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Colour gluons and scaling in a unified gauge model

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Abstract. Deep inelastic weak and electromagnetic processes are considered within the parton framework taking the partons to be integrally charged quarks and coloured gluons. Despite the participation of the spin-one gluons in these processes, scaling is shown to be maintained by treating the problem in a unified gauge model based on the group $SU(3)_{\text{colour}} \otimes SU_L(2) \otimes U(1)$. This is a consequence of the vector-dominance type of couplings between the gluons and the weak or electromagnetic vector bosons which are induced by the spontaneous breakdown of gauge symmetry. As a further consequence it is found that in the asymptotic region far above the gluon masses the colour octet parts of the weak and electromagnetic currents of the quarks are damped so that, in particular, the integrally charged quarks behave as fractionally charged quarks in this region.

Keywords. Unified gauge model; $SU_{\text{colour}}(3) \otimes SU_L(2) \otimes U(1)$; spin-one colour gluons; integrally-charged quarks; deep inelastic lepton-hadron scattering; parton model; Bjorken scaling; electron-positron annihilation.

1. Introduction

Most of the present-day models for hadrons involve two kinds of constituents—spin-half quarks and spin-one gluons. Both the quarks and gluons are presumed to participate in strong interactions. However, it is usually assumed that only the quarks have weak and electromagnetic interactions; the gluons are regarded as completely neutral with respect to these interactions. We would like to study the consequences of the alternative hypothesis that both the gluons and the quarks participate in weak and electromagnetic processes.

There is a characteristic distinction between the weak and electromagnetic interactions of quarks on one hand and of gluons on the other. This distinguishing feature is revealed in their respective contributions to the deep-inelastic structure functions of the nucleon. Whereas the contribution from the spin-half constituents calculated in the parton-model (Feynman 1972, Roy 1975) obeys Bjorken scaling, the contribution from the spin one partons violates Bjorken scaling strongly. This violation can be traced to the positive powers of momenta arising from the gluon-coupling as well as to the gluon-spin summation (Cleymens and Komen 1974, Rajasekaran and Roy 1975). It is natural to ask the following question. Is it possible for the gluons to participate in weak and electromagnetic interactions without impairing the validity of the Bjorken scaling of deep-inelastic lepton-hadron processes ? This is in fact possible provided all the interactions including the gluon-couplings are incorporated in a unified renormalisable gauge theory of weak, electromagnetic and strong interactions. Furman and Komen (1975) as well as Cheng and Wilczek (1974) have already constructed such gauge models unifying electromagnetic and strong interactions. These models provide interesting examples of scaling being achieved even for massive charged spin-one partons. The aim of the present paper is to incorporate the weak interactions also in such a unified framework. We consider a unified gauge model of weak, electromagnetic and strong interactions and analyse the resulting pattern of the various interactions in detail. Now the gluons contribute to all the deep inelastic lepton-hadron processes—electromagnetic as well as weak—and yet scaling is maintained asymptotically.

For our purpose, unified gauge models can be divided into two broad classes. In the first class (Weinberg 1973) the colour gauge group of the strong interactions is assumed to commute with the gauge group of the weak and electromagnetic interactions and the quarks are chosen to be the fractionally-charged ones of Gell-Mann and Zweig (Gell-Mann 1964, Zweig 1964). Colour in this class of theories is an exact gauge symmetry and the coloured gluons are massless. The gluons belong to the regular representation of the colour gauge group and are singlets with respect to the weak and electromagnetic gauge groups. So the gluons in this class of models cannot be seen by weak or electromagnetic probes.

Our interest lies in the second class of unified gauge models where the colour gauge group does not commute with the gauge group of the weak and electromagnetic interactions. Here colour gauge invariance is broken spontaneously, and as a consequence not only do the gluons acquire masses but also they mix with the weak and electromagnetic vector bosons. It is through this mixing that the desired weak and electromagnetic interactions of the gluons emerge. Further, as a result of this mixing phenomenon, the effective weak and electromagnetic vertices of the gluons involve vector-dominance type of diagrams. It is the $(q^2 - m^2)^{-1}$ factor, associated with these that restores scaling. The most direct way of achieving all these is to employ the integrally-charged quarks of Han and Nambu (1965). Since the electric charge operator for the Han-Nambu quarks does not commute with the generators of the colour group, the required mixing is produced.

In this paper we study a unified gauge model of weak, electromagnetic and strong interactions based on integrally-charged coloured quarks. Such a model was proposed by Pati and Salam (1973). However, certain important consequences of this type of model following from the mixing of the gluons with the weak and electromagnetic vector bosons have not been realised so for. As we have already pointed out, one consequence is the emergence of weak and electromagnetic couplings for the gluons with the important feature that these couplings preserve Bjorken scaling. As another consequence we may mention the remarkable result that even though the colour degree of freedom may be excited, the integrally-charged quarks behave asymptotically like the fractionally-charged quarks !

The unified gauge models of the first class are asymptotically free (Gross and Wilczek 1973, Politzer 1974), provided the infrared problem of massless nonabelian gauge fields can be solved. Hence there is a possibility of achieving a field-theoretic understanding of Bjorken scaling (modulo logarithmic vioations) in these models. In contrast, the unified gauge models of the second class are not asymptotically free in an exact sense, although, as we shall argue later, they may be approximately so. In any case our discussion of scaling will be confined to the parton-framework and no attempt will be made in this paper to justify the parton-model.

The gauge model is constructed in section 2 and the spontaneous breakdown of gauge symmetry is introduced in section 3. Section 4 analyses the interactions of the vector bosons and the fermions. In section 5 we calculate the effective weak and electromagnetic vertices of the gluons and the quarks. These are used in section 6 to compute the structure functions for deep inelastic lepton-hadron scattering as well as the cross section for $e^+ e^-$ annihilation. Section 7 is devoted to a summary and discussion.

2. Gauge model

We take the gauge group as $SU'(3) \otimes SU_L(2) \otimes U(1)$ for our unified gauge model† (c.f. Pati and Salam 1973). The strong interactions are primarily described in terms of the colour gauge group $SU'(3)$ and integrally-charged Han-Nambu quarks. The weak and electromagnetic interactions are related to the gaugegroup $SU_L(2) \otimes U(1)$ as in the Weinberg-Salam model (Weinberg 1967, Salam 1968). However, because of the mixing introduced by the charge operator, an exclusive association of $SU_L(2) \otimes U(1)$ with weak and electromagnetic interactions and $SU'(3)$ with strong interaction is not possible in the present model.

The quarks can be written in the form of two arrays:

$$H_{\alpha i} = \begin{pmatrix} p_1^{\ 0} & p_2^{\ +} & p_3^{\ +} \\ n_1^{\ -} & n_2^{\ 0} & n_3^{\ 0} \end{pmatrix}, \quad K_{\alpha i} = \begin{pmatrix} c_1^{\ 0} & c_2^{\ +} & c_3^{\ +} \\ \lambda_1^{\ -} & \lambda_2^{\ 0} & \lambda_3^{\ 0} \end{pmatrix}, \tag{2.1}$$

where the Latin subscript i refers to the colour indices 1, 2, 3 and the Greek subscript α spans the $SU(2)$ space and refers to p, n or c, λ. The superscripts denote the electric charges. The quarks n_i and λ_i are the Cabibbo-rotated objects:

$$\begin{aligned} n_i &= \cos\theta_c \hat{n}_i + \sin\theta_c \lambda_i, \\ \lambda_i &= -\sin\theta_c \hat{n}_i + \cos\theta_c \lambda_i, \end{aligned} \tag{2.2}$$

where $\hat{n}_i$ and λ_i are the "physical" quarks and θ_c is the Cabibbo angle. The charmed quarks c_i are required for eliminating the $|\Delta S| = 1$ neutral currents (Glashow *et al* 1970).

Both H and K belong to the 3^* representation of the colour gauge group $SU'(3)$. The 3^* representation has been chosen rather than 3 because of a technical convenience relating to the charge structure (see eqs. (2.4) and (2.5)). The transformation property under the left-handed gauge group $SU_L(2)$ depends on the chirality of the quarks. We define the left-handed and right handed quarks†† as

$$q_L = \tfrac{1}{2}(1 - \gamma_5)\, q, \quad q_R = \tfrac{1}{2}(1 + \gamma_5)\, q. \tag{2.3}$$

† Although the paper of Pati and Salam (1973) contains a description of the gauge model we find it necessary to present it in this section, chiefly because we want to disentangle the model from other features of Pati-Salam's work such as unified lepton-quark multiplets which are not germane to the present investigation.

†† Our metric and gamma matrices are the same as in Bjorken and Drell (1964).

Then, H_L and K_L belong to the doublet representation of the group $SU_L(2)$ whereas all the right-handed quarks p_{iR}, n_{iR}, c_{iR} and λ_{iR} are singlets under the same.

The electric charge operator can be written as

$$Q = I_3' + \tfrac{1}{2} Y' + I_{3L} + U \tag{2.4}$$

where I_3' and Y' are the two diagonal generators of $SU'(3)$, I_{3L} is the diagonal generator of $SU_L(2)$ and U is the generator of an abelian group $U(1)$ which commutes with $SU'(3)$ and $SU_L(2)$. For the quarks H and K, we can write

$$(I_3')_{ij} = -\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad (Y')_{ij} = -\frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix};$$

$$(I_{3L})_{\alpha\beta} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.5}$$

The matrices I_3' and Y' operate in the colour space denoted by the Latin indices, whereas I_{3L} operates in the $SU_L(2)$ space denoted by the Greek indices. Further note that the negative signs in the definitions of I_3' and Y' are due to the fact that H and K belong to 3^* rather than 3. The U quantum numbers of the various quark multiplets are the following:

$$\begin{aligned} U(H_L, K_L) &= \tfrac{1}{6}, \\ U(p_R, c_R) &= \tfrac{2}{3}, \\ U(n_R, \lambda_R) &= -\tfrac{1}{3}. \end{aligned} \tag{2.6}$$

The consistency of these assignments with the integral charges indicated as superscripts in eq. (2.1) can be easily checked.

