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Weak neutral currents in the U_3(W)-gauge theory of weak and electromagnetic interactions

L K PANDIT
Tata Institute of Fundamental Research, Bombay 400005

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Abstract. A discussion is given of the implications of the recently proposed U_3 (W)-gauge theory of weak and electromagnetic interactions (Pandit 1976) for some phenomena resulting from its weak neutral currents: (1) neutrino-electron scattering, (2) neutrino-nucleon elastic and inelastic scattering, (3) coherent neutrino-nucleus scattering (4) weak interaction effects in $e^+e^- \rightarrow \mu^+ \mu^-$ and (5) parity-violation in atomic physics. The theory agrees quite well with the available experimental results on neutrino processes. We find the coherent neutrino-nucleus cross-section for Fe^{56} to be about 6 times larger than that in the WS–GIM theory giving some hope of accounting for supernova explosion by the resulting neutrino-radiation pressure.

Keywords. U_3-gauge theory; weak neutral currents; neutrinos; leptons; quarks

1. Introduction

Recently we have proposed a unified U_3-gauge theory of weak and electromagnetic interactions (Pandit 1976; to be referred to here as paper I). The motivation was to attempt a unified spontaneously broken gauge theory description of the unusual events in the deep underground experiments at Kolar (Krishnaswami *et al* 1975), the dilepton events initiated by high energy laboratory neutrino beams, the possible production of a new heavy lepton in e^+e^--annihilation, along with the longer known weak neutral current phenomena as well as the older conventional weak and electromagnetic processes. For this purpose we had to introduce two additional flavours of quarks, called Taste ($\mathcal{T}$) and Grace ($\mathcal{G}$), besides Charm ($\mathcal{C}$) and the three old SU_3 flavours. Each quark flavour is assumed to come in three colours (red, blue and white). Correspondingly six lepton-types were introduced, where to each type corresponds a triplet of leptons. This extensive choice of fermions was made to ensure that all possible axial-vector anomalies get cancelled, and that no strangeness-changing neutral currents appear in the theory.

In paper I we had been content with making only qualitative comments on the experimental implications of the theory. In the present work we shall focus on one particular area of applications. We shall deal here with the phenomenology of the weak neutral currents according to our scheme, making comparisons with experimental results as far as possible. We shall also give comparisons with the results of the so far most promising U_2-gauge theory proposed by Weinberg

(1967) and Salam (1968) and extended, to include four quark flavours while ensuring cancellation of anomalies and banishing strangeness-changing neutral currents, by Glashow *et al* (1970). We shall refer to this theory as the WS-GIM theory. Whereas in our U_3-gauge theory we have two neutral weak intermediate vector bosons Z_1 and Z_2, the WS-GIM theory has only one called Z. In our theory only the neutral current coupled to Z_1 involves the neutrinos ν_e and ν_μ. Thus for weak neutral current phenomena involving neutrinos only the Z_1 boson mediated interaction plays a role. For phenomena such as of parity-violation in atomic physics, and weak effects in $e^+e^- \to \mu^+\mu^-$, both the Z_1 and the Z_2 mediated interactions are relevant in our theory.

In section 2 we specify the weak neutral currents according to paper I. Sections 3 to 9 are then devoted to the neutral current phenomena of common interest. We must emphasize that our purpose is not to suggest new methods of analysis or new phenomena. That has already been done in the large volume of literature that has grown in the last couple of years and very nicely documented and reviewed (Sehgal 1978, Sakurai 1978, Adler 1978, where the extensive references to the original literature are to be found). We only provide here the results of our U_3-gauge theory for such discussions.

Our results are certainly quite as encouraging as the well-known WS-GIM theory so far as the phenomena (i) of scattering of the ν_μ, $\bar{\nu}_\mu$, $\bar{\nu}_e$ off the electron, (ii) of ν_μ ($\bar{\nu}_\mu$) elastic as well as deep inelastic scattering off nucleons are concerned. The really interesting, and at times rather spectacular, differences appear in other applications. Thus the parameter a_N^2 characterising coherent neutrino-nucleus scattering (Freedman 1974), for the astrophysically interesting case of Fe^{56}, comes out in our theory to be larger by a factor of about 6 than in the WS-GIM theory, giving the hope that the resulting neutrino-radiation pressure will be adequate to account for supernova explosion (while in the WS-GIM theory it stops short of achieving this: Wilson 1974). Again there are interesting and significant differences between the results of the two theories for the weak interaction effects in $e^+e^- \to \mu^+\mu^-$ and for the parity-violating effective electron-nucleus potential leading to parity-violating effects in atomic physics. Thus it may be hoped that imminent as well as future experiments will be able to distinguish between the two theories. In view of the Kolar events, the above-mentioned astrophysical implications and the applications discussed here, we consider our theory of sufficient interest to merit further examination.

