Pramāṇa, Vol. 4, No. 5, 1975, pp 222–228. © Printed in India.

Effect of gluons on neutral-current interactions in deep inelastic neutrino scattering

G RAJASEKARAN and PROBIR ROY
Tata Institute of Fundamental Research, Bombay 400005

MS received 6 March 1975; after revision 12 May 1975

Abstract. The contribution of neutral spin-1 gluons to the deep inelastic neutral-current processes $\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) +$ hadrons is worked out in the parton model. Such a contribution violates Bjorken scaling strongly.

Keywords. Neutral current; weak interaction; spin-1 gluons; parton model.

1. Introduction

A substantial part ($\sim 50\%$) of the four-momentum of a high-energy nucleon resides in neutral isoscalar constituents. These do not interact directly with the electromagnetic or the weak charged current. However, their presence has been established indirectly through a sum-rule based on momentum-conservation involving deep inelastic inclusive reactions induced by those currents (Feynman 1972, Perkins 1972, Roy 1975). A possible choice is to identify these constituents with spin-one bosons called gluons which are supposed to mediate the strong interactions. The next question that arises is whether these gluons in a nucleon can interact directly with the neutrinos through the weak neutral-current interaction first observed by Hasert *et al* (1973). Since there is no compelling reason why such interactions should be forbidden, we may take the view that they exist. In this note, we shall investigate the influence of such interactions on the inclusive reactions: $\nu_\mu(\bar{\nu}_\mu) + N \to \nu_\mu(\nu_\mu) +$ hadrons in the deep inelastic domain.

The main features of electron-nucleon scattering as well as charged-current neutrino-nucleon scattering in the deep inelastic domain have been well accounted for in the parton model (*e.g.* Roy 1975). In fact the above-mentioned evidence concerning neutral isoscalar constituents was also obtained within the framework of the parton model. In this model, the deep inelastic lepton scattering off a nucleon is pictured as if the nucleon were composed of free, point-like constituents called partons. The lepton then scatters elastically and incoherently off each parton.

One of the important features of deep inelastic lepton scattering is Bjorken scaling and this follows in the parton model if the partons have spin 0 or spin $\frac{1}{2}$. Bjorken scaling is in very good accord with experimental data on deep inelastic electron scattering as well as the charged-current neutrino scattering. We should especially note the good evidence (Perkins 1972) on the proportionality of the total cross section of the charged-current neutrino scattering to the incident neutrino energy. This proportionality is a consequence of Bjorken scaling.

We shall take the weak neutral current to have a general vector and axial vector form. The contribution of the spin $\frac{1}{2}$ partons (quarks) to such a general weak neutral current has been already worked out (*e.g.* Rajasekaran and Sarma 1974 *a*, *b*). Our aim here is to calculate the contribution of the spin-1 partons (gluons). In contrast to the contribution from spin 0 and spin $\frac{1}{2}$ partons, the contribution from the spin-1 gluon-partons violates Bjorken scaling strongly. Consequently the total cross section for the neutral-current neutrino scattering is no longer proportional to the incident neutrino energy, but increases faster. Experimental observation of such a behaviour will be a signal for the gluon contribution.

The violation of Bjorken scaling in deep inelastic eN scattering arising from charged spin-1 partons has already been considered (Cleymens and Komen 1974). However, we are concerned with *neutral isoscalar gluons.* Clearly such objects cannot be seen in eN scattering or in (the dominant $\triangle S = 0$ isovector part of the) charged-current νN scattering. Hence, weak neutral current provides a possible agent through which their existence may be revealed.

2. The neutral current of the gluon

The effective Lagrangian density relevant for the neutral-current interaction of the gluons with the neutrinos can be written as*

$$\mathcal{L}_{\text{int}} = \frac{G}{\sqrt{2}}\, \bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu\,(aV_{\mu} + bA_{\mu}) \tag{1}$$

where G is the Fermi constant, ν stands for ν_{μ}, a and b are constants and V_{μ} and A_{μ} stand for the vector and axial vector currents formed out of gluon fields. Since we have assumed that the neutral current has vector and axial vector pieces only, the helicity of the neutrino is conserved in the interaction. Further, the neutrinos (antineutrinos) used in the experiment are produced in the well-known π or K decays (through charged current) and hence have a definite initial helicity. These two facts have allowed us to insert the factor $(1-\gamma_5)$ in eq. (1).