The leptons consist of two doublets under $SU_L(2)$:

$$E_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L; \quad M_L = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \tag{2.7}$$

and two $SU_L(2)$ singlets e_R and μ_R. Here e and μ refer to the negatively charged leptons and L and R denote the left-handed and right-handed objects defined in the same way as in eq. (2.3). All the leptons are taken to be singlets under the colour group $SU'(3)$. The U quantum numbers of the leptons are

$$\begin{aligned} U(E_L, M_L) &= -\tfrac{1}{2}, \\ U(e_R, \mu_R) &= -1. \end{aligned} \tag{2.8}$$

We can now define the fermion currents which will be useful in describing the interactions between the fermions and the gauge-bosons. They are the colour octet $j_\mu'^{l}$, the $SU_L(2)$ triplet $j_{\mu L}^k$ and the singlet current j_μ^U given by

$$j_\mu'^{l} = \sum_{q=p,n,c,\lambda} (\bar{q}_1\bar{q}_2\bar{q}_3) \frac{\lambda'^l}{2} \gamma_\mu \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}, \quad (l = 1, \ldots, 8),$$

$$j_\mu^k = \sum_{i=1}^{3} \left\{ (\bar{p}_i\bar{n}_i) \frac{\tau^k}{2} \gamma_{\mu L} \begin{pmatrix} p_i \\ n_i \end{pmatrix} + (\bar{c}_i\bar{\lambda}_i) \frac{\tau^k}{2} \gamma_{\mu L} \begin{pmatrix} c_i \\ \lambda_i \end{pmatrix} \right.$$

$$\left. + (\bar{\nu}_e\bar{e}) \frac{\tau^k}{2} \gamma_{\mu L} \begin{pmatrix} \nu_e \\ e \end{pmatrix} + (\bar{\nu}_\mu\bar{\mu}) \frac{\tau^k}{2} \gamma_{L\mu} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \right\}, \quad (k = 1, 2, 3), \tag{2.9}$$

$$j_\mu^U = \sum_{i=1}^{3} \{\tfrac{1}{6}(\bar{p}_i\gamma_{\mu L}p_i + \bar{n}_i\gamma_{\mu L}n_i + \bar{c}_i\gamma_{\mu L}c_i + \bar{\lambda}_i\gamma_{\mu L}\lambda_i)$$

$$+ \tfrac{2}{3}(\bar{p}_i\gamma_{\mu R}p_i + \bar{c}_i\gamma_{\mu R}c_i) - \tfrac{1}{3}(\bar{n}_i\gamma_{\mu R}n_i + \bar{\lambda}_i\gamma_{\mu R}\lambda_i)\}$$

$$- \tfrac{1}{2}(\bar{v}_e\gamma_{\mu L}v_e + \bar{e}\gamma_{\mu L}e + \nu_\mu\gamma_{\mu L}\nu_\mu + \bar{\mu}\gamma_{\mu L}\mu)$$

$$- (\bar{e}\gamma_{\mu R}e + \bar{\mu}\gamma_{\mu R}\mu).$$

Here $\bar{\lambda}'^l$ are the 3^* representation matrices of the $SU'(3)$ group:

$$\bar{\lambda}'^l = -(\lambda'^l)^* \tag{2.10}$$

and τ^k are the Pauli matrices. We have also defined

$$\gamma_{\mu{L \atop R}} = \tfrac{1}{2}\gamma_\mu(1 \mp \gamma_5). \tag{2.11}$$

The Lagrangian density for the fermions is

$$\mathcal{L}_F = i\sum_{i=1}^{3}\sum_{q=p,n,c,\lambda} \bar{q}_i\gamma^\mu\partial_\mu q_i + i\sum_{L=e,\mu}\bar{L}\gamma^\mu\partial_\mu L$$

$$+ f\sum_{l=1}^{8} j_\mu'^l V^{\mu l} + \tilde{g}\sum_{j=1}^{3} j_{\mu L}^i \tilde{W}^{\mu i} + \tilde{g}' j_\mu{}^U \tilde{U}^\mu. \tag{2.12}$$

We have introduced the fields $V_\mu{}^l$, $\tilde{W}_\mu{}^i$ and $\tilde{U}_\mu$ which stand for the gauge vector bosons of the groups $SU'(3)$, $SU_L(2)$ and $U(1)$ respectively and f, $\tilde{g}$ and $\tilde{g}'$ are the corresponding coupling constants. The vector boson part of the Lagrangian density is

$$\mathcal{L}_{VB} = -\tfrac{1}{4}\sum_{l=1}^{8}(V_{\mu\nu}^l + ff^{lmn}V_\mu{}^m V_\nu{}^n)^2 - \tfrac{1}{4}\sum_{i=1}^{3}(\tilde{W}_{\mu\nu}^i + \tilde{g}\epsilon^{ijk}\tilde{W}_\mu{}^j\tilde{W}_\nu{}^k)^2$$

$$- \tfrac{1}{4}\tilde{U}_{\mu\nu}^2, \tag{2.13}$$

where f^{lmn} are the structure constants of the $SU'(3)$ group and

$$V_{\mu\nu}^l = \partial_\mu V_\nu{}^l - \partial_\nu V_\mu{}^l, \quad \text{etc.}$$

Now that all the gauge-interactions have been written down, one can verify that the model is anomaly-free.

3. Breakdown of symmetry

We now introduce the spontaneous breakdown of gauge symmetry which generates the masses of all the gauge bosons except the photon and of all the fermions except the neutrinos. As a further consequence of this symmetry-breaking, the gluons mix with the weak and electromagnetic bosons. The spontaneous breakdown of symmetry is achieved through the device of the nonvanishing vacuum expectation values of the Higgs scalars.

We introduce the following two sets of Higgs scalar fields:

$$\sigma_{ai} = \begin{pmatrix} \sigma^0 & \sigma^+ & \sigma^+ \\ \sigma^- & \sigma^0 & \sigma^0 \\ \sigma^- & \sigma^0 & \sigma^0 \end{pmatrix}, \tag{3.1}$$

$$\eta_a = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}, \tag{3.2}$$

where the subscripts are to be taken in the same sense as in eq. (2.1). Under the group $SU'(3) \otimes SU_L(2) \otimes U(1)$, σ transforms as $(3^*, 2 \oplus 1, 1)$ and η transforms as $(1, 2, 1)$. The first two rows of σ_{ai} denote the two components of the $SU_L(2)$ doublet while the last row is a singlet under $SU_L(2)$. The U quantum numbers of σ are $\frac{1}{6}$ and $-\frac{1}{3}$ for the $SU_L(2)$ doublet and singlet parts respectively, whereas η has $U = \frac{1}{2}$.

The Higgs part of the Lagrangian is

$$\mathcal{L}_H = \sum_{i,a} \Big| \partial_\mu \sigma_{ai} - i \frac{f}{2} \sum_{l=1}^{8} \bar{\lambda}'^{l}_{ij} V_\mu{}^l \sigma_{aj} - i \frac{\tilde{g}}{2} \sum_{j=1}^{3} \tau^j_{a\beta} \tilde{W}_\mu{}^j \sigma_{\beta i}$$

$$- i\tilde{g}' u_{a\beta} \tilde{U}_\mu \sigma_{\beta i}{}^2 \Big|^2 + \sum_a \Big| \partial_\mu \eta_a - i \frac{\tilde{g}}{2} \sum_{j=1}^{3} \tau^j_{a\beta} \tilde{W}_\mu{}^j \eta_\beta - i \frac{\tilde{g}'}{2} \tilde{U}_\mu \eta_a \Big|^2$$

$$+ \sum_{i=1}^{3} \{(\bar{p}_i \bar{n}_i)_L (G_p \eta^c p_{iR} + G_n \eta n_{iR} + G_{n\lambda} \eta \lambda_{iR} + G_{pc} \eta^c c_{iR})$$

$$+ (\bar{c}_i \bar{\lambda}_i)_L (G_{cp} \eta^c p_{iR} + G_{\lambda n} \eta n_{iR} + G_\lambda \eta \lambda_{iR} + G_c \eta^c c_{iR}) + \text{h.c.}\}$$

$$+ G_e (\bar{\nu}_e \bar{e})_L \eta e_R + G_\mu (\bar{\nu}_\mu \bar{\mu})_L \eta \mu_R + \text{h.c.}$$

$$- V(\sigma_{ai}, \eta_a). \tag{3.3}$$

Here $\bar{\lambda}'^{l}$ is as defined in eq. (2.10), $u_{a\beta}$ denotes the U quantum numbers of the σ fields written as a matrix in the $SU_L(2)$ space ($\alpha = 1, 2$ refers to the $SU_L(2)$ doublet part of σ and $\alpha = 3$ refers to the $SU_L(2)$ singlet):

$$u_{a\beta} = \frac{1}{6} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{3.4}$$

For the Yukawa interaction of fermions with η we have introduced the coupling constants G_p, G_n, etc. and also have defined the doublet η^c (with $U = -\frac{1}{2}$) which is charge-conjugate to η:

$$\eta_a{}^c = \begin{pmatrix} \bar{\eta}^0 \\ -\eta^- \end{pmatrix} \tag{3.5}$$

Note that Yukawa interaction of fermions with σ is forbidden by invariance under $SU'(3)$. The "Higgs potential" $V(\sigma_{ai}, \eta_a)$ in eq. (3.3) represents a quartic polynomial in the scalar fields invariant under $SU'(3) \otimes SU_L(2) \otimes U(1)$.

We assume that the structure of $V(\sigma_{ai}, \eta_a)$ is such as to lead to the following nonvanishing vacuum expectation values:

$$\langle \sigma_{ai} \rangle = \langle \sigma \rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.6}$$

$$\langle \eta_a \rangle = \langle \eta \rangle \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad \langle \eta_a{}^c \rangle = \langle \eta \rangle \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{3.7}$$

where $\langle\sigma\rangle$ and $\langle\eta\rangle$ are two real constants. We replace all the scalar fields by their vacuum expectation values and ignore all other effects of the Higgs scalars. This can be justified by making the explicit assumption that the masses of all the physical scalar fields are sufficiently large. It may be further pointed out that we are using the Higgs mechanism only as a convenient device to keep track of the pattern of symmetry-breaking. Ultimately this might be replaced by the dynamical symmetry-breaking mechanism which dispenses with the elementary scalar fields altogether (*see* for instance Cornwall and Norton 1973 or Jackiw and Johnson 1973).

Thus, on replacing the scalar fields in eq. (3.3) by their vacuum expectation values given by eqs. (3.6) and (3.7), we get the boson mass terms (leaving the fermion terms for later discussion):

$$\mathcal{L}_{BM} = \tfrac{1}{4}\langle\eta\rangle^2 \{\tilde{g}^2 \mid \tilde{W}^1 \mid^2 + \tilde{g}^2 \mid \tilde{W}^2 \mid^2 + \mid -\tilde{g}\tilde{W}^3 + \tilde{g}'\, \tilde{U} \mid^2\}$$
$$+ \frac{1}{2} f^2 \langle\sigma\rangle^2 \left\{ \sum_{i=1}^{3} \left| V^i - \frac{\tilde{g}}{f}\tilde{W}^i \right|^2 \right.$$
$$\left. + \sum_{i=4}^{7} \mid V^i \mid^2 + \left| V^8 - \frac{\tilde{g}'}{\sqrt{3}f}\tilde{U} \right| \right\}, \tag{3.8}$$

where we have suppressed the Lorentz indices of the vector fields. The first curly bracket in eq. (3.8) contains the Weinberg-Salam mixing between the $SU_L(2)$ gauge bosons $\tilde{W}_\mu{}^i$ and the $U(1)$ gauge boson $\tilde{U}_\mu$ whereas the second curly bracket contains the mixing between these gauge bosons and the colour gauge bosons $V_\mu{}^i$.

