Unitarity constraints on the stabilized Randall-Sundrum scenario

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Abstract

Recently proposed stabilization mechanism of the Randall-Sundrum metric gives rise to a scalar radion, which couples universally to matter with a weak interaction ($\simeq$ 1 TeV) scale. Demanding that gauge boson scattering as described by the effective low enerrgy theory be unitary upto a given scale leads to significant constraints on the mass of such a radion.

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1 Introduction

The quest to solve the hierarchy problem that plagues the otherwise successful Standard Model (SM) has, over the years, prompted many a plausible extension. Recently, it has been proposed that quantum gravity could provide a mechanism to stabilize the Higgs mass and thereby 'solve' the problem [1,2]. Such theories argue that if the visible world were restricted to a $(3 + 1)$-dimensional hyper-surface of a larger dimensional world, then the natural scale for gravity (which propagates in the entire bulk) could, conceivably, be as low as $\mathcal{O}(1 - 10\ \text{TeV})$. Amongst these proposals is one by Randall and Sundrum [2] wherein the SM fields live on one of the two 3-branes which themselves define the ends of the world in the context of a five dimensional spacetime. The spacetime geometry is nonfactorizable and contains an exponential warp factor relating the induced metrics on each of the two branes. This warp factor clearly produces a difference in the mass scales between the two end of the world 3-branes and consequently the (low) natural scale for quantum gravity appears, to us, to be incredibly large.

An issue of particular importance in such models concerns the stability of the inter-brane distance (modulus) for this is a key to the 'solution' of the hierarchy problem. An elegant resolution was provided by Goldberger and Wise (GW) [3][5] who introduced a

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bulk scalar field into the model coupled it minimally to the bulk gravity. This simple construction provides a nontrivial potential for the radion modulus thereby stabilizing it. It was subsequently shown [6] that this mechanism, with minor modifications, serves to stabilize multibrane configurations as well.

The mass of the radion field, given by the behaviour of the above mentioned potential close to its minimum, can be quite low. Consequently, it could be expected to play a nontrivial role in low-energy phenomenology. In fact, so could the spin-2 gravitons. The implications of such interactions have been examined in the literature quite extensively [7–9]. Not unexpectedly, apart from collider phenomenology, the introduction of such a radion as well as the Kaluza-Klein tower of gravitons alters the cosmological evolution of the world to such an extent that even the familiar Hubble expansion parameter's dependence on matter density in the universe differs from the conventional one [10, 11]. However, subsequent studies [12] have shown that a stabilization mechanism, such as the one mentioned above, can also serve to reconcile the RS scenario to known cosmological observations.

In this paper we shall strive to examine the role of the radion in the context of gauge boson or heavy fermion scattering. It is normally expected that, well below the quantum gravity scale, it should be possible to treat the RS scenario as a field theory, albeit a nonrenormalizable one. The backbone of the theory is given by a renormalizable gauge theory (the SM) with the RS character manifesting itself in the form of certain additional (and potentially nonrenormalizable) terms in the effective theory. Well below the RS scale, then, the theory should look almost unitary. This is the aspect that we propose to investigate. Although some work has been done in this area [8], the choice of processes therein was not optimal and hence the bounds were rather weak.

In its simplest version, the RS scenario is described by a metric

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2 \ , \tag{1}$$

where x^μ are the ordinary 4-dimensional coordinates while $y \in [-\pi, \pi]$ parameterizes a S^1/Z_2 orbifold. The metric clearly describes a slice of AdS_5 space with a volume radius r_c and a curvature radius k^{-1}. For the above metric to be a solution of Einstein's equations, the bulk must have a negative cosmological constant and the two end-of-the-world branes at $y = 0$ and $y = \pi$ must have positive and negative tension respectively. An observer on the $y = \pi$ brane experiences a red-shift $e^{-kr_c\pi}$ for all its mass parameters with respect to an observer living at $y = 0$. Thus, if the $y = \pi$ brane is assumed to be the visible one, and if $kr_c\pi \sim 35$, the large hierarchy in the ratio $M_{\rm weak}/M_P$ could be explained naturally.

