaran

Institute of Mathematical Sciences, Chennai (Madras) 600113, India

Abstract

We consider $SU(3)_C \times SU(2)_{AL} \times SU(2)_{BL} \times U(1)_Y$ as the low-energy subgroup of supersymmetric $SU(3)^6$ unification. This may imply small deviations from quark-lepton universality at the TeV scale, as allowed by neutron-decay data. New particles are predicted with specific properties. We discuss in particular the new heavy gauge bosons corresponding to $SU(2)_{AL} \times SU(2)_{BL} \to SU(2)_L$.

1 Hexagonal SU(3) Model

The extension from $SU(3)_C \times SU(3)_L \times SU(3)_R$ trinification [1] to $SU(3)^6$ unification [2] allows for the natural anomaly-free implementation of chiral color [3] and quark-lepton nonuniversality [4, 5] at the TeV scale. In view of the fact that there is an experimental hint [6] of the latter, but not the former, we explore the possibility that the low-energy reduction of hexagonal $SU(3)$ unification is actually $SU(3)_C \times SU(2)_{AL} \times SU(2)_{BL} \times U(1)_Y$ at the TeV scale, where quarks couple to $SU(2)_{AL}$, but leptons may choose either $SU(2)_{AL}$ or $SU(2)_{BL}$ or both, and the $SU(2)_L$ of the Standard Model (SM) is the diagonal subgroup of $SU(2)_{AL} \times SU(2)_{BL}$ [7]. We show how supersymmetric unification at around 10^{16} GeV may be maintained with a suitable choice of new particle content at the TeV scale and discuss their phenomenological consequences.
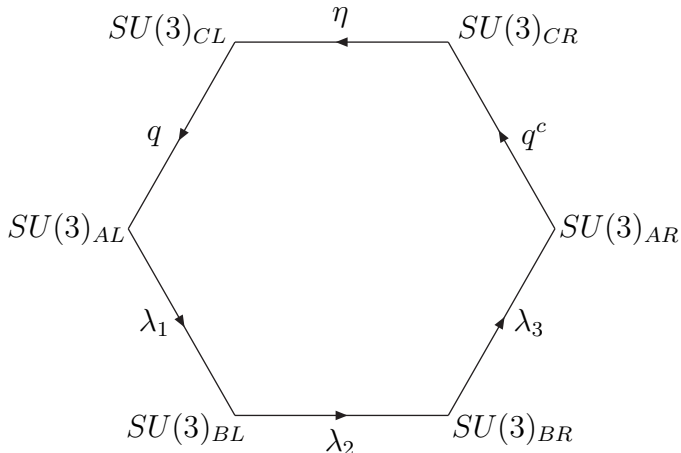

Figure 1: Moose diagram of quarks and leptons in $[SU(3)]^6$.

We start with the supersymmetric $SU(3)^6$ model of Ref. [2]. Under the gauge group $SU(3)_{CL} \times SU(3)_{AL} \times SU(3)_{BL} \times SU(3)_{BR} \times SU(3)_{AR} \times SU(3)_{CR}$, the six links of the

"moose" chain [8] are given by

$$q \sim (3, 3^*, 1, 1, 1, 1), \tag{1}$$

$$\lambda_1 \sim (1, 3, 3^*, 1, 1, 1), \tag{2}$$

$$\lambda_2 \sim (1, 1, 3, 3^*, 1, 1), \tag{3}$$

$$\lambda_3 \sim (1, 1, 1, 3, 3^*, 1), \tag{4}$$

$$q^c \sim (1, 1, 1, 1, 3, 3^*), \tag{5}$$

$$\eta \sim (3^*, 1, 1, 1, 1, 3), \tag{6}$$

as shown in Fig. 1. The electric charge is embedded into $SU(3)^6$ according to

$$Q = (I_3)_{AL} + (I_3)_{AR} - \frac{1}{2} Y_{AL} - \frac{1}{2} Y_{AR} + (I_3)_{BL} + (I_3)_{BR} - \frac{1}{2} Y_{BL} - \frac{1}{2} Y_{BR}. \tag{7}$$

Using the notation where the rows denote $(I_3, Y) = (1/2, 1/3), (-1/2, 1/3), (0, -2/3)$ and the columns denote $(I_3, Y) = (-1/2, -1/3), (1/2, -1/3), (0, 2/3)$, the particle content of this model is given in matrix form as

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}, \quad \lambda_i = \begin{pmatrix} N_i & E_i^c & \nu_i \\ E_i & N_i^c & e_i \\ \nu_i^c & e_i^c & S_i \end{pmatrix}, \tag{8}$$

and all the components of η are neutral. As shown in Ref. [2], this embedding of electric charge yields the canonical value of $3/8$ for $\sin^2 \theta_W$ at the unification scale M_U.

Whereas the quarks are unambiguously assigned in Eq. (8), the leptons are not. The left-handed doublets may be any linear combination of (ν_1, e_1) and (ν_2, e_2), while the right-handed doublets may be any linear combination of (ν_2^c, e_2^c) and (ν_3^c, e_3^c). We will see later exactly how this works. Note that if $SU(3)^6$ collapses to $SU(3)^3$ already at M_U, the leptons would then be unambigously assigned to λ_2.

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2   Gauge Coupling Unification

Above M_U, the six gauge couplings are assumed equal, maintained for example with a discrete Z_6 symmetry. At M_U, $SU(3)^6$ is assumed broken down to

$$SU(3)_C \times SU(2)_{AL} \times SU(2)_{BL} \times U(1)_Y$$

with the boundary conditions

$$\frac{1}{\alpha_C(M_U)} = \frac{1}{\alpha_{CL}(M_U)} + \frac{1}{\alpha_{CR}(M_U)} = \frac{2}{\alpha_U}, \tag{9}$$

$$\frac{1}{\alpha_{AL}(M_U)} = \frac{1}{\alpha_{BL}(M_U)} = \frac{1}{\alpha_U}, \tag{10}$$

$$\frac{3}{5\alpha_Y(M_U)} = \frac{2}{\alpha_U}. \tag{11}$$

At M_S, supersymmetry is assumed broken, together with the breaking of $SU(2)_{AL} \times SU(2)_{BL}$ to $SU(2)_L$ with the boundary condition

$$\frac{1}{\alpha_L(M_S)} = \frac{1}{\alpha_{AL}(M_S)} + \frac{1}{\alpha_{BL}(M_S)}. \tag{12}$$

Consider now the one-loop renormalization-group equations governing the evolution of the gauge couplings with mass scale:

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1}, \tag{13}$$

where $\alpha_i = g_i^2/4\pi$ and the numbers b_i are determined by the particle content of the model between M_1 and M_2. Below M_S, we assume the particle content of the SM, but with two Higgs doublets, i.e.

$$SU(3)_C: \quad b_C = -11 + (4/3)N_f = -7, \tag{14}$$

$$SU(2)_L: \quad b_L = -22/3 + (4/3)N_f + 1/3 = -3, \tag{15}$$

$$U(1)_Y: \quad b_Y = (20/9)N_f + 1/3 = 7, \tag{16}$$

where $N_f = 3$ is the number of families. Above M_S, the gauge group becomes $SU(3)_C \times SU(2)_{AL} \times SU(2)_{BL} \times U(1)_Y$ with the following minimum particle content for each family:

$$(u, d) \sim (3, 2, 1, 1/6), \quad u^c \sim (3^*, 1, 1, -2/3), \quad d^c \sim (3^*, 1, 1, 1/3), \tag{17}$$

$$(\nu_1, e_1) \sim (1, 2, 1, -1/2), \quad (e_1^c, \nu_1^c) \sim (1, 1, 2, 1/2), \tag{18}$$

$$(\nu_2, e_2) \sim (1, 1, 2, -1/2), \quad e^c \sim (1, 1, 1, 1). \tag{19}$$

The $SU(2)_{AL}$ anomalies are canceled between (u, d) and (ν_1, e_1), whereas the $SU(2)_{BL}$ anomalies are canceled between (ν_2, e_2) and (e_1^c, ν_1^c). In addition, we assume the appearance of one copy of $\eta \sim (8, 1, 1, 0)$, two copies of $(N_1, E_1; E_1^c, N_1^c) \sim (1, 2, 2, 0)$, one copy of $(N_2, E_2; E_2^c, N_2^c) \sim (1, 1, 2, \mp 1/2)$, one copy of $(N_4, E_4; E_4^c, N_4^c) \sim (1, 2, 1, \mp 1/2)$, and one copy of $(E_5^c, N_5; N_5^c, E_5) \sim (1, 2, 1, \pm 1/2)$, where $\lambda_4 \sim (1, 3, 1, 1, 3^*, 1)$ and $\lambda_5 \sim (1, 3^*, 1, 1, 3, 1)$ are extra supermultiplets to be discussed later.

The corresponding b_i's are then given by

$$SU(3)_C : \quad b_C = -9 + 2N_f + 3 = 0, \tag{20}$$

$$SU(2)_{AL} : \quad b_{AL} = -6 + 2N_f + 4 = 4, \tag{21}$$

$$SU(2)_{BL} : \quad b_{BL} = -6 + N_f + 3 = 0, \tag{22}$$

$$U(1)_Y : \quad b_Y = (13/3)N_f + 3 = 16, \tag{23}$$

Using Eqs. (9) to (16), these imply the following two constraints [2]:

$$\frac{1}{\alpha_C(M_Z)} = \frac{3}{7}\left[\frac{4}{\alpha_L(M_Z)} - \frac{1}{\alpha_Y(M_Z)}\right] + \frac{4}{7\pi}\ln\frac{M_S}{M_Z}, \tag{24}$$

$$\ln\frac{M_U}{M_Z} = \frac{\pi}{14}\left[\frac{3}{\alpha_Y(M_Z)} - \frac{5}{\alpha_L(M_Z)}\right] + \frac{2}{7}\ln\frac{M_S}{M_Z}. \tag{25}$$

Using the input [9]

$$\alpha_L(M_Z) = (\sqrt{2}/\pi)G_F M_W^2 = 0.0340, \tag{26}$$

$$\alpha_Y(M_Z) = \alpha_L(M_Z)\tan^2\theta_W = 0.0102, \tag{27}$$

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and

$$0.115 < \alpha_C(M_Z) < 0.119, \tag{28}$$

we find

$$450 \text{ GeV} > M_S > M_Z, \tag{29}$$

and

$$1.2 \times 10^{16} \text{ GeV} < M_U < 2.0 \times 10^{16} \text{ GeV}. \tag{30}$$

These are certainly acceptable values for new particles below the TeV scale and the proper suppression of proton decay.

3   Quarks, Leptons, and Other Particles

Quark masses come from the Yukawa couplings $u^c(uN_4^c - dE_4^c)$ and $d^c(uE_4 - dN_4)$ which originate from the invariant dimension-four term $q^c \eta q \lambda_4$ term in the $SU(3)^6$ superpotential. One of the η supermultiplets is assumed to have superheavy vacuum expectation values $\langle \eta_{11} \rangle = \langle \eta_{22} \rangle = \langle \eta_{33} \rangle$ which break $SU(3)_{CL} \times SU(3)_{CR}$ to $SU(3)_C$ at M_U. Thus q and q^c become triplets and antitriplets respectively under $SU(3)_C$, and an effective $q^c q \lambda_4$ term is generated.

To preserve the discrete Z_6 symmetry, $\lambda_4 \sim (1, 3, 1, 1, 3^*, 1)$ should be accompanied by $Q^c \sim (1, 1, 3, 1, 1, 3^*)$, $\bar{Q} \sim (3^*, 1, , 1, 3, 1, 1)$, $\lambda_5 \sim (1, 3^*, 1, 1, 3, 1)$, $\bar{Q}^c \sim (1, 1, 3^*, 1, 1, 3)$, and $Q \sim (3, 1, 1, 3^*, 1, 1)$. It is clear that $Q\bar{Q}$ and $Q^c \bar{Q}^c$, as well as $\lambda_4 \lambda_5$ are invariants so that all these particles are naturally superheavy. However, λ_4^3 and λ_5^3 are also invariants, so some of the components of λ_4 and λ_5 may be fine-tuned to be light.

At M_S, one of the $(1, 2, 2, 0)$ bidoublets is assumed to have vacuum expectation values $\langle N_1 \rangle = \langle N_1^c \rangle$ which break $SU(2)_{AL} \times SU(2)_{BL}$ to $SU(2)_L$. From the invariant λ_1^3 term, (ν_1, e_1) will then pair with (e_1^c, ν_1^c) to form a vector doublet under $SU(2)_L$, and from the

invariant λ_2^3 term, (ν_2, e_2) will couple to e_2^c through (N_2, E_2) to become the SM leptons, as in $SU(3)^3$ trinification. This is also the canonical case of quark-lepton nonuniversality [5] because quarks couple to $SU(2)_{AL}$ and leptons couple to $SU(2)_{BL}$.