Consistent with hermiticity and time-reversal invariance, V_{μ} can have two forms in terms of the gluon field G_{μ}:

$$V_{\mu}^{(1)} = \mathrm{i}\,(\partial_{\mu}G_{\nu}G^{\nu\dagger} - G^{\nu}\partial_{\mu}G_{\nu}^{\dagger}) \tag{2}$$

$$V_{\mu}^{(2)} = \mathrm{i}\,(\partial_{\nu}G_{\mu}^{\dagger}G^{\nu} - G^{\nu\dagger}\partial_{\nu}G_{\mu}) \tag{3}$$

The conventional "minimal" current is

$$V_{\mu} = V_{\mu}^{(1)} + V_{\mu}^{(2)} = \mathrm{i}\,(\partial_{\mu}G_{\nu}G^{\nu\dagger} - G^{\nu}\partial_{\mu}G_{\nu}^{\dagger}) + \mathrm{i}\,(\partial_{\nu}G_{\mu}^{\dagger}G^{\nu} - G^{\nu\dagger}\partial_{\nu}G_{\mu}) \tag{4}$$

However, a more general form can be written in terms of the "anomalous moment" κ as

$$\begin{aligned} V_{\mu} = {} & \mathrm{i}\,(\partial_{\mu}G_{\nu}G^{\nu\dagger} - G^{\nu}\partial_{\mu}G_{\nu}^{\dagger}) + \mathrm{i}\,(\partial_{\nu}G_{\mu}^{\dagger}G^{\nu} - G^{\nu\dagger}\partial_{\nu}G_{\mu}) \\ & + \mathrm{i}\kappa\partial^{\nu}(G_{\mu}^{\dagger}G_{\nu} - G_{\nu}^{\dagger}G_{\mu}) \end{aligned} \tag{5}$$

This is a conserved current. We shall use the more general form of eq. (5) rather than eq. (4), since eq. (5) includes the Yang-Mills current which is obtained for $\kappa = 1$.

---

* Our metric and gamma matrices are the same as in Bjorken and Drell (1964).

The form of the axial vector current, consistent with hermiticity and time-reversal invariance, is

$$A_\mu = \epsilon_{\mu\nu\lambda\rho} (G^{\nu\dagger}\partial^\lambda G^\rho + \partial^\lambda G^{\rho\dagger} G^\nu) \tag{6}$$

In contrast to the vector current, the axial vector current is not conserved.

Note the interesting fact that for self-conjugate gluons, the vector current vanishes, but the axial vector current does not. We shall however take the gluons to be non-self-conjugate so that both vector and axial vector currents exist. The new quantum number thereby introduced may be colour.

There are theoretical reasons for taking the gluon to be a colour octet in a colour SU(3) scheme (Fritzsch *et al* 1973). With an octet gluon field G^i_μ, one can construct the following V and A currents:

$$V^{(1)}_{\mu,i} = f_{ijk} G^j_\nu \partial_\mu G^{\nu,k} \tag{7}$$

$$V^{(2)}_{\mu,i} = f_{ijk} G^j_\nu \partial^\nu G^k_\mu \tag{8}$$

$$A_{\mu,i} = \epsilon_{\mu\nu\lambda\rho} d_{ijk} G^{\nu,j} \partial^\lambda G^{\rho,k} \tag{9}$$

which are in exact correspondence to eqs (2), (3) and (6). However, in our phenomenological approach, we shall work with the current given in eqs (5) and (6), although these currents may arise from suitable linear combinations of the colour SU(3) currents [eqs (7), (8) and (9)].

3. Gluon contribution to the structure functions

The inelastic structure functions $W_{1,2,3}$ are defined by (*e.g.* Llewellyn-Smith 1972)

$$\frac{1}{4\pi}\int d^4x \,[\exp(iq\cdot x)]\, \langle p \mid [J_\mu(x), J_\nu(0)] \mid p\rangle$$

$$= -g_{\mu\nu} W_1 + \frac{p_\mu p_\nu}{M^2} W_2 - \frac{i}{2M^2} \epsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta W_3 + \cdots \tag{10}$$

Here, J_μ is the weak neutral current, $|p\rangle$ is a spin-averaged nucleon state with momentum p, $W_{1,2,3}$ are real functions of $Q^2 \equiv -q^2$ and $\nu \equiv p\cdot q/M$, M is the nucleon mass and the dots stand for terms which do not concern us here.

