Pramāṇa, Vol. 19, No. 4, October 1982, pp. 315–321. © Printed in India.

Non-renormalization of the weak vertex in gauge theories with integrally charged quarks

G RAJASEKARAN and M S SRI RAM*
Department of Theoretical Physics, University of Madras, Guindy Campus, Madras 600 025, India
*Physics Department, University of Allahabad, Allahabad 211 002, India

MS received 5 July 1982

Abstract. We give current algebra arguments to show that to $O(\alpha)$ the colour octet vertices do not renormalize the effective weak vertex between colour singlet hadrons in models with broken colour symmetry. The result does not depend on the details of the mixing between colour gluons and electro-weak bosons.

Keywords. Broken colour symmetry; weak vertex; colour octet vertices; $O(\alpha)$ corrections; conservation of current.

1. Introduction

Unified gauge models of weak, electromagnetic and strong interactions based on integrally-charged quarks have been proposed (Pati and Salam 1973). The viability of these models was established by showing that the integrally-charged quarks manifest themselves as the fractionally charged quarksin high-q^2 deep inelastic experiments (Rajasekaran and Roy 1975; Pati and Salam 1976). In these models, colour symmetry of the strong interactions is also broken spontaneously and the colour gluons mix with the weak and electromagnetic gauge bosons. Due to these mixings, the weak and electromagnetic currents have colour octet terms also. Now, the radiative corrections to the weak vertex of hadrons are important in checking the Cabibbo universality of weak interactions (Sirlin 1978). It is essential to show that the colour octet contributions do not alter the results significantly as the Cabibbo universality, without these contributions taken into account is in good agreement with the present experimental evidence. By explicit calculation of the relevant diagrams it has been shown that the colour contributions to the effective weak vertex between colour singlet states at zero momentum transfer is zero up to $O(\alpha)$ (Ramachandran 1979). Here we give a simpler derivation of the same result, by showing that the effective weak current is conserved to this order. Also, the analysis is general and does not depend on the details of the mixing between gluons and electroweak bosons.

2. Models with broken colour symmetry

The models are based on the group $SU_C(3) \otimes SU_L(2) \otimes U(1)$ where $SU_C(3)$ corresponds to the colour symmetry of strong interactions and $SU_L(2) \otimes U(1)$ is the electroweak part. Consider one such model specifically (Rajasekaran and Roy 1975). Here the quarks are written in the form of two arrays:

$$H_{ai} = \begin{pmatrix} u_1^0 & u_2^+ & u_3^+ \\ d_1^- & d_2^0 & d_3^0 \end{pmatrix}, \quad K_{ai} = \begin{pmatrix} c_1^0 & c_2^+ & c_3^+ \\ s_1^- & s_2^0 & s_3^0 \end{pmatrix}. \tag{1}$$

The subscripts refer to the colour indices and superscripts refer to the electric charges; d_i and s_i are Cabibbo-rotated objects. H_L and K_L transform as (3*, 2, 1) under $\mathrm{SU}_C(3) \otimes \mathrm{SU}_L(2) \otimes \mathrm{U}(1)$. All the right-handed quarks are singlets under $\mathrm{SU}_L(2)$. Leptons are singlets under $\mathrm{SU}_C(3)$ and transform as usual under $\mathrm{SU}_L(2) \otimes \mathrm{U}(1)$.

The electric charge operator can be wrriten as,

$$Q = I_3' + \tfrac{1}{2}Y' + I_{3L} + U, \tag{2}$$

where I_3' and Y' are the two diagonal generators of $\mathrm{SU}_C(3)$, I_{3L} is the diagonal generator of $\mathrm{SU}_L(2)$ and U is the generator of U (1).

