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Higgs couplings in the integer-charge quark model

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Abstract. Colour SU(3) symmetry is broken spontaneously by the introduction of coloured Higgs scalars in the standard SU(3) × SU(2) × U(1) model, so as to make the quarks integrally charged. The resulting couplings of the Higgs bosons with the gauge bosons are worked out.

Keywords. Standard model; integrally charged quarks; broken colour symmetry; coloured Higgs bosons.

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1. Introduction

Gauge models of strong, weak and electromagnetic interactions have been proposed based on integer-charge quarks (ICQ) and broken colour symmetry by Pati and Salam (1973, 1976), Rajasekaran and Roy (1975, 1976) and others. At the phenomenological level, calculations have been made within the framework of ICQ in deep inelastic lepton-hadron scattering and jet production through e^+e^- annihilation. These give results that generally agree with experiments almost as well as the corresponding predictions of standard quantum chromodynamics, the more popular theory of strong interactions based on fractional charge quarks (FCQ) and exact colour symmetry.

The Higgs bosons in ICQ, used to break the colour symmetry spontaneously have however not been studied in sufficient detail. The various couplings between the Higgs bosons and the weak gauge bosons have not been worked out in the literature. These couplings are needed in any complete calculation of the cross-section for hadronic jet production in e^+e^- annihilation and other processes.

In this paper we consider the Higgs sector of the ICQ model of Rajasekaran and Roy and obtain the above mentioned Higgs couplings explicitly. We also complete the listing of all the gauge boson couplings.

In the next section we describe the relevant features of the ICQ model briefly. In §3 we consider the Higgs sector. In §4 we give the various couplings between the physical gauge bosons and the Higgs bosons. In §5 we summarize our results.

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2. The gauge model

We consider the SU(3) × SU(2) × U(1) model of strong, weak and electromagnetic interactions.

The quarks are assigned integer charges as proposed by Han and Nambu (1965). They belong to the 3* representation of colour SU(3). Each generation can be described by an array—for instance

$$q_{\alpha i} = \begin{pmatrix} u_1^0 & u_2^+ & u_3^+ \\ d_1^- & d_2^0 & d_3^0 \end{pmatrix}, \tag{1}$$

where the superscript in the array denotes the electric charges. The index i spans the colour space and α represents the flavour ($\alpha = 1, 2$).

Resolving the quarks $q_{\alpha i}$ into their left and right-handed components we have

$$q_{\alpha iL} = \tfrac{1}{2}(1-\gamma_5)\, q_{\alpha i} \tag{2a}$$

for the left-handed components, and

$$u_{iR} = \tfrac{1}{2}(1+\gamma_5)\, u_i \tag{2b}$$

and

$$d_{iR} = \tfrac{1}{2}(1+\gamma_5)\, d_i \tag{2c}$$

for each generation of the type given in (1). Thus the left-handed parts of $q_{\alpha i}$ transform as doublets and the right-handed components as singlets in the SU(2) space.

In this model, there is a colour octet contribution Q_8 to the total charge operator and a colour singlet contribution Q_0 (Rajasekaran and Roy 1975). The electric charge operator Q is given by

$$Q = Q_0 + Q_8 = I_{3L} + U + I_{3C} + \tfrac{1}{2} Y_C, \tag{3}$$

where

$$Q_0 = I_{3L} + U, \tag{4a}$$

and

$$Q_8 = I_{3C} + \tfrac{1}{2} Y_C. \tag{4b}$$

Here I_{3C} and Y_C are the two diagonal generators of colour SU(3), I_{3L} is the diagonal generator of weak SU(2) and U is the U(1) generator. The U quantum numbers are given by

$$U(q_L) = 1/6; \quad U(u_R) = 2/3; \quad U(d_R) = -1/3 \tag{5}$$

and similarly for other generations. The mixing phenomena revealed in the definition of the charge operator, so peculiar to this model, has led to interesting speculations in the context of monopoles in grand unified theories (Rajasekaran and Ramachandran 1983).

The leptons are doublets under the weak SU(2) group and are represented by left-handed column matrices of the type

$$E_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \tag{6}$$

for each generation. The corresponding right-handed parts e_R etc are SU(2) singlets.

Their U quantum numbers are

$$U(E_L) = -1/2; \quad U(e_R) = -1. \tag{7}$$

Here e refers to the electron and ν_e to the corresponding neutrino.

We denote by $V^p_\mu (p = 1, \ldots, 8)$, X^α_μ $(\alpha = 1, 2, 3)$ and U_μ the gauge vector bosons of the groups SU(3), SU(2) and U(1) respectively and by g_3, g'_2 and g'_1 the corresponding coupling constants. The vector boson Lagrangian L_v is given by

$$L_v = -\frac{1}{4}\sum_{k=1}^{8} (V^k_{\beta\delta} + g_3 f^{kmn} V^m_\beta V^n_\delta)^2 - \frac{1}{4}\sum_{\rho=1}^{3} (X^\rho_{\beta\delta} + g'_2 \varepsilon_{\rho\sigma\lambda} X^\sigma_\beta X^\lambda_\delta)^2 - \tfrac{1}{4} U^2_{\beta\delta} \tag{8}$$

with f^{kmn} being the structure constants of the SU(3) group and

$$V^k_{\beta\delta} = \partial_\beta V^k_\delta - \partial_\delta V^k_\beta . \tag{9}$$

Similar definitions hold for $X^p_{\beta\delta}$ and $U_{\beta\delta}$.