2. The weak neutral currents

In the U_3-gauge theory there are two neutral weak-interactions intermediate vector boson fields $Z_{1\mu}$ and $Z_{2\mu}$ coupled to two weak neutral currents $J_\mu(Z_1)$ and $J_\mu(Z_2)$ respectively:

$$L_{\rm int}(Z_1, Z_2) = f_1 Z_{1\mu} J_\mu(Z_1) + f_2 Z_{2\mu} J_\mu(Z_2). \tag{1}$$

In terms of the parameter

$$\tan\chi = 2f'/\sqrt{3}f \tag{2}$$

introduced in paper I, we have

$$f_1 = \frac{\sqrt{3}}{2} f \cos\chi, \quad f_2 = \tfrac{1}{2} f, \tag{3}$$

$$f^2 = 4\sqrt{2}\, m^2(W)\, G, \; G = 1\cdot026 \times 10^{-5}\, m_P^{-2}, \tag{4}$$

where $m(W)$ is the mass of the charged intermediate vector bosons $W^{\pm}$ and G is the standard Fermi coupling constant. Further,

$$m(Z_2) = m(W) \simeq 43 \text{ GeV}/\sin x,$$

$$m(Z_1) = m(W) \sec\chi \simeq 86 \text{ GeV}/\sin 2x. \tag{5}$$

For our phenoménological discussion we write down only the terms involving the quarks and the leptons in the neutral currents (a summation over the three colours of the quarks is always understood throughout):

$$\begin{aligned}
-iJ_\lambda(Z_1) = \frac{1}{3\cos^2\chi} & [\bar{u}\{(\tfrac{1}{2} - 2\sin^2\chi)\gamma_\lambda + \tfrac{1}{2}\gamma_\lambda\gamma_5\} u \\
& - \bar{d}\{(1 - \sin^2\chi)\gamma_\lambda + \gamma_\lambda\gamma_5\} d + (u \to c, t, g) + (d \to s) \\
& + \bar{\nu}_e \gamma_\lambda (1 + \gamma_5) \nu_e + (\nu_e \to \nu_\mu, n_1, n_2) \\
& + \bar{e}\{(-\tfrac{1}{2} + 3\sin^2\chi)\gamma_\lambda - \tfrac{1}{2}\gamma_\lambda\gamma_5\} e \\
& + (e \to \mu, E, M, l_1, l_2, L_1, L_2) \\
& + \bar{p}_1\{(1 - 3\sin^2\chi)\gamma_\lambda + \gamma_\lambda\gamma_5\} p_1 + (p_1 \to p_2) \\
& - \tfrac{1}{2}\bar{v}_1\gamma_\lambda(1 + \gamma_5) v_1 + (v_1 \to v_2, V_1, V_2)]
\end{aligned} \tag{6}$$

$$\begin{aligned}
-iJ_\lambda(Z_2) = \tfrac{1}{2} & [\bar{e}\gamma_\lambda(1 + \gamma_5) e + (e \to \mu, l_1, l_2, v_1, v_2) \\
& - \bar{E}\gamma_\lambda(1 + \gamma_5) E + (E \to M, L_1, L_2, V_1, V_2) \\
& - \bar{u}\gamma_\lambda(1 + \gamma_5) u + \bar{t}'\gamma_\lambda(1 + \gamma_5) t' + (u \to c', t' \to g')].
\end{aligned} \tag{7}$$

In eq. (7), c', t', g' are the mixtures of the charm, taste and grace carrying quarks c, t and g introduced in paper I.

It is to be noted that ν_e and ν_μ enter only the current $J_\lambda(Z_1)$. Thus for the neutral current phenomena involving the neutrinos only this current is relevant. On the other hand for the weak neutral current effects in processes such as $e^+e^- \to \mu^+\mu^-$ and parity-violation in atomic physics both the currents $J_\lambda(Z_1)$ as well as $J_\lambda(Z_2)$ are important.

Having noted down the relevant terms in the two neutral currents we may now obtain in the standard way the equivalent effective lowest order four-fermion Lagrangians for the different processes of interest.

It is also useful to introduce the standard conventional strong SU_3 octets of currents for the triplet quarks $q \equiv$ column (u, d, s):

$$\mathfrak{F}_\lambda^k = i\left(\bar{q}\frac{\lambda_k}{2}\gamma_\lambda q\right), \qquad \mathfrak{F}_{5\lambda}^k = i\left(\bar{q}\frac{\lambda_k}{2}\gamma_\lambda\gamma_5 q\right). \tag{8}$$

Then for the *hadronic* parts of the weak neutral currents we may write:

$$J_\lambda (Z_1, \text{had.}) = \frac{1}{\cos^2 \chi} \Big[(\tfrac{1}{2} - \sin^2\chi)\, \mathcal{F}_\lambda{}^3 - \frac{1}{\sqrt{3}} (\tfrac{1}{2} + \sin^2\chi)\, \mathcal{F}_\lambda{}^8$$

$$+ \tfrac{1}{2}\, \mathcal{F}^3_{5\lambda} - \frac{1}{2\sqrt{3}}\, \mathcal{F}^8_{5\lambda} \Big] + \Delta J_\lambda (1)\,, \tag{9}$$

$$J_\lambda (Z_2, \text{had.}) = -\tfrac{1}{2} [\mathcal{F}_\lambda{}^3 + \sqrt{3}\, \mathcal{F}_\lambda{}^8 + \mathcal{F}^3_{5\lambda} + \sqrt{3}\, \mathcal{F}^8_{5\lambda}] + \Delta J_\lambda (2) \tag{10}$$

In eqs (9) and (10) $\Delta J_\lambda (1)$ and $\Delta J_\lambda (2)$ stand for those parts of the currents that *do not* involve the u and the d quarks so that their matrix elements for the nucleon states will be taken as negligible in the following considerations.

