Microscopic Model for High-spin vs. Low-spin ground state in \([Ni_2M(CN)_8] (M = Mo^{V}, W^{V}, Nb^{IV})\) magnetic clusters

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Conventional superexchange rules predict ferromagnetic exchange interaction between \(Ni(II)\) and \(M (M = Mo^{V}, W^{V}, Nb^{IV})\). Recent experiments show that in some systems this superexchange is antiferromagnetic. To understand this feature, in this paper we develop a microscopic model for \(Ni(II) – M\) systems and solve it exactly using a valence bond approach. We identify the direct exchange coupling, the splitting of the magnetic orbitals and the inter-orbital electron repulsions, on the \(M\) site as the parameters which control the ground state spin of various clusters of the \(Ni(II) – M\) system. We present quantum phase diagrams which delineate the high-spin and low-spin ground states in the parameter space. We fit the spin gap to a spin Hamiltonian and extract the effective exchange constant within the experimentally observed range, for reasonable parameter values. We also find a region in the parameter space where an intermediate spin state is the ground state. These results indicate that the spin spectrum of the microscopic model cannot be reproduced by a simple Heisenberg exchange Hamiltonian.

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I. INTRODUCTION

In the expanding field of molecular magnetism, transition metal complexes have been the main focus of study. Room temperature bulk magnetization has been achieved with a hybrid organic radical/V(II) framework \(^1\) and also with the Cr(III)/V(II) Prussian blue type compound \(^2\). Besides, interesting conjectures such as the Haldane conjecture \(^3\) have been verified in molecular systems \(^4\). The interest in molecular magnetism has expanded in recent times to include low-dimensional architectures exhibiting slow relaxation of magnetization known as Single Molecule Magnets (SMMs) \(^5\), conducting magnets \(^6\) and light-triggered magnets \(^8\). The large majority of these molecular compounds are based on paramagnetic 3d metal ions, the second and third row transition metal ions have been envisaged only recently for the construction of magnetic supramolecular compounds \(^9\). These ions appear however very promising in molecular magnetism yielding magnets with fairly high refreshing temperatures \(^10\), new SMM’s \(^11\) and efficient systems with light triggered magnetization changes \(^12\). The 4d and 5d ions are characterized by spatially more extended valence orbitals, the extension following the trend \(3d < 4d < 5d\). A consequence of this ordering of the d-orbital is that the on-site electron repulsion is decreased as we go down the column in the periodic table. Besides, the metal-ligand bonds become more covalent resulting in more efficient electron delocalization. A series of experimental results obtained on simple cyano-bridged heterometallic species formed by self-assembling of an octacyanometallate \({\{Mo^{V}(CN)_{8}\}}^{3-}\) or \({\{W^{V}(CN)_{8}\}}^{3-}\) with a \(Ni^{II}L\) complex \((L = macrocyclic ligand)\) showed that the nature of the effective superexchange through the cyano-bridge depends upon the actual spin topology of the complex that is being studied. For instance, whereas significant ferromagnetic interactions are found for linear \(\{Ni – M – Ni\}\) compounds \((M = Mo^{V} or W^{V}\) in the \([M(CN)_{8}]\) unit), an antiferromagnetic behavior is observed for a cyclic tetranuclear compound (Figure 1). A related larger spin clusters of formula \([NiL]_{12}[Nb(CN)_{6}]_{6}\) exhibits an even more complex magnetic behavior \(^14\).

In this paper, we explain this puzzling nature of the superexchange interaction between two transition metal ions \(A\) and \(B\), each containing two nearly degenerate orbitals, by developing a microscopic model which naturally admits antiferromagnetic exchange interactions between them. The ion \(A\) has two electrons while \(B\) has one electron; these correspond to the case of \(Ni^{II}\) for the \(A\) ion and either \(Mo^{V}, W^{V}\) or \(Nb^{IV}\) for the \(B\) ion. The parameters entering the definition of the microscopic model are the on-site and inter-site parameters. The on-site parameters are both one and two-electron parameters. The one-electron parameters are energy differences between the active magnetic orbitals on \(A\) and \(B\), and the splitting of the two orbitals on \(B\). The two electron parameters are the electrostatic on-site Hubbard repulsion terms.