However, there is also the $\lambda_1 \lambda_2 \lambda_3 \lambda_5$ term in the $SU(3)^6$ superpotential. One of the λ_3 supermultiplets is assumed to have superheavy vacuum expectation values $\langle N_3 \rangle = \langle N_3^c \rangle = \langle S_3 \rangle$ which break $SU(3)_{AR} \times SU(3)_{BR}$ to $SU(3)_R$ at M_U. Thus (ν_1, e_1) may couple to e_2^c through (N_5^c, E_5). At the same time, one of the λ_1 supermultiplets is assumed to have a superheavy vacuum expectation value $\langle S_1 \rangle$ which breaks $SU(3)_{AL} \times SU(3)_{BL}$ to $SU(2)_{AL} \times SU(2)_{BL} \times U(1)_{YL}$ at M_U. Thus (ν_2, e_2) may also couple to e_3^c through (N_5^c, E_5). In either case, the lepton doublet and the antilepton singlet would be in different $(3, 3^*)$ reprsentations, as in two previously proposed models [10, 11]. To break $SU(3)_R \times U(1)_{YL}$ to $U(1)_Y$, we assume superheavy vacuum expectation values $\langle \nu_3 \rangle$ and $\langle S_2 \rangle$ as well. As shown in Ref. [11], having (ν, e) and (e^c, ν^c) in separate $(3, 3^*)$ representations allows ν^c to acquire a large Majorana mass, thereby realizing the canonical seesaw mechanism for very small Majorana neutrino masses. This argues for the scenario where the SM leptons are not exclusively from λ_2 as in the original $SU(3)^6$ model [2].

The new particles at M_S all have $SU(2)_L \times U(1)_Y$ invariant masses and do not contribute significantly to the S, T, U oblique parameters, thereby preserving the excellent agreement of the SM with current precision electroweak measurements [9]. The $SU(3)_C$ octet η decays in one loop to two gluons, and should be detected at the Large Hadron Collider (LHC). The $SU(3)_C$ singlets interact with one another through the terms $\lambda_1 \lambda_2 \lambda_5$ and $\lambda_4 \lambda_5$, which allow them to decay into SM particles, such as leptons and quarks as well as W and Z bosons.

In the Minimal Supersymmetric Standard Model, the leptonic doublet has to be distinguished from the Higgs doublet of the same hypercharge by R-parity to guarantee the existence of a stable particle, the Lightest Supersymmetric Particle (LSP), as a candidate

for dark matter. Here the Higgs superfields are all bidoublets and leptons doublets, so they are already distinguished by the structure of the theory and an effective R-parity exists automatically.

4 New Gauge Bosons at the TeV Scale

The salient feature of this model is of course the appearance of a second set of weak gauge bosons corresponding to the breaking of $SU(2)_{AL} \times SU(2)_{BL}$ to the $SU(2)_L$ of the SM. As a result, the left-handed quark doublet (u, d) couples to

$$g_L W + \frac{g_A^2}{\sqrt{g_A^2 + g_B^2}} W',$$

and the left-handed lepton doublet (ν, e) couples to

$$g_L W + \frac{g_A^2 \cos^2 \theta - g_B^2 \sin^2 \theta}{\sqrt{g_A^2 + g_B^2}} W',$$

where $g_L^{-2} = g_A^{-2} + g_B^{-2}$ and the SM set of $SU(2)_L$ gauge bosons W and their orthogonal combinations W' are given by

$$W = \frac{g_B W_A + g_A W_B}{\sqrt{g_A^2 + g_B^2}}, \quad W' = \frac{g_A W_A - g_B W_B}{\sqrt{g_A^2 + g_B^2}}, \tag{31}$$

with

$$\begin{pmatrix} \nu \\ e \end{pmatrix} = \cos\theta \begin{pmatrix} \nu_1 \\ e_1 \end{pmatrix} + \sin\theta \begin{pmatrix} \nu_2 \\ e_2 \end{pmatrix}. \tag{32}$$

If $\theta = 0$, then quarks and leptons interact identically with W' as well as W. If $\theta = \pi/2$, then we have the canonical case of quark-lepton nonuniversality [5].

4.1 W' coupling

In general, W can mix with W'. For illustration, let us consider the simpler case of no mixing in which the coupling of q-W'-q' is

$$i g_L \gamma^\mu P_L g_{W'qq},$$

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and the coupling of ℓ-W'-ℓ' is

$$ig_L\gamma^\mu P_L g_{W'\ell\ell}.$$

Here, g_L is the SM $SU(2)_L$ coupling, $P_L = (1 - \gamma_5)/2$, and the coefficients $g_{W'qq}$ and $g_{W'\ell\ell}$ are defined as follows:

$$
\begin{aligned}
g_{W'qq} &= \frac{g_A}{g_B}, \\
g_{W'\ell\ell} &= \frac{g_A}{g_B}\cos^2\theta - \frac{g_B}{g_A}\sin^2\theta.
\end{aligned}
\tag{33}
$$

The effective Fermi constant $G_F/\sqrt{2}$ in nuclear beta decay is then given by

$$
\begin{aligned}
\left(\frac{G_F}{\sqrt{2}}\right)_{q\ell} &= \frac{g_L^2(M_Z)}{8M_W^2} + \frac{g_L^2(M_S)}{8M_{W'}^2}g_{W'qq}g_{W'\ell\ell} \\
&= \frac{g_L^2(M_Z)}{8M_W^2}\left[1 + \frac{\alpha_L(M_S)M_W^2}{\alpha_L(M_Z)M_{W'}^2}\frac{g_A}{g_B}\left(\frac{g_A}{g_B}\cos^2\theta - \frac{g_B}{g_A}\sin^2\theta\right)\right],
\end{aligned}
\tag{34}
$$

whereas that in pure leptonic decay is

$$
\begin{aligned}
\left(\frac{G_F}{\sqrt{2}}\right)_{\ell\ell} &= \frac{g_L^2(M_Z)}{8M_W^2} + \frac{g_L^2(M_S)}{8M_{W'}^2}g_{W'\ell\ell}g_{W'\ell\ell} \\
&= \frac{g_L^2(M_Z)}{8M_W^2}\left[1 + \frac{\alpha_L(M_S)M_W^2}{\alpha_L(M_Z)M_{W'}^2}\left(\frac{g_A}{g_B}\cos^2\theta - \frac{g_B}{g_A}\sin^2\theta\right)^2\right].
\end{aligned}
\tag{35}
$$

Therefore, if $\tan^2\theta > g_A^2/g_B^2$, then $(G_F)_{q\ell} < (G_F)_{\ell\ell}$ and the neutron-decay result can be understood [5]. Furthermore, if $|g_{W'\ell\ell}| << |g_{W'qq}|$, then $(G_F)_{\ell\ell}$ is very close to G_F^{SM}, and $(G_F)_{q\ell}$ will be less than it by a small amount.