3. Neutrino-electron scattering

The Z_1-mediated neutral current interaction allows for scattering of ν_μ and $\bar{\nu}_\mu$ off e in the lowest (second) order. The scattering of ν_e and $\bar{\nu}_e$ off e is allowed by this as well as by the $W^\pm$ mediated charged current interaction. The effective four-fermion Lagrangian for these scattering processes obtained in the well-known standard manner (using a Fierz transformation on the charged current interaction) may thus be written as Sehgal 1975):

$$L_{\text{eff}} = -\frac{G}{\sqrt{2}} [\{\bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5)\, \nu_\mu\} \{\bar{e}\, \gamma_\lambda (C_V + C_A \gamma_5)\, e]$$

$$+ \{\bar{\nu}_e \gamma_\lambda (1 + \gamma_5)\, \nu_e\} \{\bar{e}\, \gamma_\lambda (C'_V + C'_A \gamma_5)\, e\}], \tag{11}$$

$$C'_V = 1 + C_V, \quad C'_A = 1 + C_A,$$

$$C_V = -\tfrac{1}{3} + 2 \sin^2\chi, \quad C_A = -\tfrac{1}{3}. \tag{12}$$

In the WS-GIM theory, $C_V = -\tfrac{1}{2} + 2\sin^2\theta_W$, $C_A = -\tfrac{1}{2}$. Thus we expect different results in the two theories, depending on the values of χ and θ_W.

With the abbreviations

$$z \equiv \sin^2 \chi,$$

$$\bar{\sigma} \equiv \sigma (\bar{\nu}_\mu e) \Big/ \Big(\frac{G^2 m_e E_\nu}{\pi}\Big), \qquad \sigma \equiv \sigma (\nu_\mu e) \Big/ \Big(\frac{G^2 m_e E_\nu}{\pi}\Big), \tag{13}$$

we have

$$\bar{\sigma} = \frac{2}{27} - \frac{4}{9} z + \frac{8}{3} z^2,$$

$$\sigma = \frac{2}{9} - \frac{4}{3} z + \frac{8}{3} z^2, \tag{14}$$

so that

$$\bar{\sigma} = \sigma + \frac{2}{27} \pm \sqrt{\frac{2\sigma}{3} - \frac{1}{27}}. \tag{15}$$

For comparison, if in the WS–GIM theory we denote also $\sin^2\theta_W$ by z, then in that theory we have

$$\bar{\sigma} = \frac{1}{6} - \frac{2}{3} z + \frac{8}{3} z^2, \qquad \sigma = \frac{1}{2} - 2z + \frac{8}{3} z^2,$$

$$\bar{\sigma} = \sigma + \frac{1}{6} \pm \sqrt{\frac{2\sigma}{3} - \frac{1}{12}}.$$

Recent experiments by Aachen-Padua collaboration (*cf.* the rapporteur talk of Gershtein 1976) have measured $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$. The results fit, within errors, the WS–GIM theory for a value of $\sin^2\theta_W \simeq 3/8$. For neutrino-electron scattering, we have the remarkable situation that our results for $\sin^2\chi \simeq 3/8$ are very close to those of the WS–GIM theory with $\sin^2\theta_W \simeq 3/8$. We shall thus adopt this value:

$$\begin{aligned} z &\equiv \sin^2\chi \simeq 3/8 \text{ (present theory)}, \\ z &\equiv \sin^2\theta_W \simeq 3/8 \text{ (WS–GIM theory)}, \end{aligned} \tag{16}$$

for citing numerical results. With this value of the parameter we then have in *units* of 10^{-42} cm^2 (E_ν/GeV):

$$\sigma(\nu_\mu e) \simeq \begin{cases} 0\cdot 83 \text{ (present theory)}, \\ 1\cdot 07 \text{ (WS–GIM theory)}, \end{cases} \tag{17}$$

$$\sigma(\bar{\nu}_\mu e) \simeq \begin{cases} 2\cdot 43 \text{ (present theory)}, \\ 2\cdot 51 \text{ (WS–GIM theory)} \end{cases} \tag{18}$$

There have also been reported recently results of $\bar{\nu}_e - e$ scattering using reactor $\bar{\nu}_e$'s (Reines *et al* 1976), *e.g.*,

$$r \equiv \frac{\sigma(\bar{\nu}_e e)}{\sigma(V - A)} = 1\cdot 70 \pm 0\cdot 44, \; (3 < E < 5 \text{ MeV}), \tag{19}$$

where $\sigma(V - A)$ is the $\bar{\nu}_e - e$ cross-section given by the conventional $V - A$ charged-current interaction, and E is the kinetic energy of the recoil electron. Theoretical values for this ratio obtained with $z \simeq 3/8$, and neglecting m_e/E_ν as a rough approximation, are:

$$r \approx \begin{cases} 1\cdot 5 \text{ (present theory)}, \\ 1\cdot 2 \text{ (WS–GIM theory)}. \end{cases} \tag{20}$$

Both the theories are thus in reasonable agreement with the present experiments on neutrino-electron scattering.