The symmetry is broken spontaneously through two sets of Higgs fields:

$$\sigma_{ai} = \begin{pmatrix} \sigma^0 & \sigma^+ & \sigma^+ \\ \sigma^- & \sigma^0 & \sigma^0 \\ \sigma^- & \sigma^0 & \sigma^0 \end{pmatrix}, \quad \underset{\sim}{\eta} = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}. \tag{3}$$

σ_{ai} transforms as (3*, $2 \oplus 1$, 1) and $\underset{\sim}{\eta}$ as (1, 2, 1) under $\mathrm{SU}_C(3) \otimes \mathrm{SU}_L(2) \otimes \mathrm{U}(1)$. The Higgs fields acquire vacuum expectation values:

$$\langle \sigma_{ai} \rangle = \langle \sigma \rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \langle \underset{\sim}{\eta} \rangle = \langle \eta \rangle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

where $\langle \sigma \rangle$ and $\langle \eta \rangle$ are two real constants. Hence, colour symmetry is also broken. It can be shown that we get the following gauge boson mass terms (Lorentz indices are omitted):

$$L_{BM} = \tfrac{1}{4} \langle \eta \rangle^2 \{g^2 \,|\, W^1|^2 + g^2 \,|\, W^2|^2 + |- gW^3 + g'\, U|^2\}$$
$$+ \tfrac{1}{2} f^2 \langle \sigma \rangle^2 \left\{ \sum_{i=1}^{3} \left| V^i - \frac{g}{f} W^i \right|^2 + \sum_{i=4}^{7} | V^i |^2 + \left| V^8 - \frac{g'}{\sqrt{3} f} U \right|^2 \right\}. \tag{5}$$

where V^i, W^i and U refer to the gauge bosons corresponding to $\mathrm{SU}_C(3)$, $\mathrm{SU}_L(2)$ and U(1) groups respectively and f, g and g' are the corresponding coupling constants. It is seen that the electroweak bosons W_μ^i and U_μ mix with the colour gauge bosons

V^i_μ and that the mixing is $O(g/f)$ for a fixed mass of the gluons. This mixing is true of other models based on broken colour symmetry, though details differ. We will only make use of the fact that the mixing is $O(g/f)$. (In this paper, we deal with the mixing in a way different from that used in Rajasekaran and Roy (1975) and Ramachandran (1979). We read off the quadratic mixing between the colour gauge bosons V^i_μ and the electroweak bosons W^i_μ, U_μ directly from (5)).

The interaction Lagrangian can be written as:

$$L_{\text{int}} = f \sum_{l=1}^{8} j^l_\mu V^{\mu l} + g \sum_{i=1}^{3} j^i_{\mu L} W^{\mu i} + g' j^u_\mu U^u,$$

$$+ \text{ [couplings with Higgs scalars]} \tag{6}$$

where j^l_μ, $j^i_{\mu L}$ and j^u_μ are the currents corresponding to the groups $SU_C(3)$, $SU_L(2)$ and U(1) respectively [Hereafter, primes will denote octet currents].

The lowest order contribution to the charge changing semileptonic processes is given by:

$$M^{(0)} = -\frac{g^2}{2} \langle P' \,|\, j^+_{\mu L}(0) \,|\, P \rangle \, D^W_{\mu\nu}(q) \, L^-_{\nu L}, \tag{7}$$

where p and p' are the momenta of the incoming and outgoing hadrons which are colour singlets, $q = p' - p$, $j^+_{\mu L} = j^1_{\mu L} + ij^2_{\mu L}$, $L^-_{\nu L}$ is the lepton current and

$$D^W_{\mu\nu}(q) = \int d^4x \, e^{iq.x} \langle 0 \,|\, T(W^+_\mu(x) \, W^-_\nu(0)) \,|\, 0 \rangle. \tag{8}$$

3. Colour octet corrections

The $O(\alpha)$ corrections to $M^{(0)}$ have been evaluated in the framework of the $SU_L(2) \otimes U(1)$ model for electro-weak interactions and unbroken colour gauge theory for strong interactions (Sirlin 1978). The corrections to $M^{(0)}$ by the colour octet vertices fall into two classes:

(I) in which the main vertex (at which leptons interact with hadron) is a colour singlet and the octet currents occur only in the loop,

(II) in which the main vertex is a colour octet and one of the vertices in the loop is also an octet, giving together a singlet contribution.

Contributions from class I are common to both unbroken and broken colour gauge theories whereas contributions of class II are peculiar to theories with broken colour symmetry only.

