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ABSTRACT

A crossing-symmetric Regge-behaved amplitude is proposed. With arbitrary Regge trajectories, it avoids ancestor and ghost difficulties and possesses second-sheet resonances which may be finite in number. The N-point and $\pi\pi$ amplitudes are given.

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A crossing-symmetric Regge-behaved dual amplitude, with second-sheet poles and correct Mandelstam boundaries, was recently proposed by Cohen-Tannoudji, Henyey, Kane and Zakrzewski [1]. This amplitude, however, possesses multiple poles which degenerate into an essential singularity when $\alpha \to \infty$; clearly a very undesirable feature. It is our purpose, in this letter, to introduce an amplitude which, while similar in many respects to the amplitude of Cohen-Tannoudji et al., avoids this difficulty. This is done by use of the technique of Van der Corput neutralizer functions [2], earlier employed by Suzuki [3] in constructing an amplitude with complex trajectories.

The model we propose possesses several features besides crossing symmetry, Regge behaviour, simple complex poles in all channels and no ancestors. It accommodates arbitrary trajectories (including trajectories with a finite number of resonances), has a factorizable N-point generalization and a $\pi\pi$-amplitude in which the Adler zero may be incorporated. The region of analyticity of the amplitude is discussed. The Mandelstam double-spectral functions vanish for $s \leqslant 4m^2$, $t \leqslant 4m^2$, and appear to be very small outside the region $1/s + 1/t \geqslant 4m^2$.

We propose the following amplitude for two-particle $\to$ two-particle spinless scattering:

$$A(s,t) = \int_0^1 x^{-(1+\alpha(t)) N(x)} (1-x)^{-(1+\alpha(s)) N(1-x)} f(sx)\, f(t(1-x))\, dx \qquad (1)$$

where

i) $f(z)$ is analytic except for a cut in Re $z \geqslant 4m^2$,

ii) $N(x)$ is a Van der Corput neutralizer satisfying [4]

$$N(0) = 1\ ,\ N(1) = 0,\ \left.\frac{d^n N(x)}{dx^n}\right|_{x=0,1} = 0, \qquad n = 1,2,\ldots \quad . \tag{2}$$

iii) $\alpha(s)$ is the Regge trajectory function with a cut for $s \geqslant 4m^2$.

Besides being manifestly crossing-symmetric, the amplitude (1) possesses several interesting features which we now proceed to discuss.

a) Regge behaviour

Write

$$A(s,t) = A_1(s,t) + A_2(s,t) \tag{3}$$

where

$$A_1(s,t) = \int_0^{\lambda} x^{-(1+\alpha(t))\,N(x)}\,(1-x)^{-(1+\alpha(s))\,N(1-x)}\,f(sx)\,f(t(1-x))\,dx \qquad 0<\lambda<1 \quad . \tag{4}$$

By a change of variable

$$A_1(s,t) = \frac{1}{s}\int_0^{\lambda s} (s^{-1}\mu)^{-(1+\alpha(t))N(s^{-1}\mu)}\,(1-s^{-1}\mu)^{-(1+\alpha(s))\,N(1-s^{-1}\mu)}\,f(\mu)\,f(t-s^{-1}t\mu)\,d\mu\ .$$

Expanding the integrand about $s^{-1} = 0$ and using (2) - noting that s^{-1} is real for all μ on the path of integration - we obtain the asymptotic behaviour

$$A_1(s,t) \sim g(t)\,s^{\alpha(t)} \qquad \text{as} \quad s\to\infty \tag{5}$$

where

$$g(t) = f(t)\int_0^{\infty} \mu^{-1-\alpha(t)}\,f(\mu)\,d\mu \qquad , \tag{6}$$

provided that $f(x)$ is chosen such that this integral converges. Under this condition the asymptotic behaviour (5) holds for all complex s. The integral (6) is, in general, over a contour in the complex μ-plane extending from the origin to the point at infinity.

For the convergence of the integral (6) we require

$$f(\mu) < \mu^{\alpha_m} \tag{7}$$

where α_m is the maximum negative value attained by $\mathrm{Re}\,\alpha(t)$ for all $t < 0$. If $\alpha(t)$ is such that the value $\alpha = -\infty$ is possible, then $f(\mu)$ must tend to zero faster than any inverse power of μ as $\mu \to \infty$.