The usual method of dealing with the mixing problem is to define a set of fields which are orthonormal to each other and which diagonalise the quadratic par of the Lagrangian completely. This would mix all the vector fields in a most cumbersome way. Instead, as elaborated below, we shall follow a more convenient procedure orignially due to t'Hooft (1971) (*see* also Furman and Komen 1975). The final results for physical matrix elements are of course the same in both methods.

We define the fields of the charged intermediate vector bosons $\tilde{W}_\mu$, $\tilde{W}_\mu{}^\dagger$, the neutral intermediate vector boson $\tilde{Z}_\mu$ and the photon $\tilde{A}_\mu$:

$$\tilde{W}_\mu = \frac{1}{\sqrt{2}}(\tilde{W}_\mu{}^1 + i\tilde{W}_\mu{}^2),$$
$$\tilde{W}_\mu{}^\dagger = \frac{1}{\sqrt{2}}(\tilde{W}_\mu{}^1 - i\tilde{W}_\mu{}^2),$$
$$\tilde{Z}_\mu = \frac{\tilde{g}\tilde{W}_\mu{}^3 - \tilde{g}'\,\tilde{U}_\mu}{\sqrt{\tilde{g}^2 + \tilde{g}'^2}}, \tag{3.9}$$
$$\tilde{A}_\mu = \frac{\tilde{g}'\,\tilde{W}_\mu{}^3 + \tilde{g}U_\mu}{\sqrt{\tilde{g}^2 + \tilde{g}'^2}}.$$

Thus the first curly bracket of eq. (3.8) becomes

$$\tfrac{1}{2}\tilde{g}^2 \langle\eta\rangle^2 \tilde{W}_\mu^\dagger \tilde{W}^\mu + \tfrac{1}{4}(\tilde{g}^2 + \tilde{g}'^2)\langle\eta\rangle^2 \tilde{Z}_\mu \tilde{Z}^\mu.$$

We next define the following gluon fields:

$$\begin{aligned} G_\mu^i &= V_\mu^i - \frac{\tilde{g}}{f}\tilde{W}_\mu^i, \quad (i = 1, 2, 3), \\ G_\mu^i &= V_\mu^i, \quad (i = 4, 5, 6, 7), \\ G_\mu^8 &= V_\mu^8 - \frac{1}{\sqrt{3}}\frac{\tilde{g}'}{f}\tilde{U}_\mu. \end{aligned} \tag{3.10}$$

This definition converts the second curly bracket of eq. (3.8) into $\tfrac{1}{2}f^2\langle\sigma\rangle^2 \sum_{i=1}^{8} G_\mu^i G^{\mu i}$. We thus have the following expression for the boson mass part of the Lagrangian:

$$\mathcal{L}_{BM} = \frac{f^2\langle\sigma\rangle^2}{2}\sum_{i=1}^{8} G_\mu^i G^{\mu i} + \frac{\tilde{g}^2\langle\eta\rangle^2}{2}\tilde{W}_\mu^\dagger \tilde{W}^\mu + \frac{\tilde{g}^2 + \tilde{g}'^2}{4}\langle\eta\rangle^2 \tilde{Z}_\mu \tilde{Z}^\mu. \tag{3.11}$$

So all the gluons have the same mass m_g. This as well as the masses of the weak vector bosons $m_{\tilde{W}}$ and $m_{\tilde{Z}}$ are given by

$$m_g^2 = f^2\langle\sigma\rangle^2\,; \quad m_{\tilde{W}}^2 = \tfrac{1}{2}\tilde{g}^2\langle\eta\rangle^2\,; \quad m_{\tilde{Z}}^2 = \tfrac{1}{2}(\tilde{g}^2 + \tilde{g}'^2)\langle\eta\rangle^2. \tag{3.12}$$

We see that the Weinberg-Salam part of eq. (3.8) has been diagonalised by the usual choice of orthogonal fields $\tilde{Z}_\mu$ and $\tilde{A}_\mu$. But the gluon fields G_μ^i for $i = 1, 2, 3, 8$ defined by eq. (3.10) are not orthogonal to the weak and electromagnetic fields $\tilde{W}_\mu^1$, $\tilde{W}_\mu^2$, $\tilde{Z}_\mu$ and $\tilde{A}_\mu$. In fact, we have avoided the complication of diagonalizing the four-fold mixing between V_μ^3, V_μ^8, $\tilde{W}_\mu^3$ and $\tilde{U}_\mu$ by the simple definition of the gluon fields made in eq (3.10). This definition renders $\mathcal{L}_{BM}$ diagonal although not all the fields are orthogonal to each other. This non-orthogonality generates quadratic couplings between the vector fields. These couplings as well as certain multiplicative renormalizations for the weak bosons and the photon will be considered in the next section.

It may be worth pointing out that this simple treatment of the mixing problem has been facilitated by the choice of the Higgs scalars that we have made. This method does not work for other choices of Higgs scalars, as for instance, the choice made by Pati and Salam (1973), namely the σ field transforming as $2 \oplus 2$ under $SU_L(2)$.

Finally we come to the fermion mass terms. The substitution of eq. (3.7) in eq. (3.3) leads to

$$\begin{aligned} \mathcal{L}_{FM} = {} & G_e\langle\eta\rangle\bar{e}e + G_\mu\langle\eta\rangle\bar{\mu}\mu + \sum_{i=1}^{3}\langle\eta\rangle\{G_p\bar{p}_ip_i + G_n\bar{n}_in_i + G_c\bar{c}_ic_i + G_\lambda\bar{\lambda}_i\lambda_i \\ & + \tfrac{1}{2}(G_{pc} + G_{cp})(\bar{p}_ic_i + \bar{c}_ip_i) + \tfrac{1}{2}(G_{pc} - G_{cp})(\bar{p}_i\gamma_5c_i - \bar{c}_i\gamma_5p_i) \\ & + \tfrac{1}{2}(G_{n\lambda} + G_{\lambda n})(\bar{n}_i\lambda_i + \bar{\lambda}_in_i) + \tfrac{1}{2}(G_{n\lambda} - G_{\lambda n})(\bar{n}_i\gamma_5\lambda_i - \bar{\lambda}_i\gamma_5n_i)\} \end{aligned} \tag{3.13}$$

where we have assumed CP invariance so that all G's are real. We may identify the lepton masses to be $m_e = G_e \langle\eta\rangle$ and $m_\mu = G_\mu \langle\eta\rangle$. The quark mass terms involve nondiagonal pieces which change parity, strangeness and charm. Diagonalisation of these will result in the mass terms

$$\sum_{i=1}^{3} \{m_p \bar{\hat{p}}_i \hat{p}_i + m_n \bar{\hat{n}}_i \hat{n}_i + m_c \bar{\hat{c}}_i \hat{c}_i + m_\lambda \bar{\hat{\lambda}}_i \hat{\lambda}_i\},$$

where $\hat{p}_i, \hat{n}_i, \hat{c}_i$ and λ_i denote the physical quark fields which are related to the old fields p_i, n_i, c_i and λ_i through Cabibbo type of rotation. We shall not go into the details here except to mention that there are actually four " Cabibbo angles ", two for the left-handed quarks and two for the right-handed quarks and the conventional Cabibbo angle can be written in terms of the former (*see* for instance Rajasekaran, 1972). Thus, as we already mentioned in section 2, the quark-fields entering the interaction Lagrangian given by eq. (2.7) are in fact Cabibbo rotated quarks.

4. The interactions among the physical vector bosons and fermions

In this section we shall rewrite all the interactions in terms of the ' physical ' vector particles [*i.e.*, gluons, weak bosons and the photons c.f. eqs (3.9) and (3.10)] which emerged as a result of symmetry breaking. However, before that, we have to define some new quantities.

4.1. *Definitions of new fields and coupling constants*

It is convenient to introduce a nomenclature for the gluons. The gluons $G_\mu{}^i$ for $i = 1, 2, 3, 8$ which have colour-hypercharge $Y' \cdot 0$ will be called $\rho_\mu{}^i$ whereas $G_\mu{}^i$ for $i = 4, 5, 6, 7$ with $Y' \neq 0$ will be called $K_\mu{}^i$. Further, let us define (suppressing the vector index)

$$\rho = \frac{1}{\sqrt{2}}(G_1 + iG_2), \qquad K^\pm = \frac{1}{\sqrt{2}}(G_4 \mp iG_5),$$

$$\rho^\dagger = \frac{1}{\sqrt{2}}(G_1 - iG_2), \qquad K^0 = \frac{1}{\sqrt{2}}(G_6 - iG_7),$$

$$\rho^{3,8} = G^{3,8}, \qquad \bar{K}^0 = \frac{1}{\sqrt{2}}(G_6 + iG_7). \tag{4.1}$$

Note that ρ, $\rho^\dagger$, K^+ and K^- are charged gluons while ρ^3, ρ^8, K^0 and $\bar{K}^0$ are neutral. It should be stressed that ρ's and K's stand for colour octet gluons and they are not to be confused with the known hadrons usually denoted by these symbols.

As we already pointed out in the last section, in addition to the cubic and quartic interactions between the vector bosons contained in eq. (2.13), we have quadratic interactions. These latter arise due to the fact that not all of our vector boson fields as defined in eqs (3.9) and (3.10) are orthogonal to each other. All these interactions (quadratic, cubic and quartic) can be obtained by substituting eqs (3.9) and (3.10) into eq. (2.13). The resulting expressions, which have a rather complicated appearance, simplify if we redefine the fields of the weak vector bosons and the weak coupling constants by certain multiplicative " renormalization " factors.

