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A See-Saw Model for Atmospheric and Solar Neutrino Oscillations

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Abstract

We have constructed an explicit see-saw model containing two singlet neutrinos, one carrying a $(B - 3L_e)$ gauge charge with an intermediate mass scale of $\sim O(10^{10})$ GeV along with a sterile one near the GUT (grand unification theory) scale of $\sim O(10^{16})$ GeV. With these mass scales and a reasonable range of Yukawa couplings, the model can naturally account for the near-maximal mixing of atmospheric neutrino oscillations and the small mixing matter-enhanced oscillation solution to the solar neutrino deficit.

The super-Kamiokande experiment has recently provided convincing evidence for the atmospheric neutrino oscillation [1] as well as confirmed earlier results on solar neutrino oscillation [2]. The atmospheric neutrino oscillation data seem to require a large mixing angle between ν_μ and ν_τ,

$$\sin^2 2\theta_{\mu\tau} > 0.82 \tag{1}$$

and

$$\Delta M^2 = (0.5 - 6) \times 10^{-3} \mathrm{eV}^2. \tag{2}$$

On the other hand, the solar neutrino oscillation data can be explained by the small mixing-angle matter-enhanced solution between ν_e and a combination of ν_μ/ν_τ with [3]

$$\sin^2 2\theta_{e-\mu/\tau} = 10^{-2} - 10^{-3} \tag{3}$$

and

$$\Delta m^2 = (0.5 - 1) \times 10^{-5} \mathrm{eV}^2. \tag{4}$$

This represents the most conservative solution to the solar neutrino anomaly although one can get equally good solutions with large mixing-angle matter-enhanced and vacuum oscillations as well. One would naturally expect a near-maximal mixing between ν_μ and ν_τ (1), as required by the atmospheric neutrino data, if they were almost degenerate Dirac partners with a small mass difference given by (2). In the context of a three-neutrino model however, the solar neutrino solution (4) would then require the ν_e to show a much higher level of degeneracy with one of these states, which is totally unexpected. Therefore, it is more natural to consider the three neutrino mass states as nondegenerate with

$$m_1 = (\Delta M^2)^{1/2} \simeq 0.05 \mathrm{eV}, \quad m_2 = (\Delta m^2)^{1/2} \simeq 0.003 \mathrm{eV}, \quad m_3 << m_2. \tag{5}$$

There is broad agreement on this point in the current literature on neutrino physics [4], much of which is focussed on the question of reconciling this hierarchical structure of neutrino masses with at least one large mixing angle (1).

The cannonical mechanism for generating neutrino masses and mixings is the so called see-saw model involving heavy right-handed singlet neutrinos [5]. It naturally leads to small hierarchical masses for the three doublet neutrinos, but with small mixing angles. Alternatively one can generate the small neutrino masses radiatively via the Zee model [6, 7] or the R-parity breaking supersymmetric model [8]. Instead of heavy right-handed neutrinos, one needs here an expanded scalar sector in the $\leq$ TeV region, as extra Higgs multiplets in the former case and as squarks and sleptons in the latter. The radiative mechanism offers more flexibility to reconcile hierarchical neutrino masses with at least one large mixing angle. In fact, explicit models for neutrino masses and mixing have been constructed recently to explain the atmospheric and solar neutrino data in terms of these two radiative mechanisms [7, 8]. It should be noted however that the presence of extra scalars in the $\leq$ TeV region in these models represents a potential problem with large flavour-changing neutral-current (FCNC) effects. Moreover, these extra scalars can either be detected or ruled out in future colliders. On the other hand, the see-saw model is less vulnerable to FCNC effects and collider search, although it is harder to reconcile hierarchical neutrino masses with a large mixing in this case. The present work is devoted to this exercise. As we shall see below, this model can naturally reconcile hierarchical neutrino masses with a large mixing angle (1). Moreover, the low-energy ($\leq$ TeV) spectrum of this model is identical to the standard model, so that it has no potential problem with flavour-changing neutral currents.