To be treated as a field theory of gravitation, the metric of eq.(1) needs to be promoted to space time dependent fields. The substitution $\eta_{\mu\nu} \to g_{\mu\nu}(x)$ incorporates both the massless 4-dimensional graviton and its Kaluza-Klein (KK) counterparts, while $r_c \to T(x)$ describes the spin-0 modulus field[6] The volume radius r_c is thus nothing but the expectation value of the modulus field $T(x)$. Explaining the hierarchy between the Planck scale and the electroweak scale thus requires $\langle T(x) \rangle \sim 35/\pi k$. Goldberger and Wise achieve this naturally by postulating an extra bulk scalar field coupled minimally

[6]The components $g_{5\mu}$ of the full metric, on KK reduction, result in vector fields which, however, do not couple to the SM fields at the lowest order.

to gravity [3]. The quantization of the modulus is best done in terms of a redefined field

$$\varphi \equiv \langle\varphi\rangle e^{-k\pi(T-r_c)} \qquad \langle\varphi\rangle = \sqrt{\frac{24M^3}{k}} e^{-k\pi r_c} \tag{2}$$

where M is the Plank mass in the 5-dimensional theory. Apart from stabilising the modulus, the GW potential has the additional consequence that the mass of the radion field φ is much smaller than that of the lowest lying KK-excitation of the graviton. Thus, in such a scenario, the radion is more likely to play a significant role in weak-scale phenomenology than the graviton excitations.

It is easy to see that, at the lowest order, the radion field couples to the SM matter only through the trace of the energy momentum tensor [4, 10]. For massive fields, then, the relevant interaction terms in the effective theory Lagrangian are given by

$$\mathcal{L}_{\rm int} = \frac{\varphi}{\langle\varphi\rangle} T^\mu_\mu \tag{3}$$

For massive fermions and vector fields, this reduces to

$$\mathcal{L}_{\rm int} = \frac{\varphi}{\langle\varphi\rangle} \left[m_\psi \bar{\psi}\psi + m_V^2 V_\mu V^\mu \right] \tag{4}$$

The radion coupling to ordinary matter is clearly analogous to that of the SM Higgs, albeit with a different coupling strength. The radion, thus, could be looked for in observables wherein the Higgs plays an important role. These range from direct production (in associated Bjorken process or gg fusion) to radiative effects such as in the electroweak precision data.

A light radion could also signal its presence in $t\bar{t}$ or vector boson scattering. It is well known that, within the SM, the Higgs plays an essential role in restoring the perturbative unitarity of such scattering processes. With the radion playing the role of an additional Higgs-like state, it is conceivable that the extra contribution due to a radion exchange could destroy the high-energy behaviour of such an amplitude.

Unitarity of gauge boson scattering in the SM has been well studied in the literature [13, 14]. For longitudinally polarized gauge bosons, the s-wave amplitudes are given by

$$\begin{aligned}
\mathcal{M}^{\rm (SM)}(Z_L Z_L \to Z_L Z_L) &= \frac{-is}{v^2}\, g_{ZZ}(\tilde{h}) \\
\mathcal{M}^{\rm (SM)}(W_L^+ W_L^- \to W_L^+ W_L^-) &= \frac{-is}{v^2}\, g_{WW}(\tilde{h}) \\
\mathcal{M}^{\rm (SM)}(Z_L Z_L \to W_L^+ W_L^-) &= \mathcal{M}^{\rm (SM)}(W_L^+ W_L^- \to Z_L Z_L) \\
&= \frac{-is}{v^2}\, g_{ZW}(\tilde{h}) \\
v &\equiv \langle H\rangle \approx 246\,\text{GeV}
\end{aligned} \tag{5}$$

where we have neglected terms of $\mathcal{O}(m_W^2/s, m_Z^2/s)$ and

$$\begin{aligned} g_{ZZ}(x) &= \frac{x}{16\pi}\left[3 + \frac{x}{1-x} - 2x\ln\left(\frac{1+x}{x}\right)\right], \\ g_{WW}(x) &= \frac{x}{16\pi}\left[2 + \frac{x}{1-x} - x\ln\left(\frac{1+x}{x}\right)\right], \\ g_{ZW}(x) &= \frac{x}{16\pi}\,\frac{1}{1-x} \\ \tilde{h} &= \frac{m_H^2}{s} \end{aligned} \tag{6}$$

Clearly these amplitudes grow with m_H to the extent that a SM Higgs heavier than approximately 900 GeV renders them nonunitary.