Taking the fixed number λ to be very close to unity, we obtain for $A_2(s,t)$ as s tends to ∞

$$A_2(s,t) = \int_\lambda^1 x^{-(1+\alpha(t))N(x)} (1-x)^{-(1+\alpha(s))N(1-x)} F(sx)\, f(t(1-x))\, dx$$

$$< s^{\alpha_m} \int_\lambda^1 x^{-(1+\alpha(t))N(x)} (1-x)^{-(1+\alpha(s))N(1-x)} x^{\alpha_m} f(t(1-x))\, dx$$

$$= s^{\alpha_m} \sum_{n=0}^{\infty} h_n(t) \int_\lambda^1 (1-x)^{-1-\alpha(s)+n} dx + s^{\alpha_m} \int_\lambda^1 k(x)\, dx$$

$$= s^{\alpha_m} \left\{ \sum_{n=0}^{\infty} \frac{(1-\lambda)^{n-\alpha(s)}}{n-\alpha(s)} + \int_\lambda^1 k(x)\, dx \right\}, \qquad \mathrm{Re}\,\alpha(s) < 0 \quad .$$

$K(x)$ is a function that tends to zero faster than any power of $(1-x)$ as $x \to 1$. When λ is made to approach unity this term vanishes provided that the limit in s is taken such that $\mathrm{Re}\,\alpha(s) < 0$. Thus we obtain, for fixed $t < 0$

$$A(s,t) \sim g(t)\, s^{\alpha(t)} \quad \text{as} \quad s \to \infty \quad , \ \mathrm{Re}\,\alpha(s) < 0 \quad . \tag{8}$$

This asymptotic behaviour may now be continued, in the α-plane, to $\mathrm{Re}\,\alpha > 0$, provided that we exclude all the points $\alpha = n$, $n = 0,1,2,\ldots$. In the s-plane we see that if $\mathrm{Re}\,\alpha(s) \to \infty$ as $|s| \to \infty$ in some direction, then such a direction must be excluded in taking the limit for large s. Otherwise, the asymptotic behaviour (8) is valid for all directions in the s-plane.

b) <u>Second-sheet resonances</u>

The poles in t are generated as end-point singularities at $x = 0$. Expanding the integrand in (1) about $x = 0$, following a trivial decomposition, one obtains

$$A(s,t) = \sum_{n=0}^{\infty} \frac{R_n(s,t)}{n - \alpha(t)} + B(s,t) \, , \qquad \mathrm{Re}\,\alpha(t) < 0 \tag{9}$$

where

$$B(s,t) = \int_0^1 \left\{ x^{-(1+\alpha(t))\,N(x)} \, (1-x)^{-(1+\alpha(s))\,N(1-x)} - x^{-(1+\alpha(t))} \right\} f(sx)\, f(t(1-x))\, dx \tag{10}$$

and the functions $R_n(s,t)$ are polynomials in s of order n given by

$$R_n(s,t) = \frac{1}{n!} \left\{ s^n f^{(n)}(0)\, f(t) + n\, s^{n-1} f^{(n-1)}(0)\, (-t)\, f'(t) + \ldots + f(0)\, (-t)^n f^{(n)}(t) \right\} . \tag{11}$$

The residue at $\alpha = n$ is $R_n(s,t_n)$ where t_n is such that $\alpha(t_n) = n$. We observe that:

i) The poles in the sum in Eq.(9) are complex poles in t.

ii) Although the trajectory function is essentially arbitrary, no ancestors are introduced.

iii) The number of resonances is not necessarily infinite.

iv) The residues are completely determined by the function $f(x)$ and are independent of any particular choice we make for the neutralizer $N(x)$.

v) The function $B(s,t)$ contains no poles in t and does not contribute to the asymptotic behaviour (8) as $s \to \infty$. Thus only the first term in (9), containing all the resonances in t, gives the dominant Regge pole behaviour as $s \to \infty$. It is in this sense that the amplitude is dual [5].

vi) It appears to be possible to eliminate daughters, at least from the four-point function, by a suitable choice of $F(x)$.

c) Analyticity boundary

Because of the presence of $\alpha(t)$ and $\alpha(s)$ in the integrand in (1), the domain where the double-spectral function is non-zero is the sharp-corner region $s \geq 4m^2$, $t \geq 4m^2$. We note, however, that for both the high-energy region and the neighbourhood of resonances, we again almost have a curved boundary of analyticity. Thus for large s, for example, our amplitude is for all practical purposes given by

$$A(s,t) \approx \int_0^1 x^{-1-\alpha(t)} f(sx)\, f(t(1-x))\, dx \tag{12}$$

which has the boundary $1/s + 1/t \geq 1/(4m^2)$, as in Ref. 1. The same form also gives the amplitude in the low t region where the resonances dominate. In fact, one may directly verify that the contribution of the factor

$$x^{-(1+\alpha(t))\,N(x)}\,(1-x)^{-(1+\alpha(s))\,N(1-x)}$$

to the integrand for $\rho(s,t)$ tends to zero faster than any power. Thus to a good approximation $\rho(s,t)$ vanishes outside $1/s + 1/t \geqslant 1/(4m^2)$, for any choice of neutralizer function $N(x)$.

d) N-point function

Generalization of the amplitude to the N-point function is straightforward and is similar to the corresponding situation in the Veneziano model. We write

$$A_N = \int \prod_{P} du_P\,(u_P)^{-(1+\alpha(s_P))\,N(u_P)} \quad f(s_P(1-u_P) \prod_{P' \neq (1,j)} \delta\Big(u_{P'} - \prod_{\bar{P}'} u_{\bar{P}'} - 1\Big) \qquad (13)$$

where P labels the various possible partitions [6], s_P the corresponding energy variable and $\bar{P}$ denotes channels dual to P. The factorized form of the integrand ensures both the bootstrap principle and the general factorization for residues. To see this, notice that as $u_P \to 0$, all variables $u_{\bar{P}} \to 1$. The residue of the pole at $\alpha(s_P) = 0$ is obtained from the integrand as $u_P \to 0$. On choosing a suitable normalization one gets

$$\text{Res. of } A_N\Big|_{\alpha(s_P)=0} = A_{N-m+1}\,A_{m+1} \qquad ,$$

where the partition P separates the N interacting particles into two groups of m and $N-m$ particles, respectively.