4. Elastic $(\nu_\mu p)$-scattering

The effective Z_1-mediated Lagrangian for ν_μ-nucleon scattering is given by

$$L_{\text{eff}} = i \frac{G}{\sqrt{2}} \bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \nu_\mu \, [g_{V3} \mathfrak{F}_\lambda^{3} + g_{V8} \mathfrak{F}_\lambda^{8} + g_{A3} \mathfrak{F}_{5\lambda}^{3} + g_{A8} \mathfrak{F}_{5\lambda}^{8} + \cdots] \tag{21}$$

where the dots stand for terms *not* involving the u and d quarks and thus having negligible matrix elements for the nucleon. We have in eq. (21)

$$g_{V3} = 1 - 2\sin^2\chi,\ g_{A3} = 1,$$

$$g_{V8} = -\frac{1}{\sqrt{3}}(1 + 2\sin^2\chi), \quad g_{A8} = -\frac{1}{\sqrt{3}}. \tag{22}$$

The values of eq. (22) are to be contrasted with the corresponding values in the WS–GIM theory: $g_{V3} = 1 - 2\sin^2\theta_W$, $g_{V8} = -(2/\sqrt{3})\sin^2\theta_w$, $g_{A3} = 1$, $g_{A8} = 0$. It is thus hoped that experiments will eventually be able to distinguish clearly between the two theories differing in *isoscalar* currents.

The invariant matrix element for ν_μ-nucleon scattering (momenta $k + p \to k' + p'$, $q \equiv k - k'$, $Q^2 \equiv -q^2$, M = nucleon mass) may be written as

$$\mathcal{M} = -\frac{G}{\sqrt{2}}\,\bar{u}(k')\,\gamma_\rho\,(1 + \gamma_5)\,u(k)\,\bar{u}(p')\,\Gamma_{n\rho}\,u(p), \tag{23}$$

$$\Gamma_{n\rho} \equiv \left(\gamma_\rho F_{nV}(Q^2) + i\sigma_{\rho\lambda}\, q_\lambda \frac{F_{nM}(Q^2)}{2M} + \cdots\right) + (\gamma_\rho\gamma_5 F_{nA}(Q^2) + \cdots), \tag{24}$$

where the subscript n stands for neutral current. An important parameter of interest is

$$R_{\mathrm{el}} \equiv \left[\frac{d\sigma\ (\nu_\mu p \to \nu_\mu p)/dq^2}{d\sigma\ (\nu_\mu n \to \mu^- p)/dq^2}\right]_{q^2=0} = \frac{[F_{nV}(0)]^2 + [F_{nA}(0)]^2}{[F_V(0)]^2 + [F_A(0)]^2}, \tag{25}$$

where F_V and F_A are the corresponding form factors for the charged current. Neglecting the Cabibbo-angle, and using SU_3-symmetry for relating matrix elements of the current octets we have

$$F_{nV}(0) = \frac{1}{2}g_{V3} + \frac{\sqrt{3}}{2}g_{V8} = -2\sin^2\chi, \tag{26}$$

and (taking for the axial-vector current octet the values $F \simeq 0\cdot 45$, $D \simeq 0\cdot 80$),

$$F_{nA}(0) = \frac{1}{2}(F + D)\,g_{A3} + \frac{1}{2\sqrt{3}}(3F - D)\,g_{A8} \simeq 0\cdot 533. \tag{27}$$

Thus

$$R_{\mathrm{el}} \simeq \frac{4\sin^4\chi + (0\cdot 533)^2}{1 + (1\cdot 25)^2}. \tag{28}$$

For $z \equiv \sin^2\chi \simeq 3/8$, we find $R_{\mathrm{el}} \simeq 0\cdot 23$. According to the WS–GIM theory for $\sin^2\theta_w \simeq 3/8$, $R_{\mathrm{el}} \simeq 0\cdot 18$. Rather indirectly related to R_{el} are the recently reported model dependent evaluations of elastic scattering experiments: $\sigma\ (\nu_\mu\ p$

$\rightarrow \nu_\mu p)/\sigma(\nu_\mu n \rightarrow \mu^- p) = 0\cdot 23 \pm 0\cdot 09$ (Lee *et al* 1976) and $0\cdot 17 \pm 0\cdot 05$ (Cline *et al* 1976). These experiments are clearly not yet in a position to distinguish between the two theories.