Let us first consider the atmospheric neutrino oscillation. It is clear from (1), (2) and (5) that it requires the heaviest neutrino state to be a roughly equal mixture of $\nu_\mu - \nu_\tau$ with mass ~ 0.05 eV. In the simplest see-saw model, this requires one heavy singlet neutrino, having a Dirac coupling to this equal mixture of $\nu_\mu - \nu_\tau$ [9]. Such a heavy neutrino can be motivated in a U(1) extension of the standard model, with the U(1) gauge charge corresponding to $(B - 3L_i)$, where one needs one right-handed singlet neutrino carrying an L_i number of 1 for anomaly cancellation [10]. This is analogous to the left-right symmetric model, corresponding to the U(1) gauge charge $(B - L_e - L_\mu - L_\tau)$, where one needs three right-handed neutrino singlets for anomaly cancellation. Such a U(1) extension of the standard model was recently constructed for the U(1) gauge charge $(B - 3L_\tau)$ [10] and its phenomenological implications studied [11]. Although one can make the right-handed singlet neutrino of this model to couple to a roughly equal mixture of $\nu_\mu - \nu_\tau$ by adjusting the model parameters, it will not be a natural feature of this model. To achieve this naturally, we must treat the μ and τ flavours on equal footing and distinguish them from e. Accordingly we shall consider the U(1) extension of the standard model, corresonding to the U(1) gauge charge,

$$Y' = B - 3L_e. \tag{6}$$

Moreover, we shall introduce a reflection symmetry via a multiplicative quantum number, N–parity, in order to avoid the coupling of the singlet neutrino with ν_e [12].

The leptons and Higgs scalars of the model are listed below with their $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ gauge charges, where the negative N–parity states have been identified by the subscript.

$$\begin{array}{ll}
\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L \sim (2,-1/2;-3) & \nu^c_{eL} \sim (1,0;3)_- \\
\begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}_L \sim (2,1/2;0) & \begin{pmatrix}\eta_1^+ \\ \eta_1^0\end{pmatrix}_L \sim (2,1/2;3) \\
\begin{pmatrix}\eta_2^+ \\ \eta_2^0\end{pmatrix}_L \sim (2,1/2;-3)_- & \chi^0 \sim (1,0;-6) \\
\zeta^0 \sim (1,0;-3) & \nu_S \sim (1,0;0)
\end{array} \tag{7}$$

Here one extra singlet neutrino ν_S with no Y' charge has been added for explaining the solar neutrino data. Since the mass M of this sterile neutrino is not protected by any symmetry, it is expected to be very high, going up to the GUT scale (10^{16} GeV). All the other new particles will acquire masses at an intermediate scale, corresponding to the spontaneous breaking of the $U(1)_{Y'}$ gauge symmetry. The resulting mass matrix for the five neutrino states in the basis $[\nu_e \;\; \nu_\mu \;\; \nu_\tau \;\; \nu_e^c \;\; \nu_S]$ is given by,

$$\begin{pmatrix}
0 & 0 & 0 & 0 & f'' <\eta_1> \\
0 & 0 & 0 & f_1 <\eta_2> & f_1' <\phi> \\
0 & 0 & 0 & f_2 <\eta_2> & f_2' <\phi> \\
0 & f_1 <\eta_2> & f_2 <\eta_2> & f <\chi> & 0 \\
f'' <\eta_1> & f_1' <\phi> & f_2' <\phi> & 0 & M
\end{pmatrix} \tag{8}$$

Note that the scalar ζ^0 does not contribute to the mass matrix. However, it provides a soft N–parity breaking term in the Lagrangian, which allows the model to avoid a potential domain-wall problem.