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and the inter-orbital Coulomb and exchange integrals. The inter-site one-electron parameters are the transfer integrals and the inter-site two-electron Coulomb interactions in the zero differential overlap approximation. We identify the critical interaction parameters that control the sign of the superexchange interaction and present a quantum phase diagram that shows a crossover in the ground state from effective ferromagnetic interaction to effective antiferromagnetic interaction between the ions. We show this behavior in different organizations of the AB system such as $A - B$, linear $A - B - A$, $B - A - B$, $A - B - A - B$, and cyclic $A - B - A - B$ clusters. In the next section we introduce the microscopic model for superexchange in the $Ni^{II}_{S=1} - M^{V}_{S=1/2}$ system. This is followed by a section in which we describe the valence bond (VB) method for solving the model. The last section deals with the results and their discussion.

II. MODEL HAMILTONIAN FOR SUPEREXCHANGE IN AB SYSTEMS

The nature of exchange interaction between two ions is the outcome of a competition between delocalization of the electrons which reduces the kinetic energy and direct exchange interactions which reduces the electron-electron repulsion energy for parallel spin alignment. Usually, the kinetic energy is decreased if the electrons are in a low-spin state; in the high spin state, the delocalization of electrons is blocked because of the Pauli exclusion. Besides, low-spin configuration also usually affords more phase space for electron delocalization. This is why the kinetic exchange is usually antiferromagnetic in nature. However, when there is a near degeneracy of the partially occupied orbitals on an ion, then delocalization is possible even when the spins are aligned parallel and indeed, the direct exchange interaction favours parallel electron spin alignment on such degenerate site. In this case, the final outcome cannot be easily predicted and depends upon the actual values of the interaction parameters. Thus, a microscopic model for explaining observed sign of the exchange interaction should include direct exchange interactions between nearly degenerate orbitals on a given site and the relative splitting of the degenerate orbitals. Besides these terms, another interaction term of considerable import is the strength of the intra-orbital electron repulsions relative to the strength of on-site inter-orbital electron repulsions. This is because, if the inter-orbital intra-site electron repulsions is weak compared to the intra-orbital electron repulsion, it will favour single occupancy of two orbitals on the same site over double occupancy of one of the orbitals. Occupancy of the orbitals controls the nature of the superexchange process, thus the relative strengths of on-site inter and intra orbital repulsions become very important. The other interaction terms which are comparable in strength to these repulsion integrals are the intersite electron-electron interactions within the zero differential overlap approximation and the intra-site electron repulsion between a charge density in one orbital and a charge density in the overlap cloud of two orbitals, which we call the $W$ term. The model Hamiltonian for investigating the superexchange interaction can be written as:

$$
\hat{H} = \sum_i \epsilon_i \hat{n}_i + \sum_{<ij>} t_{ij} (\hat{E}_{ij} + \hat{E}_{ji}) + \sum_i \frac{U_i}{2} \hat{n}_i (\hat{n}_i - 1) + \\
\sum_{i,i'} (U_{i,i'} \hat{n}_i \hat{n}_{i'} + \frac{W_{i,i'}}{2} [(E_{i,i'} + E_{i',i})(\hat{n}_i + \hat{n}_{i'}) + (\hat{n}_i + \hat{n}_{i'})(E_{i,i'} + E_{i',i}) - 2((E_{i,i'} + E_{i',i}) + \\
\frac{J_{i,i'}}{2} (E_{i,i'} E_{i',i'} + E_{i',i} E_{i',i}) + E_{i',i} E_{i,i'} + E_{i,i'} E_{i',i}) + E_{i,i'} E_{i',i} - \hat{n}_i - \hat{n}_{i'})] + \sum_{<ij>} \frac{V_{ij}}{2} \hat{n}_i \hat{n}_j;
$$

where, the operators $a_{i,\sigma}^\dagger$ ($a_{i,\sigma}$) creates (annihilates) an
electron in orbital $i$ with spin $\sigma$. The line first corresponds to the non-interacting part of the Hamiltonian with $\epsilon_i$ being the energy of the $i^{th}$ orbital, and $t$ the transfer integral between an orbital on one site and another on the neighboring site. All values of $t$ are assumed to be the same and the orbital energy of the A type atoms are fixed at zero, while the orbital energies of the B type atoms are $-\Delta$ and $(\Delta + \delta_B)$ as shown in Fig. 2. The second line corresponds to the intra-orbital interaction term, with $U_i$ being the Hubbard parameter. The remaining lines except the last represent the inter-orbital on-site electron repulsion terms, $U_{i,i'} = [ii'ii'] = \int \int \phi_i^*(1)\phi_i(1)\phi_{i'}(2)\phi_{i'}(2)d^3r_1d^3r_2$, $W_{ii'} = [ii'ii'] = \int \int \phi_i^*(1)\phi_i(1)\phi_{i'}(2)\phi_{i'}(2)d^3r_1d^3r_2$ and $J_{ii'} = [ii'ii'] = \int \int \phi_i^*(1)\phi_i(1)\phi_{i'}^*(2)\phi_{i'}(2)d^3r_1d^3r_2$, where $J_{ii'}$ is the exchange integral. Since we deal with only two types of ions A and B, and only two orbitals on each ion, we write these as $U_A$, $U_{A-A}$, $W_A$ and $J_A$ for the A ions and similarly for the B ions. The last line corresponds to the inter-site inter-orbital repulsion integral $V_{ij} = [iijj] = \int \int \phi_i^*(1)\phi_i(1)\phi_j^*(2)\phi_j(2)d^3r_1d^3r_2$ and is parametrized using Ohno parametrization [16] and appropriate scaling [17].