The radion contributions to these amplitudes are quite analogous to the Higgs contributions and are easily calculated. Concentrating again on the s-wave amplitudes, we find that these are given by

$$\begin{aligned} \mathcal{M}^{(\varphi)}(Z_L Z_L \to Z_L Z_L) &= \frac{-is}{\langle\varphi\rangle^2}\, f_{ZZ}(\tilde{\varphi}) \\ \mathcal{M}^{(\varphi)}(W_L^+ W_L^- \to W_L^+ W_L^-) &= \frac{-is}{\langle\varphi\rangle^2}\, f_{WW}(\tilde{\varphi}) \\ \mathcal{M}^{(\varphi)}(Z_L Z_L \to W_L^+ W_L^-) &= \mathcal{M}^{(\varphi)}(W_L^+ W_L^- \to Z_L Z_L) \\ &= \frac{-is}{\langle\varphi\rangle^2}\, f_{ZW}(\tilde{\varphi}) \end{aligned} \tag{7}$$

where

$$\begin{aligned} f_{ZZ}(x) &= \frac{1}{16\pi}\left[\frac{1}{1-x} + (2x-1) - 2x^2\ln\left(\frac{1+x}{x}\right)\right], \\ f_{WW}(x) &= \frac{1}{16\pi}\left[\frac{1+3x}{2(1-x)} - x^2\ln\left(\frac{1+x}{x}\right)\right], \\ f_{ZW}(x) &= \frac{1}{16\pi}\,\frac{1}{1-x} \\ \tilde{\varphi} &= \frac{m_\varphi^2}{s} \end{aligned} \tag{8}$$

and again terms of $\mathcal{O}(m_W^2/s, m_Z^2/s)$ have been ignored.

We plot the functions $f_i(x)$ and $g_i(x)$ in Fig. 1. The sharp rise as $x \to 1$ is clearly symptomatic of the s-channel resonance. Beyond $x = 1$, the curves would fall off with the asymptotic behaviour being $\sim 1/x$. However, the region $x > 1$ is of no interest to us. A few points need to be noted here

- The SM amplitudes and the radion contributions have the *same sign* and hence there is no scope of destructive interference.

- For large x (i.e. $x \lesssim 1$), the functions $g_i(x)$ typically dominate $f_i(x)$. To this information, one has to couple the fact that the pre-factor $\langle\varphi\rangle^{-2}$ is expected to be smaller than v^{-2}. Thus, for similar Higgs and radion masses, it is obvious that

