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Gamma-Ray Emission from Pulsars

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Abstract. We have attempted to devise a scheme by which it may be possible to identify pulsars which are likely to be γ–ray pulsars. We apply this test to a representative population of pulsars and identify the likely candidates for γ emission . We also discuss some individual cases including the Crab and Vela pulsars.

Key words: pulsars, γ–rays

1. Introduction

Emission of radiation from pulsars is a very complex phenomenon. Several models have been proposed to explain this phenomenon, but none are entirely satisfactory. One of the more successful models is the one due to Ruderman & Sutherland (1975). The main feature of this model is the development of polar 'gaps' across which there exist potential differences of the order of 10^{12} V. The self-consistent height of the gap is such that a charged particle traversing the gap picks up enough energy from the potential difference across the gap that it can end up producing an avalanche of electron-positron ($e^- - e^+$) pairs which short the electric field and thereby close the gap. In the Ruderman-Sutherland (RS) scheme the $e^- - e^+$ plasma is essential as it initiates processes which ultimately produce radio emission. In recent years the work of Jones (1985) has cast some doubt on the validity of the RS model. Nevertheless, at present this model is perhaps our best hope of understanding a host of phenomena associated with pulsars.

It has been suggested recently that extremely strong magnetic fields present near the surface of a neutron star do not allow high energy photons to create free $e^- - e^+$ pairs, but convert them instead into bound $e^- - e^+$ pairs, or positronia (Shabad & Usov 1982, 1985; Herold, Ruder & Wunner 1985). The effect of this trapping of photons on the RS model was investigated by us (Bhatia, Chopra & Panchapakesan 1987,1988). We showed that the trapping was energy-dependent and that the photons of low as well as high energy escaped this fate. These photons were available for the production of radio emission. We also showed that the self consistent height of the gap becomes somewhat larger if trapping is taken into account and this actually improves agreement of the RS model with the observations.

In this paper we investigate, within the framework of the RS model, a scheme by which it may be possible to find out whether a given pulsar will be a γ-ray pulsar or not. An alternative model of emission of γ-rays is due to Cheng, Ho & Ruderman (1986) involving the outer gaps in the magneto spheres of pulsars. We also calculate the yield of γ-rays from pulsars which are likely to emit them. Our calculation takes into account the conversion of photons into positronia by the strong magnetic fields of pulsars. Similar calculations performed by Zhao *et al.* (1989) are flawed on two counts. Firstly, the possibility of photon capture is not taken into account. Secondly, the radius of curvature of the magnetic field near the neutron star surface is taken so large ($\sim 10^8$ cm for a 1-sec pulsar) that it will prevent the formation of gaps, without which there is no RS model. Actually, the radius of curvature adopted by Zhao *et al.* is valid only at large distances from the star (Equation (58) of RS paper). On the other hand, the expression adopted for the critical photon energy above which a photon is absorbed and creates a pair, is true only near the surface of the star. This inconsistency reduces the value of the calculations of Zhao *et al.* Nevertheless, there are some useful ideas in their work, such as the cascading of photons into $e^- - e^+$ pairs.

In Section 2 we discuss briefly the process of conversion of photons into positronia and derive the criterion that a pulsar must satisfy if it is to emit γ-rays. In Section 3 we apply this criterion to a sample population of pulsars and discuss some special cases such as the Crab and the Vela pulsars and millisecond pulsars in binary systems.

2. Criterion for the emission of γ-rays

In our earlier papers we derived the expression for the self-consistent gap height taking into account the conversion of photons into positronia. It is well known that in the presence of a magnetic field both energy and momentum conservation can be satisfied for the conversion of a photon into an $e - e^+$ pair. If the magnetic field be considered along the z-direction, then the momentum conservation relation may be written as

$$k_\perp = (eB/c)(y^- - y^+) \tag{1}$$

where $k_\perp$ is the momentum of the photon perpendicular to B and y^- and y^+ are the y-coordinates of the Landau orbits of e^- and e^+. Equation (1) also holds for the conversion of the photon into a bound $e^- - e^+$ pair if the Coulomb interaction between the pair particles can be considered as perturbation. Since the photon energy is proportional to $k_\perp$ and the positronium energy depends mildly on $k_\perp$, there is a possibility of the dispersion relations, *i.e.*, $(\omega, k_\perp)$ curves, of the two intersecting and therefore the two having a quantum state with the same energy. However, vacuum polarization correction keeps the two states separated, and this separation is well marked for the ground state of the positronium (Herold, Ruder & Wunner 1985; Shabad & Usov 1985). The gap between the two states is given by