We first introduce the coupling constants $\tilde{e}$, $\tilde{h}$ and $\tilde{k}$ defined by

$$\tilde{e}=\frac{\tilde{g}\tilde{g}'}{(\tilde{g}^2+\tilde{g}'^2)^{1/2}}\,;\qquad \tilde{h}=\frac{\tilde{g}^2}{(\tilde{g}^2+\tilde{g}'^2)^{1/2}}\,;\qquad \tilde{k}=\frac{\tilde{g}'^2}{(\tilde{g}^2+\tilde{g}'^2)^{1/2}} \tag{4.2}$$

We then define the "renormalized" fields W_μ, Z_μ and A_μ:

$$\begin{aligned} W_\mu &= \left(1+\frac{\tilde{g}^2}{f^2}\right)^{1/2}\tilde{W}_\mu,\\ A_\mu &= \left(1+\frac{4}{3}\frac{\tilde{e}^2}{f^2}\right)^{1/2}\tilde{A}_\mu,\\ Z_\mu &= \left(1+\frac{\tilde{h}^2}{f^2}+\frac{\tilde{k}^2}{3f^2}\right)^{1/2}\tilde{Z}_\mu \end{aligned} \tag{4.3}$$

and the "renormalized" coupling constants g, e, h and k:

$$\begin{aligned} g &= \left(1+\frac{\tilde{g}^2}{f^2}\right)^{-1/2}\tilde{g}; & e &= \left(1+\frac{4}{3}\frac{\tilde{e}^2}{f^2}\right)^{1/2}\tilde{e};\\ h &= \left(1+\frac{\tilde{h}^2}{f^2}+\frac{\tilde{k}^2}{3f^2}\right)^{-1/2}\tilde{h}; & k &= \left(1+\frac{\tilde{h}^2}{f^2}+\frac{\tilde{k}^2}{3f^2}\right)^{-1/2}\tilde{k}. \end{aligned} \tag{4.4}$$

Although we have introduced four semi weak coupling constants, g, e, h and k in terms of which we shall write down all the interactions, only two of these are independent. Thus, for instance, h and k can be expressed in terms of g and e:

$$\begin{aligned} h &= \left(g^2-e^2-\frac{1}{3}\frac{e^2g^2}{f^2}\right)D,\\ k &= e^2\left(1-\frac{g^2}{f^2}\right)D,\\ D &= \left\{g^2-e^2-\frac{8}{3}\frac{e^2}{f^2}(g^2-e^2)+\frac{g^2e^2}{f^4}\left(g^2-\frac{8}{9}e^2\right)\right\}^{-1/2}. \end{aligned} \tag{4.5}$$

It may also be useful to note from eq. (4.4) that all these renormalised coupling constants g, e, h and k are bounded from above by the strong coupling constant f. We shall always regard g, e, h and k as small compared to f.

Combining eqs (3.9) and (3.10) and using eqs (4.2)–(4.4), the fields V_μ^i for $i=1, 2, 3, 8$ can be written as

$$\begin{aligned} V_\mu^i &= \rho_\mu^i+\frac{\tilde{g}}{f}\tilde{W}_\mu^i=\rho_\mu^i+\frac{g}{f}W_\mu^i, \quad i=1,2\\ V_\mu^3 &= \rho_\mu^3+\frac{\tilde{h}}{f}\tilde{Z}_\mu+\frac{\tilde{e}}{f}\tilde{A}_\mu=\rho_\mu^3+\frac{h}{f}Z_\mu+\frac{e}{f}A_\mu,\\ V_\mu^8 &= \rho_\mu^8-\frac{\tilde{k}}{\sqrt{3}f}\tilde{Z}_\mu+\frac{\tilde{e}}{\sqrt{3}f}\tilde{A}_\mu=\rho_\mu^8-\frac{k}{\sqrt{3}f}Z_\mu+\frac{e}{\sqrt{3}f}A_\mu. \end{aligned} \tag{4.6}$$

We now use the set of eqs (4.1)–(4.6) along with eq. (3.9) in the Lagrangian of section 2 and thus get all the physical interactions.

4.2. *Interactions among the physical vector bosons*

The quadratic part of eq. (2.13) is

$$\mathcal{L}^{(2)}_{VB} = -\tfrac{1}{4}\left(\sum_{l=1}^{8}(V^{l}_{\mu\nu})^2 + \sum_{i=1}^{3}(\tilde{W}^{i}_{\mu\nu})^2 + (\tilde{U}_{\mu\nu})^2\right)$$

$$= -\tfrac{1}{4}\left(\sum_{i=1}^{8}(G^{l}_{\mu\nu})^2 + 2W^{\dagger}_{\mu\nu}W^{\mu\nu} + (Z_{\mu\nu})^2 + (A_{\mu\nu})^2\right)$$

$$-\frac{1}{2f}\left\{g\,(\rho^{\dagger}_{\mu\nu}W^{\mu\nu} + \rho^{\mu\nu}W^{\dagger}_{\mu\nu}) + h\,\rho^{3}_{\mu\nu}Z^{\mu\nu} + e\,\rho^{3}_{\mu\nu}A^{\mu\nu}\right. \tag{4.7}$$

$$\left.-\frac{k}{\sqrt{3}}\rho^{8}_{\mu\nu}Z^{\mu\nu} + \frac{e}{\sqrt{3}}\rho^{8}_{\mu\nu}A^{\mu\nu} + \frac{e}{f}\left(h-\frac{k}{3}\right)Z_{\mu\nu}A^{\mu\nu}\right\}.$$

Hence, in addition to the kinetic terms, we have vector-dominance type of couplings between the weak vector bosons and the gluons: $W\rho$, $Z\rho^{3,\,8}$, $A\rho^{3,\,8}$ with coupling constants $= \frac{1}{f}$ (appropriate semiweak coupling constant). There is also a ZA coupling which is of higher order in the semiweak coupling constant.

Let us next consider the cubic interactions contained in eq. (2.13):

$$\mathcal{L}^{(3)}_{VB} = -\frac{f}{2}\sum_{l,m,n=1}^{8} f^{lmn}\, V^{l}_{\mu\nu} V^{\mu m} V^{\nu n} - \frac{\tilde{g}}{2}\sum_{i,j,k=1}^{3}\epsilon^{ijk}\,\tilde{W}^{i}_{\mu\nu}\tilde{W}^{\mu j}\tilde{W}^{\nu k}$$

$$= i\left[\frac{f}{2}\left\{(\rho^{\dagger}\rho\rho^3) - \frac{1}{\sqrt{2}}(\rho\bar{K}\tau'_{-}K) - \frac{1}{\sqrt{2}}(\rho^{\dagger}\bar{K}\tau_{+}'K) - \frac{1}{2}(\rho^{3}\bar{K}\tau_{3}'K) - \frac{\sqrt{3}}{2}(\rho^{8}\bar{K}K)\right\}\right.$$

$$+g\left\{(\rho^{\dagger}W\rho^3) + (\rho W^{\dagger}\rho^3) - \frac{1}{\sqrt{2}}(W\bar{K}\tau'_{-}K) - \frac{1}{\sqrt{2}}(W^{\dagger}\bar{K}\tau'_{+}K)\right\}$$

$$+h\left\{(\rho^{\dagger}\rho Z) - \frac{1}{2}(Z\bar{K}\tau_{3}'K) + \frac{k}{2h}(Z\bar{K}K) + (W^{\dagger}WZ)\right\}$$

$$+e\left\{(\rho^{\dagger}\rho A) - \frac{1}{2}(A\bar{K}\tau_{3}'K) - \frac{1}{2}(A\bar{K}K) + (W^{\dagger}WA)\right\}$$

$$\left.+\frac{1}{f}\left\{gh\,(\rho^{\dagger}WZ) + gh\,(\rho W^{\dagger}Z) + ge\,(\rho^{\dagger}WA) + ge\,(\rho W^{\dagger}A) + g^2\,(W^{\dagger}W\rho^3)\right\}\right], \tag{4.8}$$

where $\tau_{\pm}' = \frac{1}{2}(\tau_1' \pm i\tau_2')$, the prime is used to denote matrices acting in colour space and $K = \begin{pmatrix}K^{+}\\K_0\end{pmatrix}$. Moreover, we have introduced the following convenient notation for the symmetric Yang-Mills coupling of three vector fields:

$$(ABC) \equiv A_{\mu\nu}B^{\mu}C^{\nu} + B_{\mu\nu}C^{\mu}A^{\nu} + C_{\mu\nu}A^{\mu}B^{\nu}. \tag{4.9}$$

The first line of eq. (4.8) is the cubic strong interaction among the gluons. The next three lines of the same equation describe the emission or absorption of a single weak vector boson W or Z or a photon A by the gluons, all of which are characterised by an appropriate semiweak coupling constant. There are also mutual interactions among the weak-electromagnetic vector bosons of the type $(W^\dagger WZ)$ and $(W^\dagger WA)$. The last line contains the cubic interactions which are of higher order in the semiweak coupling constant.

We shall now examine the quartic interaction:

$$\mathcal{L}_{VB}^{(4)} = -\tfrac{1}{4} f^2 \Sigma f^{lmn} f^{lpq} V_\mu^m V_\nu^n V^{\mu p} V^{\nu q}$$
$$- \tfrac{1}{4} g^2 \Sigma \epsilon^{ijk} \epsilon^{ial} \tilde{W}_\mu^j \tilde{W}_\nu^k \tilde{W}^{\mu a} \tilde{W}^{\nu b}. \qquad (4.10)$$

By following the same procedure again, the quartic interactions can be seen to be of the following types:

(*a*) $f^2 G^4$;

(*b*) $gfWG^3$, $hfZG^3$, $efAG^3$, etc.;

(*c*) $g^2 W^2 G^2$, $h^2 Z^2 G^2$, $e^2 A^2 G^2$, $ehAZG^2$, etc.;

(*d*) $g^3 f^{-1} W^3 G$, $g^2 ef^{-1} W^2 AG$, $g^2 hf^{-1} W^2 ZG$,
$ge^2 f^{-1} WA^2 G$, $gehf^{-1} WAZG$, etc.;

(*e*) $g^2 W^4$, $e^2 W^2 A^2$, $h^2 W^2 Z^2$, $eh W^2 AZ$, etc.

Here (*a*) refers to the quartic gluon coupling, (*b*) to couplings linear in the weak or electromagnetic boson fields, (*c*) to couplings quadratic in the photon and/or weak bosons, (*d*) to couplings cubic in the photon and/or weak bosons and (*e*) to quartic couplings among weak and electromagnetic bosons. We choose not to write these interactions in detail since, as will be explained later, these are not needed for our present purpose.