The eight gauge bosons V^k_β transform as an octet under the colour group. In the ICQ model there are two pairs of charged bosons and the remaining are neutral, analogous to the octet of vector mesons of flavour SU(3). The U(1) gauge boson U_μ is neutral. Suitable combinations of the X^α_μ $(\alpha = 1, 2, 3)$ and U_μ will produce, as in the standard electroweak model, two charged weak bosons, a neutral weak boson and the photon.

We do not give the fermionic part of the Lagrangian and the interaction between the fermions and the vector bosons. These have been given in detail by Rajasekaran and Roy (1975).

3. The Higgs sector

All gauge bosons except the photon and all fermions except the neutrino get masses through the Higgs mechanism.

The colour SU(3) symmetry is broken by the introduction of the coloured scalars:

$$\sigma_{\alpha i} = \begin{pmatrix} \sigma^0_{11} & \sigma^+_{12} & \sigma^+_{13} \\ \sigma^-_{21} & \sigma^0_{22} & \sigma^0_{23} \\ \sigma^-_{31} & \sigma^0_{32} & \sigma^0_{33} \end{pmatrix} \tag{10}$$

a set of nine complex scalar fields with $\alpha = 1, 2, 3$ and i spanning the colour space. The superscripts $+$, $-$ and 0 are used to denote the charges of the scalars. σ transforms as $(3^*, 2, 1/6) + (3^*, 1, -1/3)$ under SU(3) × SU(2) × U(1). Note that the flavour index α taking values 1 and 2 refer to the SU(2) doublet whereas $\alpha = 3$ corresponds to the SU(2) singlet.

To break the SU(2) × U(1) symmetry we retain the complex Higgs doublet η_α of the standard electroweak model

$$\eta_\alpha = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} \quad (\alpha = 1, 2), \tag{11}$$

where η_α transforms as $(1, 2, \frac{1}{2})$ under SU(3) × SU(2) × U(1).

The Higgs Lagrangian L_H is then given by

$$L_H = \sum_{i,\alpha} \left| \partial_\mu \sigma_{\alpha i} - \frac{ig_3}{2} \sum_{k=1}^{8} \bar{\lambda}^k_{ij} V^k_\mu \sigma_{\alpha j} - \frac{ig'_2}{2} \sum_{\delta=1}^{3} \tau^\delta_{\alpha\beta} X^\delta_\mu \sigma_{\beta i} - ig'_1 u_{\alpha\beta} U_\mu \sigma_{\beta i} \right|^2 + \sum_\alpha \left| \partial_\mu \eta_\alpha - \frac{ig'_2}{2} \sum_{\rho=1}^{3} \tau^\rho_{\alpha\beta} X^\rho_\mu \eta_\beta - \frac{ig'_1}{2} U_\mu \eta_\alpha \right|^2 + V(\sigma, \eta) + Y(\sigma, \eta, q, l). \tag{12}$$

In the above equation $\tau^\rho_{\alpha\beta}$ $(\rho = 1, 2, 3)$ are the Pauli matrices and $\bar{\lambda}^k = -(\lambda^k)^*$, where λ^k are the usual Gell-Mann matrices. We have chosen the 3* representation of the SU(3) matrices because σ transforms as a 3* under SU(3). $u_{\alpha\beta}$ denotes the U quantum numbers of the σ fields written as a matrix in the $\mathrm{SU}_L(2)$ space ($\alpha = 1, 2$ refers to the $\mathrm{SU}_L(2)$ doublet part of σ and $\alpha = 3$ refers to the $\mathrm{SU}_L(2)$ singlet):

$$u_{\alpha\beta} = \frac{1}{6} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{13}$$

$V(\sigma, \eta)$ is a quartic polynomial in σ, η which is so chosen as to give the required vacuum expectation values:

$$\langle \sigma_{\alpha i} \rangle = \langle \sigma \rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \qquad \langle \eta_\alpha \rangle = \langle \eta \rangle \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{14}$$

where $\langle \sigma \rangle$ and $\langle \eta \rangle$ are real constants. We shall not describe the precise form of $V(\sigma, \eta)$. Y contains the Yukawa couplings among the fermions and scalars. The Yukawa couplings with η are the same as in the standard model. Yukawa couplings with σ are not possible if baryon number is assumed to be conserved. Here we shall assume baryon number conservation. However, parenthetically we may note that the colour triplet scalars σ present in the ICQ model provide a new source of baryon number violation. The consequences of this have been discussed elsewhere (Rajasekaran 1982).