In general, $M_{W'}$ and $\sin\theta$ are independent parameters. But in order to explain the neutron-decay result [6], we should have

$$1 - \frac{(G_F)_{q\ell}}{(G_F)_{\ell\ell}} \simeq \frac{\alpha_L(M_S)M_W^2}{\alpha_L(M_Z)M_{W'}^2}g_{W'\ell\ell}\left(g_{W'\ell\ell} - g_{W'qq}\right) \simeq 0.0023 \pm 0.0014.
\tag{36}$$

Here we have used the latest value of $|V_{us}| = 0.2262(23)$ [12] instead of the 2004 PDG value of $0.2200(26)$. This reduces significantly the possible descrepancy of the neutron-decay result from universality. Using Eqs. (21) and (22) as well as $M_S/M_Z = 2.2$ and $M_U/M_Z = 1.7\times10^{14}$
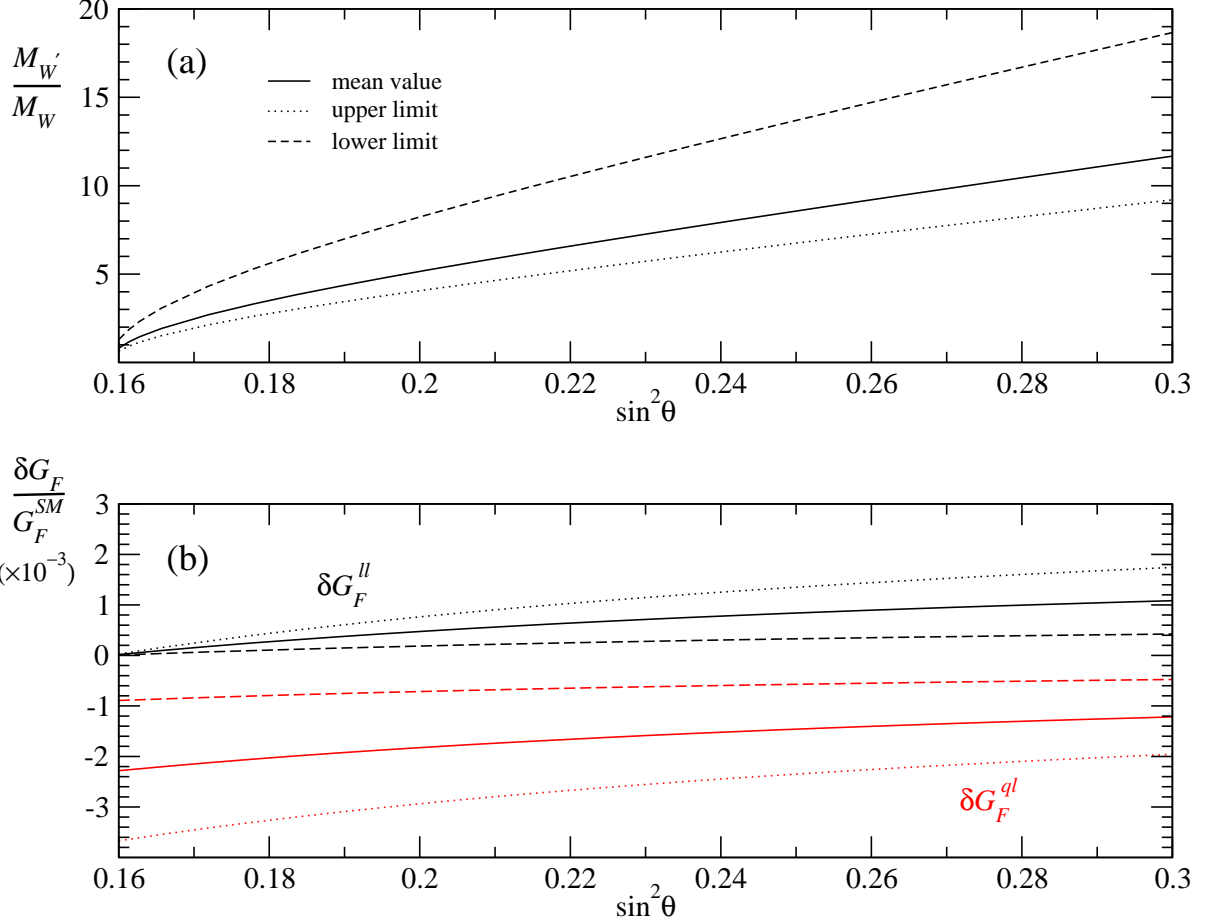
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Figure 2: (a) $M_{W'}/M_W$ as a function of $\sin^2\theta$, cf. Eq. (37). (b) The corresponding deviations of $(G_F)_{ql}$ and $(G_F)_{ll}$ from G_F^{SM} as functions of $\sin^2\theta$, cf. Eqs. (34) and (35). The solid curve is obtained from the mean value while the dotted and dashed curves are obtained from the upper and lower values respectively.

from $\alpha_C(M_Z) = 0.117$, we find $\alpha_A(M_S) = 0.040$ and $\alpha_B(M_S) = 0.212$. Hence we obtain the following relation between $M_{W'}$ and $\sin\theta$:

$$\frac{M_W^2}{M_{W'}^2}\sin^2\theta(\sin^2\theta - 0.1587) \simeq 3.11 \pm 1.89 \times 10^{-4}. \tag{37}$$