5. Deep inelastic inclusive $(\nu_\mu, \bar{\nu}_\mu)$-N scattering

In this section by N we shall mean the isospin-averaged nucleon. Then the standard experimental parameters of interest are (X standing for 'anything'):

$$R_\nu \equiv \sigma_0/\sigma_-, \; R_{\bar{\nu}} \equiv \bar{\sigma}_0/\sigma_+, \tag{29}$$

where

$$\begin{aligned} \sigma_0 &\equiv \sigma(\nu_\mu + N \rightarrow \nu_\mu + X), \; \bar{\sigma}_0 \equiv (\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + X), \\ \sigma_- &\equiv \sigma(\nu_\mu + N \rightarrow \mu^- + X), \; \sigma_+ \equiv (\bar{\nu}_\mu + N \rightarrow \mu^+ + X). \end{aligned} \tag{30}$$

Neglecting the Cabibbo angle, and denoting by A, V and I the parts arising from the axial-vector current, that from the vector current, and that from the interference of the axial-vector and vector currents, we may write for the charged current processes:

$$\sigma_- \equiv A + V + I, \quad \sigma_+ \equiv A + V - I, \tag{31}$$

so that for the neutral current processes we have (in the notation of section 4)

$$\begin{aligned} \sigma_0 &= \tfrac{1}{2}[g_{A3}^2 A + g_{V3}^2 V + g_{A3} g_{V3} I] + S, \\ \bar{\sigma}_0 &= \tfrac{1}{2}[g_{A3}^2 A + g_{V3}^2 V - g_{A3} g_{V3} I] + \bar{S}, \end{aligned} \tag{32}$$

where S and $\bar{S}$ stand for the isospin $I = 0$ contributions. From the data on the inclusive charged current processes (at least at the lower energies) we have $A : V : I \simeq 1 : 1 : 1$, so that we obtain

$$\begin{aligned} R_\nu &\geqslant \tfrac{1}{6}[g_{A3}^2 + g_{V3}^2 + g_{A3}\, g_{V3}], \\ R_{\bar{\nu}} &\geqslant \tfrac{1}{2}[g_{A3}^2 + g_{V3}^2 - g_{A3} g_{V3}]. \end{aligned} \tag{33}$$

Using the values of g_{A3} and g_{V3} given in eq. (22), and putting $z \equiv \sin^2\chi$, we have

$$\begin{aligned} R_\nu &\geqslant \tfrac{1}{6}[1 + (1 - 2z)^2 + (1 - 2z)], \\ R_{\bar{\nu}} &\geqslant \tfrac{1}{2}[1 + (1 - 2z)^2 - (1 - 2z)]. \end{aligned} \tag{34}$$

Note that these expressions are formally the same as for the WS-GIM theory with z standing for $\sin^2\theta_w$. Taking the value $z \simeq 3/8$, we obtain

$$R_\nu \gtrsim 0\cdot 2, \; R_{\bar{\nu}} \gtrsim 0\cdot 4. \tag{35}$$

These are quite consistent with the three sets of currently available experimental values (the rapporteur talk by Gershtein 1976):

(i) $R_\nu = 0{\cdot}26 \pm 0{\cdot}04$, $R_{\bar\nu} = 0{\cdot}39 \pm 0{\cdot}06$ (Gargamelle); (ii) $R_\nu = 0{\cdot}24 \pm 0{\cdot}02$, $R_{\bar\nu} = 0{\cdot}34 \pm 0{\cdot}09$ (CITF), and (iii) $R_{\bar\nu} = 0{\cdot}29 \pm 0{\cdot}04$, $R_{\bar\nu} = 0{\cdot}39 \pm 0{\cdot}10$ (HPWF).

Instead of the above theoretical inequalities, we may derive rough equalities, if we make the approximation of treating the nucleon in the " valence-quark parton model " (Sehgal 1975). The effective four-fermion interaction Lagrangian relevant here is

$$L_{\text{eff}} = -\frac{G}{\sqrt{2}}\,\bar\nu_\mu \gamma_\rho\,(1+\gamma_5)\,\nu_\mu\,[\bar u\,\gamma_\rho\,(C_V + C_A\gamma_5)\,u + \bar d\,\gamma_\rho\,(C'_V + C'_A\,\gamma_5)\,d] \tag{36}$$

with

$$C_V = \tfrac{1}{3} - \tfrac{4}{3}\sin^2\chi,\ C_A = \tfrac{1}{3};$$

$$C'_V = -\tfrac{2}{3}\cos^2\chi,\ C'_A = -\tfrac{2}{3}\,. \tag{37}$$

The values of eq. (37) are to be contrasted with the corresponding values in WS–GIM model:

$$C_V = \tfrac{1}{2}\,(1 - \tfrac{8}{3}\sin^2\theta_W),\ C'_V = -\tfrac{1}{2}\,(1 - \tfrac{4}{3}\sin^2\theta_W),\ C_A = -C'_A = \tfrac{1}{2}.$$

Now we obtain:

$$R_\nu = \frac{1}{9}\left(5 - 8z + \frac{20}{3}\,z^2\right),\qquad R_{\bar\nu} = \frac{1}{9}\,(5 - 8z + 20z^2) \tag{38}$$

in terms of $z \equiv \sin^2\chi$. For WS–GIM, we would instead have $R_\nu = \frac{1}{2} - z + (20/27)\,z^2$ and $R_{\bar\nu} = \frac{1}{2} - z + (20/9)\,z^2$ with z standing for $\sin^2\theta_w$. With the value $z \simeq 3/8$, eq. (38) gives $R_{\bar\nu} \approx 0{\cdot}3$ and $R_\nu \approx 0{\cdot}5$ ($0{\cdot}2$ and $0{\cdot}4$ in WS–GIM model), in reasonable agreement, in view of the rough approximation of using the valence-quark parton model, with the experimental values quoted above.