Consider the lowest order contributions to class I diagrams shown in figure 1. It is given by the expression,

$$M^I(p, p') = -\frac{g^2}{2} \langle p' \,|\, j^{+I}_{\mu L}(0) \,|\, p \rangle \, D^W_{\mu\nu}(q) \, L^-_{\nu L}, \tag{9}$$

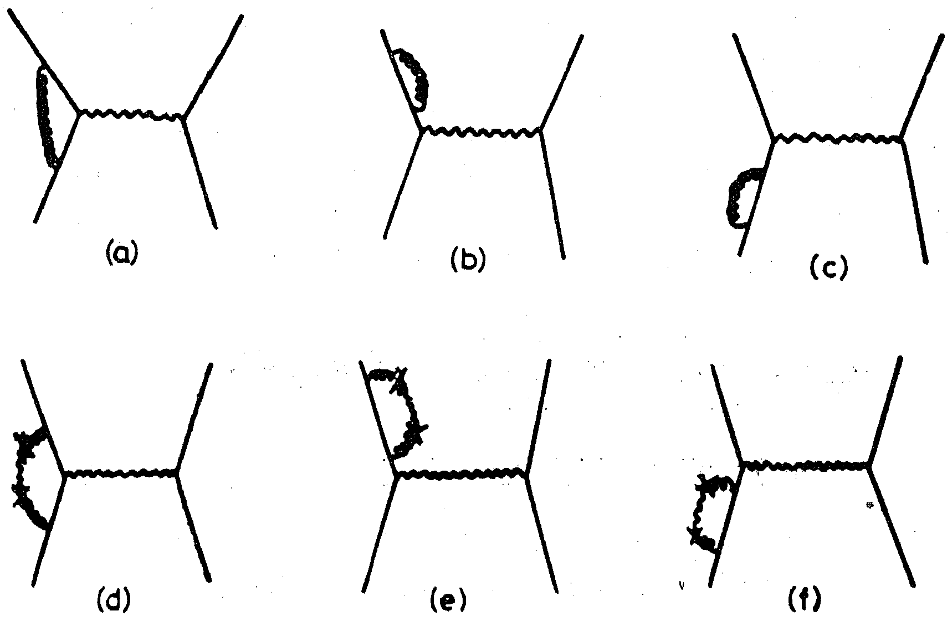

Figure 1. Diagrams with colour singlet main vertex. The hadron vertex is on the left while the lepton vertex is on the right. Curly lines denote colour octet gluons and the wavy lines denote W-bosons. Mixing between the two is indicated by a cross.

where
$$j^{+I}_{\mu L}(x) = \frac{f^2}{2}\int \mathrm{d}y\,\mathrm{d}z\, T\,[j'^{+}_{\lambda}(y)\, j^{+}_{\mu L}(x)\, j'^{-}_{\rho}(z)] \times \langle 0 \mid T\,[V^{\lambda +}(y)\; V^{\rho -}(z)] \mid 0\rangle. \tag{10}$$

Because of the mixing between gluons and electroweak bosons, $\langle 0 \mid T\,[V^{\lambda+}(y)\, V^{\rho-}(z)] \mid 0\rangle$ can be expanded in a series of powers of $(g/f)^2$ and hence the right side of (10) contains terms of $O(f^2)$, $O(g^2)$ etc. Figures 1(a), 1(b) and 1(c) represent the $O(f^2)$ corrections and figures 1(d), 1(e) and 1(f) represent $O(g^2) = O(\alpha)$ corrections to $M^{(0)}$.

Comparing (9) with (7) we see that $j^{+}_{\mu L}(x)$ essentially acquires a correction $j^{+I}_{\mu L}(x)$. In the following we will show that $j^{+I}_{\mu L}$ is conserved. Consider the divergence of the 3-current correlation function.

$$\begin{aligned}\partial^{\mu}_{x}\, T\,[j'^{+}_{\lambda}(y)\, j^{+}_{\mu L}(x)\, j'^{-}_{\rho}(z)] &= T\,[j'^{+}_{\lambda}(y)\, \partial^{\mu} j^{+}_{\mu L}(x)\, j'^{-}_{\rho}(z)] \\ &+ \delta(x_0 - y_0)\, T\,(j'^{-}_{\rho}(z)\, [j^{+}_{0L}(x), j'^{+}_{\lambda}(y)]) \\ &+ \delta(x_0 - z_0)\, T\,(j'^{+}_{\lambda}(y)\, [j^{+}_{0L}(x),\; j'^{-}_{\rho}(z)]).\end{aligned} \tag{11}$$