4.3. *Interactions between the fermions and the physical vector bosons*

We rewrite the fermion-vector boson interaction given in eq. (2.12) in terms of physical vector bosons introduced in sections 3 and 4.1. Using eqs (3.9) and (4.1–4.6), the following form is obtained for this interaction:

$$\mathcal{L}_F^{\text{INT}} = f \sum_{l=1}^{8} j'^{\,l}_\mu G^{\mu l} + e j_\mu^{\text{EM}} A^\mu$$
$$+ \frac{g}{\sqrt{2}} (j_\mu^{W-} W^\mu + j_\mu^{W+} W^{\mu\dagger}) + h j_\mu^Z Z^\mu. \qquad (4.11)$$

The currents appearing here are all related to the original $SU'(3)$ and $SU_L(2)$ currents of eq. (2.9) as follows:

$$j_\mu^{W\pm} = j_\mu^{\pm} + j'^{\,\pm}_\mu = (j_\mu^1 \pm i j_\mu^2) + (j'^{\,1}_\mu \pm i j'^{\,2}_\mu),$$
$$j_\mu^{W3} = j_\mu^3 + j'^{\,3}_\mu,$$
$$j_\mu^{EM} = (j_\mu^3 + j_\mu^U) + \left(j'^{\,3}_\mu + \frac{1}{\sqrt{3}} j'^{\,8}_\mu\right), \qquad (4.12)$$
$$j_\mu^Z = \sec^2\theta\,(j_\mu^{W3} - \sin^2\theta\, j_\mu^{EM})$$
$$= (j_\mu^3 - \tan^2\theta\, j_\mu^U) + \left(j'^{\,3}_\mu - \frac{\tan^2\theta}{\sqrt{3}} j'^{\,8}_\mu\right),$$

where θ is the Weinberg-Salam mixing angle given by

$$\tan^2 \theta = \frac{k}{h}. \tag{4.13}$$

Equation (4.11) exhibits the strong, electromagnetic and weak interactions of the quarks and the leptons. It is clear that the strong interaction is colour invariant whereas the weak and electromagnetic interactions break colour symmetry. The weak and electromagnetic currents defined by eq. (4.12) contain a colour singlet part as well as a colour octet part (denoted by a prime). All the low-lying hadrons are taken to be colour singlets and so any matrix element of the colour octet part of the currents taken between the low-lying hadron states vanishes. The colour octet part of the currents will contribute to physical processes only above the *colour threshold*-namely only when colour nonsinglet hadrons are produced.

It may be worthwhile to write down the colour-singlet and colour-octet parts of the electromagnetic current separately:

$$\begin{aligned}
(j_\mu{}^{EM})_{\substack{\text{colour}\\ \text{singlet}}} &= j_\mu{}^3 + j_\mu{}^U \\
&= \sum_{i=1}^{3}{}' \left(\tfrac{2}{3}\,\bar{p}_i\gamma_\mu p_i - \tfrac{1}{3}\,\bar{n}_i\gamma_\mu n_i - \tfrac{1}{3}\,\bar{\lambda}_i\gamma_\mu\lambda_i + \tfrac{2}{3}\bar{c}_i\gamma_\mu c_i\right) \\
&\quad - (\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu) \qquad (4.14) \\
&= \sum_{i=1}^{3}{}' \sum_{q=p,n,c,\lambda} Q_0(q_i)\,\bar{q}_i\gamma_\mu q_i - (\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu), \\
(j_\mu{}^{EM})_{\substack{\text{colour}\\ \text{octet}}} &= j'_\mu{}^3 + \frac{1}{\sqrt{3}}\,j'_\mu{}^8 \\
&= \sum_{q=p,n,c,\lambda} \left(-\tfrac{2}{3}\,\bar{q}_1\gamma_\mu q_1 + \tfrac{1}{3}\,\bar{q}_2\gamma_\mu q_2 + \tfrac{1}{3}\,\bar{q}_3\gamma_\mu q_3\right) \\
&= \sum_{i=1}^{3} \sum_{q=p,n,c,\lambda} Q_8(q_i)\,\bar{q}_i\gamma_\mu q_i,
\end{aligned}$$

where $Q_0(q_i)$ and $Q_8(q_i)$ denote respectively the colour-singlet and colour-octet parts of the electric charge of the quark q_i. One can see that

$$Q_0(q_i) = \left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, \tfrac{2}{3}\right) \quad \text{for} \quad (p_i, n_i, \lambda_i, c_i) \text{ respectively,}$$

$$Q_8(q_i) = \left(-\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right) \text{ for } (q_1, q_2, q_3) \text{ respectively.}$$

Hence, as far as the colour singlet part of the electromagnetic current is concerned (*i.e.*, below colour threshold), the quarks p, $n\lambda$ and c behave like fractionally charged Gell-Mann-Zweig quarks. Only if both the colour singlet and the colour octet parts of electromagnetic current contribute equally, do the quarks behave—through $Q_0(q_i) + Q_8(q_i)$—as integrally charged Han-Nambu quarks. This point will be discussed furthe: in section 6.

It should be noted that the colour octet currents $j'_\mu{}^l$ ($l = 1, \ldots, 8$) are pure vector currents whereas $j_\mu{}^a$ ($a = 1, 2, 3$) are pure V–A currents; on the other hand $j_\mu{}^U$ contains both V–A and $V + A$ pieces (*vide* eq. 2.9). It is now clear from eq. (4.12) that the charged weak currents which are purely left-handed below colour threshold lose this feature when that threshold is crossed. The neutral weak current has the Weinberg-Salam mixture of V and A below colour threshold, but gets an additional vector piece above colour threshold.

4.4 *The complete interaction Lagrangian*

Finally, it is convenient to write down all the interactions—those of the vector bosons as well as of the fermions—in the same form as in eq. (4.11). Hence we write:

$$\mathcal{L}^{\text{INT}} = f \sum_{l=1}^{8} J'^{l}_{\mu,\,\text{strong}}\, G^{\mu l} + eJ_{\mu}^{EM} A^{\mu} + \frac{g}{\sqrt{2}} (J_{\mu}^{W-} W^{\mu} + J_{\mu}^{W+} W^{\mu\dagger}) + hJ_{\mu}^{Z} Z^{\mu}. \tag{4.15}$$

In eq. (4.15)

$$\begin{aligned} J_{\mu}^{W\pm} &= J_{\mu}^{\pm} + J'^{\pm}_{\mu}, \\ J_{\mu}^{W3} &= J_{\mu}^{3} + J'^{3}_{\mu}, \\ J_{\mu}^{EM} &= (J_{\mu}^{3} + J_{\mu}^{U}) + \left(J'^{3}_{\mu} + \frac{1}{\sqrt{3}} J'^{8}_{\mu}\right), \\ J_{\mu}^{Z} &= \sec^2\theta\, (J_{\mu}^{W3} - \sin^2\theta\, J_{\mu}^{EM}) \\ &= (J_{\mu}^{3} - \tan^2\theta\, J_{\mu}^{U}) + \left(J'^{3}_{\mu} - \frac{\tan^2\theta}{\sqrt{3}} J'^{8}_{\mu}\right). \end{aligned} \tag{4.16}$$

We have used J_{μ} to denote the total current which is the sum of the corresponding j_{μ} defined in eq. (4.12) and the vector boson current. The latter consists of linear, quadratic and cubic terms which can be read off from eqs (4.7), (4.8) and (4.10) respectively. We shall not write these terms explicitly since for our subsequent calculations we can directly make use of the interactions as given in eqs (4.7)–(4.10). We do, however, note the difference between J'^{l}_{μ} of eqs (4.16) and $J'^{l}_{\mu,\,\text{strong}}$ of eq. (4.15), namely, J'^{l}_{μ} includes the linear term in the gluon fields whereas $J'^{l}_{\mu,\,\text{strong}}$ does not.

Equation (4.16) shows that each of the total weak and electromagnetic currents can be split up into a colour singlet part (unprimed) plus a clour octet part (primed). By examining the way in which these interactions have been generated from the original gauge invariant interactions of eqs. (2.12) and (2.13) one can see that whereas, as we already noted, the weak and electromagnetic currents of quarks have both colour singlet and colour octet parts, the corresponding currents of gluons are pure colour octets and further that the currents of leptons, weak bosons and photons are pure colour singlets.

The masses of the gluons and the weak vector bosons have been already given in eq. (3.12). (The transition from $\tilde{W}$, $\tilde{Z}$ to W, Z does not cause any drastic change). The masses m_W and m_Z are greater than 37 GeV as in the original Weinberg-Salam model. The gluon mass m_g does not suffer from any such constraint and we shall not try to fix it. If there are other colour-nonsinglet hadrons much lighter than the gluons, then the colour threshold could be considerably lower than m_g.

On the other hand, from the point of view of the parton framework, the effective mass of the gluon-parton may be much smaller than the masses of the colour-non-singlet hadrons just as the effective mass of the quark-parton is believed to be small compared to the usual hadronic masses. If this is the case, colour threshold could be much higher than the gluon-mass m_g

5. Effective weak and electromagnetic vertices

In this section we calculate the effective vertices for the weak and electromagnetic interactions of the gluons and the quarks to the lowest order of the semiweak coupling constant, using the interaction Lagrangian given in the last section. Because of the quadratic coupling of the weak bosons with the gluons, this generally involves the addition of a vector-dominance-type of diagram to the direct coupling diagram.

For the current matrix element $\langle \rho^3 \mid gJ_\mu^{W-} \mid \rho^+ \rangle$, which corresponds to the absorption of the weak boson W^- by the gluon ρ^+, in order g we have to add the two diagrams (*a*) and (*b*) shown in figure 1. By using eq. (4.8) and comparing with eq. (4.15), the contribution of the direct diagram (*a*) is seen to be

$$-\sqrt{2}\, g \epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta}, \tag{5.1}$$

where ϵ and ϵ' are the polarization vectors of the initial and final gluons while $V_{\mu\alpha\beta}$ is the symmetric Yang-Mills vertex:

$$V_{\mu\alpha\beta} \equiv (p + p')_\mu\, g_{\alpha\beta} + (q - p)_\beta\, g_{\mu\alpha} - (p' + q)_\alpha\, g_{\mu\beta}. \tag{5.2}$$

The initial and final momenta are p and p' respectively and $q = p' - p$. In order to calculate the contribution of diagram (1 *b*), we need the ρ propagator:

$$-\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_g^2}\right)(q^2 - m_g^2)^{-1} \tag{5.3}$$

and the ρW vertex read off from eq. (4.7):

$$\frac{g}{f}(q^2 g_{\mu\nu} - q_\mu q_\nu). \tag{5.4}$$

Using these, the contribution of diagram (1 *b*) can be calculated to be

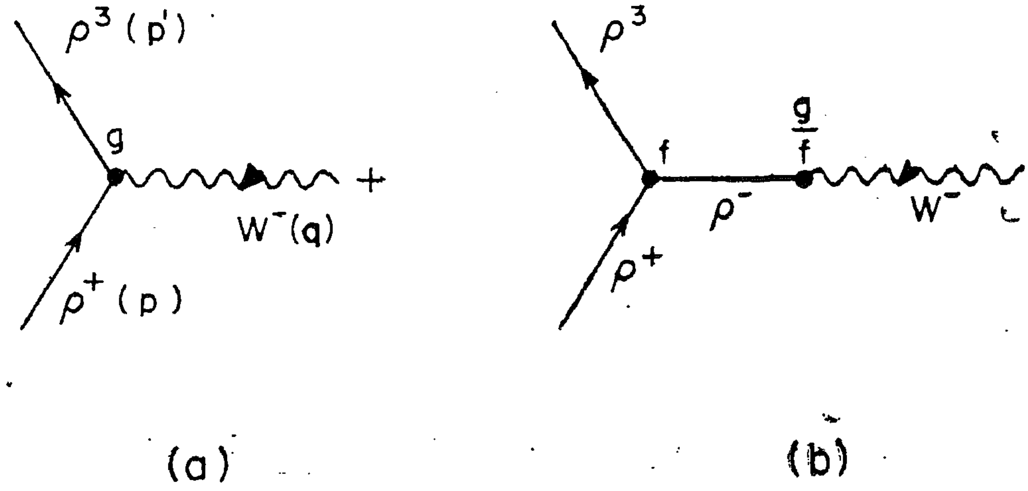

Figure 1. The two diagrams contributing to the effective vertex for the absorption of the weak vector boson W^- by the gluon ρ^+.

