THE YOUNG'S MODULUS OF DIAMOND

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Introduction

THE only elastic property of diamond which has been experimentally studied up to this time is its bulk modulus, the value of which as determined by Adams[1] and Williamson[2] are respectively $6 \cdot 25 \times 10^{12}$ and $5 \cdot 56 \times 10^{12}$ dynes per sq. cm. In the present note an account of an experimental determination of the Young's modulus of diamond for a rod, whose axis is nearly parallel to the octahedral cleavage of the crystal is given.

The Young's modulus E for any cubic crystal along a direction whose direction cosines are l, m, n with respect to the crystallographic axes, is given by[3]

$$\frac{1}{\mathrm{E}} = \frac{c_{12}}{(c_{11} - c_{12})(c_{11} + 2c_{12})} - \frac{1}{2c_{44}} - \left(\frac{1}{c_{11} - c_{12}} - \frac{1}{2c_{44}}\right)(l^4 + m^4 + n^4),$$

where c_{11}, c_{12}, c_{44} are the three elastic constants. The surface described by a radius vector of magnitude $1/\mathrm{E}$ and direction cosines l, m, n, gives a picture of the variation of the Young's modulus with direction. It can be easily shown that for any direction lying in the octahedral plane of any cubic crystal.

$$l^4 + m^4 + n^4 = \tfrac{1}{2}.$$

Hence the section of the elastic surface by the (111) plane is a circle. Its radius is equal to the reciprocal of the Young's modulus along the diagonal of a face of the cube, for, the direction cosines of this diagonal are

$$l = \frac{1}{\sqrt{2}}, \quad m = \frac{1}{\sqrt{2}}, \quad \text{and} \quad n = 0,$$

thus making

$$l^4 + m^4 + n^4 = \tfrac{1}{2}.$$

Thus all rods cut *with their lengths parallel to this plane*, have the same Young's modulus.

The rod used in the present determination has its axis inclined at an angle of about 1° to the (111) plane as found by an X-ray examination of the specimen carried out by Dr. P. Nilakantan. But the broad faces of the rectangular specimen make considerable angles (13° and 16°) with the (111) plane. The Young's modulus was determined by bending the rod, the neutral surface being parallel to the broad faces. Since during bending the elongations and contractions of the various filaments take place in a direction parallel to the axis of the rod, *i.e.*, in a direction practically parallel to the (111) plane, the Young's modulus determined is the one along this plane.

For a direction normal to the (111) plane, the values of the direction cosines are

$$l = m = n = \frac{1}{\sqrt{3}},$$

which makes the value of E greater than that in a direction lying in the octahedral plane. For a direction along the cubic axis,

$$l = 1, \text{ and } m = n = 0,$$

which makes the value of E smaller than that in a direction lying in the octahedral plane. So the value obtained in this determination will be between the maximum and minimum values.

2. *Method*

The principle of the method employed is the same as that of the 'Scale and Telescope method' of Voigt.[4] But to ensure greater accuracy, the scale was replaced by a fine narrow illuminated slit and the telescope by a long focus camera,—the shift of the image being registered photographically.

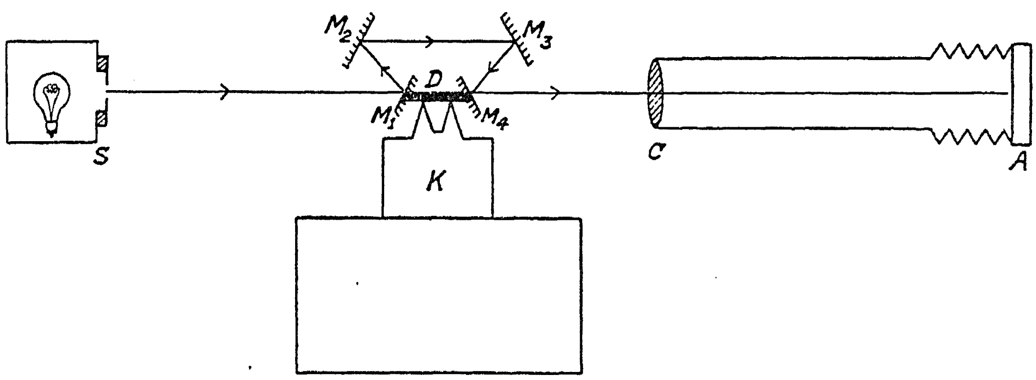

FIG. 1

The specimen D was supported horizontally on two parallel knife edges 0·393 cm. apart, cut out on a block of tool steel which was tempered after working the knife edges. The bottom of the block was ground plane and made to sit perfectly on the plane surface of a heavy iron anvil.