6. Pion production in $\nu_\mu(\bar\nu_\mu)$ nucleon collision

We consider the experimentally interesting neutral current processes

$$\nu_\mu + p \to \nu_\mu + p + \pi^0, \tag{39}$$

$$\nu_\mu + n \to \nu_\mu + n + \pi^0, \tag{40}$$

along with (for the sake of comparison) the charged-current process

$$\nu_\mu + n \to \mu^- + p + \pi^0. \tag{41}$$

Define the parameter

$$R_0 \equiv \frac{\sigma\,(\nu_\mu + p \to \nu_\mu + p + \pi^0) + \sigma\,(\nu_\mu + n \to \nu_\mu + n + \pi^0)}{2\sigma\,(\nu_\mu + n \to \mu^- + p + \pi^0)}\,. \tag{42}$$

Our theory has significant *isoscalar* neutral current (§4) of importance, *e.g.*, in $N^*_{1/2}$ production, in contrast with the WS–GIM model. In the region where the final πN system is in the P_{33} (1232) (or Δ) resonance region only the isovector parts of the weak current will be relevant, whereby denoting

$$\sigma(\nu_\mu + n \to \mu^- + p + \pi^0) \equiv \sigma_V + \sigma_A + \sigma_I, \tag{43}$$

where σ_V, σ_A, σ_I are the contributions of the vector interaction, the axial-vector interaction, and that of their interference, we obtain (ignoring the Cabibbo angle) in the notation of section 4:

$$R_0 = \frac{g_{V3}^2 \sigma_V + g_{A3}^2 \sigma_A + g_{V3} g_{A3} \sigma_I}{\sigma_V + \sigma_A + \sigma_I}. \tag{44}$$

Using the estimates (Lee 1972) for Δ-production in the static model (at incident $E = 1$ GeV):

$$\sigma_V : \sigma_A : \sigma_I = 0\cdot263 : 0\cdot202 : 0\cdot235, \tag{45}$$

we obtain

$$R_0 = 0\cdot376\, g_{V3}^2 + 0\cdot289\, g_{A3}^2 + 0\cdot336\, g_{A3}\, g_{V3}. \tag{46}$$

In the same manner we obtain for the parameter appropriate to $\bar{\nu}_\mu$ reactions:

$$\begin{aligned}\bar{R}_0 &\equiv \frac{\sigma(\bar{\nu}_\mu + p \to \bar{\nu}_\mu + p + \pi^0) + \sigma(\bar{\nu}_\mu + p \to \bar{\nu}_\mu + p + \pi^0)}{2\sigma(\bar{\nu}_\mu + p \to \mu^+ + n + \pi^0)} \\ &= \frac{g_{V3}^2 \sigma_V + g_{A3}^2 \sigma_A - g_{V3} g_{A3} \sigma_I}{\sigma_V + \sigma_A - \sigma_I} \\ &= 1\cdot14\, g_{V3}^2 + 0\cdot878\, g_{A3}^2 - 1\cdot02\, g_{V3}\, g_{A3}.\end{aligned} \tag{47}$$

We have [(22)] $g_{V3} = 1 - 2\sin^2\chi$, $g_{A3} = 1$. These are the *same* formally as the values in the WS–GIM theory if χ is replaced by θ_W. For the value $z \equiv \sin^2\chi \simeq 3/8$, we find $\bar{R}_0 \simeq 0\cdot397$ and $R_0 \simeq 0\cdot694$, apart from corrections due to the non-resonant background and the nuclear physics effects (Adler 1975). Experiments on $N^*_{1/2}$ production will be more crucial.

7. Coherent neutrino-nucleus scattering, neutrino-radiation pressure and astrophysics

With the advent of the weak neutral current interaction of the neutrinos, a coherent scattering of neutrinos by nuclei becomes a possibility (Freedman 1974), for $E \approx$ 50–100 MeV. Recall the notation of eqs (21) and (22) and note that in the non-relativistic limit

$$\langle \text{nucleus} \mid -i\mathscr{F}^k_{5\lambda}(0) \mid \text{nucleus} \rangle = \begin{cases} 0, & \text{for } \lambda = 4 \\ \propto \text{nuclear spin}, & \text{for } \lambda = 1, 2, 3; \end{cases} \tag{48}$$

$$\langle \text{nucleus} \mid -i\mathscr{F}^3_\lambda(0) \mid \text{nucleus} \rangle = \begin{cases} \tfrac{1}{2}(Z - N), & \text{for } \lambda = 4, \\ \propto, & \text{for } \lambda = 1, 2, 3; \end{cases} \tag{49}$$

$$\langle \text{nucleus} \mid -i\mathscr{F}_\lambda^8(0) \mid \text{nucleus} \rangle = \begin{cases} (\sqrt{3}/2)\,A, \text{ for } \lambda = 4, \\ 0, \text{ for } \lambda = 1, 2, 3, \end{cases} \tag{50}$$

where Z is the proton number, N the neutron number and $A = Z + N$ for the nucleus. Thus, since for a nucleus only the valence nucleons have unpaired spins, we expect a big coherent effect (proportional to the number of protons and neutrons) only for the vector-coupling. The coherent cross-section is then given by (Adler 1975):

$$\frac{d\sigma^{\text{coh}}}{d\cos\theta} = a_0^2 \frac{E_\nu^2 G^2}{2\pi} A^2 (1 + \cos\theta), \tag{51}$$

where the important parameter is

$$a_0^2 = \left(\frac{\sqrt{3}}{2} g_{V8} + \frac{1}{2} g_{V3} \frac{Z - N}{A}\right)^2. \tag{52}$$