III. SOLUTION OF THE SUPEREXCHANGE MODEL

The Hamiltonian in eqn. 4 is nonrelativistic and hence conserves total spin. Since we are interested in the solution of the model in different total spin sectors, we use the valence bond technique for solving the model. In this technique, the complete and linearly independent basis states with a given total spin can be generated using explicit spin pairings [10]. For example, if singly occupied sites $i$ and $j$ are spin paired, then a line is drawn between sites $i$ and $j$ in the VB diagram to indicate the spin pairing, $(\alpha_i\beta_j - \beta_i\alpha_j)/\sqrt{2}$, $i < j$. We say that the line begins at site $i$ and ends at site $j$. If the spins at sites $i_1, i_2 \cdots i_l$ are not paired, we pass an arrow through these sites in the VB diagram and this denotes the state $\alpha_{i_1}\alpha_{i_2} \cdots \alpha_{i_l}$. Because the Hamiltonian conserves $S^z$, it is sufficient to work in the subspace $M_S = S$. In the $S = 0$ subspace, a VB diagram has either empty and doubly occupied sites (represented by dots and crosses respectively) or lines between singlet paired singly occupied sites. A VB diagram involving $N$ orbitals can be drawn by arranging the orbitals at the vertices of a regular $N$-gon and drawing straight lines between vertices, the electrons at whose orbitals are singlet paired.

The Rumer-Pauling rule states that all such VB diagrams with no intersecting lines form a complete and linearly independent set [19]. The complete and linearly independent set of VB states in nonzero spin space can be obtained by taking recourse to modified Rumer-Pauling rules [13]. Some typical VB diagrams are shown in Fig. 3. Each orbital in the VB picture has one of four possibilities: (i) the orbital is empty, (ii) the orbital is doubly occupied, (iii) a line begins at the orbital or (iv) a line ends at the orbital. It is possible to associate these four possibilities of an orbital in a VB diagram with the four states of two bits in an integer, identified with the orbital. Thus, for a eight orbital system, the VB diagrams are represented by a sixteen bit integer and these integers can be generated in an ascending order. In any given spin space, the effect of the operator term $E_{ij}$ in the Hamiltonian on a basis state is to alter the orbital occupancy of orbitals $i$ and $j$ (subject to Pauli principle) and pair the spins in the orbitals that were involved to yield a new VB diagram with a known amplitude. If the new VB diagram violates Rumer-Pauling rules, it is trivially possible to express it as a linear combination of the basis VB states.

Using the above procedure, the Hamiltonian matrix can be set-up in the chosen total spin sector. The resulting Hamiltonian matrix is sparse (since the number of terms in the Hamiltonian is far smaller than the dimension of the complete VB space) and nonsymmetric and can be partially diagonalized to obtain a few of the low-lying eigenstates, using Rettrups modification of the Davidson algorithm, when the Hilbert space dimensional-
The Hamiltonian matrix is resorted to in each spin sector encountered is of dimensionality 1512 for the S=1 subspace and the value of the integer I, representing the VB diagram, numbers 1 through 8 are the site indices and the bits representing the site in the integer I. The bit-pattern values at the bottom against each site gives the bit state of the sites and crosses (×) represent doubly occupied sites. In each VB diagram, numbers 1 through 8 are the site indices and the values at the bottom against each site gives the bit state of the bits representing the site in the integer I. The bit-pattern and the value of the integer I, representing the VB diagram are also shown.

FIG. 3: Representative VB diagrams in (a) S=0, (b) S=1, (c) S=2 and (d) S=3 subspaces. Dots (●) represent empty sites and crosses (×) represent doubly occupied sites. In each VB diagram, numbers 1 through 8 are the site indices and the values at the bottom against each site gives the bit state of the bits representing the site in the integer I. The bit-pattern and the value of the integer I, representing the VB diagram are also shown.