We take the deep inelastic limit: $\nu \to \infty$, $Q^2 \to \infty$ such that $x \equiv Q^2/2M\nu$ is finite. In this limit, the functions W_1, $\nu W_2/M$ and $\nu W_3/M$ will be called F_1, F_2 and F_3 respectively. We calculate these structure functions using the parton model for the currents of eqs (5) and (6). We consider only massive spin-1 gluons for which the spin-summation is $-g_{\mu\nu} + p_\mu p_\nu/\mu^2$, where μ is the mass of the gluon. It is the second term which leads to scaling violation. Throughout, $g(x)$ [$(\bar{g}(x)$] is the probability-averaged population-density of gluons [antigluons] with a fraction x of the longitudinal momentum of the nucleon in the infinite momentum frame. Since the anomalous moment term in eq. (5) gives a qualitatively different result, we discuss the two cases (*a*) $\kappa = 0$ and (*b*) $\kappa \neq 0$ separately.

Case (*a*): $\kappa = 0$

$$F_1(Q^2, x) = \frac{1}{3}\left[a^2\left(1 + \frac{Q^2}{4\mu^2}\right) + 2b^2\left(\frac{2\mu^2}{Q^2} + 1 + \frac{Q^2}{8\mu^2}\right)\right][g(x) + \bar{g}(x)]$$

$$F_2(Q^2, x) = x\left[a^2\left(1 + \frac{Q^2}{6\mu^2}\right) + \frac{2}{3}b^2\left(1 + \frac{Q^2}{4\mu^2}\right)\right][g(x) + \bar{g}(x)]$$

$$F_3(Q^2, x) = -\frac{4ab}{3}\left(1 + \frac{Q^2}{4\mu^2}\right)[g(x) - \bar{g}(x)] \tag{11}$$

Thus, Bjorken scaling is violated linearly in Q^2. In the deep inelastic region, if we impose the condition $Q^2 \gg \mu^2$ and keep only the leading terms, then we have

$$F_1(Q^2, x) \rightarrow (a^2 + b^2)\frac{Q^2}{12\mu^2}[g(x) + \bar{g}(x)]$$

$$F_2(Q^2, x) \rightarrow (a^2 + b^2)\frac{Q^2 x}{6\mu^2}[g(x) + \bar{g}(x)]$$

$$F_3(Q^2, x) \rightarrow -ab\frac{Q^2}{3\mu^2}[g(x) - \bar{g}(x)] \tag{12}$$

It is interesting to note that although Bjorken scaling is violated, the leading scale-violating terms in eq. (12) satisfy the Callan-Gross relation originally derived for spin-$\frac{1}{2}$ constituents:

$$F_2 \rightarrow 2xF_1$$

Thus, there seems to be a close correspondence between spin-1 partons with $\kappa = 0$ and spin-$\frac{1}{2}$ partons.

Case (*b*) : $\kappa \neq 0$

$$F_1(Q^2, x) = \frac{1}{3}\left[a^2(1 + \kappa)^2\left(1 + \frac{Q^2}{4\mu^2}\right) + 2b^2\left(\frac{2\mu^2}{Q^2} + 1 + \frac{Q^2}{8\mu^2}\right)\right]$$
$$\times [(g(x) + \bar{g}(x)]$$

$$F_2(Q^2, x) = x\left[a^2\left\{1 + (1 + \kappa^2)\frac{Q^2}{6\mu^2} + \frac{\kappa^2 Q^4}{12\mu^4}\right\} + \frac{2}{3}b^2\left(1 + \frac{Q^2}{4\mu^2}\right)\right]$$
$$\times [g(x) + \bar{g}(x)]$$

$$F_3(Q^2, x) = -\frac{4ab}{3}(1 + \kappa)\left(1 + \frac{Q^2}{4\mu^2}\right)[g(x) - \bar{g}(x)] \tag{13}$$