The coherent scattering gives rise to a significant neutrino-radiation pressure of great importance in astrophysics. For example, consider the astrophysically interesting case of Fe^{56}, for which $(Z - N)/A = -(4/56)$ is negligibly small so that we may take $a_0^2 \simeq [(\sqrt{3}/2)\, g_{v8}]^2$. Now, in the present theory, $g_{v8} = -(1/\sqrt{3})(1 + 2\sin^2\chi)$. Taking the value $z \equiv \sin^2\chi \simeq 3/8$, we obtain $a_0^2 \simeq 0{\cdot}77$. This is *larger by a factor 5·5 than the value obtained in the WS–GIM theory,* where $g_v = -(2/\sqrt{3})\sin^2\theta_w$, so that with $\sin^2\theta_W \simeq 3/8$, a_0^2 (WS–GIM) $\simeq 0{\cdot}14$. According to calculations of Wilson (1974) the WS–GIM value $a_0^2 \simeq 0{\cdot}14$ is rather inadequate for achieving supernova explosion. Our value, $a_0^2 \simeq 0{\cdot}77$, might possibly turn out large enough to accomplish this important phenomenon of astrophysics. If so, then this will be a most encouraging result for the present theory.

8. Effects of the Z_1 and Z_2 exchange in $e^+ e^- \to \mu^+ \mu^-$

We now address ourselves to the effects of exchanging (in the annihilation channel) the weak neutral vector bosons Z_1 and Z_2 in the process $e^+e^- \to \mu^+\mu^-$. In contrast the WS-theory has only one neutral boson available for this purpose. The relevant effective Lagrangian may now be written as:

$$L_{\text{eff}} = -\frac{G}{\sqrt{2}} [h_{VV} (\bar{e}\gamma_\rho e)(\bar{\mu}\gamma_\rho \mu) + h_{AA} (\bar{e}\gamma_\rho\gamma_5 e)(\bar{\mu}\gamma_\rho\gamma_5\mu)$$
$$+ h_{VA} \{(\bar{e}\gamma_\rho\gamma_5 e)(\bar{\mu}\gamma_\rho\mu) + (\bar{e}\gamma_\rho e)(\bar{\mu}\gamma_\rho\gamma_5\mu)\}], \tag{53}$$

where

$$h_{VV} = \tfrac{1}{2} + \tfrac{2}{3}(\tfrac{1}{2} - 3\sin^2\chi)^2,\; h_{AA} = \tfrac{2}{3},\; h_{VA} = \tfrac{2}{3} - \sin^2\chi. \tag{54}$$

Note that, on account of their being two exchanged bosons,

$$h_{VA}^2 \neq h_{VV}\, h_{AA}, \tag{55}$$

in contrast with the WS–theory, where $h_{VV} = \frac{1}{2}(1 - 4\sin^2\theta_W)^2$, $h_{VA} = \frac{1}{2}(1 - 4\sin^2\theta_W)$ and $h_{AA} = \frac{1}{2}$. Thus we expect significant differences between the two theories to show up that may be decided upon by sufficiently refined future experiments.

In e^+e^- colliding beam storage ring facilities available at present and projected for the future, the e^-, e^+ in circular motion attain a polarization (due to synchrotron radiation) so that the spins $\vec{S}(e^-)$, $\vec{S}(e^+)$ align oppositely normal to the orbit plane. Let us denote by θ the polar angle of the produced μ^- relative to the incident e^- momentum $\vec{p}(e^-)$, and by ϕ the azimuthal angle of the μ^- relative to the $[\vec{p}(e^-), \vec{S}(e^-)]$-plane. Various weak effects may be studied following existing proposals (Cung *et al* 1972, Godine and Hankey 1972, Love 1972). Thus, for the *forward-backward asymmetry* we have:

$$\frac{\frac{d\sigma}{d\Omega}(\theta, \phi) - \frac{d\sigma}{d\Omega}(\pi - \theta, \phi)}{\frac{d\sigma}{d\Omega}(\theta, \phi) + \frac{d\sigma}{d\Omega}(\pi - \theta, \phi)}$$

$$= C(s) \frac{\cos\theta}{1 + \cos^2\theta - P^2(1 - \cos^2\theta)\cos 2\phi}, \quad 0 \leqslant \theta \leqslant \pi/2,$$

where $\boldsymbol{P}$ stands for the magnitude of the initial polarization of e^-, and

$$C(s) = -\frac{2Gh_{AA}}{\sqrt{2}\,\pi\alpha} s. \tag{56}$$

Here α is the fine-structure constant and s is the square of the centre of mass energy of the colliding e^+e^-. In the present theory, as noted above, $h_{AA} = \frac{2}{3}$, while in the WS-theory $h_{AA} = \frac{1}{2}$, so that (independent of the values of $\sin^2\chi$ or $\sin^2\theta_W$ respectively) we find

$$C(s) \simeq -\frac{10^{-3}}{3}\left(\frac{s}{\text{GeV}^2}\right), \text{ (WS–GIM theory)},$$

$$C(s) \simeq -\frac{4}{3} \times \frac{10^{-3}}{3}\left(\frac{s}{\text{GeV}^2}\right), \text{ (present theory)}. \tag{57}$$