IV. RESULTS AND DISCUSSION

The range of values on-site electron repulsion integrals take is known for the transition metal ions and it is reasonable to assume that $U_A$, when A represents Ni$^{2+}$ is smaller than $U_B$ when B corresponds to M (Mo$^V$, W$^V$, Nb$^V$), since the higher charge on the M ion should make the orbitals more compact, resulting in larger intra-orbital electron repulsion integrals. It is reasonable to assume $U_A$ to be 6 eV and $U_B$ to be 8 eV. Indeed, we have also verified that the results we present do not change qualitatively when these parameters are allowed to vary by up to 2 eV about the mean values. The parameters $U_{A-B}$ and $U_{B-B}$ are slightly smaller than the corresponding intra-orbital repulsion integral. The integrals $W_A$ and $W_B$ are much smaller than the $U_{A-A}$ or the $U_{B-B}$ integrals as they involve overlap charge densities and both $W_A$ and $W_B$ have been fixed at 1 eV. The exchange integrals $J_A$ and $J_B$ are somewhat smaller than the integrals $W_A$ and $W_B$. We also note that since it is experimentally known that the spin on the A ion ($N_i^{2+}$) is always one we fix $J_A$ at a somewhat large value of 0.7 eV. Large $J_A$ reduces the repulsion between electrons with parallel spin alignment compared to antiparallel spin alignment in the two orbitals on site $A$ leading to a spin-1 object on site $A$ and this holds true even when the degeneracy of the two orbitals on the $A$ site is slightly lifted. For this reason, we have assumed the two orbitals on $A$ to be degenerate. In each case, we have also verified that the expectation value of $\hat{S}_A^z$ operator is nearly 2.0 confirming that the spin on the $A$ site is very nearly one in all cases. The total spin of the system in the ground state is sensitive to the parameters $\delta_B$, $J_B$ and $U_{B-B}$. A large $\delta_B$ would result in an antiferromagnetic exchange interaction since the virtual state with a doubly occupied lower orbital on the $B$ site has a lower energy than spin one state on the $B$ ion. A large $J_B$ would however lower the energy of the virtual state in which the $B$ ion has a spin one configuration. A small $U_{B-B}$ would also favour a spin one virtual state on the $B$ ion by favouring single occupancy of the two active $B$ ion orbitals. We have solved the model Hamiltonian over a wide range of the parameters and have obtained the quantum phase diagrams for demarcating the low-spin and high-spin ground states in this parameter space.

In Figures 4, 5 and 6 we have presented the contours of superexchange values ($J_S$) in these systems for various values of the model parameters obtained by fitting the low-spin - high-spin gap to a spin Heisenberg Hamiltonian involving spin-1’s at $A$ sites and spin-1/2’s at $B$ sites. The solid line corresponding to $J_S = 0$ provides the phase boundary between the high-spin and low-spin ground states. The contours corresponding to fixed $J_S$ values and represented by dotted lines are obtained by spline interpolation using MATLAB. We note that small $U_{B-B}$, small $\delta_B$ and large $J_B$ values promote a high spin ground state while large $U_{B-B}$, large $\delta_B$ and small $J_B$ values promote a low spin ground state. We also note that the superexchange $J_S$ values, for the same model parameters, are larger for smaller system sizes.

A. Phase diagram in the $U_{B-B} - J_B$ plane

In the $U_{B-B} - J_B$ plane, we should expect to see the high-spin ground state for large $J_B$ and small $U_{B-B}$. This is because the large exchange integral on the $B$ site will favour parallel alignment of the electrons on the $B$
site, if the virtual transfer of the electron does not lead to a doubly occupied $B$ site orbital. The latter requirement is guaranteed if $U_{B-B}$ is small. In Fig. 4 we see that in all the cases, the ground state corresponds to the high spin state for small $U_{B-B}$ and large $J_B$. We also note from the shift in the phase line to the right, that with increase in system size the high-spin state is favoured for larger $U_{B-B}$ values. From the gap between the high spin and the low spin states, we have also calculated the effective superexchange parameter. We find that the largest ferromagnetic superexchange $J$ value is 101.42 K (70.49 cm$^{-1}$) for $U_{B-B} = 5$ eV and $J_B = 0.5$ eV, while the largest antiferromagnetic superexchange $J$ value corresponds to -33.36 K (-23.19 cm$^{-1}$) for $U_{B-B} = 7$ eV and $J_B = 0.1$ eV in the linear $A-B$ cluster. Another interesting feature to note is that in the $B - A - B$ cluster, the low-spin ground state appears for a much smaller $U_{B-B}$ value, for a given $J_B$ than in the $A - B - A$ cluster.