Both ζ^0 and χ^0 are expected to acquire large vacuum expectation values and masses at the scale of $U(1)_{Y'}$ breaking. In contrast, the $SU(2)$ doublets η_1 and η_2 are required to have positive mass-squared terms at this scale, so that they would have large masses but very small *vevs* $<\eta_1>$ and $<\eta_2>$ [13]. For example, $<\eta_2>$ can be estimated from the relevant part of the scalar potential,

$$m_2^2 \eta_2^\dagger \eta_2 + \lambda(\eta_2^\dagger \eta_2)(\zeta^\dagger \zeta) + \lambda'(\eta_2^\dagger \eta_2)(\chi^\dagger \chi) - \mu \phi^\dagger \eta_2 \zeta^\dagger \tag{9}$$

where the last term is the N–parity breaking soft term mentioned above. Although we start with a positive mass-squared term for the field η_2, after minimisation of the potential we find that this field acquires a small non-zero *vev* given by,

$$<\eta_2> = \frac{\mu <\phi><\zeta>}{M_2^2} \tag{10}$$

where $M_2^2 = m_2^2 + \lambda < \zeta^0 >^2 + \lambda' < \chi^0 >^2$ represents the physical mass of η_2 and $< \phi > \simeq 10^2$ GeV. One expects a similar value for $< \eta_1 >$. The size of the soft term can be anywhere up to the spontaneous symmetry breaking scale, *i.e.*, $\mu \leq M_2$.

In order to account for the desired neutrino masses and mixing we shall require the size of the *vev*s to be

$$< \eta_1 > \sim < \eta_2 > \sim 1\text{GeV}. \tag{11}$$

This would correspond to assuming $\mu \sim < \zeta > /100$ in (10). Alternatively one can get this with $\mu \sim < \zeta >$ and $M_2 \simeq m_2 \simeq 10 < \zeta >$. In either case one can get the required *vev* with reasonable choice of the mass parameters around the scale of the spontaneous symmetry breaking.

We shall now proceed to calculate the masses and mixing angles of the three light left-handed neutrinos by diagonalising the 5×5 mass matrix. Since we have added two singlet neutrinos, one of the doublet neutrinos will remain massless. This is also clear from the fact that the determinant of the mass-matrix (8) is zero. Let

$$a_1 = \frac{f_1 < \eta_2 >}{\sqrt{f < \chi >}}, \qquad a_2 = \frac{f_2 < \eta_2 >}{\sqrt{f < \chi >}},$$
$$b_1 = \frac{f_1' < \phi >}{\sqrt{M}}, \qquad b_2 = \frac{f_2' < \phi >}{\sqrt{M}}$$
$$c = \frac{f'' < \eta_1 >}{\sqrt{M}} \quad . \tag{12}$$

We then take the approximation $a_{1,2} \gg b_{1,2} \gg c$, which will be true for our parameter space of interest. The two nonzero light mass eigenvalues are now

$$m_1 \simeq a_1^2 + a_2^2, \tag{13}$$
$$m_2 \simeq \frac{(a_1 b_2 - a_2 b_1)^2}{a_1^2 + a_2^2}. \tag{14}$$

The $f'_{1,2}$ are Yukawa couplings of the standard model Higgs boson to ν_μ, ν_τ. Assuming them to be similar in size to the top quark Yukawa coupling as in SO(10) grand unified theories implies

$$f'_{1,2} \sim 1. \tag{15}$$

On the other hand, assuming them to be similar in size to the τ Yukawa coupling would imply

$$f'_{1,2} \sim 10^{-2}. \tag{16}$$

Comparing (5), (12), and (14), we see that the Yukawa couplings of (15) give

$$M \sim 10^{16} \mathrm{GeV} (\sim M_{GUT}), \tag{17}$$

while the Yukawa couplings of (16) imply

$$M \sim 10^{12} \mathrm{GeV}, \tag{18}$$

which is also a reasonable value.

Thus one can get the right mass for the matter-enhanced solution to solar neutrino oscillations for M in the range of 10^{12-16} GeV. Moreover we see from (12) and (13) that

$$m_1 \sim \left(\frac{f_1^2 + f_2^2}{f < \chi >} \right) \mathrm{GeV}. \tag{19}$$

Here the Yukawa couplings appearing in the numerator and denominator correspond to the scalars η_2 and χ^0 respectively. Assuming them to be of similar size, we see that any value of this Yukawa coupling in the range of (15) – (16) will give the required m_1 of equation (5) for

$$< \chi > \sim < \zeta > \sim 10^{8-10} \mathrm{GeV}. \tag{20}$$

Thus we can have the right mass for atmospheric neutrino oscillations for a reasonable scale of the $U(1)_{Y'}$ symmetry breaking and a reasonable range of the Yukawa couplings.