The residues at other integer values of $\alpha(s_P)$ are obtained from the coefficients of the corresponding powers of u_P in the expansion of the integrand about $u_P = 0$. One notes that, due to the properties of the function $N(x)$, such an expansion arises entirely from the function $f(x)$. It may therefore be possible to curtail severely degeneracies of the poles by a judicious choice of $f(x)$.

e) $\pi\pi$-amplitude

A simple modification in the amplitude makes it suitable for trajectories with positive intercept. The problem is to eliminate the pole at $\alpha(s) = 0$ without changing either the high-energy behaviour or crossing symmetry. We propose

$$A(s,t) = \int_0^1 dx\, x^{-(1+\alpha(s))\,N(x)}\,(1-x)^{-(1+\alpha(t))\,N(1-x)}\, f(s(1-x),1-x)\; f(tx,x) \quad , \tag{14}$$

where we have made f depend upon two variables. If we now impose $f(0,0) = 0$, the power series expansion of the integrand starts with the term $x^{-\alpha(s)}$ and the first pole is situated at $\alpha(s) = 1$. High-energy behaviour is not affected. The N-point generalization of this amplitude is immediate.

The Adler condition $A(m_\pi^2, m_\pi^2) = 0$ requires that the integral

$$\int_0^1 x^{-(1+\alpha(m_\pi^2))\,N(x)}\,(1-x)^{-(1+\alpha(m_\pi^2))\,N(1-x)}\, f(m_\pi^2 x, x)\; f(m_\pi^2(1-x), 1-x)\; dx \tag{15}$$

vanishes. The integrand in (15) is symmetric about $x = \frac{1}{2}$ and behaves like $x^{-\alpha(m_\pi^2)}$ near $x = 0$ (using $f(0,0) = 0$). For the integral to vanish, it is therefore necessary that $f(m_\pi^2 x_0, x_0) = 0$ for at least one point $x_0 \in (0,\frac{1}{2})$. Such a condition can easily be satisifed and it is then clearly possible to incorporate the Adler zero in the present model [7].

f) Remarks

We wish to emphasise that the use of the neutralizer function has prevented the occurrence of terms like [1] $\sum_{k=0}^{\infty} \frac{s^j}{[j-\alpha(t) + n+k]^{k+1}}$ which - besides introducing multipoles - have the undesirable feature that a non-linear trajectory α ends up as an essential singularity as $\alpha \to \infty$. By using the neutralizer function, which kills all multipoles, we have retained most of the nice features of the model of Ref.1 and at the same time avoided this catastrophe.

There is in our model sufficient freedom to ensure correct threshold behaviour and perhaps some unitarity effects without affecting the crossing symmetry and asymptotic behaviour of the amplitude. As argued in Ref.1, such arbitrariness is expected since unitarity is not yet fully imposed. However, since amplitude (1) has second-sheet resonance poles, correct Regge asymptotic behaviour for essentially arbitrary α, non-vanishing double-spectral functions and a factorizable N-point generalization, it may be expected to serve as a starting point in a search for an exactly unitary amplitude.

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1) G. Cohen-Tannoudji, F. Henyey, G.L. Kane and W.J. Zakrzewski, Phys. Rev. Letters 26, 112 (1971).

2) See, E.T. Copson, Asymptotic Expansion (CUP, London 1965).

3) M. Suzuki, Phys. Rev. Letters 23, 205 (1969).

4) There exist many functions which satisfy the conditions in Eq.(2). An example of such a function is

$$\frac{1}{c} \int_x^1 \exp\left(-\frac{1}{u} - \frac{1}{1-u}\right) du, \quad c = \int_0^1 \exp\left(-\frac{1}{u} - \frac{1}{1-u}\right) du \quad .$$

However, the main features of our amplitude do not depend on the precise form of $N(x)$.

5) Obviously, duality in the sense that the sum over t-channel resonances equals the sum over s-channel resonances cannot be maintained in an amplitude that allows for a finite number of resonances.

6) We use the notation of H-M. Chan, Proc. Roy. Soc. (London) A318, 379 (1970).

7) It is clear that in our approach the Adler condition is a dynamical requirement imposed on $f(x,y)$ such that the integral (15) vanishes. Other similar approaches that produce an Adler zero do so either by an explicit kinematical restriction [see A.I. Bugrij, L.L. Jenkovsky and N.A. Kobylinsky, Kiev preprint, ITF-71-28E (1971)], or in the context of an amplitude for scalar particles [see Ref.1].

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