$$\sqrt{2}\, g\epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta}\, \frac{q^2}{q^2 - m_g^2}\,. \tag{5.5}$$

The $q_\mu q_\nu$ terms occurring in (5.3) and (5.4) do not contribute because of the form of the three-point vertex (5.2). Hence the effective vertex, which is the sum of expressions (5.1) and (5.5), is

$$\langle \rho^3 (p') \,|\, gJ_\mu^{W-} \,|\, \rho^+ (p)\rangle = \sqrt{2}\, g\epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta}\, \frac{m_g^2}{q^2 - m_g^2}\,. \tag{5.6 a}$$

By following the same procedure, we get all the other vertices:

$$\langle \rho^{\pm} (p') \,|\, eJ_\mu^{EM} \,|\, \rho^{\pm} (p)\rangle = \pm\, e\epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta}\, \frac{m_g^2}{q^2 - m_g^2}\,, \tag{5.6 b}$$

$$\langle \rho^{\pm} (p') \,|\, hJ_\mu{}^{Z} \,|\, \rho^{\pm} (p)\rangle = \pm\, h\epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta}\, \frac{m_g^2}{q^2 - m_g^2}\,, \tag{5.6 c}$$

$$\langle K (p') \,|\, gJ_\mu^{W-} \,|\, K (p)\rangle = -g\epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta} \bar{\xi}_f \tau'_- \xi_i\, \frac{m_g^2}{q^2 - m_g^2}\,, \tag{5.6 d}$$

$$\langle K (p') \,|\, eJ_\mu^{EM} \,|\, K (p)\rangle = -\frac{e}{2}\, \epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta} \bar{\xi}_f\, (\tau_3' + 1)\, \xi_i\, \frac{m_g^2}{q^2 - m_g^2}\,, \tag{5.6 e}$$

$$\langle K (p') \,|\, hJ_\mu{}^{Z} \,|\, K (p)\rangle = -\frac{h}{2}\, \epsilon'^{\beta}\, \epsilon^{\alpha}\, V_{\mu\alpha\beta} \bar{\xi}_f\, (\tau_3' - \tan^2 \theta)\, \xi_i\, \frac{m_g^2}{q^2 - m_g^2}\,. \tag{5.6 f}$$

For the electromagnetic and the neutral current vertices of the ρ-gluons given in eq. (5.6 *b*) and (5.6 *c*), ρ^3 alone contributes in the intermediate state. On the other hand, for the same vertices of the *K*-gluons given in eqs. (5.6 *e*) and (5.6 *f*) both ρ^3 and ρ^8 intermediate states occur. In eqs (5.6 *d*)–(5.6 *f*), τ'_- and τ_3' denote the "colour isospin" matrices which are sandwiched between the final and initial spinors ξ_f and ξ_i of the *K*-gluons which are defined to be $\begin{pmatrix}1\\0\end{pmatrix}$ for K^+ and $\begin{pmatrix}0\\1\end{pmatrix}$ for K^0. Eqs (5.6 *d*)–(5.6 *f*) are applicable, with an overall change of sign, to $\bar{K}$-gluons also.

One should note that the quartic vertices considered in the last section are not needed here because we restrict ourselves to the lowest order in the semiweak coupling constants and because calculations are done within the parton framework. In the parton model, the constitutents of the hadrons are treated as free and hence the strong coupling constant *f* should be taken as zero effectively. One can see that the quartic vertices involve either higher order semiweak coupling constants or positive powers of *f*; hence none of these enters our calculations.

We next discuss the effective vertices for the quarks which we shall denote by *Q* For the example shown in figure 2, we get

$$\begin{aligned} &\langle Q (p') \,|\, gJ_\mu^{W+} \,|\, Q (p)\rangle \\ &= g\bar{u}\,(p') \left\{ (\tau_+ \gamma_{\mu L} + \bar{\lambda}'_+ \gamma_\mu) - \bar{\lambda}'_+ \gamma_\mu\, \frac{q^2}{q^2 - m_g^2} \right\} u\,(p) \\ &= g\bar{u}\,(p') \left((\tau_+ \gamma_{\mu L} - \bar{\lambda}'_+ \gamma_\mu\, \frac{m_g^2}{q^2 - m_g^2} \right) u\,(p), \end{aligned} \tag{5.7 a}$$

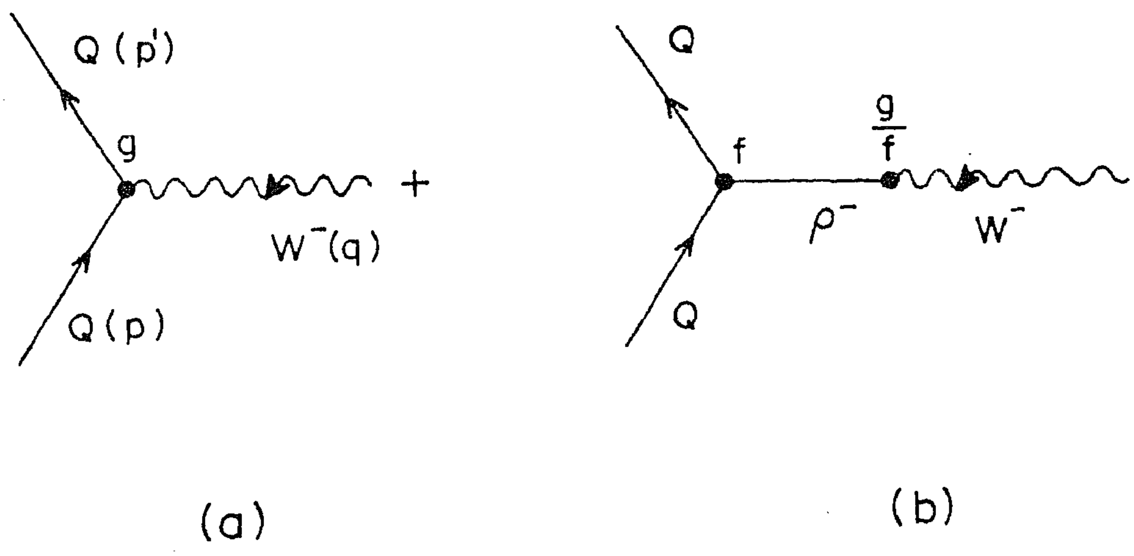

Figure 2. The two diagrams contributing to the effective vertex for the absorption of the weak vector boson W^- by the quark.

where $\bar{\lambda}'_{\pm} = \frac{1}{2}(\bar{\lambda}_1' \pm i\bar{\lambda}_2')$ and u denotes the quark spinor which includes all the internal symmetry structure [both $SU'(3)$ and $SU_L(2)$]. In the first line of eq. (5.7 *a*) we have given the two contributions from diagrams (*a*) and (*b*) respectively. The $q_\mu q_\nu$ terms in the vector dominance contribution do not contribute when sandwiched between the spinor wavefunctions of the quarks. The second line of eq. (5.7 *a*) exhibits the vertex as a sum of two parts—a colour singlet part which is not modified by the vector-dominance diagram and a colour octet part which gets a q^2—dependent contribution from that diagram.

The same phenomenon occurs for all the vertices. Each of the vertices can be written as a colour singlet part which is the same as in the Weinberg-Salam model and a q^2-dependent colour octet part:

$$\langle Q(p') \mid eJ_\mu^{EM} \mid Q(p)\rangle$$
$$= \frac{e}{2}\,\bar{u}(p')\left\{\left(\tau_3 + \frac{1}{3}\right)\gamma_\mu - \left(\bar{\lambda}_3' + \frac{\bar{\lambda}_8'}{\sqrt{3}}\right)\frac{m_g^2}{q^2 - m_g^2}\right\} u(p), \qquad (5.7\,b)$$

$$\langle Q(p') \mid hJ_\mu^Z \mid Q(p)\rangle$$
$$= \frac{h}{2}\,\bar{u}(p')\left\{\left(\tau_3\gamma_{\mu L} - \tan^2\theta\,(\tau_3\gamma_{\mu R} + \frac{1}{3}\gamma_\mu\right)\right\}$$
$$- \left(\bar{\lambda}_3' - \tan^2\theta\,\frac{\bar{\lambda}_8'}{\sqrt{3}}\right)\frac{m_g^2}{q^2 - m_g^2}\Bigg]\, u(p). \qquad (5.7\,c)$$

In writing these equations, it may be helpful to note

$$\langle Q(p') \mid J_\mu^U \mid Q(p)\rangle = \tfrac{1}{2}\,\bar{u}(p')\,(\tau_3\,\gamma_{\mu R} + \tfrac{1}{3}\,\gamma_\mu)\,u(p).$$

Again, both ρ^3 and ρ^8 contribute in the intermediate states of the vertices corresponding to eqs (5.7 *b*) and (5.7 *c*).

The set of eqs (5.6) and (5.7) summarises the essential content of the present unified gauge model as far as the weak and electromagnetic interactions of the quark-partons and gluon-partons are concerned. We see that all the partons are endowed with a structure arising from the vector-dominance denominator. However, this structure should be distinguished from the usual structure of hadrons, for it remains even when the strong coupling constant f is put zero.

If we had followed the alternative procedure of defining the orthonormal set of vector fields which diagonalises the quadratic part of the Lagrangian completely, then the quadratic coupling between weak vector bosons and gluons would of course have been absent. In this approach, the vector-domainance denominator in the complete matrix element of lepton-hadron scattering would arise from direct lepton-gluon couplings. Thus, the vector-dominance structure is a characteristic feature of this class of models.