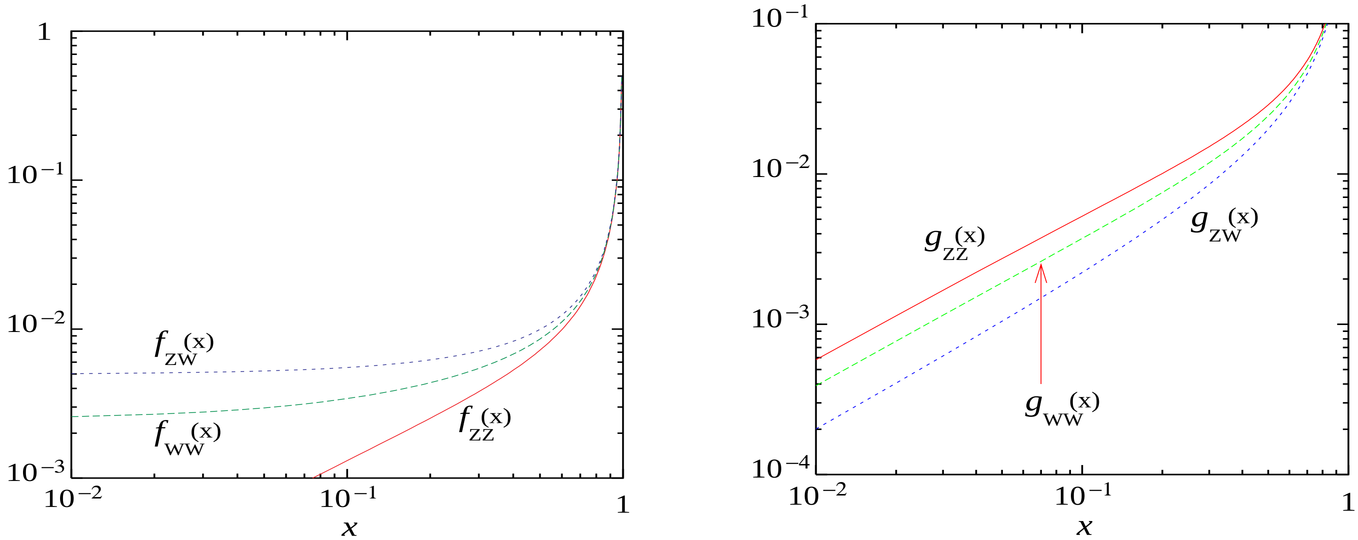

Figure 1: *The functions appearing in the expressions for the $J = 0$ amplitudes.* **(left)** *radion contribution;* **(right)** *SM contribution.*

the SM amplitudes are bigger, for this range of $\sqrt{s}$, than the radion contributions. In other words, unitarity bounds, if any, would be primarily driven by the SM dynamics.

- For small x, on the other hand, $f_i(x)$ easily dominate $g_i(x)$ and the situation regarding possible unitarity bounds gets reversed. This is but a reflection of the fact that, for a light Higgs, partial wave unitarity is respected by the SM, while it might be suspect in the theory with radions.

- For small x, $f_{WW}(x)$ and $f_{ZW}(x)$ reach constant values of $1/(32\pi)$ and $1/(16\pi)$ respectively, while $f_{ZZ}(x)$ falls off as $3x/(16\pi)$. Hence, for a given radion mass m_φ, the last three amplitudes grows with the center of mass energy $\sqrt{s}$ and thus stand to violate partial wave unitarity. The amplitude $Z_L Z_L \to Z_L Z_L$ though goes over to a constant value.

Looking at the arguments listed above, it becomes clear that the *most conservative* bounds on the radion parameter space would emanate for a light Higgs. In our analysis, therefore, we shall consider the smallest mass allowed to the SM Higgs, namely 114 GeV. In fact, there are even some preliminary signals of a Higgs with a very similar mass [17]. Although it is only the $\mathcal{M}^{(\varphi)}(W_L^+ W_L^- \to W_L^+ W_L^-)$ and $\mathcal{M}^{(\varphi)}(Z_L Z_L \to W_L^+ W_L^-)$ that are expected to give the strongest bounds, one must remember that these channels are coupled.
For completeness, we consider not only coupled $W_L W_L$, $Z_L Z_L$ channels but also couple

them to hh-channel. To this end, the complete set of $J = 0$ partial wave amplitudes is:

$$
\begin{aligned}
\mathcal{M}(hh \to hh) &= \frac{-\iota}{16\pi\langle\varphi\rangle^2 s}\Bigg[\left(3m_\varphi^2 + 16m_h^2 + \frac{(2m_h^2+m_\varphi^2)^2}{s-m_\varphi^2}\right)\ (s-4m_h^2) \\
&- 2(2m_h^2+m_\varphi^2)^2\ \ln\left(\tfrac{s+m_\varphi^2-4m_h^2}{m_\varphi^2}\right)\Bigg] \\
\mathcal{M}(Z_LZ_L \to Z_LZ_L) &= \frac{-\iota}{16\pi\langle\varphi\rangle^2 s}\Bigg[\left(3m_\varphi^2 - 8M_Z^2 + \frac{(m_\varphi^2-2M_Z^2)^2}{s-m_\varphi^2}\right)\ (s-4M_Z^2) \\
&- 2(m_\varphi^2-2M_Z^2)^2\ \ln\left(\tfrac{s+m_\varphi^2-4M_Z^2}{m_\varphi^2}\right)\Bigg] \\
\mathcal{M}(W_LW_L \to W_LW_L) &= \frac{-\iota}{16\pi\langle\varphi\rangle^2 s}\Bigg[\left(s + 2m_\varphi^2 - 8M_W^2 + \frac{(m_\varphi^2-2M_W^2)^2}{s-m_\varphi^2}\right)\ (s-4M_W^2) \\
&- \tfrac{1}{2}(s-4M_W^2)^2 - (m_\varphi^2-2M_W^2)^2\ \ln\left(\tfrac{s+m_\varphi^2-4M_W^2}{m_\varphi^2}\right)\Bigg] \\
\mathcal{M}(Z_LZ_L \to W_LW_L) &= \mathcal{M}(W_LW_L \to Z_LZ_L) \\
&= \frac{-\iota}{16\pi\langle\varphi\rangle^2 s}\left[\frac{(s-2M_Z^2)(s-2M_W^2)}{s-m_\varphi^2}[(s-4M_Z^2)(s-4M_W^2)]^{\frac{1}{2}}\right] \\
\mathcal{M}(hh \to V_LV_L) &= \mathcal{M}(V_LV_L \to hh) \\
&= \frac{-\iota}{16\pi\langle\varphi\rangle^2 s}\left[\frac{(s-2M_V^2)(s+2m_h^2)}{s-m_\varphi^2}[(s-4M_V^2)(s-4m_h^2)]^{\frac{1}{2}}\right]
\end{aligned}
\tag{9}
$$

with $V \; = \; Z \; or \; W$.

Hence, we need to consider the eigenvalues of the matrix

$$
\begin{pmatrix}
\mathcal{M}_{WW} & \mathcal{M}_{WZ} & \mathcal{M}_{Wh} \\
\mathcal{M}_{ZW} & \mathcal{M}_{ZZ} & \mathcal{M}_{Zh} \\
\mathcal{M}_{hW} & \mathcal{M}_{hZ} & \mathcal{M}_{hh}
\end{pmatrix}
$$

where the amplitudes $\mathcal{M}_{ij}$ follow an obvious notation.
These amplitudes are all real and hence the unitarity constraint on these amounts to demanding that the magnitude of the highest eigenvalue, λ_{max} satisfy:

$$
|\lambda_{max}| \le \frac{1}{2}\ . \tag{10}
$$

Although the matrix elements obviously contain the full amplitudes (SM as well as radion contributions), it turns out that neglecting the SM contribution is a good approximation, particularly for small Higgs masses.
Before we actually embark on analysing the unitarity bounds, let us, briefly, consider these eigenvalues. These are obviously functions of the three independent quantities $\sqrt{s}$, m_φ and $\langle\varphi\rangle$. However, in the approximation that we are working in, the eigenvalues are dependent on $\langle\varphi\rangle$ only by a direct multiplicative factor $\langle\varphi\rangle$. Our results thus can be conveniently plotted as $\langle\varphi\rangle^2|\lambda_{max}|$ as a function of $\sqrt{s}$ for various choices of m_φ and these are shown in Fig. 2. The unitarity bounds for various choices of $\langle\varphi\rangle^2$ are then horizontal lines as shown therein.