The zeroth order current $j^{+}_{\mu L}(x)$ is conserved *i.e.*

$$\partial^{\mu}\, j^{+}_{\mu L}(x) = 0.$$

Also, the colour currents j'^{l}_{μ} commute with the weak currents

$$[j^{+}_{0L}(x),\ j'^{\pm}_{\lambda}(y)]\,|_{x_0=y_0}=0.$$

Hence, $$\partial^{\mu}_{x}\ T\,[j'^{+}_{\lambda}(y)\ j^{+}_{\mu L}(x)\ j'^{-}_{\rho}(z)]=0. \tag{12}$$

From (10) and (12) we see that

$$\partial^{\mu}\ j^{+\mathrm{I}}_{\mu L}(x)=0. \tag{13}$$

Then the familiar CVC arguments (see for example, Marshak *et al* 1969) will lead to the conclusion that the weak vertex is not renormalized by these diagrams.

The above argument is actually the same as the usual proof of non-renormalization of the weak vertex by the strong chromodynamic interactions ($O(f^2)$). Our only point is that in theories with broken colour, the same argument also applies to the $O(g^2)$ corrections coming from class-I diagrams.

Consider the class-II diagrams (shown in figure 2). These are peculiar to theories with broken colour symmetry only. The contribution from these diagrams is given by:

$$M^{\mathrm{II}}=\frac{ig^2}{\sqrt{2}}\langle p'\mid j^{+(\mathrm{II})}_{\mu L}(0)\mid p\rangle\ D^{V-W}_{\mu\nu}(q)\,L^{-}_{\nu L}, \tag{14}$$

where $$D^{V-W}_{\mu\nu}(q)=\int d^4x\exp(iq\cdot x)\,\langle 0\mid T\,[V^{+}_{\mu}(x)\ W^{-}_{\nu}(0)]\mid 0\rangle,$$

$$\sim O(g/f) \tag{15}$$

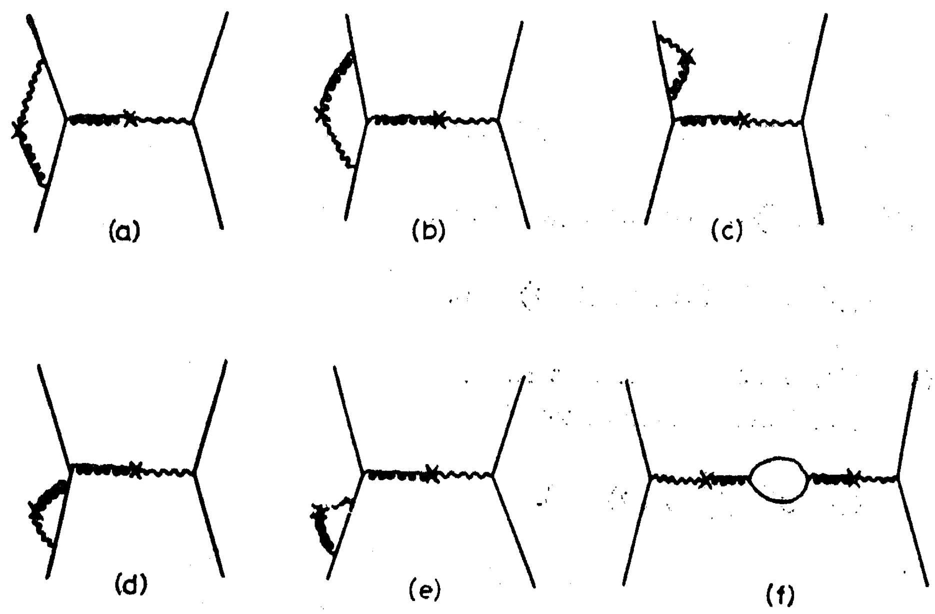

Figure 2. Diagrams with colour octet main vertex.

and $$\langle p' j^{+\mathrm{II}}_{\mu L}(x) | p \rangle = \frac{-i g f^2}{2\sqrt{2}} \int \mathrm{d}y\mathrm{d}z \langle p' | j^{+}_{\lambda L}(y)\, j'^{+}_{\mu}(x)\, j'^{-}_{\rho}(z) | p \rangle$$

$$x \langle 0 | \; T\,[W^{\lambda+}(y)\; V^{\rho-}(z)]\, | 0 \rangle. \tag{16}$$

Note that $M^{\mathrm{II}} = O(g^4)$. This implies that we are considering $O(g^2) = O(\alpha)$ corrections to the weak vertex. Also we consider strong interactions to the zeroth order only.