6. Deep inelastic lepton-hadron scattering and $e^+ e^-$ annihilation

6.1. *Lepton-hadron scattering*

Let us first define the inelastic structure functions of the nucleon arising solely from the colour octet parts of the currents:

$$\frac{1}{4\pi} \int d^4 x\, e^{iqx} \langle p \mid [J'^l_\mu (x), J'^m_\mu (0)] \mid p \rangle$$

$$= \delta^{lm} \left(-g_{\mu\nu} W_1' + \frac{p_\mu p_\nu}{m_N^2} W_2' + \ldots \right). \qquad (6.1)$$

In eq. (6.1) $\mid p \rangle$ is a spin-averaged nucleon state with four momentum p and the dots stand for terms which do not concern us here. The occurrence of the Kronecker delta in the colour $SU(3)$ indices l and m in the RHS of eq. (6.1) as well as the lack of any dependence of W_1' and W_2' on the colour indices follow from the colour singlet property of the nucleon. Note that the colour octet current J'^l_μ is a pure vector and so there is no additional structure function such as W_3' corresponding to parity-violation. The structure functions W_1' and W_2' are real functions of $Q^2 = -q^2$ and $\nu = p \,.\, q/m_N$. In the deep inelastic limit when ν, $Q^2 \to \infty$ in such a way that $x = Q^2/2m_N\nu$ stays finite, the functions W_1' and $\nu W_2'/m_N$ will be called F_1' and F_2' respectively. According to the hypothesis of Bjorken scaling, F_1' and F_2' are functions of x only.

In view of eqs. (4.16) where the physical currents are split up into their colour singlet and colour-octet parts, we can write the deep inelastic structure functions relevant for charged current (CC), neutral current (NC) and electromagnetic current (EM) scattering as a sum of $F\,(WS)$, given by the Weinberg-Salam theory, plus a colour contribution. Defining the structure functions for the complete weak and electromagnetic currents as well as their colour singlet parts by equations analogous to eq. (6.1), we get

$$\begin{aligned} F^{CC}_{1,2} &= F^{CC}_{1,2}\,(WS) + 2F'_{1,2}, \\ F^{NC}_{1,2} &= F^{NC}_{1,2}\,(WS) + (1 + \tfrac{1}{3}\tan^4\theta)\, F'_{1,2}, \\ F^{EM}_{1,2} &= F^{EM}_{1,2}\,(WS) + \tfrac{4}{3}\, F'_{1,2}, \qquad (6.2) \\ F^{CC}_{3} &= F^{CC}_{3}\,(WS), \\ F^{NC}_{3} &= F^{NC}_{3}\,(WS). \end{aligned}$$

It should be remembered that in the colour-singlet part of the electromagnetic current the quarks behave like Gell-Mann-Zweig quarks so that $F\,(WS)$ should be calculated with the fractional charge assignment. The Weinberg-Salam contribution $F(WS)$ has been worked out by a number of authors (*e.g.*, Palmer 1973, Sehgal 1974) and so we shall concentrate on the colour-contribution $F'_{1,2}$ alone†.

To calculate $F'_{1,2}$ we need the matrix elements of J'^{l}_{μ} between the gluon partons G^{m} as well as between the quark partons Q. Although these are contained in eqs (5.6) and (5.7), the following compact equations will be more convenient:

$$\langle G^{n}(p') \mid J'^{l}_{\mu} \mid G^{m}(p) \rangle = i f^{lmn} \epsilon'^{\beta} \epsilon^{\alpha} V_{\mu\alpha\beta} \frac{m_g^{2}}{q^{2} - m_g^{2}}, \tag{6.3}$$

$$\langle Q(p') \mid J'^{l}_{\mu} \mid Q(p) \rangle = - \bar{u}(p') \frac{\lambda'^{l}}{2} \gamma_{\mu} u(p) \frac{m_g^{2}}{q^{2} - m_g^{2}}, \tag{6.4}$$

where the quark spinors include all the internal symmetry structure of $SU'(3)$ and $SU_L(2)$. In fact only the colour components $l = 1, 2, 3, 8$ are relevant for the weak and electromagnetic currents.

Using the vertices given in eqs (6.3) and (6.4) it is straightforward to calculate the structure functions $F'_{1,2}$ within the parton-model framework. We deem it sufficient to present the results:

$$F'_1(x, Q^2) = 4\left(1 + \frac{Q^2}{4m_g^2}\right)\frac{m_g^4}{(Q^2 + m_g^2)^2} g(x) + \frac{1}{4}\frac{m_g^4}{(Q^2 + m_g^2)^2} \sum_{q=p,n,c,\lambda} \{q(x) + \bar{q}(x)\}, \tag{6.5}$$

$$F_2'(x, Q^2) = x\left(3 + \frac{Q^2}{m_g^2} + \frac{Q^4}{4m_g^4}\right)\frac{m_g^4}{(Q^2 + m_g^2)^2} g(x) + \frac{x}{2}\frac{m_g^4}{(Q^2 + m_g^2)^2} \sum_{q=p,n,c,\lambda} \{q(x) + \bar{q}(x)\}. \tag{6.6}$$

In these equations $g(x)$ is the probability function for any one of the octet of gluons to have a fraction x of the longitudinal momentum of the nucleon in the infinite momentum frame. It is of course the same for each of the eight gluons since the nucleon is a colour singlet. Similarly the quark-contribution contains the probability functions for the quarks $p(x)$, $n(x)$, $c(x)$ and $\lambda(x)$ as well as for the antiquarks $\bar{p}(x)$, $\bar{n}(x)$, $\bar{c}(x)$ and $\bar{\lambda}(x)$. Again, these are independent of colour—*e.g.*, $p_1(x) = p_2(x) = p_3(x)$.

The factor $m_g^4 (Q^2 + m_g^2)^{-2}$ in eqs (6.5) and (6.6) owes its existence to the vector-dominance type of diagrams; without it both F_1' and F_2' become infinite in the deep inelastic limit $Q^2 \to \infty$. In other words, the contribution from the direct coupling of the spin-one gluons violates Bjorken scaling strongly (Cleymens and Komen 1974, Rajasekaran and Roy 1975). But now in the unified gauge theory there naturally arises the suppression factor $m_g^4 (Q^2 + m_g^2)^{-2}$ and thus

† This colour contribution should be added only above colour threshold. The success of the fractionally-charged quark parton model in the interpretation of the electron-scattering experiments at SLAC and the neutrino-scattering experiments at CERN suggests that these regions of energy are below colour threshold. However, we keep an open mind about the regions now being explored at Fermilab.

Bjorken scaling is preserved asymptotically. Indeed as $Q^2 \to \infty$, the gluon contribution to F_1' vanishes and that* to F_2' becomes $\frac{1}{4}\, xg(x)$. Thus gluons contribute a term $\frac{1}{4}\, xg(x)$ which is universal for all lepton-hadron scattering—apart from some numerical coefficients and the Weinberg-Salam mixing angle given in eq. (6.2). Further, the colour octet parts of the quark contributions to eqs (6.5) and (6.6) vanish as $Q^2 \to \infty$. Hence, in the limit $Q^2 \gg m_g^2$ we obtain:

$$
\begin{aligned}
&F_1^{CC}(x) \to F_1^{CC}(WS), \quad F_2^{CC}(x) \to F_2^{CC}(WS) + \frac{x}{2}\, g(x),\\
&F_1^{NC}(x) \to F_1^{NC}(WS), \quad F_2^{NC}(x) \to F_2^{NC}(WS) + \left(1 + \frac{1}{3}\tan^4\theta\right) \frac{x}{4}\, g(x),\\
&F_1^{EM}(x) \to F_1^{EM}(WS), \quad F_2^{EM}(x) \to F_2^{EM}(WS) + \frac{x}{3}\, g(x),\\
&F_3(x) \to F_3(WS).
\end{aligned}
\tag{6.7}
$$

Although in the far asymptotic region, F_1' and F_2' do scale, in the intermediate energy region (above the colour threshold, but with $|q^2| \sim m_g^2$) scaling is violated as is clear from eqs (6.5) and (6.6). Whether the form of the scaling-violation given by these equations is physically relevant will depend on the applicability of the parton model in this intermediate region.

It is well known (*e.g.*, Lipkin 1972) that the integrally charged Han-Nambu quarks behave like fractionally charged Gell-Mann-Zweig quarks below the threshold for colour excitation. The integrally charged nature of the Han Nambu quarks is expected to become manifest once the colour threshold is crossed. We have seen above, however, that in the present unified gauge model the colour octet parts of the quark contributions to the structure functions contain a damping factor $m_g^4 (Q^2 + m_g^2)^{-2}$ and hence vanish for $Q^2 \gg m_g^2$. So we reach the interesting conclusion that *even above colour threshold*, once the scaling regime is reestablished (*i.e.*, $Q^2 \gg m_g^2$) the Han-Nambu quarks behave as though they were fractionally charged. The importance of this conclusion perhaps warrants the more detailed statement given below.

From eq. (5.7 *b*) we may define an "effective" electric charge of the quark q_i:

$$
Q_{\rm eff}(q_i) = Q_0(q_i) - Q_8(q_i)\, \frac{m_g^2}{q^2 - m_g^2}. \tag{6.8}
$$

In eq. (6.8) the constants $Q_0(q_i)$ and $Q_8(q_i)$ are as defined in eqs (4.14). One can now distinguish between three cases in so far as the q^2 dependence of $Q_{\rm eff}$ is concerned.

(*a*) *Below the threshold for colour excitation*: The colour octet part of the current is inoperative and hence

$$
Q_{\rm eff}(q_i) = Q_0(q_i).
$$

* This asymptotically non-vanishing contribution to F'_2 arises from the third term containing Q^4 in eq. (6.6), which in turn can be traced to the 'anomalous moment' term in the gluon interaction (*see* Rajasekaran and Roy 1975). Note that the Yang-Mills coupling fixes the 'anomalous moment' x to be unity.

In other words, the effective charges are the same as in the Gell-Mann-Zweig model.

(*b*) *Above colour threshold but with* $|q^2| \ll m_g^2$: From eq. (6.8)

$$Q_{eff}(q_i) = Q_0(q_i) + Q_8(q_i).$$

Referring to eq. (4.14) we see that these are just the charges for the Han-Nambu quarks.

(*c*) *Above colour threshold and with* $|q^2| \gg m_g^2$: In this case we again get

$$Q_{eff}(q_i) = Q_0(q_i).$$

Thus, despite the manifestation by the quarks of their integral charges in the intermediate region, for asymptotic $|q^2|$ they again behave like fractionally charged objects.

Since the colour octet parts of the quark contributions to the weak structure functions also are damped out for $|q^2| \gg m_g^2$, in this asymptotic region the charged current regains its V–A nature and the neutral current becomes the usual Weinberg-Salam mixture of V and A, as far as the quark contributions are concerned.