In the rest of this section we show that with the form (14) for $\langle \sigma_{\alpha i} \rangle$, ten coloured Higgs scalars survive after colour symmetry is spontaneously broken. For this, we work in the unitary gauge. We can then write

$$\sigma_{\alpha k} = \sum_{j,\beta} \left[\exp i \sum_{p=1}^{8} \frac{\bar{\lambda}^p}{2} \theta_p \right]_{kj} \left[\exp i \left(\sum_{\mu=1}^{3} T^\mu \theta'_\mu + U \theta_U \right) \right]_{\alpha\beta} \rho_{\beta j}, \tag{15}$$

where θ_p, θ'_μ and θ_U are real scalar fields with vanishing expectation values. ρ and σ are each 3×3 matrices and U is the generator of the $U(1)$ group. T^μ are the SU(2) generators written as the following 3×3 matrices:

$$T^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2} \lambda^1; \tag{16a}$$

$$T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\lambda^2; \tag{16b}$$

$$T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\lambda^3; \tag{16c}$$

The generator U is given by

$$U = \frac{1}{6}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \frac{1}{2\sqrt{3}}\lambda^8. \tag{16d}$$

(Though T^μ and U have been written in terms of the SU(3) λ matrices, it must be kept in mind that they operate on the flavour space). Thus (15) becomes

$$\sigma_{\alpha k} = \sum_{j,\beta}\left[\exp \sum_{p=1}^{8} \frac{\bar{\lambda}^p}{2}\theta_p\right]_{kj}\left[\exp i\left(\sum_{\delta=1}^{3}\frac{\lambda^\delta}{2}\theta'_\delta + \frac{\lambda^8}{2\sqrt{3}}\theta_U\right)\right]_{\alpha\beta}\rho_{\beta j} \tag{17}$$

The condition that there can be no quadratic couplings between the gauge fields and the scalar fields in the unitary gauge, leads to the following constraints on the scalar fields $\rho_{\beta j}$:

$$\sum_{\alpha,i,j}\left[\langle\rho_{\alpha i}\rangle^* \lambda^k_{ij}\,\rho_{\alpha j} - \rho^*_{\alpha i}\,\lambda^k_{ij}\,\langle\rho_{\alpha j}\rangle\right] = 0 \quad (k = 1, 2, \ldots, 8), \tag{18}$$

$$\sum_{\alpha,\beta,i}\left[\langle\rho_{\alpha i}\rangle^* \lambda^\delta_{\alpha\beta}\,\rho_{\beta i} - \rho^*_{\alpha i}\,\lambda^\delta_{\alpha\beta}\,\langle\rho_{\beta i}\rangle\right] = 0 \quad (\delta = 1, 2, 3) \tag{19}$$

and

$$\sum_{\alpha,\beta,i}\left[\langle\rho_{\alpha i}\rangle^* \lambda^8_{\alpha\beta}\,\rho_{\beta i} - \rho^*_{\alpha i}\,\lambda^8_{\alpha\beta}\,\langle\rho_{\beta i}\rangle\right] = 0. \tag{20}$$

In (18) to (20) the vacuum expectation values $\langle\rho_{\alpha i}\rangle$ are chosen to be

$$\langle\rho_{\alpha i}\rangle = \langle\sigma_{\alpha i}\rangle = \langle\sigma\rangle\,\delta_{\alpha i} \tag{21}$$

from (14). This choice of vacuum expectation value leads to a breakdown of the local SU(3) × SU(2) × U(1) gauge symmetry to the level of electromagnetic U(1) while an approximate global SU(3) symmetry is also preserved. Hence the masses m_g of all the eight gluons are equal, in the zeroth order in electroweak couplings.

Corresponding to the transformation (17), the gauge boson, quark and lepton fields also get transformed. However, the new Lagrangian with $\rho_{\alpha i}$ replacing $\sigma_{\alpha i}$ and the new gauge boson, quark and lepton fields replacing the old ones resembles the old Lagrangian in its form.

We write

$$\rho_{\alpha i} = \langle\rho_{\alpha i}\rangle + \rho'_{\alpha i} \tag{22a}$$

with

$$\langle \rho'_{\alpha i} \rangle = 0. \tag{22b}$$

On substituting (21) and (22) into (18) and remembering that α, β and i go from 1 to 3, in the summation we obtain

$$\mathrm{Tr}\,(\lambda^k \rho') - \mathrm{Tr}\,(\rho'^{\dagger} \lambda^k) = 0 \quad (k = 1, \ldots, 8). \tag{23}$$

Substitution of (21) and (22) into (19) and (20) will give us

$$\mathrm{Trace}\,(\lambda^{\delta} \rho') - \mathrm{Trace}\,(\rho'^{\dagger} \lambda^{\delta}) = 0 \quad (\delta = 1, 2, 3) \tag{24}$$

and

$$\mathrm{Trace}\,(\lambda^8 \rho') - \mathrm{Trace}\,(\rho'^{\dagger} \lambda^8) = 0. \tag{25}$$

Thus we see that the constraints given by (24) and (25) are contained in (23). Hence even if θ'_{δ} and θ_U were set equal to zero in (17) we will still obtain the complete set of constraints. This is clearly because of the type of the Higgs scalars σ chosen and the choice of the vacuum expectation values, given in (21). It is the factor $\delta_{\alpha i}$ that is responsible for collapse of the constraints (19) and (20) in the electroweak sector to those in the colour sector (18). This also explains why the result of the symmetry breaking was the same in an earlier work (Rajasekaran and Rindani 1982) where the weak SU(2) was ignored as an approximation and a model based on SU(3) × U(1) was considered.