$$\Delta k = \alpha mc^2[\ln\{B^2/(4\alpha^2 B_c^2)\}]^{1/2}(B/B_c)^{1/2}\exp(-B_c/B), \tag{2}$$

assuming $B \ll B_c$, where B_c is the so called critical magnetic field ($B_c = m^2c^3/eh \sim 4.4 \times 10^{13}$ G). In the above equation α is the fine-structure constant. Since the energy of the positronium at the cross-over point, the point where the two dispersion curves crossed (now the degeneracy having been removed by vaccum polarization), is less than the energy of the photon by the amount Δk, the reconversion of the positronium into two or more photons is prohibited. The conversion of a photon into a bound pair takes a time $h/\Delta k$. During this time the photon, travelling straight while the lines of force of the magnetic field curl away, gains in momentum perpendicular to the magnetic field. If the energy gained by the photon is larger than Δk, then the photon does not get converted into a positronium and will continue as a photon. Thus, the condition for the photon to avoid conversion becomes

$$(\hbar/\Delta k)(\hbar\omega c/\rho) > \Delta k$$

or

$$\omega > (\rho/c\hbar^2)(\Delta k)^2, \tag{3}$$

where ρ is the radius of curvature of the magnetic field and ω is the frequency of the photon. Since the photons are due to curvature radiation, it'is customary to identify ω with the characteristic frequency of curvature radiation, namely,

$$\omega \sim \omega_c = (3/2)(\gamma^3 c/\rho), \tag{4}$$

where γ is the energy in units of mc^2 of the charged particle acquired in travelling a distance d:

$$\gamma = e\Omega B d^2/mc^3, \tag{5}$$

where $\Omega = 2\pi/P$, P being the period of the pulsar in sec. Following RS theory, we identify d with the gap height for self-consistency. Then, from Equations (2)–(5) we get the expression for the gap height in the presence of the photon $\Rightarrow$ positronium conversion. It is given by

$$d > (2/3)^{1/6}(P/2\pi)^{1/2}(c^2\rho^2\hbar/m)^{1/6}[2\ln(B/2\alpha B_c)]^{1/6} \times (\alpha B_c/B)^{1/3}\exp(-B_c/3B) \tag{6}$$

where B now is the surface magnetic field of the pulsar. It has been argued by RS that in the neighbourhood of the star the magnetic field is likely to be a mixture of dipole and higher multipole fields and a reasonable value of ρ is 10^6cm. It may be noticed that d has a weak dependence on ρ and therefore a small factor multiplying 10^6 will have not much effect on d. The surface magnetic field of a pulsar is given by the well known expression:

$$B_{12} = 1.01(P\dot{P}_{15})^{1/2}G, \tag{7}$$

where $B_{12} = B/10^{12}$ and $\dot{P}_{15} = \dot{P}/10^{-15}$. After traversing the gap height d, the charged particle (electron or positron) will acquire energy with a typical Lorentz factor given by Equation (5). As these particles move along the magnetic field lines, they emit curvature radiation with photon frequency typically centred at ω_C given by Equation (4). In terms of P and $\dot{P}$, the energy of the curvature photon becomes

$$E_c = 2.61 \times 10^5(P\dot{P}_{15})^{1/2}\ln(2.461P\dot{P}_{15}) \times \exp[-87.32/(P\dot{P}_{15})^{1/2}]\,\text{erg}. \tag{8}$$

If this energy exceeds a critical energy E_a given by

$$E_a = 1.5 \times 10^{-2}(P/\dot{P}_{15})^{1/2}(r/R)\,\text{erg}, \tag{9}$$

then according to Hardee (1977) the photon will be absorbed at a distance r from the centre of the star of radius R at the magnetic axis and will be transformed into a $e^- - e^+$ pair. This expression holds near the surface of the star, therefore $(r/R) \sim 1$. If $E_c \gg E_a$, then the pair particles will also acquire sufficient energy to radiate γ- photons. Zhao *et al.* have estimated that if $E_c > 20E_a$, then it is possible for γ photons to escape the star and make the pulsar a γ–ray pulsar. Thus, for a pulsar to be a potential γ-emitter the condition $E_c > nE_a(n \sim 20)$ should be satisfied. In the $P - \dot{P}$ plane this criterion takes the form

$$(1.7 \times 10^6/n)\dot{P}_{15} \ln(2.461\, P\dot{P}_{15}) \exp[-87.32/(P\dot{P}_{15})^{1/2}] = 1, \tag{10}$$

where we have used Equations (8) and (9)