However, as we have already noted, the colour octet parts do not vanish completely. There is a nonvanishing contribution† in the far asymptotic region coming from the spin-one gluons given by eq. (6.7). As some specific consequences of this contribution, we may mention here the following: (1) The Callan-Gross relation (*see e.g.*, Roy 1975) $2xF_1 = F_2$ is no longer valid. In particular, we get:

$$\lim \frac{\sigma_L}{\sigma_T} = \frac{F_2^{EM} - 2xF_1^{EM}}{2xF_1^{EM}} \rightarrow \frac{1}{9} \frac{g(x)}{\sum_{q=p,n,c,\lambda} Q_0^2(q)\{q(x) + \bar{q}(x)\}} \tag{6.9}$$

where $\sigma_{L,T}$ are the longitudinal, transverse virtual photon cross-sections (*e.g.*, Roy 1975) and $Q_0(q)$ are the fractional charges of the quarks. (2) The high energy limit of $\sigma^{cc}(\bar{\nu}N)/\sigma^{cc}(\nu N)$—*i.e.*, the ratio of the cross-sections of anti-neutrino and neutrino scattering off an isospin-averaged nucleon target *via* charged current interaction—is no longer 1/3 even if the quark-antiquark " sea " contribution is neglected. In these circumstances one may write (putting the Cabibbo angle $\theta_c = 0$ and now defining the quark probabilities with respect to a proton target):

$$\frac{\sigma^{cc}(\bar{\nu}N)}{\sigma^{cc}(\nu N)} \rightarrow \frac{\int_0^1 dx\, x\{n(x) + p(x) + \frac{1}{4}g(x)\}}{\int_0^1 dx\, x\{3n(x) + 3p(x) + \frac{1}{4}g(x)\}} \tag{6.10}$$

(3) The result

$$F_2^{eN} = \frac{5}{18} F_2^{\nu N(CC)}$$

† Since an application of the momentum conservation sum-rule to the CERN and SLAC data on lepton-hadron scattering already shows (*e.g.*, Roy 1975) that $\int_0^1 dx x g(x)$ is non-zero, $g(x)$ cannot vanish identically.

based on the fractionally-charged quark model in the absence of the " sea " contribution, is no longer true. Again, ignoring the " sea " and with $\theta_c = 0$, we now have:

$$F_2^{eN} = \tfrac{5}{6}\, x\{p(x) + n(x)\} + \tfrac{1}{3} x g(x),$$

$$F_2^{\nu N(CC)} = 3x\{p(x) + n(x)\} + \tfrac{1}{2}\, x g(x). \tag{6.11}$$

Thus, because of the gluon terms, we see that

$$F_2^{eN} \neq \frac{5}{18} F_2^{\nu N(CC)}$$

Most other sum-rules and quark parton model relationships, based on fractionally-charged quarks, remain unaffected.

6.2. *Electron-Positron annihilation*

If the parton model is applicable to the process of a high-energy electron-positron pair annihilating into hadrons through one-photon intermediate state, then the corresponding crosssection may be written as

$$\sigma(e^+e^- \to \text{hadrons}) = \sum_{i=1}^{3} \sum_{q=p,n,c,\lambda} \sigma(e^+e^- \to q_i\bar{q}_i)$$
$$+ \sigma(e^+ e^- \to \rho^+ \rho^-) + \sigma(e^+ e^- \to K^+ K^-). \tag{6.12}$$

Here $q_i\bar{q}_i$ refers to the quark-antiquark pair and ρ^+, ρ^-, K^+, K^- refer to the charged gluons (not to the usual hadrons denoted by these symbols). Using the form and matrix elements of the electromagnetic current already discussed [*cf.* eqs (5.6 *b*), (5.6 *e*), (5.7 *b*), (6.8)], we have:

$$\sigma(e^+ e^- \to q_i\bar{q}_i) = \frac{4\pi\alpha^2}{3S}\left(1 + \frac{2m_q^2}{S}\right)\left(1 - \frac{4m_q^2}{S}\right)^{\frac{1}{2}}$$
$$\left\{Q_0(q_i) - Q_8(q_i) \times \frac{m_g^2}{S - m_g^2}\right\}^2, \tag{6.13}$$

$$\sigma(e^+ e^- \to \rho^+ \rho^-) = \sigma(e^+ e^- \to K^+K^-)$$
$$= \frac{\pi\alpha^2}{12S}\left(1 - \frac{4m_g^2}{S}\right)^{3/2}\left(\frac{S^2}{m_g^4} + 20\,\frac{S}{m_g^2} + 12\right)\frac{m^4}{(S - m_g^2)^2}, \tag{6.14}$$

where S is the square of the total $e^+ e^-$ energy in the CM system, m_q the effective quark mass and α the fine structure constant. For comparison, we may also give

$$\sigma(e^+ e^- \to \mu^+ \mu^-) = \frac{4\pi\alpha^2}{3S}\left(1 + \frac{2m_\mu^2}{S}\right)\left(1 - \frac{4m_\mu^2}{S}\right)^{\frac{1}{2}} \tag{6.15}$$

and define

$$R \equiv \frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)}. \tag{6.16}$$

When $S \to \infty$, the quark crosssection of eq. (6.13) as well as the gluon crosssection of eq. (6.14) behave like $1/S$, so that R goes to a constant. Here again the vector dominance factor $m_g^4 (S - m_g^2)^{-2}$ of the gluon contributions is crucial for the asymptotic constancy of R.

The detailed S-dependence contained in the above formulae may not be physically relevant. However, the crosssection for large enough S may reflect some interesting features of the underlying physics. Let us take the CM energy to be much larger than the effective quark mass. Let us further assume, for simplicity, that S is sufficiently large so that all the four quark degrees of freedom p, n, λ, c have been excited; in other words the centre of mass energy is above the charm-anti-charm threshold. Again, we may envisage three cases:

(*a*) *CM energy below the threshold for colour excitation*:
In this region

$$\sigma(e^+ e^- \to q_i \bar{q}_i) = \frac{4\pi\alpha^2}{3S} Q_0^2(q_i),$$

$$\sigma(e^+ e^- \to \rho^+ \rho^-) = \sigma(e^+ e^- \to K^+ K^-) = 0,$$

$$R = 3\tfrac{1}{3}. \tag{6.17}$$

(*b*) *CM energy above colour threshold but* $S \ll m_g^2$: Now

$$\sigma(e^+ e^- \to q_i \bar{q}_i) = \frac{4\pi\alpha^2}{3S} \{Q_0(q_i) + Q_8(q_i)\}^2,$$

$$\sigma(e^+ e^- \to \rho^+ \rho^-) = \sigma(e^+ e^- \to K^+ K^-) = 0 \tag{6.18}$$

and so

$$R = 6.$$

(*c*) *CM energy very much larger than the gluon mass*:

Here $$\sigma(e^+ e^- \to q_i \bar{q}_i) = \frac{4\pi\alpha^2}{3S} Q_0^2(q_i),$$

$$\sigma(e^+ e^- \to \rho^+ \rho^-) = \sigma(e^+ e^- \to K^+ K^-) = \frac{\pi\alpha^2}{12S},$$

$$R = 3\tfrac{1}{3} + \tfrac{1}{8} = 3\tfrac{11}{24}. \tag{6.19}$$

Hence, provided that the colour threshold is much lower than the gluon mass, R starts with the well-known value $3\frac{1}{3}$ for three fractionally charged quartets of quarks, increases to 6 (which is characteristic of the Han-Nambu quartets), perhaps rises† to even higher values in the region $S \simeq m_g^2$ [*vide* eq. (6.13)] and then falls back to $3\frac{11}{24}$ which is not very different from the first number. On the other hand, if there is no intermediate region of energy corresponding to case (*b*), then the high values $R \geqslant 6$ may not be attained. If we are below the charm-anti-charm threshold, the values of R given in eqs (6.17), (6.18) and (6.19) should be replaced by 2, 4 and $2\frac{1}{8}$ respectively.

7. Summary and discussion

The point of view that weak and electromagnetic interactions being universal should be shared by gluons also has led us to study a unified gauge model based on the group $SU'(3) \otimes SU(2)_L \otimes U(1)$ with integrally charged quarks. We

† Of course, parton model may not be trusted in this region.

have worked out the weak and electromagnetic interactions of both the gluons and the quarks in this model. When these gluons and quarks are treated within the parton framework, we recover scaling in the asymptotic region. The contributions of both gluons and quarks to deep inelastic lepton-hadron scaling as well as to $e^+ e^-$ annihilation have been calculated. In the asymptotic region, the weak and electromagnetic currents of the quarks recover their colour-singlet nature. In particular, the integrally charged quarks behave as though they are fractionally charged.

Although in this paper, we have confined our attention to the $SU'(3) \otimes SU(2)_L \otimes U(1)$ model, it is clear that the main conclusions would follow in a large class of models. The scaling behaviour of the gluon interactions as well as the asymptotic colour singlet nature of the quark currents would follow in all models in which the colour gauge bosons mix with the photon and weak vector bosons.

The above-mentioned conclusions are consequences of the vector-dominance type of denominators which arise due to the mixing of the gluons with the weak and electromagnetic vector bosons. Vector-dominance of currents is an old idea (Nambu 1957) and it was given a gauge-theoretic basis by Sakurai (1960). Matrix elements of currents were taken to be dominated by the low-lying hadronic vector mesons (such as the ρ meson of mass 770 MeV) which also played the role of the strong gauge bosons. Vector-dominance of this type has been incorporated within a unified gauge model by Bars, Halpern and Yoshimura (1973). Although the model studied in the present paper also leads to the vector-dominance of the currents, we are now concerned with the coloured currents and our strong gauge bosons are coloured gluons. The choice of coloured gluons as the gauge bosons is preferable to the identification of the low-lying hadronic vector mesons as the strong gauge bosons of a unified theory, for in the latter case, there is no 'natural' mechanism of ensuring the absence of order α violation of parity and strangeness (*see* Weinberg 1973).

We had mentioned in the introduction that in the present model strong interactions are not asymptotically free. There are two factors which militate against asymptotic freedom, namely the presence of Higgs scalars and the mixing of the strong interaction with the $U(1)$ interaction. However, neither of these appears to be an insurmountable obstacle towards the construction of an asymptotically free model. For, the elementary Higgs scalar fields can be avoided if the symmetry is broken dynamically. The mixing with the $U(1)$ interaction is not a serious factor, since the corresponding coupling constant g' is small enough for the interaction to be treated perturbatively. Alternatively one may perhaps embed $SU'(3) \otimes SU_L(2) \otimes U(1)$ in a larger non-abelian group with no abelian factor. To sum up, the models of the type considered in this paper can be expected to be asymptotically free, at least approximately.

We have avoided detailed comparison with experiments since, at the present moment, there is already a great deal of discussion in the literature concerning the relevance of the new degrees of freedom such as colour and charm in the context of the recent experimental developments in $e^+ e^-$ annihilation and lepton-hadron scattering. We have restricted ourselves to presenting certain interesting consequences pertaining to these processes following from the unified gauge model based on integrally charged quarks. Hopefully, this will help to sharpen the confrontation between various theoretical models and experimental data.

Note added

After this paper was completed, we received a preprint by Pati and Salam (ICTP preprint IC|75|95) in which the authors have arrived at the same results as ours regarding the quark contribution. However, contrary to their conclusion, we find that colour does manifest itself through the gluons. The gluon contribution scales and does not vanish in the asymptotic region.

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