Equation (23) implies the following set of constraints:

$$\rho'_{ij} = \rho'^{*}_{ji} \; (i \neq j) \tag{26}$$

and

$$(\rho'^{*}_{11} - \rho'_{11}) = (\rho'^{*}_{22} - \rho'_{22}) = (\rho'^{*}_{33} - \rho'_{33}). \tag{27}$$

These amount to 8 constraints and so, out of the 18 real fields $\rho_{\alpha i}$, only 10 are independent physical scalar fields. These survive as 10 massive Higgs particles. The 8 massless Goldstone bosons have been eaten by 8 gauge bosons which become massive. These are the gluons. The ten Higgs fields can be written in terms of new independent fields ψ_i $(i = 0, 1, 2, \ldots, 8)$ which is a colour nonet and ϕ a colour singlet i.e. we write,

$$\rho'_{ij} = \frac{1}{2} \sum_{k=0}^{8} (\lambda^k)_{ij} \psi_k + \frac{i}{\sqrt{6}} \delta_{ij} \phi. \tag{28}$$

Here λ^k are the usual SU(3) matrices for $k = 1$ to 8 and $\lambda^0 = (2/3)^{\frac{1}{2}} I$ where I is the 3 × 3 unit matrix. Because of the conditions (26) and (27), ψ_k and ϕ are real. Thus, apart from the neutral Higgs particle of the standard model, in the present model the Higgs sector consists of a colour nonet ψ_k and a colour singlet ϕ.

4. Couplings among gauge bosons and Higgs bosons

The replacement of all the scalar fields in the Lagrangian by their vacuum expectation values leads to some combination of gauge fields acquiring masses. The resulting boson mass terms in the Lagrangian are given by

$$\frac{1}{2} g_3^2 \langle\sigma\rangle^2 \sum_{i=1}^{8} G_\mu^i G^{\mu i} + \frac{g_2'^2}{2} \langle\eta\rangle^2 \tilde{W}_\mu^\dagger \tilde{W}^\mu + \frac{(g_2'^2 + g_1'^2)}{4} \langle\eta\rangle^2 \tilde{Z}_\mu \tilde{Z}^\mu. \tag{29}$$

Here,

$$G_\mu^i = V_\mu^i - \frac{g_2'}{g_3} X_\mu^i \quad (i = 1, 2, 3), \tag{30a}$$

$$G_\mu^i = V_\mu^i \qquad (i = 4, 5, 6, 7), \tag{30b}$$

$$G_\mu^8 = V_\mu^8 - \frac{g_2'}{\sqrt{3} g_3} U_\mu, \tag{30c}$$

$$\tilde{W}_\mu = \frac{1}{\sqrt{2}} (X_\mu^1 + iX_\mu^2), \tag{30d}$$

$$\tilde{W}_\mu^\dagger = \frac{1}{\sqrt{2}} (X_\mu^1 - iX_\mu^2), \tag{30e}$$

$$\tilde{Z}_\mu = \frac{g_2' X_\mu^3 - g_1' U_\mu}{(g_2'^2 + g_1'^2)^{1/2}}. \tag{30f}$$

We also define

$$\tilde{A}_\mu = \frac{g_1' X_\mu^3 + g_2' U_\mu}{(g_2'^2 + g_1'^2)^{1/2}}. \tag{30g}$$

The gluon fields G_μ^i $(i = 1, \ldots 8)$ are not orthogonal to the weak and electromagnetic fields $\tilde{W}_\mu$, $\tilde{W}_\mu^\dagger$, $\tilde{A}_\mu$ and $\tilde{Z}_\mu$. Due to this definition of the gauge boson fields, quadratic couplings between the vector boson fields are introduced (Rajasekaran and Roy 1975).

The kinetic energy terms in L_v are given by

$$-\frac{1}{4}\left[\sum_{t=1}^{8} (V_{\beta\lambda}^t)^2 + \sum_{i=1}^{3} (X_{\beta\lambda}^i)^2 + U_{\beta\lambda}^2\right]. \tag{31}$$

The cubic terms are

$$-\frac{g_3}{2} \sum_{t,m,n=1}^{8} f^{tmn} V_{\beta\lambda}^t V_\beta^m V_\lambda^n - \frac{g_2'}{2} \sum_{i,j,k=1}^{3} \varepsilon_{ijk} X_{\beta\lambda}^i X_\beta^j X_\lambda^k. \tag{32}$$

In order to define the physical gauge boson fields and coupling constants we first define the following quantities:

$$B = (1 + g_2'^2/g_3^2)^{1/2}, \tag{33a}$$

$$C = \left[1 + \frac{4 g_1'^2 g_2'^2}{3 g_3^2 (g_1'^2 + g_2'^2)}\right]^{1/2}, \tag{33b}$$

and

$$D = \left[1 + \frac{g_2'^4}{g_3^2 (g_1'^2 + g_2'^2)} + \frac{g_1'^4}{3 g_3^2 (g_1'^2 + g_2'^2)}\right]^{1/2}. \tag{33c}$$

Then the physical electroweak gauge fields are given by

$$W_\mu = B\tilde{W}_\mu; \quad A_\mu = C\tilde{A}_\mu; \quad Z_\mu = D\tilde{Z}_\mu \tag{34}$$

and the physical electroweak coupling constants are

$$g_2 = g_2'/B,$$
$$e = \frac{g_1' g_2'}{C(g_1'^2 + g_2'^2)^{1/2}}. \tag{35}$$

Also $$e = g_2 \sin\theta_w, \tag{36}$$

θ_w being the Weinberg angle. The physical gluon fields are G^i_μ $(i = 1, \ldots 8)$ and the strong coupling constant is g_3.