We will now show that the effective current $j^{+(\mathrm{II})}_{\mu L}$ is also conserved, when sandwiched between colour singlet states:

Consider,

$$\begin{aligned} &\partial^{\mu}_{x}\, T\,(j^{+}_{\lambda L}(y)\; j'^{+}_{\mu}(x)\; j'^{-}_{\rho}(z)) \\ &= T\,(j^{+}_{\lambda L}(y)\; \partial^{\mu} j'^{+}_{\mu}(x)\, j'^{-}_{\rho}(z)) \\ &+ \delta\,(x_0 - y_0)\, T\,(j'^{-}_{\rho}(z)\; [j'^{+}_{0}(x),\, j^{+}_{\lambda L}(y)]) \\ &+ \delta\,(x_0 - z_0)\; T\,(j^{+}_{\lambda L}(y)\; [j'^{+}_{0}(x),\, j'^{-}_{\rho}(z)]) \end{aligned} \tag{17}$$

The zeroth order octet current is conserved,

$$\partial^{\mu}\, j'^{+}_{\mu}(x) = 0 \tag{18}$$

Also, $$[j'^{+}_{0}(x),\, j^{+}_{\lambda L}(y)]\,|_{x_0 = y_0} = 0 \tag{19}$$

and $$[j'^{+}_{0}(x),\, j'^{-}_{\rho}(z)]\,|_{x_0 = z_0} = 2\delta^3\,(\mathbf{x} - \mathbf{z})\; j'^{3}_{\rho}(z).$$

Hence,

$$\partial^{\mu}_{x}\, T\,[j^{+}_{\lambda L}(y)\; j'^{+}_{\mu}(x)\; j'^{-}_{\rho}(z)] = 2\,\delta^4\,(x - z)\;\, T\,[j^{+}_{\lambda L}(y)\; j'^{3}_{\rho}(z)]. \tag{20}$$

Now if $|p\rangle$ and $|p'\rangle$ are colour singlet states,

$$\langle p' |\; T(j^{+}_{\lambda L}(y)\; j'^{3}_{\rho}(z))\; |p\rangle = 0. \tag{21}$$

as $j^{+}_{\lambda L}(y)\; j'^{3}_{\rho}(z)$ is a colour octet operator.

From equations (16) to (21) it follows that

$$\langle p' |\; \partial^{\mu}\, j^{+II}_{\mu L}(x)\; |p\rangle = 0. \tag{22}$$

Hence the effective current is conserved. As in the case of class I diagrams, this will imply that the weak vertex is not renormalized by these diagrams.

This argument is valid for processes involving neutral currents also.

4. Conclusions

In integrally-charged quark models, the colour symmetry is broken and the colour gluons and electro-weak bosons mix. We have shown that the colour octet contributions to the weak vertex is zero to $0(\alpha)$ at zero momentum transfer, by proving that the effective weak current is conserved to this order.

We have considered class II diagrams only to the zeroth order in the strong coupling constant. Sirlin (1978) has shown that the strong interaction effects are small when the underlying strong interaction theory is asymptotically free. The models of the type considered in this paper can be expected to be asymptotically free, at least approximately (Rajasekaran and Roy, 1975, see especially the last section). Then the arguments of Sirlin may be expected to go through in this case also and contributions from diagrams formally of higher order in f will be small.

Acknowledgement

This work forms part of a research project supported by the University Grants Commission whose assistance is gratefully acknowledged.

References

Marshak R E, Riazuddin and Ryan C P 1969 *Theory of weak interactions in particle physics*, (New York: Wiley Interscience)

Pati J and Salam A 1973 *Phys. Rev.* **D8** 1240

Pati J and Salam A 1976 *Phys. Rev. Lett.* **36** 11

Rajasekaran G and Roy P 1975 *Pramana* **5** 303

Ramachandran R 1979 *Pramana* **12** 227

Sirlin A 1978 *Rev. Mod. Phys.* **50** 573 (and references therein)