For illustration, we show $M_{W'}/M_W$ as a function of $\sin^2\theta$ in Fig. 2(a). The solid curve is obtained from the central value of the right-hand side of Eq. (36) while the dotted and dashed curves are obtained from the upper and lower values respectively. For a given value
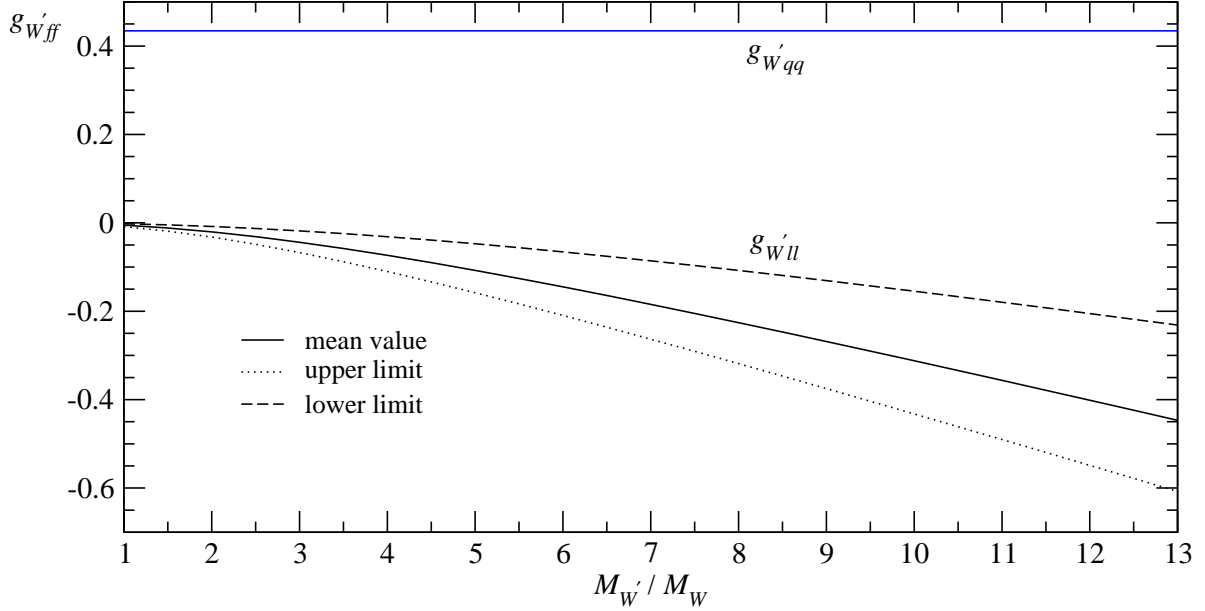
Figure 3: The coupling strengths $g_{W'qq}$ and $g_{W'\ell\ell}$ as a function of $M_{W'}/M_W$.

of $\sin^2\theta$, $M_{W'}$ lies within a range of values as shown. Correspondingly, the deviations of $(G_F)_{q\ell}$ and $(G_F)_{\ell\ell}$ from G_F^{SM} are also correlated with $\sin^2\theta$. We present these deviations as functions of $\sin^2\theta$ in Fig. 2(b). Since $(G_F)_{\ell\ell}$ has been measured very preicsely, smaller values of $\sin^2\theta$ are preferred.

In the following, we will choose the mass of the W' boson as an input parameter rather than the mixing angle θ. Since the effective coupling strength $g_{W'\ell\ell}$ is a function of $\sin^2\theta$, cf. Eq. (33), it is also a function of $M_{W'}$. Of course this dependence is not intrinsic to the model; it is simply due to the empirical constraint of Eq. (37). For illustration, the effective coupling strengths $g_{W'qq}$ and $g_{W'\ell\ell}$, as functions of $M_{W'}$, are shown in Fig. 3. Again, the dotted curve is obtained from the upper limit and the dashed curve from the lower limit. We note that both couplings are suppressed compared to a SM-like coupling for which $g_{W'qq} = 1$ and $g_{W'\ell\ell} = 1$. Furthermore, the magnitude of $g_{W'\ell\ell}$ is highly suppressed for a light W' boson and grows graduately with increasing $M_{W'}$. The difference between $g_{W'qq}$ and $g_{W'\ell\ell}$ has a very important impact on the phenomenology of W' which will be addressed below.

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4.2   Decay of W' boson

Similar to the W boson decay in the Standard Model, the W' boson of this model can decay also into lepton pairs and quark pairs. [Its decay into SM gauge bosons is negligible in the absence of mixing.] Taking into account the masses of the decay products, the W' partial decay width is given by

$$
\begin{aligned}
\Gamma\left(W' \to f\bar{f}'\right) &= N_C \frac{g^2 M_{W'}}{48\pi} g_{W'ff}^2 \lambda^{1/2}\left(1, \gamma_f, \gamma_{f'}\right) \\
&\times \left[1 - \frac{1}{2}\gamma_f - \frac{1}{2}\gamma_{f'} - \frac{1}{2}\left(\gamma_f - \gamma_{f'}\right)^2\right],
\end{aligned}
\tag{38}
$$

where $\gamma_f = m_f^2/M_{W'}^2$, $\gamma_{f'} = m_{f'}^2/M_{W'}^2$ and $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. Here, N_C is the color factor of the fermion and $g_{W'ff}$ denotes either $g_{W'qq}$ or $g_{W'\ell\ell}$, as defined in Eq. (33). Of the hadronic modes, we need to consider only the decays $W' \to u\bar{d}$, $W' \to c\bar{s}$, and $W' \to t\bar{b}$ because

$$
|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1
$$

and all other CKM matrix elements are small. Since the W' boson is very heavy, we can treat all its decay products as massless particles except for the top quark. The leptonic decay width of W' boson can now be simplified as

$$
\Gamma\left(W' \to \ell\bar{\ell}'\right) = \frac{g_L^2}{48\pi} M_{W'} g_{W'\ell\ell}^2,
\tag{39}
$$

where $\ell\ell' = e\nu_e, \mu\nu_\mu, \tau\nu_\nu$. If $M_{W'} < m_t$, the W' boson can only decay into light quark pairs,

$$
\Gamma\left(W' \to q\bar{q}'\right) = N_C \frac{g_L^2}{48\pi} M_{W'} g_{W'qq}^2,
\tag{40}
$$

where $qq' = ud,\ cs$. If $M_{W'} > m_t$, the tb decay channel opens up and the partial decay width becomes