The kinetic energy and cubic interaction terms of L_v given in (31) and (32) have been recast in terms of the physical gauge fields in earlier work (Rajasekaran and Roy 1975) which also contains a discussion of the significance of the quadratic vector-dominance couplings in the ICQ model.

The quartic terms are given by

$$-\tfrac{1}{4} g_3^2 f^{tmn} f^{tpq} V^m_\mu V^n_\nu V^{\mu p} V^{\nu q} - \tfrac{1}{4} g_2^2 \varepsilon^{\alpha\beta\gamma} \varepsilon^{\alpha\delta\sigma} X^\beta_\mu X^\gamma_\nu X^{\mu\delta} X^{\nu\sigma}. \tag{37}$$

In terms of the physical vector fields these are given below (earlier literature does not contain these): (a) The quartic terms with 4 gluons are the same as in standard QCD. (b) The quartic terms with 3 gluons are

$$-g_3 f^{tmn} f^{tpq} G^m_\mu G^n_\nu G^{\nu q}\Big[g_2(\delta^{p1} W^{\mu 1} + \delta^{p2} W^{\mu 2})$$
$$+ e\left(\delta^{p3} + \frac{1}{\sqrt{3}}\delta^{p8}\right)A^\mu + \left(a\delta^{p3} - \frac{b}{\sqrt{3}}\delta^{p8}\right)Z^\mu\Big], \tag{38}$$

where we have defined $W_\mu = \dfrac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu)$

$$a = e\cot\theta_w \quad \text{and} \quad b = e\tan\theta_w. \tag{39}$$

(c) Quartic terms with 2 gluons are

$$\frac{g_2^2}{2} f^{tmn} f^{tpq} \{G^m_\beta G^n_\tau (\delta^{p1} W^{\beta 1} + \delta^{p2} W^{\beta 2})(\delta^{q1} W^{\tau 1} + \delta^{q2} W^{\tau 2})$$
$$+ G^n_\tau G^{\tau q}(\delta^{m1} W^{\beta 1} + \delta^{m2} W^{\beta 2})(\delta^{p1} W^1_\beta + \delta^{p2} W^2_\beta) + G^n_\beta G^p_\tau (\delta^{q1} W^{\beta 1}$$
$$+ \delta^{q2} W^{\beta 2})(\delta^{m1} W^{\tau 1} + \delta^{m2} W^{\tau 2})\} - \frac{e^2}{2} f^{tmn} f^{tpq}\left(\delta^{m3} + \frac{1}{\sqrt{3}}\delta^{m8}\right)$$
$$\left\{G^n_\beta G^{\beta q} A_\tau A^\tau \left(\delta^{p3} + \frac{1}{\sqrt{3}}\delta^{p8}\right) + G^n_\beta G^p_\tau A^\beta A^\tau \left(\delta^{q3} + \frac{1}{\sqrt{3}}\delta^{q8}\right)\right\}$$
$$-\frac{1}{2} f^{tmn} f^{tpq}\left(a\delta^{m3} - \frac{b}{\sqrt{3}}\delta^{m8}\right)\left\{G^n_\beta G^{\beta q} Z^\tau Z^\tau \left(a\delta^{p3} - \frac{b}{\sqrt{3}}\delta^{p8}\right)\right.$$

$$+G^n_\beta G^p_\tau Z^\beta Z^\tau\left(a\delta^{q3}-\frac{b}{\sqrt{3}}\delta^{q8}\right)\Big\}$$

$$-eg_2 f^{tmn} f^{tpq}\Big\{G^m_\beta G^n_\tau A^\beta\left(\delta^{p3}+\frac{1}{\sqrt{3}}\delta^{p8}\right)(\delta^{q1}W^{\tau 1}+\delta^{q2}W^{\tau 2})$$

$$+G^n_\beta G^{\beta q} A_\tau\left(\delta^{p3}+\frac{1}{\sqrt{3}}\delta^{p8}\right)(\delta^{m1}W^{\tau 1}+\delta^{m2}W^{\tau 2})$$

$$+G^n_\beta G^p_\tau A^\beta\left(\delta^{q3}+\frac{1}{\sqrt{3}}\delta^{q8}\right)(\delta^{m1}W^{\tau 1}+\delta^{m2}W^{\tau 2})\Big\}$$