**B. Phase diagram in the $U_{B-B} - \delta_B$ plane**

The phase diagram in $U_{B-B} - \delta_B$ plane for all the four systems is shown in Fig. 4. We note that the ground state spin is extremely sensitive to the $\delta_B$ value. Even a small splitting of the $B$ site orbital forces the system into a low-spin ground state. This is because the lifting of the degeneracy of the $B$ site orbital favours doubly occupied lower energy orbital in the virtual state, which would result in stabilization of the low-spin state. Here again, we note that the high spin ground state is more robust in the larger clusters. In the cyclic $A - B - A - B$ cluster, for $U_{B-B} = 5.5$ eV, the ground state shifts to low-spin state for $\delta_B = 0.063$ eV while in the $A - B - A$ cluster, this occurs at $\delta_B = 0.04$ eV and in the $A - B$ system the ground state ceases to be the high spin state for $\delta_B > 0.016$ eV. The highest ferromagnetic superexchange constant $J$ is observed for $\delta_B = 0$ and $U_{B-B} = 5$ eV while the highest antiferromagnetic $J$ is observed for $\delta_B = 0.1$ eV and $U_{B-B} = 7$ eV in all the
clusters. We also can note that for a given $\delta_B$ value, we have a low spin ground state in the $B - A - B$ system, at much smaller value of $U_{B-B}$ value than in the $A - B - A$ system.

C. Phase diagram in the $J_B - \delta_B$ plane

Large values of $\delta_B$ has the effect of promoting the antiferromagnetic superexchange while a large $J_B$ favours ferromagnetic superexchange. Thus, we see from Fig. 5 that the high spin state is the ground state below the phase line while the low-spin state is the ground state above the phase line. The phase line shifts higher in the $J_B - \delta_B$ plane as the system size is increased thereby showing that the high spin ground state is more stable to lifting of the degeneracy of the orbitals on the $B$ site for larger system sizes. In this plane, the behaviour of the $A - B - A$ and the $B - A - B$ systems are almost identical.

One very interesting feature not clearly seen in the phase diagrams of the cyclic $A - B - A - B$ cluster is the appearance of the intermediate spin $S=2$ ground state over a very narrow range of parameter values between the $S=1$ and the $S=3$ ground states. The intermediate spin ground state is seen only in the largest cluster we have studied. In fact, in the linear $A - B - A - B$ cluster, this region extends over a wider parameter values, as seen in Fig 6. Thus, it may be possible to synthesize high nuclearity complexes in intermediate spin ground states. The physics behind the existence of such ground states is however somewhat different from the frustrated magnetic exchanges present in SMMs. In our case, it is difficult to identify the exchange interaction between two sites as either ferro or antiferromagnetic and magnetism can be viewed as a whole and not pair-wise, as is usually the case. Besides, it is also not possible to map our model onto a simple Heisenberg exchange Hamiltonian.

To conclude, we have developed a model Hamiltonian which admits both low-spin and high-spin ground states.
for small changes in the values of the model parameters. The only model parameter that can perhaps be determined directly for a system is $\delta_B$, from spectroscopic data. Other parameters can only be inferred indirectly either from electron spectroscopic studies on simpler systems or from ab initio model system calculations. Our model can explain the observed antiferromagnetic exchange in the Nb$_6$Ni$_{12}$ and related systems, contrary to superexchange rules [13, 14]. The model also yields reasonable effective superexchange constants for model parameters in the accepted range.

![FIG. 6: Contours of the effective superexchange constants ($J_S$ cm$^{-1}$) of (a) $A - B$, (b) $A - B - A$, (c) $B - A - B$ chains and (d) cyclic $A - B - A - B$ systems as a function of $J_B$ and $\delta_B$. The phase diagrams are obtained for $t=0.1$ eV, $\Delta=0.0$ eV; $U_A=6$ eV; $U_B=8$ eV; $U_{A-A}=4$ eV; $U_{B-B}=5.5$ eV; $J_A=0.7$ eV; $W_A=W_B=1$ eV. All the systems display high-spin ground state at lower $\delta_B$ and higher $J_B$ values.](image)

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FIG. 7: Phase diagrams of linear $A-B-A-B$ system in (a) $U_{B-B} - J_B$ (b) $U_{B-B} - \delta_B$ and (c) $J_B - \delta_B$ planes. $S_{GS}$ is the ground state spin of the system. It is interesting to see the appearance of intermediate spin ground state $S_{GS} = 2$ for a wide range of parameter values, which is not prominent in the cyclic $A-B-A-B$ system.