$$
\Gamma\left(W' \to t\bar{b}\right) = N_C \frac{g_L^2}{48\pi} M_{W'} g_{W'qq}^2 \left(1 - \frac{3}{2}\gamma_t + \frac{1}{2}\gamma_t^3\right).
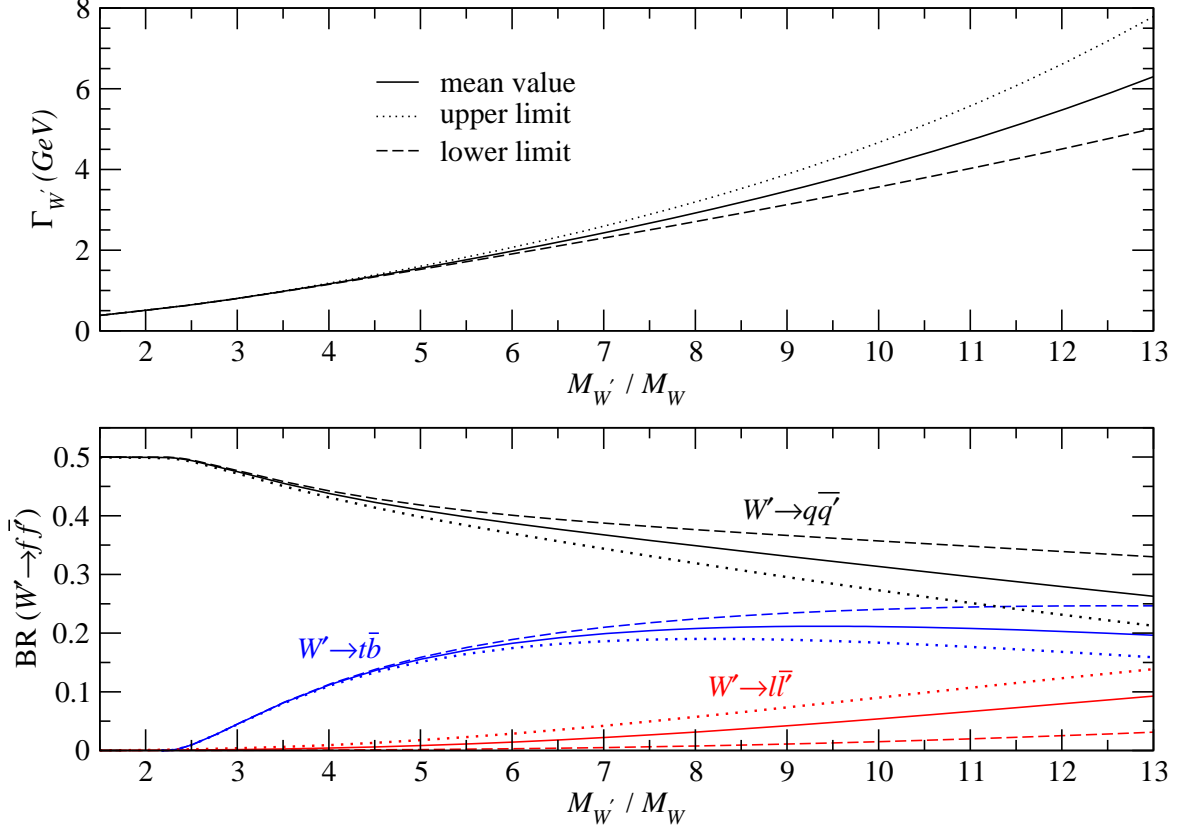\tag{41}
$$

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Figure 4: (a) The total decay width and (b) the decay branching ratios of the W' boson as functions of its mass $M_{W'}$.

Therefore, the total decay width of W' is

$$\Gamma_{W'}^{tot}\left(M_{W'} < m_t\right) = \frac{g_L^2}{48\pi} M_{W'}\left(3g_{W'\ell\ell}^2 + 6g_{W'qq}^2\right), \tag{42}$$

$$\Gamma_{W'}^{tot}\left(M_{W'} > m_t\right) = \frac{g_L^2}{48\pi} M_{W'}\left[3g_{W'\ell\ell}^2 + 9g_{W'qq}^2\left(1 - \frac{1}{2}\gamma_t + \frac{1}{6}\gamma_t^3\right)\right]. \tag{43}$$

In Fig. 4 we present the total decay width of the W' boson and its decay branching ratios (BR) as functions of $M_{W'}$. Here, we have separated the light quark decay modes (dashed-line) from the heavy quark (tb) mode (dotted-line). It clearly shows that in the region of small $M_{W'}$ ($1.5\,M_W < M_{W'} < 2.5\,M_W$) the light quark decay mode dominates over the other modes. This is due to the suppression of the $g_{W'\ell\ell}$, cf. Fig. 3. As a result, the detection of W' through its leptonic decay in the small $M_{W'}$ region is more difficult to achieve and the

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current experimental data cannot rule out the existence of this W'. In the medium mass region, the heavy quark decay channel opens. As a result, the decay branching ratio of the light quark mode decreases but is still larger than the heavy quark mode. Both hadronic decay modes become comparable with increasing $M_{W'}$. Again, the leptonic decay mode is negligible due to the suppression of $g_{W'\ell\ell}$. In the region of very heavy W', say $M_{W'} > 10\, M_W$, the leptonic decay branching ratio becomes larger because $g_{W'qq}$ and $g_{W'\ell\ell}$ are of the same order.

4.3 Discovery potential in hadron collision

In this study, we will examine the discovery potential of the W' boson of this model at the Fermilab Tevatron and CERN Large Hadron Collider (LHC). Many direct searches for a W' boson in its various decay modes have been performed at the Tevatron and produced lower limits on its mass. The leptonic decay mode is the best choice for disentangling the W' event from the copious QCD background. Searches using the decay mode $W' \rightarrow e\nu$ exclude a W' boson with mass $< 754\,\mathrm{GeV}$ at 95% C.L. [13, 14], while similar searches considering the decay mode $W' \rightarrow \mu\nu$ have excluded a W' boson with mass $< 660\,\mathrm{GeV}$ at 95% C.L. [15]. Combining both leptonic channels, the most stringent limit was obtained, excluding a W' boson with mass $< 768\,\mathrm{GeV}$ at 95% C.L. [14]. These mass limits all assume that the new vector boson's couplings to leptonic final states are as given by the Standard Model, which predicts that the total width of the boson increases linearly with its mass. In addition to the leptonic mode, a search using the light quark decay mode $W' \rightarrow q\bar{q'}$ excludes a W' boson in the range $300 < M_{W'} < 420\,\mathrm{GeV}$ at 95% C.L. [16], while a search using the decay mode $W' \rightarrow t\bar{b}$ excludes a W' boson in the range $225 < M_{W'} < 536\,\mathrm{GeV}$ for $M_{W'} \gg m_{\nu_R}$ and $225 < M_{W'} < 566\,\mathrm{GeV}$ for $M_{W'} < m_{\nu_R}$ [17].