$$-g_2 f^{tmn} f^{tpq}\Big\{G^m_\beta G^n_\tau Z^\beta\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)(\delta^{q1}W^{\tau 1}+\delta^{q2}W^{\tau 2})$$

$$+G^n_\beta G^{\beta q} Z_\tau\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)(\delta^{m1}W^{\tau 1}+\delta^{m2}W^{\tau 2})$$

$$+G^n_\beta G^p_\tau Z^\beta\left(a\delta^{q3}-\frac{b}{\sqrt{3}}\delta^{q8}\right)(\delta^{m1}W^{\tau 1}+\delta^{m2}W^{\tau 2})\Big\}$$

$$-e f^{tmn} f^{tpq}\Big\{G^q_\beta G^{\beta n} A_\tau Z^\tau\left(\delta^{m3}+\frac{1}{\sqrt{3}}\delta^{m8}\right)\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)$$

$$+G^n_\beta G^p_\tau A^\beta Z^\tau\left(\delta^{m3}+\frac{1}{\sqrt{3}}\delta^{m8}\right)\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{m8}\right)\Big\}. \qquad (40)$$

(d) Quartic terms with one gluon are

$$-\frac{eg_2}{g_3} f^{tmn} f^{tpq}\Big\{Z_\beta A^\beta G^n_\tau(\delta^{q1}W^{\tau 1}+\delta^{q2}W^{\tau 2})\left(\delta^{m3}+\frac{1}{\sqrt{3}}\delta^{m8}\right)\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)$$

$$+Z_\beta A^\beta G^q_\tau\left(\delta^{m3}+\frac{1}{\sqrt{3}}\delta^{m8}\right)(\delta^{n1}W^{\tau 1}+\delta^{n2}W^{\tau 2})\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)$$

$$+Z_\beta A_\tau G^{\tau n}(\delta^{p1}W^{\beta 1}+\delta^{p2}W^{\beta 2})\left(\delta^{q3}+\frac{1}{\sqrt{3}}\delta^{q8}\right)\left(a\delta^{m3}-\frac{b}{\sqrt{3}}\delta^{m8}\right)$$

$$+Z_\beta A_\tau G^{\beta p}(\delta^{n1}W^{\tau 1}+\delta^{n2}W^{\tau 2})\left(\delta^{q3}+\frac{1}{\sqrt{3}}\delta^{q8}\right)\left(a\delta^{m3}-\frac{b}{\sqrt{3}}\delta^{m8}\right)\Big\}$$

$$-\frac{g_2^2}{g_3} f^{tmn} f^{tpq}\Big\{Z_\beta G^n_\tau(\delta^{q1}W^{\tau 1}+\delta^{q2}W^{\tau 2})(\delta^{m1}W^{\beta 1}+\delta^{m2}W^{\beta 2})\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)$$

$$+Z_\beta G^{\beta n}(\delta^{m1}W^1_\tau+\delta^{m2}W^2_\tau)(\delta^{p1}W^{\tau 1}+\delta^{p2}W^{\tau 2})\left(a\delta^{q3}-\frac{b}{\sqrt{3}}\delta^{q8}\right)$$

$$+Z_\beta G^n_\tau(\delta^{q1}W^{\tau 1}+\delta^{q2}W^{\tau 2})(\delta^{p1}W^{\beta 1}+\delta^{p2}W^{\beta 2})\left(a\delta^{m3}-\frac{b}{\sqrt{3}}\delta^{m8}\right)\Big\}$$

$$-\frac{g_2^2}{g_3} f^{tmn} f^{tpq}\Big\{A_\beta G^n_\tau(\delta^{q1}W^{\tau 1}+\delta^{q2}W^{\tau 2})(\delta^{m1}W^{\beta 1}+\delta^{m2}W^{\beta 2})\left(e\delta^{p3}+\frac{e}{\sqrt{3}}\delta^{p8}\right)$$

$$+A_\beta G^{\beta n}(\delta^{m1} W^{\tau 1}+\delta^{m2} W^{\tau 2})(\delta^{p1} W^1_\tau+\delta^{p2} W^2_\tau)\left(e\delta^{q3}+\frac{e}{\sqrt{3}}\delta^{q8}\right)$$

$$+A_\beta G^n_\tau(\delta^{q1} W^{\tau 1}+\delta^{q2} W^{\tau 2})(\delta^{p1} W^{\beta 1}+\delta^{p2} W^{\beta 2})\left(e\delta^{m3}+\frac{e}{\sqrt{3}}\delta^{m8}\right)\Big\}$$

$$-\frac{e^2 g_2}{g_3} f^{tmn} f^{tpq}\Big\{A_\beta A^\beta G^n_\tau(\delta^{q1} W^{\tau 1}+\delta^{q2} W^{\tau 2})\left(\delta^{m3}+\frac{1}{\sqrt{3}}\delta^{m8}\right)\left(\delta^{p3}+\frac{1}{\sqrt{3}}\delta^{p8}\right)$$

$$+A_\beta A_\tau G^{\tau n}\left(\delta^{q3}+\frac{1}{\sqrt{3}}\delta^{q8}\right)(\delta^{p1} W^{\beta 1}+\delta^{p2} W^{\beta 2})\left(\delta^{m3}+\frac{1}{\sqrt{3}}\delta^{m8}\right)\Big\}$$