Once again, keeping the leading terms alone, we get

$$F_1(Q^2, x) \rightarrow [a^2(1 + \kappa)^2 + b^2]\frac{Q^2}{12\mu^2}[g(x) + \bar{g}(x)]$$

$$F_2(Q^2, x) \rightarrow \kappa^2 a^2 \frac{Q^4}{12\mu^4} x[g(x) + \bar{g}(x)]$$

$$F_3(Q^2, x) \rightarrow -ab(1 + \kappa)\frac{Q^2}{3\mu^2}[g(x) - \bar{g}(x)] \tag{14}$$

Now the F_2 term violates Bjorken scaling quadratically in Q^2, although the other terms do so only linearly, so that even the Callan-Gross relation is lost.

4. Gluon contribution to cross sections

Defining $y \equiv \nu/E$, where E is the energy of the incident neutrino in the laboratory system, the differential cross sections for the reactions

$$\nu^{\pm} + N \longrightarrow \nu^{\pm} + \text{anything}$$

in the deep inelastic limit may be written as (Llewellyn-Smith 1972)

$$\frac{d^2\sigma}{dxdy}(\nu^{\pm}N) = \frac{G^2ME}{\pi}\left[(1-y)F_2 + y^2 xF_1 \mp y\left(1-\tfrac{1}{2}y\right)xF_3\right] \tag{15}$$

where ν^+ and ν^- refer to neutrino and antineutrino respectively. Putting in the gluon-contributions to $F_{1,2,3}$ given in section 3 and using $Q^2 = 2MExy$ we can get the differential cross section. We shall write down the results, keeping the leading terms only.

Case (*a*): $\kappa = 0$

$$\frac{d^2\sigma}{dxdy}(\nu^{\pm}N) = \frac{G^2}{6\pi}\frac{M^2E^2}{\mu^2}x^2 y\left[\{(a\pm b)^2 + (a\mp b)^2(1-y)^2\}g(x) + \{(a\mp b)^2 + (a\pm b)^2(1-y)^2\}\bar{g}(x)\right] \tag{16}$$

For the sake of comparison we may give here the corresponding formula for the spin-$\frac{1}{2}$ (quark) contribution:

$$\frac{d^2\sigma}{dxdy}(\nu^{\pm}N) = \frac{G^2}{2\pi}MEx\left[\{(c\pm d)^2 + (c\mp d)^2(1-y)^2\}q(x) + \{(c\mp d)^2 + (c\pm d)^2(1-y)^2\}\bar{q}(x)\right] \tag{16 a}$$

where q and $\bar{q}$ are the population-densities for quarks and antiquarks and c and d are the vector and axial-vector coupling constants occurring in the interaction of the quark-field ψ:

$$\mathcal{L}_{\text{int}} = \frac{G}{\sqrt{2}}\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu\,(c\bar{\psi}\gamma_{\mu}\psi + d\bar{\psi}\gamma_{\mu}\gamma_5\psi)$$

The y-distribution arising from the spin-1 gluon contribution is thus a cubic in y, in contrast to the spin-$\frac{1}{2}$ quark contribution which is a quadratic in y. However, it should be noted that the quantity within the square brackets in eq. (16), is precisely of the same form as that arising from spin-$\frac{1}{2}$ partons (eq. 16 *a*). This result is due to the fact that the relationship between (the leading terms of) F_1, F_2 and F_3 for spin-1 partons with $\kappa = 0$ is exactly the same as for spin-$\frac{1}{2}$ partons. Part of this correspondence was already pointed out in section 3.

The total cross section is

$$\sigma(\nu^{\pm}N) = \frac{G^2}{72\pi}\frac{M^2E^2}{\mu^2}\left[7(a^2+b^2)\int_0^1 dx\,x^2\{g(x)+\bar{g}(x)\} \pm 10ab\int_0^1 dx\,x^2\{g(x)-\bar{g}(x)\}\right] \tag{17}$$

which increases quadratically with the incident energy E, as compared to the usual linear rise. From eq. (17), one may also obtain the bound:

$$\frac{1}{6} \leqslant \frac{\sigma(\bar{\nu}N)}{\sigma(\nu N)} \leqslant 6 \tag{18}$$

This should be compared to the well-known result:

$$\frac{1}{3} \leqslant \frac{\sigma(\bar{\nu}N)}{\sigma(\nu N)} \leqslant 3$$

which is valid for quark-partons.