At a hadron collider the W' bosons are predominantly produced through the charge-

current Drell-Yan process:

$$q\bar{q}' \rightarrow W'^{+} \rightarrow f\bar{f}',$$

where q and q' denote the light up-type quarks ($q = u,\ c$) and down-type quarks ($q' = d,\ s$) respectively. The total cross section for this process at a hadron collider is

$$\sigma\left(P_1 P_2 \rightarrow f\bar{f}'\right) = \sum_{q,\bar{q}'} \int \mathrm{d}x_1 \mathrm{d}x_2 \left[f_{q/P_1}\left(x_1, \mu\right) f_{\bar{q}'/P_2}\left(x_2, \mu\right) \hat{\sigma}\left(q\bar{q}' \rightarrow f\bar{f}'\right) + (x_1 \leftrightarrow x_2) \right], \tag{44}$$

where P_1, P_2 represent the hadronic initial state, $f_{q/P}(x, \mu)$ is the parton distribution function (PDF). We take the factorization scale (μ) to be the invariant mass of the constituent process in our numerical calculation. The parton-level cross section $\hat{\sigma}$ is given by

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int \mathrm{d}\Pi_2 \sum_{\substack{\text{spin} \\ \text{color}}} \overline{\left| \mathcal{M}\left(q\bar{q}' \rightarrow f\bar{f}'\right) \right|^2}, \tag{45}$$

where the bar over the $|\mathcal{M}|^2$ denotes averaging over the initial-state spin and color, $\mathrm{d}\Pi_2$ represents 2-body final-state phase space, and the squared matrix element reads

$$\overline{|\mathcal{M}|^2} = \frac{N_C^f}{12} \frac{g_L^4 \left|V_{ud}\right|^2}{64\pi^2 \hat{s}} \frac{\hat{u}^2}{\left(\hat{s} - M_{W'}^2\right)^2 + \left(M_{W'}\Gamma_{W'}\right)^2} g_{W'qq}^2 g_{W'\ell\ell}^2, \tag{46}$$

where the explicit factor $1/12$ results from the average over the quark spins and colors, and N_C^f is the number of color state of decay products:

$$N_C^f = \begin{cases} 1 & f = \ell, \\ 3 & f = q. \end{cases} \tag{47}$$

Here, the Mandelstam variables are defined by

$$\hat{s} = \left(p_u + p_d\right)^2, \quad \hat{t} = \left(p_d - p_\ell\right)^2, \quad \hat{u} = \left(p_u - p_\ell\right)^2, \tag{48}$$

where p_i denotes the momentum of particle i.

In Fig. 5 we present the inclusive cross sections of W' production and decay through the process $u\bar{d} \rightarrow W' \rightarrow f\bar{f}'$ at the Tevatron and the LHC. For comparison, we also present the
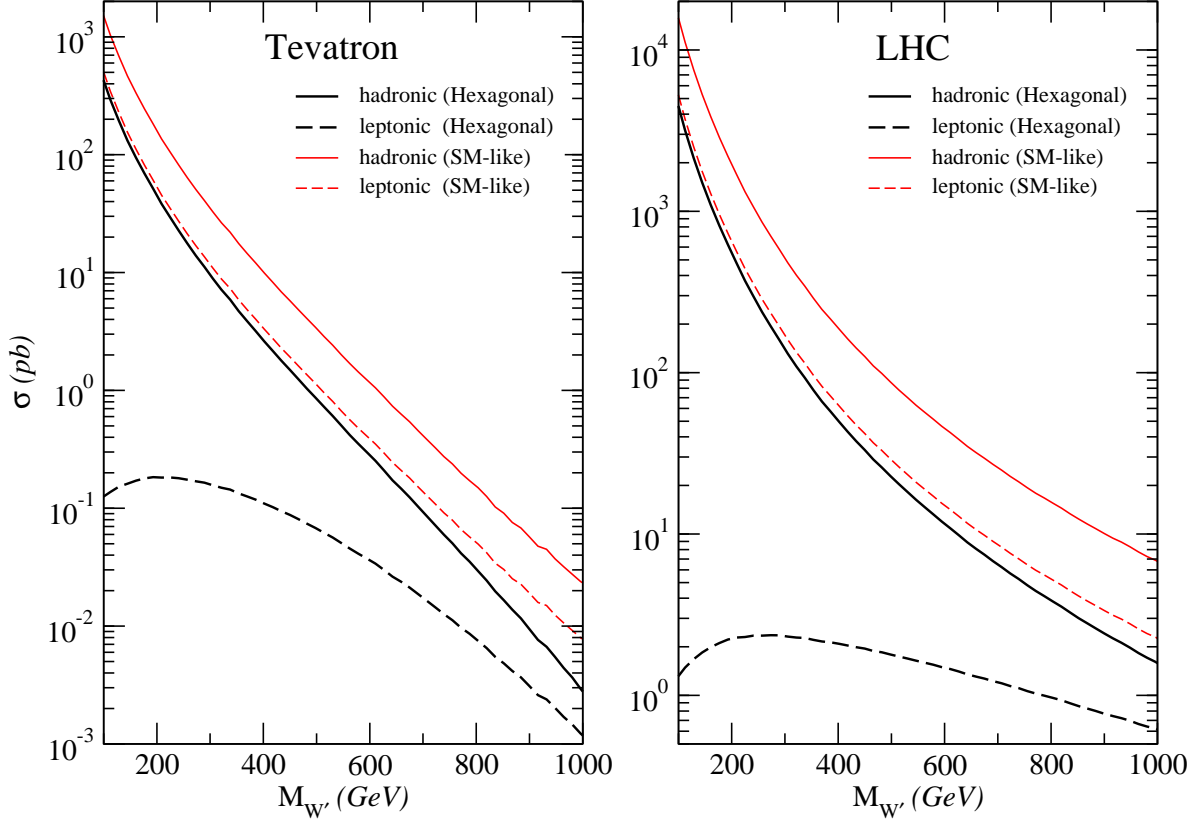
Figure 5: Inclusive cross sections of W' production and decay through $q\bar{q}' \to W' \to f\bar{f}'$ as functions of $M_{W'}$ at the Tevatron and the LHC.

inclusive cross sections of the same process with the assumption that all the couplings are as in the Standard Model. For our numerical calculation, we use the leading-order parton distribution function set CTEQ6L [18]. The value of the relevant electroweak parameters are $\alpha = 1/137.0459895$, $G_\mu = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $m_t = 178\,\mathrm{GeV}$, $M_W = 80.33\,\mathrm{GeV}$, $M_Z = 91.1867\,\mathrm{GeV}$, and $\sin^2\theta_W = 0.231$. Thus, the square of the weak gauge coupling is $g^2 = 4\sqrt{2}M_W^2 G_\mu$. Here, we focus our attention on the positively charged W' boson only.