3. Results and discussion

The usefulness of Equation (10) is obvious. One can check whether a given pulsar is expected to be a γ-pulsar or not. We plot its observed P against observed $\dot{P}$. If this point lies above the curve of Equation (10), the pulsar is expected to be a γ-emitter. This curve is shown in Figure 1 and is labelled as curve 1. In this figure we have also shown a number of pulsars with $B > 3 \times 10^{12}$G, the lower limit of the field for conversion of photons into positronia (Bhatia, Chopra & Panchapakesan 1987), taken from the data collected by Manchester & Taylor (1981). The pulsars which lie above the curve are the likely candidates for γ-ray pulsars. The γ-luminosity of these pulsars can be estimated from the expression (Ruderman & Sutherland 1975; Harding 1981),

$$L_\gamma = \dot{N}\gamma mc^2, \tag{11}$$

where γ is given by Equation (5) and N is the net charged particle flux from the polar cap given by

$$\dot{N} = \pi R_p{}^2 B/(eP)$$

Table 1. List of pulsars likely to be γ-ray pulsars.

Designation (PSR)	P (sec)	$\dot{P}_{15}$	B_{12}	Predicted γ-luminosity ($h\nu > 1$ MeV) erg sec^{-1}
1831 – 03	0.686	41.5	5.39	1.2×10^{32}
0959 – 54	1.436	51.66	8.70	5.1×10^{32}
1524 – 39	2.417	19.07	6.85	4×10^{32}
1558 – 50	0.864	69.57	7.83	8.3×10^{32}
1727 – 47	0.829	163.67	11.76	8.5×10^{33}
1822 – 09	0.768	52.32	6.40	3.2×10^{32}
1846 – 06	1.451	45.7	8.22	3.9×10^{32}
1916 + 14	1.180	211.4	15.95	3.8×10^{33}
1844 – 04	0.597	51.9	5.62	2.2×10^{32}
1845 – 19	4.308	23.31	10.12	8.6×10^{30}
2002 + 31	2.111	74.57	12.67	2.3×10^{32}
1802 – 23	0.112	110	3.53	1.7×10^{32}
1509 – 58	0.150	1490	15.09	6.5×10^{36}
0531 + 21	0.033	422.4	7.37	1.5×10^{35}*
0833 – 45	0.089	124.7	6.57	1.0×10^{34}*

PSR 1802 – 23 reported by Raubenheimer *et al.* (1986) as emitting high energy γ-rays. PSR 1509-58 observed by Nel *et al.* (1990) at Tev γ-rays with the observed luminosity of 4.7×10^{34} erg sec^{-1}.

*Estimated luminosities of Crab and Vela pulsars with revised magnetic fields (see text).

following the RS theory. Here $R_p = (2/3)^{3/4} R(\Omega R/c)^{1/2}$ is the radius of the polar cap. These luminosities are shown in Table 1. It is worth noting that all the pulsars in Table 1 (which are the ones likely to be γ-ray pulsars) have surface magnetic fields $> 5 \times 10^{12}$G. This is perhaps as well because the minimum strength of the magnetic field for the conversion of a photon into positronium is $\sim 3 \times 10^{12}$G. In this connection it is interesting to note that the pulsars suggested by Zhao *et al.* as possible γ-emitters are not found to be so if we apply the above criterion as they all lie below the curve in Fig. 1. These pulsars are PSR 1951 + 32, PSR 1356–60, PSR 1754–24 and PSR 0740–28. As a matter of fact, none of these have been confirmed as γ–emitters, lending weight to the argument advanced by us.

The Crab and Vela pulsars are also shown in Fig. 1. Both these lie below the curve. Since both these are established γ-emitters, this seems anomalous. The magnetic fields for these pulsars quoted in the literature, $\sim 2 - 3 \times 10^{12}$G, are rather weak. However, it must be remembered that the magnetic fields of pulsars are found from P and $\dot{P}$ by