$$-\frac{g_2}{g_3} f^{tmn} f^{tpq}\Big\{Z_\beta Z^\beta G^{\tau n}(\delta^{q1} W^1_\tau+\delta^{q2} W^2_\tau)\left(a\delta^{p3}-\frac{b}{\sqrt{3}}\delta^{p8}\right)\left(a\delta^{m3}-\frac{b}{\sqrt{3}}\delta^{m8}\right)$$

$$+Z_\beta Z_\tau G^{\tau n}(\delta^{p1} W^{\beta 1}+\delta^{p2} W^{\beta 2})\left(a\delta^{q3}-\frac{b}{\sqrt{3}}\delta^{q8}\right)\left(a\delta^{m3}-\frac{b}{\sqrt{3}}\delta^{m8}\right)\Big\}$$

$$-\frac{3g_2^3}{g_3} f^{tmn} f^{tpq}\Big\{G^n_\beta(\delta^{m1} W^1_\tau+\delta^{m2} W^2_\tau)(\delta^{q1} W^{\beta 1}$$

$$+\delta^{q2} W^{\beta 2})(\delta^{p1} W^{\tau 1}+\delta^{p2} W^{\tau 2})\Big\}. \tag{41}$$

(e) Quartic terms with no gluons are identical to that in FCQ and are therefore not given.

We now consiuer the interactions between the gauge bosons and the Higgs bosons. We have two sets of Higgs particles (ψ_i, ϕ) and η. Couplings between η and the gauge bosons are identical to that in the standard model. We do not give them here. The couplings between the coloured Higgs (ψ_i, ϕ) and the gauge bosons are given below:

(i) The scalar-scalar-vector couplings are

$$\left(-\frac{g_3}{2} f_{ijk}\,\partial_\mu \psi_i \psi_j-\frac{g_3}{\sqrt{6}}\partial_\mu \psi_k \phi+\frac{g_3}{\sqrt{3}}\partial_\mu \phi \psi_k\right) G^k_\mu-\left(a\delta^{n3}-\frac{b}{\sqrt{3}}\delta^{n8}\right)$$

$$f_{ijn}\,\partial_\mu \psi_i \psi_j Z^\mu-\left(e\delta^{n3}+\frac{e}{\sqrt{3}}\delta^{n8}\right) f_{ijn}\,\partial_\mu \psi_i \psi_j A^\mu$$

$$-g_2 f_{ij\sigma}\,\partial_\mu \psi_i \psi_j W^{\mu\sigma}, \tag{42}$$

where $\sigma = 1, 2$ and $i, j, k = 1, \ldots, 8$. The couplings respect $SU(3)\times SU(2)\times U(1)$ invariance as expected of gauge couplings.

(ii) The only vector-vector-scalar coupling is

$$\frac{g_3^2}{2}\langle\sigma\rangle G^i_\mu G^{\mu j} \psi^k d_{ijk}. \tag{43}$$

We notice that this also respects global $SU(3)\times SU(2)\times U(1)$ symmetry. The ψ_i do not couple to the electroweak gauge bosons. Hence this expression for vector-vector-scalar couplings is identical to what we would have even if we ignore weak interactions.

(iii) Vector-vector-scalar-scalar couplings are given below:

$$\frac{g_3^2}{8} d_{pmk}\, d_{stk}\, \psi_s\, \psi_t\, G_\beta^p\, G^{\beta m} + \frac{g_3^2}{12} \phi^2\, G_\beta^t\, G^{\beta t}$$

$$+\frac{g_2}{2} g_3\, G_\beta^p\, \psi_s\, \psi_t f_{psk} f_{tk\rho}\, (W^{\beta 1}\, \delta^{\rho 1} + W^{\beta 2}\, \delta^{\rho 2})$$

$$-\frac{g_2}{6} g_3\, G_\beta^p\, \phi\psi_s f_{sp\rho}\, (W^{\beta 1}\, \delta^{\rho 1} + W^{\beta 2}\, \delta^{\rho 2})$$

$$+\frac{g_3}{2} \psi_s\, \psi_t\, G_\beta^p\, Z^\beta f_{psm} f_{tm\rho} \left(a\delta^{\rho 3} - \frac{b}{\sqrt{3}}\, \delta^{\rho 8} \right)$$

$$-\frac{g_3}{\sqrt{6}} \phi\psi_s\, G_\beta^p\, Z^\beta f_{sp\rho} \left(a\delta^{\rho 3} - \frac{b}{\sqrt{3}}\, \delta^{\rho 8} \right)$$

$$+\frac{eg_3}{2} \psi_s\, \psi_t\, G_\beta^p\, A^\beta f_{psm} f_{tm\rho} \left(\delta^{\rho 3} + \frac{1}{\sqrt{3}}\, \delta^{\rho 8} \right)$$

$$-\frac{e}{\sqrt{6}} g_3\, \phi\psi_s\, G_\beta^p\, A^\beta f_{sp\rho} \left(\delta^{\rho 3} + \frac{1}{\sqrt{3}}\, \delta^{\rho 8} \right)$$