Two cover glass slips M_1 and M_4 of good optical quality were selected by observation of the Haidinger's rings given by them and silvered by the evaporation process. The mirrors were fixed to the ends of the specimen rigidly by pitch. M_2 and M_3 were two other fixed mirrors also of good optical quality. S was a narrow slit illuminated by a 100 watt lamp. **CA** was the long focus camera of which the lens **C** was the achromatic objective of a one metre telescope.

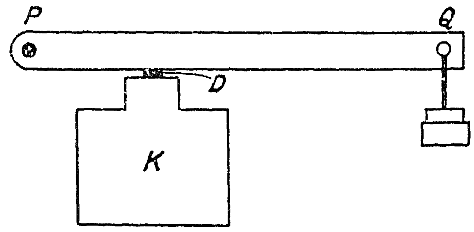

FIG. 2

Fig. 2 illustrates the method of applying the load on the specimen. PQ was a rigid steel lever pivoted about a smooth horizontal axle passing through P, the length of the lever being at right angles to the length of the specimen D. That part of the lever, which rested on D, was ground into a fine knife edge. The load was applied at Q, thus producing a magnified vertical force on D.

The image of the slit after successive reflections at the mirrors M_1, M_2, M_3 and M_4 was focussed by the camera lens **C** on the photographic plate **A**. The positions of the image before and after loading the specimen were recorded photographically on the same plate. The shift of the image was measured accurately using a Hilger micrometer reading to thousandths of a millimetre.

3. *Theory*

If θ be the angular tilt of each of the two mirrors M_1 and M_4 produced by the bending of the specimen due to an effective central load of W, we have

$$\theta = \frac{W l^2}{16 E I},$$

where l is the distance between the knife edges, E is the Young's modulus and I is the moment of inertia of the cross-section about the neutral surface.

If s be the shift of the image on the photographic plate whose distance from the optic centre of the lens is **C A**, the angle ϕ through which the final ray is turned is given by $s/\mathbf{CA}$. Denoting the distance of M_1 from the slit by **D**, the distance $M_1 M_2$ by a, the distance $M_2 M_3$ by b, the distance $M_3 M_4$ by c, and the distance $M_4 \mathbf{C}$ by d, we have

$$\phi\,(\mathbf{D} + a + b + c + d) = \{4\,\mathbf{D} + 2\,(a + b + c)\}\,\theta.$$

4. *Measurements*

As mentioned before, the distance between the knife edges was 0·393 cm. The moment of inertia of area of cross-section, *i.e.*, the value of I for the specimen was 0·0000273 cm.[4] In the experiment the distance **C A** was 150·1 cm. and the distance **D** was 245 cm. The mean shift on the photographic plate, for an effective load of 1,765 gm. on the specimen was 0·0484 cm.

The value of the Young's modulus calculated from the mean shift is $5 \cdot 5 \times 10^{12}$ dynes per sq. cm. The accuracy claimed is only 1%.

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5. *Summary*

It is shown that the section of the elastic surface of a cubic crystal by an octahedral plane is a circle. Voigt's 'scale and telescope' method for the Young's modulus determinations of crystals is improved by the use of a narrow slit and a long focus camera, the shift being recorded photographically and measured by a Hilger micrometer. The experimental value for the Young's modulus of diamond in any direction lying in the octahedral plane is found to be $5 \cdot 5 \times 10^{12}$ dynes/cm.^{2} The maximum value of the Young's modulus of a cubic crystal being in a direction normal to this plane, we can expect a higher value for a rod cut normal to the octahedral plane.

REFERENCES

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