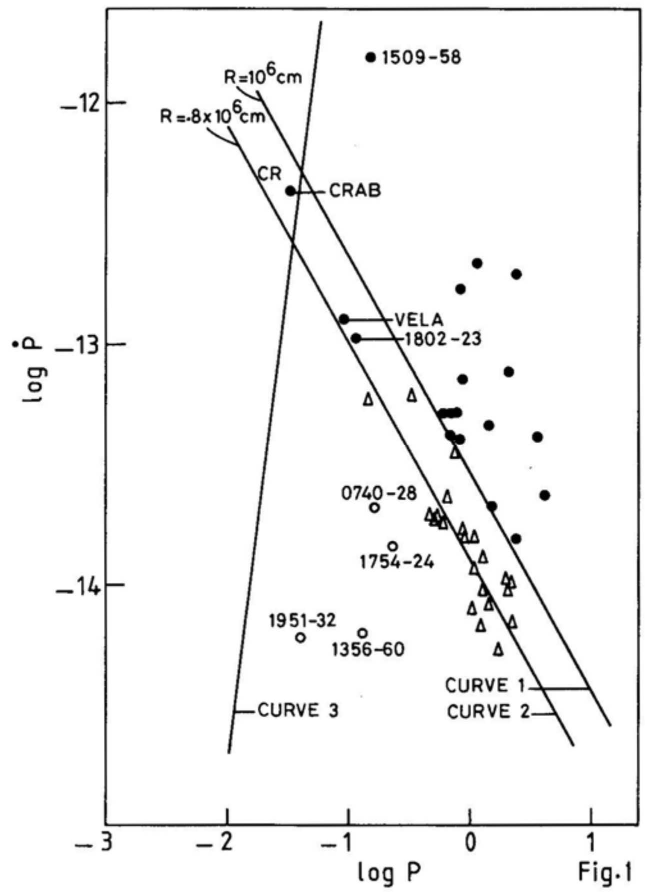

Figure 1. Curve obtained from Equation (10) in the $P - \dot{P}$ plane. The labelling of curves, 1, 2 or 3, is explained in the text. ● – likely emitters of γ-rays, Δ – not likely to emit γ-rays, o – pulsars suggested by Zhao *et al.* as possible emitters of γ-rays but not confirmed so far and unlikely to emit γ-rays if we apply our criterion.

using the following expression derived on the basis of classical dipole radiation,

$$B^2 = [3Ic^3/(8\pi^2 R^6)](P\dot{P}) \tag{12}$$

where I is the moment of inertia of the star. It is customary to adopt $R = 10^6$ cm and $I = 10^{45}$ gm-cm^2 for all stars. But all the stars do not have this precise radius. The various equations of state of the neutron star matter give the radius in the range 7–15 km (see Table 2 in Cutler, Lindblom & Splinter 1990). If we take the radius on the lower side of the range, say, 7 km, and calculate the magnetic fields of the Crab and Vela pulsars, we find them increased by a factor of about two. The point to be emphasized is that the magnetic fields of pulsars estimated by Equation (12) may be uncertain to within a factor of two. If we redraw the curve of Equation (10) with the magnetic field enhanced by a factor of two (the curve is labelled as 2) then we find that both the Crab and the Vela pulsars lie above the curve and their case is no longer anomalous. In fact, the estimated luminosities of these pulsars with the revised magnetic fields are of the same order of magnitude as observed (see Table 1). However, the position of the pulsars mentioned above and suggested by Zhao *et al.* as possible γ-ray emitters does not improve even with respect to the modified curve. It may be pointed out that we have argued elsewhere also (Bhatia, Chopra & Panchapakesan 1987) that the magnetic fields of the Crab and the Vela pulsars may have been underestimated by a factor of 2.

In Fig. 1 we have also drawn the curve one would get from the RS theory in case the conversion of photons into positronia is not operational, that is the magnetic fields are weak. For this curve (labelled as curve 3) we have adopted $\rho = 10^6$ cm (and $n = 20$) to make it consistent with the RS scheme unlike Zhao *et al.* who take $\rho \sim 10^8$ cm. The pulsars likely to emit γ-rays should lie to the left of this curve. In our sample there is hardly any such pulsar, although there could be such cases. So, one could conclude that for a pulsar to be γ-ray emitter it must generally lie above the curve I or it must lie on the left of the curve 3. With the ongoing search for γ-ray pulsars with instruments of decreasing thresholds this prediction is open to verification.

The millisecond pulsars found in binary systems are spun-up pulsars. Here the pulsar is resurrected from a run-down neutron star by accreting matter from its companion. The resulting contraction of the star gives rise to a millisecond period neutron star. These pulsars differ from the usual pulsars as far as the strength of the magnetic field is concerned. The magnetic field of the spun-up pulsars is $\sim 10^8$ G compared to $\sim 10^{12}$ G for the usual pulsars. This low magnetic field does not allow the formation of polar gaps in the manner suggested by the RS theory. Hence the mechanism discussed above for γ-ray emission appears to be inadequate to explain the γ-emission of spun-up pulsars. One will have to seek some other explanation, such as that involving outer gaps (Cheng, Ho & Ruderman 1986).

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