$$+\frac{g_2^2}{2} \psi_s\, \psi_t f_{\rho sk} f_{tk\beta}\, (W_\lambda^1\, \delta^{\rho 1} + W_\lambda^2\, \delta^{\rho 2})\, (W^{\lambda 1}\, \delta^{\beta 1} + W^{\lambda 2}\, \delta^{\beta 2})$$

$$-\frac{g_2^2}{\sqrt{6}} \phi\psi_s\, (W_\lambda^1\, \delta^{\beta 1} + W_\lambda^2\, \delta^{\beta 2})(W^{\lambda 1}\, \delta^{\rho 1} + W^{\lambda 2}\, \delta^{\rho 2}) f_{s\rho\beta}$$

$$+ g_2\, \psi_s\, \psi_t\, Z_\lambda f_{\rho sm} f_{tm\beta}\, (W^{\lambda 1} \delta^{\beta 1} + W^{\lambda 2}\, \delta^{\beta 2}) \left(a\delta^{\rho 3} - \frac{b}{\sqrt{3}}\, \delta^{\rho 8} \right)$$

$$+ eg_2\, \psi_s\, \psi_t\, A_\lambda f_{\rho sm} f_{tm\beta}\, (W^{\lambda 1}\, \delta^{\beta 1} + W^{\lambda 2}\, \delta^{\beta 2}) \left(\delta^{\rho 3} + \frac{1}{\sqrt{3}}\, \delta^{\rho 8} \right)$$

$$+ a^2\, Z_\lambda\, Z^\lambda\, \psi_s\, \psi_t \left\{ \frac{1}{4\sqrt{3}}\, d_{st8} - \frac{1}{4}\, d_{3sm}\, d_{tm3} + \frac{1}{4} f_{3sm} f_{tm3} \right\}$$

$$+ b^2\, Z_\lambda\, Z^\lambda\, \psi_s\, \psi_t \left\{ -\frac{1}{12\sqrt{3}}\, d_{st8} - \frac{1}{12}\, d_{8sm}\, d_{tm8} + \frac{1}{12} f_{8sm} f_{tm8} \right\}$$

$$+ ab\, Z_\lambda\, Z^\lambda\, \psi_s\, \psi_t \left\{ -\frac{1}{6}\, d_{st3} + \frac{1}{2\sqrt{3}}\, d_{3sm}\, d_{tm8} - \frac{1}{2\sqrt{3}} f_{8sm} f_{tm3} \right\}$$

$$+ e^2\, A_\lambda\, A^\lambda\, \psi_s\, \psi_t \left\{ \frac{1}{6\sqrt{3}}\, d_{st8} + \frac{1}{6}\, d_{st3} - \frac{1}{2\sqrt{3}}\, d_{3sm}\, d_{tm8} - \frac{1}{12}\, d_{8sm}\, d_{tm8} \right.$$

$$-\frac{1}{4}\, d_{3sm}\, d_{tm3} + \frac{1}{2\sqrt{3}} f_{8sm} f_{tm3} + \frac{1}{12} f_{8sm} f_{tm8}$$

$$\left. +\frac{1}{4} f_{3sm} f_{tm3} \right\} + ea\, Z_\lambda\, A^\lambda\, \psi_s\, \psi_t \left\{ \frac{1}{2\sqrt{3}}\, d_{ts8} - \frac{1}{2\sqrt{3}}\, d_{3sm}\, d_{tm8} \right.$$

$$\left. +\frac{1}{6}\, d_{st3} - \frac{1}{2}\, d_{3sm}\, d_{tm3} + \frac{1}{2} f_{3sm} f_{tm3} + \frac{1}{2\sqrt{3}} f_{3sm} f_{tm8} \right\}$$

$$+eb\,Z_\lambda\,A^\lambda\,\psi_s\,\psi_t\left\{\frac{1}{6\sqrt{3}}\,d_{ts8}+\frac{1}{2\sqrt{3}}\,d_{3sm}\,d_{tm8}+\frac{1}{6}\,d_{st3}\right.$$
$$\left.-\frac{1}{2}\,d_{8sm}\,d_{tm8}-\frac{1}{6}f_{8sm}f_{tm8}-\frac{1}{2\sqrt{3}}f_{3sm}f_{tm8}\right\}. \tag{44}$$

5. Conclusions

We have given in this paper a more complete description of the ingredients of the ICQ model (Rajasekaran and Roy 1975, 1976) than that available in previous literature. All the interactions between the Higgs bosons and the gauge bosons have been worked out here.

These couplings together with the quartic couplings between the gauge bosons (also worked out fully in this paper) are very important in studying several aspects of weak interaction phenomenology. These studies are of relevance currently, since the discovery of the weak bosons in the laboratory has opened up the possibility of carrying out various experiments related to the electroweak theory.

The Higgs bosons of the ICQ model also play an important role in the cancellation of mass singularities in physical processes like $Z \to 3$ jets and $e^+e^- \to 3$ jets (Rajasekaran and Rindani 1982; Lakshmibala *et al* 1981). A study of these phenomena requires knowledge of the couplings derived here. The results of these studies will be reported elsewhere.

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