For the *longitudinal polarization* of the μ^-, we have

$$P_L(\mu^-) = \frac{G}{\sqrt{2}\pi\alpha} h_{VA}\, s \left[1 + \frac{2\cos\theta}{1 + \cos^2\theta - P^2\sin^2\theta\cos 2\phi}\right]. \tag{58}$$

Using the expression quoted above for h_{VA}, we note ($z \simeq 3/8$):

$$h_{VA} \simeq \begin{cases} +0{\cdot}29 \text{ (present theory)}, \\ -0{\cdot}25 \text{ (WS–GIM theory)}. \end{cases} \tag{59}$$

Thus we have a *significant contrast* (*of sign*) between the two theories.

Finally, to test for the value of h_{VV}, we note that the magnitude of the $\mu^+\mu^-$-pair production cross-section is expected to deviate from QED, and this deviation is sensitive to h_{VV}. Thus

$$\frac{\Delta\sigma}{\sigma_{QED}} = \frac{G}{\sqrt{2}\,\pi\alpha}\, h_{VV}\, s \simeq 3\cdot 3 \times 10^{-4}\, h_{VV} \left(\frac{s}{\text{GeV}^2}\right). \tag{60}$$

For h_{VV} we find ($z \simeq 3/8$):

$$h_{VV} \simeq \begin{cases} 0\cdot 74 \text{ (present theory)}, \\ 0\cdot 125 \text{ (WS–GIM theory)}. \end{cases} \tag{61}$$

Again there is a significant difference between the two theories, of nearly a factor 6.

The present energies, however, do not seem to be enough to show up the various effects discussed above. Future machines (Petra, Pep, etc.) should hopefully lead to their detection.

9. Parity-violation in atomic physics

Finally, we consider briefly the parity-violating effect in atomic physics (Bouchiat and Bouchiat 1974) expected in our theory due to the electron-nucleon weak interactions mediated by the weak neutral bosons Z_1 and Z_2. The relevant effective interaction Lagrangian here is

$$L_{\text{eff}} = -\, i \frac{G}{\sqrt{2}} \,(\bar{e}\gamma_\rho\gamma_5 e) \left[(1 - \sin^2\chi) \left(\mathscr{F}_\rho^{\,3} + \frac{1}{\sqrt{3}}\, \mathscr{F}_\rho^{\,8}\right)\right] \tag{62}$$

Remembering eqs (48), (49) and (50), we have for the parity-violating effective electron nucleus potential

$$V_{\text{eff}} = \frac{G}{4\sqrt{2}m_e} [\vec{\sigma}\cdot\vec{p}\, \delta^3(\vec{r}) + \delta^3(\vec{r})\, \vec{\sigma}\cdot\vec{p}]\, Q(Z, N), \vec{p} \equiv (1/i)\, \vec{\nabla}_r, \tag{63}$$

where Z is the proton-number and N the neutron-number of the atomic nucleus and

$$Q(Z, N) = \begin{cases} 2Z(1 - \sin^2\chi), \text{ (present theory)}, \\ (1 - 4\sin^2\theta_W)\, Z - N, \text{ (WS–GIM theory)}. \end{cases} \tag{64}$$

Note the contrast in the two theories: whereas $Q(Z, N)$ involves both Z and N in the WS-theory, it is independent of N in the present theory. The relevant matrix element for parity-violating atomic transition is

$$\langle nS_{1/2} \mid V_{\text{eff}} \mid n'p_{1/2} \rangle = Z^2 Q(Z, N)\, K_r \times \text{(other factors)}, \tag{65}$$

where K_r is a relativistic correction factor. As example, consider the interesting case of *Cesium* ($Z = 55$, $N = 78$). For this atom $K_r = 2\cdot 8$. So that (taking again both $\sin^2\chi$ and $\sin^2\theta_W \simeq 3/8$)

$$Z^2 Q(Z, N)\, K_r \simeq \begin{cases} 0\cdot 6 \times 10^6 \text{ (present theory)}, \\ -\, 0\cdot 9 \times 10^6 \text{ (WS–GIM theory)}. \end{cases} \tag{66}$$

For an experiment of current interest (*see* the review of Adler 1975) using a dye laser, thus the two theories predict a parity-violating effect differing in sign and magnitude. To test for their further differences different heavy atoms have to be studied in order to look for the different Z and N dependences in the two theories.

As a concluding remark we may say that the results of the present work appear sufficiently interesting to encourage further examination of the $U_3(W)$-gauge theory.

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