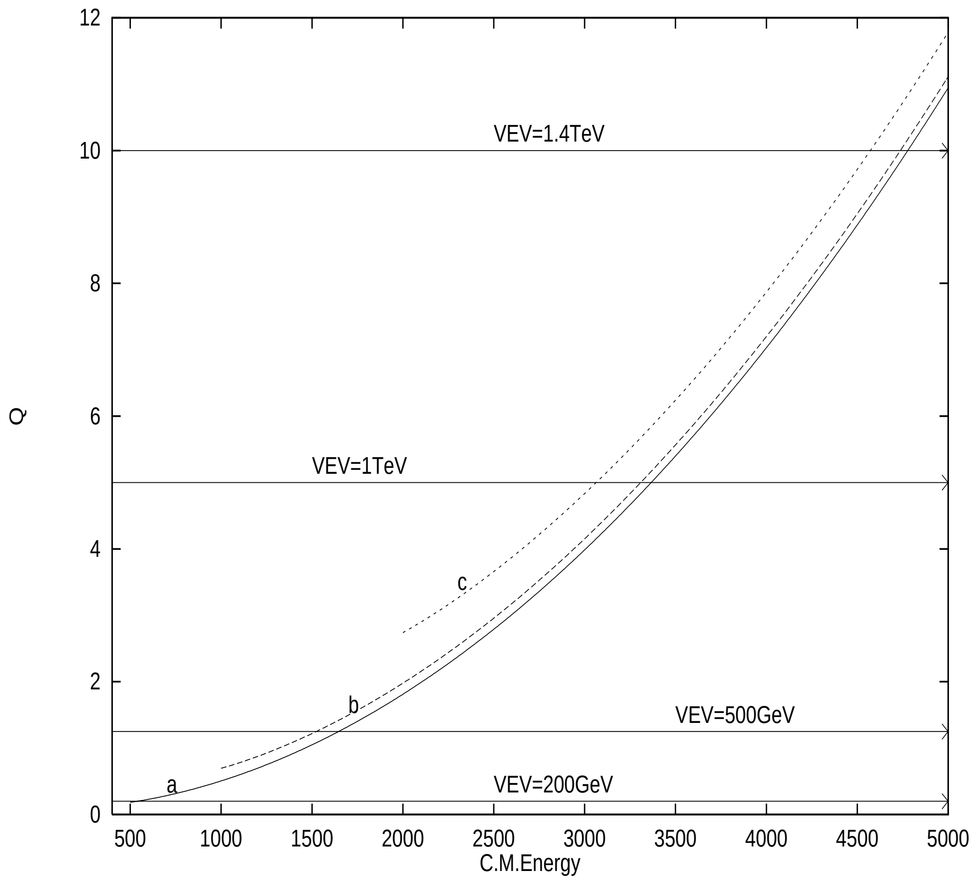

Figure 2: $Q = \langle\varphi\rangle^2|\lambda_{max}| \times 10^{-5}\ (GeV)^2$ *vs* $\sqrt{s}\ (GeV)$ *curves for different choices of* m_φ. *Curves a, b and c correspond to* $m_\varphi =$ *250, 500 and 1000* GeV *respectively.*

A very relevant feature to benoted for the present problem is that the amplitudes and hence the eigenvalues, increase asymptotically with $\sqrt{s}$. This is unlike the SM case where because of cancellations between various contributing graphs, the amplitudes approached constant values at large $\sqrt{s}$ but proportional to m_h^2. This last feature allowed one to put bounds on m_h^2 by putting perterbative unitarity restrictions on the amplitudes. We cannot directly follow the same procedure here but use a procedure followed in [16]. For an effective theory to be reasonable, it should be valid at least till energies comparable and somewhat above the particle masses in the theory. As a rule then, we can demand that perterbative unitarity be valid till energies $\sqrt{s}$ equal to $2m_\varphi$. With this, we see that if the radion coupling is weak, e.g. $\langle\varphi\rangle^2 \sim 2\ (TeV)^2$, no violation is seen even for very heavy radion mass $\sim$ 1 TeV and thus no meaningful limit is obtained. At the other end, for strongly coupled radion as has been considered in [15], typically $\langle\varphi\rangle = 200$ GeV, there will be a limit on m_φ, about 300 GeV, above which the conditions discussed above will be violated. at intermediate $\langle\varphi\rangle$, typically $\langle\varphi\rangle = 500$ GeV, the corresponding limit on m_φ becomes higher.