Case (*b*): $\kappa = 0$

Again keeping the leading terms alone, we have

$$\frac{d^2\sigma}{dxdy}(\nu^{\pm} N) = \frac{G^2}{3\pi} \frac{M^3 E^3}{\mu^4} a^2 \kappa^2 (1-y) y^2 x^3 [g(x) + \bar{g}(x)], \tag{19}$$

$$\sigma(\nu^{\pm}N) = \frac{G^2}{36\pi} \frac{M^3 E^3}{\mu^4} a^2 \kappa^2 \int_0^1 dx x^3 [g(x) + \bar{g}(x)] \tag{20}$$

Therefore, the y-distribution remains a cubic although the total cross section now increases as E^3.

The gluon-contribution to the neutral-current cross sections given in eqs (16), (17), (19) and (20) can be added to the well-known quark-contributions. However, in the high-energy limit, only the gluon-contributions survive.

These characteristic signals of the spin-1 gluon—namely, y^3 terms and cross sections increasing faster than E—may be looked for, when detailed experimental data on the inclusive neutral current processes become available.

5. Comments on gluons in gauge-models

As far as the gluon-participation in the neutral-current weak interactions is concerned, the gauge models can be divided into two broad classes. In the first class (Weinberg 1973, Gross and Wilczek 1973, Politzer 1973), the gauge group of the strong interactions is assumed to commute with the gauge group of the weak and electromagnetic interactions. These models are asymptotically free. The gluons, which are perhaps massless, belong to the regular representation of the strong gauge group and are completely neutral with respect to the electromagnetic and weak interactions. In other words, the gluons in this class of models cannot be seen by the electromagnetic or the weak interactions (both charged as well as the neutral current variety).

In the second class of gauge models (Furman and Komen 1975, Cheng and Wilczek 1974) which have been constructed for strong and electromagnetic interactions, the gluons are charged and hence contribute to the electromagnetic current. Although these models are not asymptotically free in an exact sense, they are approximately so. Here one has the interesting example of scaling being achieved even for massive charged spin-1 partons. These models can perhaps be extended to include weak interactions also.

What we have considered in this note is, in contradistinction to both the above classes of models, a possibility of scaling-violation in weak neutral-current interaction arising from spin-1 gluons. For the present, this should be regarded primarily as a phenomenological possibility to be confronted with experiment. The relevance of gauge theories to observed phenomena is still an open question. So, we consider it worthwhile to study the consequences of various hypotheses from a phenomenological point of view.

References

Bjorken J D and Drell S D 1964 *Relativistic Quantum Mechanics* (New York: McGraw Hill)

Cheng T P and Wilczek F 1974 *Phys. Lett.* **53B** 269

Cleymens J and Komen G J 1974 *Nucl. Phys.* **B78** 396

Feynman R P 1972 *Photon Hadron Interactions* (New York: Benjamin)

Fritzsch H, Gell-Mann M and Leutwyler H 1973 *Phys. Lett.* **47B** 365

Furman M A and Komen G J 1975 *Nucl. Phys.* **B84** 323

Gross D J and Wilczek F 1973 *Phys. Rev. Lett.* **30** 1343

Hasert F J, Kabe S, Krenz W, *et al* 1973 *Phys. Lett.* **46B** 138

Llewellyn-Smith C H 1972 *Physics Reports* **3** 262

Perkins D H 1972 *Proc. XVI Int. Conf. High Energy Phys.* (National Accelerator Laboratory, Batavia) Vol. 4 189

Politzer H D 1973 *Phys. Rev. Lett.* **30** 1346

Rajasekaran G and Sarma K V L 1974*a* *Pramāṇa* **2** 62, E 225

Rajasekaran G and Sarma K V L 1974*b* *Pramāṇa* **3** 44

Roy P 1975 *Theory of Lepton-Hadron Processes at High Energies* (Oxford: Clarendon Press)

Weinberg S 1973 *Phys. Rev. Lett.* **31** 494