Due to the suppression of the effective couplings ($g_{W'qq}$ and $g_{W'\ell\ell}$) compared to those of the Standard Model, the inclusive cross section predicted by this model is much smaller than that of the SM, thereby shifting the limits of $M_{W'}$ to lower values. For example, W' couples to quarks with a suppression factor of $g_{W'qq} = g_A/g_B \simeq 0.43$, hence its production cross

section will be a factor of 5 smaller than expected for a corresponding gauge boson of the same mass in the SM. After using the kinematics cuts listed in Ref. [16] for the light-quark mode, we compare the W' cross section of our model with present data and conclude that $M_{W'}$ should be larger than 310 GeV. The leptonic mode is suppressed so much in our model that the W' boson satifies all the current experimental constraints, but it also means that one cannot detect this extra vector boson in this mode in the future. At the LHC, as shown in Fig. 5, the production of W' boson with its subsequent decay can be observed by studying events with two hard jets. Again, the leptonic decay mode is not very competitive. Detailed ananlysis of these two modes together with various backgrounds will be presented elsewhere. We note also that the hadronic decay channel exhibits a completely different behavior from the leptonic decay channel, especially for a light W' boson. This is a consequence of the difference between the effective coupling strengths (cf. Fig. 3), and can be explained as follows. Since the width of the W' boson is very small compared to its mass, we can write the parton-level cross section $\hat{\sigma}$ in Eq. (45) as

$$\hat{\sigma}\left(q\bar{q}' \to f\bar{f}'\right) = \hat{\sigma}\left(q\bar{q}' \to W'^{+}\right) \times Br\left(W'^{+} \to f\bar{f}'\right) \tag{49}$$

under the narrow-width approximation. As an s-channel process, the cross section $\hat{\sigma}(q\bar{q}' \to W'^{+})$ drops off rapidly with increasing $\hat{s}$ as $\hat{\sigma} \propto 1/\hat{s}$. On the other hand, due to the large suppression of $g_{W'\ell\ell}$, the decay branching ratio of $W' \to \ell\bar{\ell}'$ is very tiny when $M_{W'}/M_W \leq 5$ and increases with increasing $M_{W'}$. These two effects compete with each other and leave the bump in the inclusive cross section (cf. bold dashed curve in Fig. 5).

Since (W'^{+}, Z', W'^{-}) is a triplet under $SU(2)_L$, Z' has the same mass as W' and the same couplings to quarks and leptons, assuming no mixing with the SM gauge bosons. As usual, one can use the leptonic decay mode to distinguish W' from Z'. The W' boson decays into one charged lepton and one neutrino which has the collider signature of a charged lepton plus missing energy, while the Z' boson decays into two detectable charged leptons. In our model,

however, we have to use the hadronic decay mode to detect these extra vector bosons, due to the suppresssion of the leptonic decay mode discussed above. As far as the light-quark mode is concerned, both W' and Z' will have the collider signature of two hard jets. Since both W' and Z' couple to quarks via the left-handed gauge interaction, the two hard jets in the final state will have exactly the same kinematics distributions, it is thus impossible to distinguish one from the other. On the other hand, one can easily separate them by using the heavy-quark mode. For example, the W' boson will decay into a $t\bar{b}$ pair with the top quark subsequently decaying into $\ell b \nu$ while the Z' will decay into a $t\bar{t}$ pair with the top-quark pair subsequently decaying into $\ell\bar{\ell}b\bar{b}\nu\bar{\nu}$.

5   Conclusion

In this paper we have proposed a supersymmetric gauge extension of the Standard Model, where $SU(2)_L$ is enlarged to $SU(2)_{AL} \times SU(2)_{BL}$ at the TeV scale. This model is motivated by (1) the possibility of $SU(3)^6$ hexagonal unification and (2) the possibility of small deviations from quark-lepton universality as allowed by neutron decay.

The distinguishing feature of our model is that quarks couple to $SU(2)_{AL}$ while leptons couple to a linear combination of $SU(2)_{AL}$ and $SU(2)_{BL}$ with mixing angle θ. The gauge couplings g_A and g_B are fixed from $SU(3)^6$ unification, and the mass of the (W'^+, Z', W'^-) $SU(2)_L$ triplet is related to the angle θ from neutron decay. We have discussed in this paper the possible production and decay of this new W' boson. Using present Tevatron data, we set the lower limit of 310 GeV on $M_{W'}$ through its possible decay into quarks. [The leptonic mode turns out to be very much suppressed.] Since $M_{W'}$ is expected to be no more than a few times M_W in this particular theoretical context, it should become observable at the LHC.

Acknowledgements

References

[1] A. De Rujula, H. Georgi, and S. L. Glashow, in Proc. of Fifth Workshop on Grand Unification, ed. by K. Kang *et al.*, (World Scientific, Singapore, 1984), p. 88.

[2] E. Ma, Phys. Lett. **B593**, 198 (2004).

[3] P. H. Frampton and S. L. Glashow, Phys. Lett. **B190**, 157 (1987); see also G. Zoupanos, ibid. **B129**, 315 (1983).

[4] H. Georgi, E. E. Jenkins, and E. H. Simmons, Phys. Rev. Lett. **62**, 2789 (1989).

[5] X.-Y. Li and E. Ma, Mod. Phys. Lett. **A18**, 1367 (2003).

[6] H. Abele *et al.*, Phys. Rev. Lett. **88**, 211801 (2002); H. Abele *et al.*, Eur. Phys. J. **C33**, 1 (2004).

[7] This symmetry has also been considered recently by P. Langacker and B. D. Nelson, Phys. Rev. **D72**, 053013 (2005). The earliest discussion was by V. Barger, W. Y. Keung, and E. Ma, Phys. Rev. Lett. **44**, 1169 (1980).

[8] H. Georgi, Nucl. Phys. **B266**, 274 (1986).

[9] S. Eidelman *et al.*, Particle Data Group, Phys. Lett. **B592**, 1 (2004).

[10] K. S. Babu, E. Ma, and S. Willenbrock, Phys. Rev. **D69**, 051301(R) (2004).

[11] E. Ma, Mod. Phys. Lett. **A20**, 1953 (2005).

[12] E. Blucher, Talk at CKM2005.

[13] D0 Collaboration, S. Abachi *et al.*, Phys. Rev. Lett. **76**, 3271 (1996).

[14] CDF Collaboration, T. Affolder *et al.*, Phys. Rev. Lett. **87**, 231803 (2001).

[15] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **84**, 5716 (2000).

[16] CDF Collaboration, F. Abe *et al.*, Phys. Rev. **D55**, R5263 (1997).

[17] CDF Collaboration, D. Acosta *et al.*, Phys. Rev. Lett **90**, 081802 (2003).

[18] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky and W. K. Tung, JHEP **0207**, 012 (2002).