Admittedly, the conclusions reached here do not have a very definitive character as compared to the one reached in [13] because unlike the VEV of the higgs field, the corresponding radion VEV is unknown. The energies upto which the unitarity restrictions are not violated represent an effective upper limit of energies for which the underlying

theory/interactions can be considered as an "effective theory".
We make a brief digression here to consider other possible interactions wherein radion exchange may play a significant role. Clearly any such process should involve only heavy particles. Apart from the gauge bosons (and the Higgs), the only other heavy particle within the SM is the top quark. As pointed out earlier, a process involving $t\bar{t}$ could also serve to be a signal for the presence of the radion. One such process is $Z_L Z_L \to t\bar{t}$. The radion contribution to $J = 0$ amplitude for the $++$ $(= --)$ helicities of the $t\bar{t}$ final state is given by

$$\mathcal{M}(Z_L Z_L \to t\bar{t}) \;=\; \frac{i m_t \sqrt{s}}{16\pi \langle\varphi\rangle^2} \frac{1}{1-\tilde{\varphi}} \tag{11}$$

while the cross helicity amplitudes vanish. The high energy behaviour for $Z_L Z_L \to t\bar{t}$ is better behaved as compared to the same for gauge-boson scattering, thus leading to much weaker constraints. A similar statement holds for $W_L^+ W_L^- \to t\bar{t}$ as well.

Equation (10) imposes an inequality in a space spanned by $\sqrt{s}$, m_φ and $\langle\varphi\rangle$. Note that while the authors of Ref. [16] prefer $x_c = 1/2$, we have chosen to be more general.

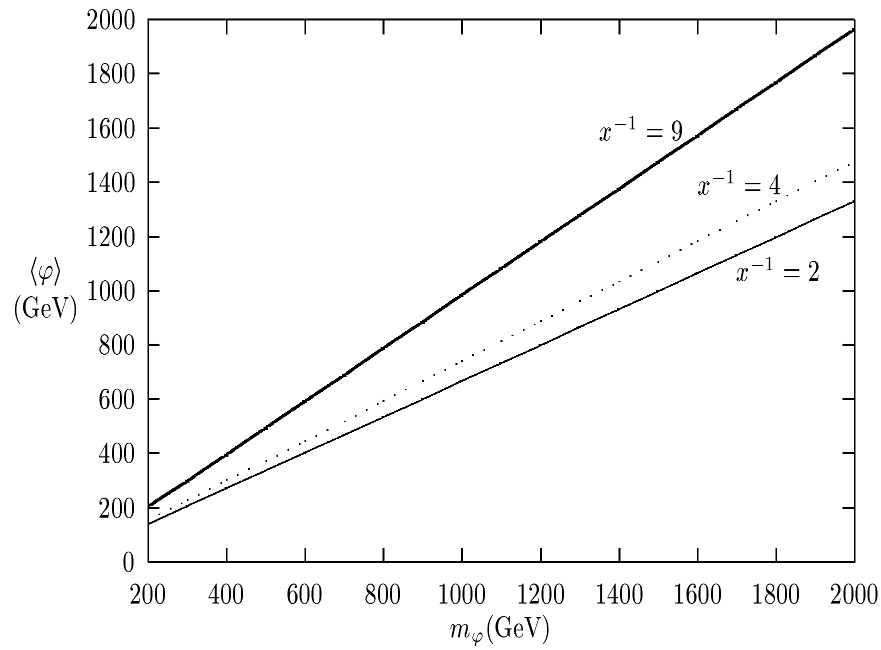

Figure 3: *Constraints on the parameter space obtained by demanding that the $J = 0$ amplitudes for longitudinal gauge-boson scattering respect unitarity bounds upto an energy scale given by $s = x^{-1} m_\varphi^2$. The region* above *the curves are ruled out.*

In conclusion, the present investigation, unlike the parallel one for Higgs mass, thus yields no bound on the radion mass but only constrains the 'allowed' region in the m_φ–$\langle\varphi\rangle$ plane. However, what makes this result a little more interesting is the fact that this allowed region is somewhat complimentary to the ones obtained by Kim et.al [9], in their investigation relating to neutral current data. Since only the intersection of such allowed domains is truly permissible, the present study could prove rather useful in defining future search strategies for the radion.

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Note added: As this manuscript was being finalised, a very similar work [18] was published. Although the two papers share a few common points, the primary focus are somewhat different.

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