Let us now look at the mixing matrix connecting the neutrino flavour eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ to the mass eigenstates (ν_3, ν_2, ν_1), written in increasing order of mass. Because of the structure of (8) with a guaranteed zero mass eigenvalue, we can express this mixing matrix as a product of two matrices U_1 and U_2 corresponding to the atmospheric and solar neutrino mixing angles respectively, *i.e.*,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_1 U_2 \begin{pmatrix} \nu_3 \\ \nu_2 \\ \nu_1 \end{pmatrix}. \tag{21}$$

We get

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}, \tag{22}$$

where

$$\tan\theta_1 = \frac{\sin\theta_1}{\cos\theta_1} \simeq \frac{a_1}{a_2} \left(= \frac{f_1}{f_2} \right).$$

Note that $\tan\theta_1$ is simply the ratio of the Yukawa coulings of η_2 to ν_μ and ν_τ which are expected to be of similar size. Assuming them to be equal implies $\tan\theta_1 = 1$; *i.e.*, maximal mixing for atmospheric neutrino oscillation, $\sin^2 2\theta_1 = 1$. Moreover any value of this ratio in the range

$$0.64 < \frac{f_1}{f_2} < 1.56 \tag{23}$$

will ensure the near-maximal mixing condition of equation (1). Thus we can get the required mixing angle for atmospheric neutrino oscillations without any fine tuning of the Yukawa couplings.

Finally we get

$$U_2 = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{24}$$

where

$$\sin\theta_2 = \frac{c\sqrt{a_1^2 + a_2^2}}{a_1 b_2 - a_2 b_1} \simeq \frac{c}{b_{1,2}}.$$

Substituting (12) in (24) gives

$$\sin\theta_2 = \frac{f''}{f'_{1,2}} \frac{<\eta_1>}{<\phi>}. \tag{25}$$

This is to be compared with the required angle (3), which corresponds to

$$\sin\theta_2 = (1.6 - 5) \times 10^{-2}. \tag{26}$$

Assuming the Yukawa couplings to be of similar size, equation (25) gives the required mixing angle for

$$<\eta_1> \sim 1\text{GeV} \tag{27}$$

as mentioned earlier. There is a contribution to this angle from the charged lepton sector, which is however relatively small, as we see below.

We have been working in the basis where the charged-lepton mass matrix, arising from their couplings to the standard-model Higgs boson ϕ, is diagonal. However, there will be non-diagonal terms introduced by the Yukawa couplings of η_1 to $e\mu$ and $e\tau$ [7]; *i.e.*

$$M_\ell = \begin{pmatrix} m_e & 0 & 0 \\ f''_1 <\eta_1> & m_\mu & 0 \\ f''_2 <\eta_1> & 0 & m_\tau \end{pmatrix} \tag{28}$$

Its contribution to the ν_e mixing angle is

$$\sin\theta_2 = f_1'' < \eta_1 > \frac{m_e}{m_\mu^2} \sim f_1''\ 10^{-1} \leq 10^{-3}, \tag{29}$$

since the Yukawa coupling in this case is at most $\sim 10^{-2}$.

In summary, we have constructed a see-saw model containing two heavy singlet neutrinos. One of them carries a $(B - 3L_e)$ gauge charge and acquires an intermediate scale mass, corresponding to the spontaneous breaking of this gauge symmetry. The associated scalars have also masses at this scale. The other singlet neutrino is sterile and has a very heavy mass near the GUT scale. With these two mass scales and a reasonable range of Yukawa couplings, the model can naturally account for the near maximal mixing solution to the atmospheric neutrino oscillation and the small mixing MSW solution to the solar neutrino oscillation.

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